Heat Transfer in Homogeneous Gas-cooled Reactors

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The heat source distribution in a nuclear reactor is a function of neutron flux, fuel loading, and core temperature, each of which is a function of position. In certain types of solid fuel gas-cooled reactors, the fuel distribution is such that the reactor may be regarded as homogeneous. This paper describes the method of calculating the temperature distribution in the core of such a reactor. A cylindrical core is assumed, and the variations of coolant flow and outlet temperature with radius are found. As a first approximation the dependence of neutron flux on temperature is taken to be small, and the flux is assumed to have been calculated on the basis of uniform core temperature. Using this flux distribution, the first core temperature distribution is found. Subsequent flux distributions can be calculated to refine the values of temperature.

INTRODUCTION

The heat source distribution in a nuclear reactor is a function of neutron flux, fuel distribution and core temperature, each of which is a function of position. A basic assumption of the following calculations is that the fuel distribution is sufficiently uniform for the reactor to be regarded as homogeneous. Calculation of core temperatures is an iterative process. A temperature distribution must be assumed, on the basis of which a flux and heat source distribution are found; a new temperature distribution is calculated, and the process repeated as many times as necessary. The errors involved in assuming that flux is independent of temperature are sufficiently small for initial assessment purposes. The obvious extension of the method to dispense with this assumption is briefly described. A further assumption which may reasonably be made at first is that no heat is lost from the core into the reflector, etc. A cylindrical core is considered.

SUMMARY OF METHOD

Because of the non-uniform flux distribution in the core, the heat source distribution, which is assumed to follow the flux,+ varies from one coolant channel to another. The coolant must distribute itself between the channels so that there is equal pressure drop in all channels, and the pressure drop is a function of the amount of heat added to the fluid. The coolant flow rate through all channels will therefore not be the same. This non-uniform distribution may be prevented by creating artificial pressure drops in the cooler channels. Unless the temperature distribution is very severe, however, this is not regarded as being necessary, or even advisable, as it results in the overall pressure drop being increased.

The method consists first of determining the distribution of mass flow with radius, as a function of the pressure drop in a channel and of the heat input to the coolant per channel.

The total mass flow is determined by the power of the reactor and the specified coolant temperature rise. The actual pressure drop and mass flows can thence be calculated.

The second step is to determine the temperatures of the coolant and fuel rods throughout the reactor from the flux distribution (i.e., heat source distribution) and the mass flow distribution.

PRINCIPAL NOTATIONS

A_1	coolant channel inlet area
C_p	coolant specific heat at constant pressure
D	hydraulic mean diameter
f	friction factor
H	enthalpy
h	heat transfer coefficient
L	a length of channel
P	stagnation pressure
p	static pressure
P_c	heat transfer perimeter
Q	quantity of heat
Q_{av}	heat output per coolant channel
Q_o	heat output of central channel
Q_r	heat output of channel at radius r
Q_{tot}	total reactor heat output
q	velocity head
r	radius
r_o	core outer radius
T	temperature
V	coolant velocity
W_{tot}	total coolant mass flow
w_r	mass flow distribution with radius
z	axial position
z_o	overall core length
α_r	radial form factor
α_z	axial form factor
ρ	coolant density
σ	core voidage
ϕ	thermal neutron flux
ϕ_o	flux at $r=0$
ϕ_r	flux distribution with radius r
$\phi_{r_{av}}$	average value of ϕ_r for a given z
ϕ_z	flux distribution with axial position
$\phi_{z_{av}}$	average value of ϕ_z for a given r

CALCULATION OF CORE PRESSURE LOSS

The pressure drop in the coolant from inlet header to outlet header must be found. The coolant

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+ This assumption is, of course, in error, due *inter alia*, to (a) non-uniform fuel distribution, (b) gamma-heating, (c) reactions such as (n, α), (n, p), etc. The first is the major source of error and, if the non-uniformity is small, the other errors may also be neglected. A further source of error, concerning which our knowledge is considerably less complete, is that the fission rate is not proportional to the local moderator temperature, but to the local neutron temperature, which is not necessarily the same. Such an assumption as this is, however, normally made and must, for the present, suffice.

channel consists generally of a series of sections of various lengths and cross-sectional areas, and a step-by-step method of calculation of the pressure drop must be adopted.

The pressure drop for turbulent flow is given, in the unheated portion of a passage of constant area, by the usual equation

$$\Delta p = \Delta P = 4f\frac{L}{D}\,\frac{\rho V^2}{2g} = 4fLq/D. \qquad \ldots (1)$$

Allowance must be made for contractions, enlargements, etc.

For a contraction

$$\Delta P = K_c\frac{\rho V^2}{2g} \qquad \ldots\ldots\ldots (2)$$

where V is the mean velocity downstream of the contraction, and K_c is a coefficient which varies with the diameter ratio, as follows:

D_2/D_1	0·8	0·6	0·4	0·2	0
K_c	0·13	0·28	0·38	0·45	0·50

For gently rounded or narrow conical entrances, as a general approximation, $K_c = 0\cdot05$.

For a sudden enlargement, the usual expression for the pressure loss is

$$\Delta P = \left\{1 - \frac{D_1^2}{D_2^2}\right\}^2 \frac{\rho V^2}{2g}, \qquad \ldots (3)$$

where V is the mean velocity at D_1, the smaller diameter.

In the heated portion of a constant area duct, the pressure drop may be calculated by the method of Woodrow (1955) or of Pinkel, Noyes and Valerino (1950). When the flow may be regarded as incompressible, as is generally the case (Mach number less than, say, 0·2), a relatively simple expression can be derived from the energy equation

$$dH + dQ + \frac{VdV}{gJ} = 0, \qquad \ldots\ldots (4)$$

and the momentum equation

$$\frac{dp}{\rho} + \frac{VdV}{g} + \frac{2fV^2dz}{gD}. = 0 \qquad \ldots (5)$$

Integration of (4) between stations *i* (inlet) and *o* (outlet) gives

$$C_p(T_i - T_o) + Q_{io} + \frac{V_i^2 - V_o^2}{2gJ} = 0,$$

or

$$T_o = T_i + \frac{V_i^2}{2gJC_p} + \frac{Q_{io}}{C_p} - \frac{V_o^2}{2gJC_p}$$

$$= \bar{T} - \frac{V_o^2}{2gJC_p} \text{ say} \qquad \ldots\ldots\ldots\ldots (6)$$

where

$\bar{T}$ = outlet stagnation temperature.

From (5) and the continuity equation $w = \rho AV = \text{constant}$, we have

$$dp + \frac{w}{Ag}dV + \frac{2wf}{gAD}Vdz = 0 \qquad \ldots\ldots (7)$$

To integrate (7), it is necessary to assume the manner of variation of V with z. It has been shown by Woodrow (1955) that the outlet conditions of the fluid are virtually independent of the variation of Q with z. This implies that any reasonable variation of V with z will be satisfactory, and the simplest is a linear relation. Assuming, therefore, that $V = V_i + \frac{z}{z_o}(V_o - V_i)$, we obtain from (7) that

$$\Delta p = p_i - p_o$$

$$= \frac{w}{Ag}(V_o - V_i) + \frac{wfz_o}{gAD}(V_o + V_i)$$

$$= \frac{w^2}{A^2g}\left\{\frac{1}{\rho_o} - \frac{1}{\rho_i}\right\} + \frac{w^2fz_o}{gA^2D}\left\{\frac{1}{\rho_o} + \frac{1}{\rho_i}\right\}$$

$$= \frac{\rho_iV_i^2}{2g}\left\{2\left[\frac{\rho_i}{\rho_o} - 1\right] + \frac{2fz_o}{D}\left[\frac{\rho_i}{\rho_o} + 1\right]\right\}. \ldots (8)$$

Now

$$\frac{\rho_i}{\rho_o} = \frac{p_i}{p_o}\cdot\frac{T_o}{T_i} = \frac{p_i}{p_i + \Delta p}\cdot\frac{T_i + \Delta T}{T_i}$$

$$= \frac{1 + \frac{\Delta T}{T_i}}{1 + \frac{\Delta p}{p_i}}$$

$$\simeq \left(1 + \frac{\Delta T}{T_i} - \frac{\Delta p}{p_i}\right)$$

Thus

$$\Delta p \simeq \frac{\rho_iV_i^2}{2g}\left\{2\left(\frac{\Delta T}{T_i} - \frac{\Delta p}{p_i}\right) + \frac{2fz_o}{D}\left(2 + \frac{\Delta T}{T_i} - \frac{\Delta p}{p_i}\right)\right\} \qquad \ldots\ldots\ldots\ldots (9)$$

whence

$$\Delta P = \Delta p + \frac{\rho_iV_i^2}{2g} - \frac{\rho_oV_o^2}{2g}$$

$$\simeq \frac{\rho_iV_i^2}{2g}\left\{\frac{\Delta T}{T_i} - \frac{\Delta p}{p_i} + \frac{2fz_o}{D}\left(2 + \frac{\Delta T}{T_i} - \frac{\Delta p}{p_i}\right)\right\}. \qquad \ldots\ldots\ldots\ldots (10)$$

In general, $V_i \simeq V_o$ so that $\Delta P \simeq \Delta p$; the pressure drop can then be found directly. Otherwise, it is clear than ΔP can be obtained from (6) and (10). Since $(\Delta p/p_i)$ will generally be much smaller than $(\Delta T/T_i)$, (10) can be simplified to

$$\Delta P \simeq \frac{\rho_iV_i^2}{2g}\left\{\frac{\Delta T}{T_i} + \frac{2fz_o}{D}\left(2 + \frac{\Delta T}{T_i}\right)\right\}. \quad \ldots (11)$$

The pressure drops in the various sections of the channel may conveniently be expressed in terms of the inlet velocity head q_1. The change in velocity head accompanying a change in channel area can be related to the voidage σ by:

$$q \propto \sigma^2$$

for adiabatic flow; and in a constant-area channel,

$$q \propto T$$

for flow with heat transfer.

Thus, the successive pressure drops may be summed in terms of q_1 to give an expression of the form:

$$\Delta P = q_1\left(C_1 + C_2\frac{T_2}{T_1}\right),$$

where C_1, C_2 are functions of a given coolant channel geometry, and T_1, T_2 are the coolant temperatures at channel inlet and outlet.

Now if w_r = mass flow of coolant through a channel at radius r,

and Q_r = heat input rate to a channel at radius r,

then $w_rC_p(T_2 - T_1) = Q_r$

or
$$\frac{T_2}{T_1}=1+\frac{Q_r}{w_rC_pT_1}$$

Then
$$\frac{\Delta P}{\frac{1}{2g}\rho_1V_1^2}=C_1+C_2\left(1+\frac{Q}{wC_pT_1}\right)$$
$$=C_3+C_2\frac{Q}{wC_pT_1}\text{ say.}$$

Now
$$\frac{1}{2g}\rho_1V_1^2=\frac{1}{2g}\ \frac{1}{\rho_1}\left(\frac{w}{A_1}\right)^2.$$

Therefore
$$w^2+\left\{\frac{Q}{C_pT_1}\cdot\frac{C_2}{C_3}\right\}w-\left\{\frac{2g\Delta P\rho_1A_1^2}{C_3}\right\}=0.$$

Whence, finally,
$$w=\left\{-\frac{Q}{2C_pT_1}\cdot\frac{C_2}{C_3}\right\}+\left\{\left[\frac{Q}{2C_pT_1}\cdot\frac{C_2}{C_3}\right]^2+\frac{2g\rho_1A_1^2}{C_3}\Delta P\right\}^{\frac{1}{2}}.$$

Thus $w_r=w(Q_r, \Delta P)$; if inlet conditions and coolant channel geometry are fixed, and for a known function Q_r, values of w_r may be found for any value of ΔP.

An average temperature rise ΔT_c of the coolant and a total reactor power Q_{tot} fix the total coolant mass flow W_{tot}:
$$W_{tot}=\frac{Q_{tot}}{C_p\Delta T_c}.$$

If the core is considered to be homogeneous with fractional voidage σ_1 at inlet, where the channel area is A_1, the number of channels in an annulus of width dr at radius r is
$$2\pi r\,dr\frac{\sigma_1}{A_1}.$$

The total mass flow is thus
$$\int_0^{r_0}\frac{\sigma_1}{A_1}\,.\,2\pi r\,.\,w_r\,.\,dr.$$

The condition that this value must equal W_{tot} determines ΔP and w_r.

CALCULATION OF Q_r

To determine w_r as above, Q_r must be known. This is found by assuming (as discussed above) that the heat source distribution Q_r is proportional to the thermal neutron flux distribution ϕ_r, and independent of local moderator temperature.

Let Q_{av} = average heat input throughout the core per channel,

and Q_o = heat input to the central channel.

Then
$$Q_r=\frac{Q_r}{Q_o}\times\frac{Q_o}{Q_{av}}\times Q_{av}$$
$$=\frac{\phi_r}{\phi_o}\times\frac{\phi_o}{\phi_{r_{av}}}\times Q_{av}$$
$$=\frac{\phi_r}{\phi_o}\times\alpha_rQ_{av}.$$

Here, ϕ_o denotes the flux on the centre line of the core at the same axial height as ϕ_r. On the assumption that the fission cross section is constant throughout the core, the ratio ϕ_r/ϕ_o is independent of z. $\phi_{r_{av}}$ denotes the average flux for all r at the particular value of z. The ratio $\phi_o/\phi_{r_{av}}$, the radial form factor, is also independent of z, and may be found from the radial flux distribution. It is clearly
$$\alpha_r=\frac{\phi_o}{\phi_{r_{av}}}=\frac{r_o^2}{2\int_0^{r_o}\frac{\phi_r}{\phi_o}r\,dr}$$

The flux distribution can be expressed as $\frac{\phi_r}{\phi_o}$. Thus, $\phi_o/\phi_{r_{av}}$, and thence Q_r, may be found.

CALCULATION OF TEMPERATURES

In general, three coolant channels are of interest: that at the centre line of the reactor, that in which the temperature rise is greatest, and that in which the temperature rise is least. The temperatures of the coolant in, and of the fuel and moderator adjacent to, these channels will indicate the temperatures obtaining throughout the core of the reactor.

It is necessary to determine from the flux distribution the axial form factor α_z:
$$\alpha_z=\frac{\phi_o}{\phi_{z_{av}}}$$
$$=\frac{z_o}{\int_0^{z_o}\frac{\phi_z}{\phi_o}dz}$$

For the particular channel, then, Q_r and w_r are known. The total coolant temperature rise is:
$$\Delta T_c=\frac{Q_r}{w_rC_p}$$

At the mean coolant temperature $T_1+\frac{1}{2}\Delta T_c$, the Reynolds number $(w_rD/\mu A)$ and the Prandtl number $(C_p\mu/k)$ are found. For any coolant channel cross-section which is sufficiently near to a circle (any equilateral polygon, for instance), the heat transfer coefficient h may be found from*:
$$Nu=\frac{hD}{k}=0{\cdot}023\,Pr^{0\cdot4}\,Re^{0\cdot8}.$$

It is not likely that consideration of the variation of h with T along the channel will be warranted. This may be done, however, without difficulty.

The total amount of heat absorbed by the coolant in the channel is Q_r. The average heat flux at the channel surface is therefore
$$\frac{Q_r}{z_o}\text{ per unit length,}$$
and the heat flux at the centre of the channel is
$$\frac{\alpha_zQ_r}{z_o}\text{ per unit length.}$$

Therefore the coolant temperature rise from inlet to any point z is
$$T_z-T_1=\frac{\alpha_zQ_r}{z_ow_rC_p}\int_0^z\left(\frac{\phi}{\phi_o}\right)_z dz$$

* Under other conditions, the appropriate expressions can be found in standard texts.

and the heat transfer temperature difference, giving the material surface temperature, is

$$\Delta T_s = \frac{\alpha_z Q_r}{h P_c z_0} \left(\frac{\phi}{\phi_0} \right)_z ,$$

where P_c is the heat transfer perimeter.

Calculation of temperatures within the fuel rod itself will, of course, depend upon the particular design, but follows directly from the foregoing, using the usual heat transfer methods.

CALCULATION OF REVISED TEMPERATURE DISTRIBUTION

Initial flux calculations in reactor feasibility studies are normally carried out on the basis of a uniform temperature throughout the core. Bearing in mind the proviso contained in the footnote on page 1, a refined flux distribution may be found on the basis of moderator temperatures calculated as above. In these circumstances the flux can no longer be represented simply by two expressions for ϕ_r and ϕ_z. ϕ_r/ϕ_0 and $\phi_{r_{av}}/\phi_0$ are in fact functions of z, and different curves or values for these quantities are required for all values of z. The concepts of radial and axial form factor acquire rather less significance, and it is necessary to form a hypothetical mesh of adequately fine structure throughout the core, at each mesh point of which the calculation of local heat generation may be made. The temperature assigned to each mesh point for the purpose of flux calculation can be used in conjunction with the calculated flux to determine local values for Q. These values are integrated with respect to z at given values of r to determine values of Q_r which may then be used, as described earlier, to calculate a refined temperature distribution.

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