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SPECTRUM CALCULATIONS FOR NEUTRONS SLOWING DOWN BY ELASTIC COLLISIONS

by

J. P. POLLARD

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Abstract

A method is suggested for obtaining the collision density as a function of lethargy for neutrons slowing down in an infinite homogeneous moderator containing uniformly distributed sources of fission neutrons. The only reactions considered are elastic scattering, spherically symmetric in the centre of mass system, and absorption. The solution, which is exact for single element moderators when the only neutron reaction is elastic scattering, extends the well known Greuling – Goertzel approximation and shows that it is in error by no more than 13 per cent. The method is also exact for a mixture of nuclides, provided the cross section ratios are energy independent, and a useful approximation otherwise.

CONTENTS

1. NOTATION

E = neutron energy in Mev;

E_M = energy corresponding to the origin of lethargy (12 Mev say) – there are a negligible number of neutrons produced from fission with energies in excess of E_M;

u = neutron lethargy,
= $\ln(E_M/E)$;

A_i = mass ratio of the i^{th} nuclide to the neutron;

α_i = maximum fractional energy that may be lost by a neutron on collision with the i^{th} nuclide,
= $(A_i - 1)^2/(A_i + 1)^2$;

u_i = maximum lethargy that may be gained by a neutron on collision with the i^{th} nuclide,
= $\ln(1/\alpha_i)$;

$h_i(u)$ = probability that a neutron of lethargy u will be elastically scattered by the i^{th} nuclide,
= $\Sigma_{si}(u)/\Sigma_{tot}(u)$;

$g(u)$ = probability that a neutron of lethargy u will be absorbed,
= $\Sigma_{atot}(u)/\Sigma_{tot}(u)$;

$s(u)$ = fission spectrum per unit lethargy, (Cranberg, et al., 1956)
= $0.45270\, E\, e^{-\frac{E}{0.965}} \sinh (2.29\, E)^{1/2}$;

S = total number of neutrons produced by fission per cm^3 per sec ;

$\phi(u)$ = neutron flux per unit lethargy;

$F(u)$ = collision density per unit lethargy for unit fission source,
= $\Sigma_{tot}(u)\, \phi(u)/S$;

$q(u)$ = slowing down density for unit fission source;

ξ_i = average increase of lethargy for elastic collisions with the i^{th} nuclide,
= $1 - \frac{\alpha_i}{1-\alpha_i} u_i$.

2. INTRODUCTION

The collision density of neutrons being elastically scattered in an infinite medium, which contains a uniform spatial distribution of sources of fission neutrons, is given by the method described in this report. The only neutron reactions considered are elastic scattering, spherically symmetric in the centre of mass system, and absorption. The solution is exact for a mixture of nuclides provided the cross sections all have the same energy dependence. The collision density is also given exactly for single element materials for which neutron absorption is negligible and extends the well known Greuling– Goertzel solution. For single element materials, or for mixtures of nuclides, which have energy dependent cross section ratios, the method of this report was found to be a useful approximation. The method was found to give a reasonable check on more elaborate spectra calculations using digital computer programmes designed to solve slowing down integral equations directly for D_2O, H_2O and Be neglecting (n, 2n) production.

3. GENERAL THEORY

3.1 Expression for q(u) as a Taylor's series

An analysis of the contribution of elastic scattering reactions with n possible different nuclides for neutrons less lethargic than u, to the slowing down density at u, results in the following equation (Glasstone and Edlund, 1952)

$$q(u) = \sum_{i=1}^{n} \frac{1}{1-\alpha_i} \int_{u-u_i}^{u} h_i(u')\, F(u')(e^{u'-u} - \alpha_i)\, du' . \qquad (1)$$

Following Glasstone and Edlund (ibid.) expand $f_i(u')$ ($= h_i(u')\, F(u')$) as a Taylor's series about lethargy u

$$f_i(u') = \sum_{j=1}^{m+1} D^{j-1} f_i(u)\,(-U)^{j-1}/(j-1)! \quad ,$$

where $U = u-u'$

$D = \frac{d}{du}$,

and higher derivatives of $f_i(u)$ above m are assumed to vanish.

The slowing down density is then given by

$$q(u) = \sum_{i=1}^{n} \sum_{j=1}^{m+1} D^{j-1} f_i(u)\,(-1)^{j-1}\, \theta_j(u_i) , \qquad (2)$$

$$\text{where } \theta_j(u_i) = \frac{1}{(j-1)!} \int_0^{u_i} U^{j-1}(e^{-U} - \alpha_i)\, dU/(1-\alpha_i). \qquad (3)$$

3.2 Some properties of $\theta_j(u_i)$

From equation (3), integrating by parts and remembering the $e^{-u_i} = \alpha_i$ obtain

$$\theta_j(u_i) = \frac{1}{j!} \int_0^{u_i} U^j\, e^{-U}\, dU/(1-\alpha_i), \qquad (4)$$

$$\theta_j(u_i) = \theta_{j+1}(u_i) + \left(\frac{\alpha_i}{1-\alpha_i}\right)\frac{u_i^{j+1}}{(j+1)!} ,$$

$$\therefore \ \theta_j(u_i) = \theta_{j-1}(u_i) - \left(\frac{\alpha_i}{1-\alpha_i}\right)\frac{u_i^{j}}{j!} . \qquad (5)$$

That $\theta_0(u_i) = 1$ follows directly from equation (4),

$$\therefore \ \theta_j(u_i) = \left(1-\alpha_i \sum_{k=0}^{j} \frac{u_i^k}{k!}\right) / (1-\alpha_i), \qquad (6)$$

$$\text{or} \quad \theta_j(u_i) = \left(\frac{\alpha_i}{1-\alpha_i}\right)\sum_{k=j+1}^{\infty} \frac{u_i^k}{k!} . \qquad (7)$$

In particular equation (6) shows that

$$\theta_1(u_i) = \xi_i ,$$

$$\theta_2(u_i) = \xi_i - \left(\frac{\alpha_i}{1-\alpha_i}\right)\frac{u_i^2}{2} = (\gamma\xi)_i \text{ of Greuling and Goertzel,}$$

and $\lim_{j\to\infty} \theta_j(u_i) = 0$.

It is of interest to examine the limit of $\theta_j(u_i)$ corresponding to the two extremes, (i) $u_i \to \infty$ (hydrogen) and (ii) $u_i \to 0$ (heavy scatterer).

For convenience $\zeta_j(u_i)$ is introduced, being defined by

$$\zeta_j(u_i) = \theta_j(u_i) / [\theta_1(u_i)]^j . \qquad (8)$$

(i) Equation (5) gives

$$\theta_j(\infty) = 1 ,$$

and therefore $\zeta_j(\infty) = 1$, which both hold for all values of j.

(ii) $u_i \to 0$ corresponds to $\delta \to 0$, where $\delta = 1 - \alpha$. Equation (7) yields

$$\theta_j(\delta) \simeq \delta^j/(j+1)! , \text{ as } u_i \simeq \delta ,$$

$$\text{and } \zeta_j(\delta) \simeq 2^j/(j+1)! . \qquad (9)$$

Therefore in the limit as $\delta \to 0$

$$\theta_j(0) = 0 ,$$

and $\zeta_j(0) = 2^j/(j+1)!$.

3.3 Matrix expression for q(u) and its derivatives

The assumption is now made that $h_i(u)$ is a slowly varying function of lethargy

$$\therefore \quad D^j f_i(u) = h_i(u)\, D^j F(u) .$$

The calculation then proceeds by defining

$$T_j(u) = \sum_{i=1}^{n} h_i(u)\, \theta_j(u_i) , \tag{10}$$

which is a slowly varying function of lethargy from the assumption made above. Equation (2) may then be written

$$q(u) = \sum_{j=1}^{m+1} (-1)^{j-1}\, T_j\, D^{j-1}\, F(u) . \tag{11}$$

One method for inverting this equation is by repeated differentiation with respect to u up to the m^{th} derivative and by solving the resultant $(m+1)$ equations for F(u). The methods of matrix algebra may be used by defining the $(m+1)^{th}$ order column vectors

$$\tilde{q} = (q(u),\, D\,q(u),\, D^2\, q(u),\, \ldots,\, D^m\, q(u)\,)' ,$$

and $\tilde{F} = (F(u),\, D\,F(u),\, D^2 F(u),\, \ldots,\, D^m\, F(u)\,)'$,

then equation (11) and its successive derivatives may be written

$$\tilde{q} = \mathcal{T}\, \tilde{F} , \tag{12}$$

where $\mathcal{T}$ is the $(m+1) \times (m+1)$ triangular matrix given by

$$\mathcal{T} = \begin{pmatrix} T_1 & -T_2 & T_3 & \ldots & T_{m+1}(-1)^m \\ 0 & T_1 & -T_2 & \ldots & T_m\ (-1)^{m-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \ldots & T_1 \end{pmatrix} .$$

Inversion of the above matrix is readily carried out and therefore

$$\tilde{F} = \mathcal{T}^{-1}\, \tilde{q} . \tag{13}$$

The first few elements of the inverse matrix are

$$\mathcal{T}^{-1} = \begin{pmatrix} t_1 & t_2 & t_3 & t_4 & \ldots \\ 0 & t_1 & t_2 & t_3 & \ldots \\ 0 & 0 & t_1 & t_2 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \end{pmatrix} , \tag{14}$$

where $t_1 = \dfrac{1}{T_1}$, $t_2 = \dfrac{T_2}{T_1^2}$, $t_3 = \dfrac{T_2^2 - T_1 T_3}{T_1^3}$, $t_4 = \dfrac{T_2^3 - 2T_1 T_2 T_3 + T_1^2\, T_4}{T_1^4}$, etc.

3.4 Alternative expression for q(u)

A further expression for the slowing down density can also be obtained from the definition of q(u) as being the number of neutrons that slow down past lethargy u per cm^3 per sec from sources emitting one neutron per cm^3 per sec. Since all fission neutrons less lethargic than u will eventually slow down past u except for those absorbed during the slowing down process

$$q(u) = \int_{-\infty}^{u} s(u')\,du' - \int_{-\infty}^{u} g(u')\,F(u')\,du' \quad . \tag{15}$$

Repeated differentiation of equation (15) provides a further connection between the column vectors $\tilde{q}$ and $\tilde{F}$ when the assumption is made that g(u) is a slowly varying function of lethargy, namely

$$\tilde{q} = \tilde{s} - g\,\mathfrak{L}\,\tilde{F} - \tilde{F}_g \quad , \tag{16}$$

where the $(m+1)^{th}$ order column vectors introduced are defined by

$$\tilde{s} = \left(\int_{-\infty}^{u} s(u')\,du'\,,\ s(u)\,,\ Ds(u)\ ,\ \ldots\ D^{m-2}s(u)\right)' \ ,$$

$$\tilde{F}_g = \left(\int_{-\infty}^{u} g(u')\,F(u')\,du'\,,\ 0,\ 0,\ \ldots,\ 0\right)' \quad ,$$

and the $(m+1) \times (m+1)$ shift matrix is defined by

$$\mathfrak{L} = \left(\begin{array}{c|c} 0 \ldots 0 & 0 \\ \hline & 0 \\ \mathfrak{I}_m & \vdots \\ & 0 \end{array}\right) ;$$

$\mathfrak{I}_m$ being the m x m unit matrix.

The column vector $\tilde{q}$ may then be eliminated from equation (13) to give

$$\tilde{F} = \mathcal{C}\ (\tilde{s} - \tilde{F}_g), \tag{17}$$

$$\text{where } \mathcal{C} = (\mathcal{J} + g\mathfrak{L})^{-1} \quad . \tag{18}$$

The above definition may be written

$$\mathcal{C} = (\mathfrak{I}_{m+1} + g\mathcal{J}^{-1}\mathfrak{L})^{-1}\mathcal{J}^{-1} ,$$

which can be expanded in powers of g to give

$$\mathcal{C} = \sum_{k=o}^{\infty} (-1)^k g^k (\mathcal{J}^{-1}\mathfrak{L})^k \mathcal{J}^{-1} \quad .$$

Using equation (14) and the definition

$$R_j = T_j/(T_1^{\,j}) , \tag{19}$$

which is to be compared with equation (8), the first few elements of $\mathcal{C}$ are given by

$$\mathcal{C}_{11} = \frac{1}{T_1} \{ 1 - R_2 g + (2R_2^2 - R_3)g^2 - (5R_2^3 - 5R_2R_3 + R_4) g^3 + \ldots \} ,$$

$$\mathcal{C}_{12} = R_2 - (2R_2^2 - R_3) g + (5R_2^3 - 5R_2R_3 + R_4) g^2 -$$

$$- (14R_2^4 - 21 R_2^2 R_3 + 6 R_2 R_4 + 3 R_3^2 - R_5) g^3 + \ldots ,$$

etc. (20)

It should be noted that the elements of $\mathcal{C}$ are, in general, functions of lethargy. For the analysis to hold they should, however, be only slowly varying.

3.5 F(u) expressed in terms of the fission spectrum functions

The first element of the column vector $\tilde{F}$ is the collision density, F(u). Equation (17) therefore gives

$$F(u) = F^*(u) - \mathcal{C}_{11} \int_{-\infty}^{u} g(u') F(u') du' , \tag{21}$$

where $$F^*(u) = \mathcal{C}_{11} \int_{-\infty}^{u} s(u') du' + \mathcal{C}_{12} s(u) + \mathcal{C}_{13} \frac{d}{du} s(u) + \ldots . \tag{22}$$

Equation (21) is solved numerically by a step by step calculation which starts at u = 0 (it is assumed that F(u) = 0 for u < 0).
$\int_{-\infty}^{u} g(u') F(u') du'$ is obtained by numerical integration using the spectrum as it becomes available in its evaluation. $\mathcal{C}_{11} \int_{-\infty}^{u} g(u') F(u') du'$ is then used as a correction term to F*(u) to give F(u) using equation (21).

When absorption is negligible, i.e. g(u) = 0 for all u, then the numerical integration procedure is no longer necessary and equation (21) becomes

$$F(u) = \frac{1}{T_1} \int_{-\infty}^{u} s(u') du' + \frac{T_2}{T_1^2} s(u) + \left(\frac{T_2^2 - T_1 T_3}{T_1^3} \right) \frac{d}{du} s(u) + \ldots . \tag{23}$$

For single element materials for which neutron absorption is negligible equation (23) gives

$$F(u) = \frac{1}{\theta_1} \int_{-\infty}^{u} s(u') du' + \frac{\theta_2}{\theta_1^2} s(u) + \left(\frac{\theta_2^2 - \theta_1 \theta_3}{\theta_1^3} \right) \frac{d}{du} s(u) + \ldots , \tag{24}$$

which is exact since the scattering probability does not vary with lethargy (i.e. h(u) = 1 for all u). It should be noted that the first two terms correspond to the Greuling – Goertzel solution. Figure 3 shows the maximum effect of the third term of equation (24) as a function of θ_1 and A for materials which are assumed to have negligible neutron absorption.

The calculation of the fission spectrum functions required in equations (22), (23) and (24) is assisted by Table I which lists

$$\int_{-\infty}^{u} s(u')\,du' \;,\; s(u) \text{ and } \frac{d}{du}\, s(u)$$

for the energy range 0.1 (0.1) 10.0 Mev; the first two quantities were extracted from a compilation of reactor physics constants (Argonne National Laboratory, 1958).

4. SINGLE ELEMENT MODERATORS

4.1 Simplification of preceding equations

The preceding equations may be expressed directly in the scattering parameters θ_j and ζ_j which are not functions of lethargy.

Equation (10) becomes

$$T_j(u) = h(u)\,\theta_j \quad ,$$

and equation (20) gives

$$\mathcal{C}_{11} = \frac{1}{h\theta_1}\,\{ 1 - \zeta_2\, g - (\zeta_2 - 2\,\zeta_2^{\,2} + \zeta_3)\, g^2 -$$

$$-(\zeta_2 - 4\,\zeta_2^{\,2} + 5\,\zeta_2^3 + 2\,\zeta_3 - 5\,\zeta_2\,\zeta_3 + \zeta_4)\, g^3 - \;\ldots\; \} \;,$$

$$\mathcal{C}_{12} = \frac{1}{h}\,\{ \zeta_2 - (2\,\zeta_2^{\,2} - \zeta_3) g - (2\,\zeta_2^2 - 5\,\zeta_2^3 - \zeta_3 + 5\,\zeta_2\,\zeta_3 - \zeta_4)\, g^2 -$$

$$-(2\,\zeta_2^2 - 10\,\zeta_2^3 + 14\,\zeta_2^4 - \zeta_3 + 3\,\zeta_3^2 + 10\,\zeta_2\,\zeta_3 -$$

$$-21\,\zeta_2^2\,\zeta_3 - 2\,\zeta_4 + 6\,\zeta_2\,\zeta_4 - \zeta_5)\, g^3 - \;\ldots\; \} \;,$$

$$\text{etc.} \qquad (25)$$

The following relation was used in deriving the above;

$$(g/h)^n = g^n (1-g)^{-n} = g^n\,(1 + ng + n(n+1)\,g^2/2! + \;\ldots) \;.$$

For a heavy scatterer, equation (9) may be used to give

$$\mathcal{C}_{11} = \frac{1}{h\,\theta_1}\,(1 - \tfrac{2}{3}\,g - \tfrac{1}{9}\,g^2 - \tfrac{8}{135}\,g^3 - \tfrac{14}{405}\,g^4 - \;\ldots) \;,\; \text{etc.} \qquad (26)$$

The coefficients in this expansion are within 1 per cent. of the coefficients corresponding to uranium 238.

4.2 Graphite

Absorption is negligible and therefore the collision density is given exactly by equation (24). Figure 3 shows the Greuling – Goertzel solution to be in error by at most 1.3 per cent. This follows from the fact that the third term in equation (24) has the greatest effect on F(u) at E=4.4 Mev.

4.3 Beryllium

As an example of the inclusion of absorption a study has been made of the effect on the slowing down spectrum of the loss of neutrons due to the (n, α) and (n,2n) reactions of neutrons with Be. An accurate estimate of the spectrum (Pollard, unpublished) using a computer technique is compared with the solution obtained using the method suggested in this report – see Figure 1. This spectrum treats the (n, 2n) reaction as simply absorption. However, it was obtained in order that an estimate could be made of the minimum possible enhancement of the slowing down density due to the (n, 2n) reaction. Figure 1 also contains $F^*(u)$ and the exact F(u), completely neglecting absorption, for comparison. The latter was obtained using the programme written by Hines (1959) but with α = 0.637899.

5. MIXTURES

5.1 Light Water

Absorption in hydrogen was neglected and the spectrum obtained using equation (23) is compared with the spectrum obtained from solving the slowing down integral equation using a digital computer (Duncan, et al., in press) – see Figure 2. The approximate spectrum is seen to be in error in the regions where the oxygen has large peaks in the scattering cross section. This is obviously to be expected, since the assumption that $D^k h_0 \simeq 0$ is far from being satisfied for $k > 0$. The oxygen scattering cross section was assumed to be smoothed out slightly to remove the sharp peaks and hence improve the assumption $D^k h_0 \simeq 0$.

5.2 Heavy Water

The (n, 2n) effect in deuterium was neglected. Figure 2 shows the approximate spectrum and spectrum obtained using a digital computer (Duncan, et al., ibid.) for comparison. Again the effect of the wild fluctuations of the oxygen scattering cross section is evident.

Mixtures of D_2O and H_2O could be treated in the same manner as for D_2O and H_2O.

6. ASYMPTOTIC REGION

The asymptotic region corresponds to

$$\int_{-\infty}^{u} s(u')du' = 1 \quad \text{and } s(u) = 0 ,$$

which holds for $u > u_T$, say ($E < 0.01$ Mev).

Equations (21) and (22) give

$$F(u) = \mathcal{C}_{11} \left(1 - \int_{-\infty}^{u} g(u')\, F(u')\, du' \right). \tag{27}$$

When absorption is negligible this further simplifies to

$$F(u) = 1/T_1 = 1/\bar{\xi} ,$$

which is a standard formula derived by Glasstone and Edlund (1952).

If $g(u) = 0$ for $u \leq u_T$ then equation (27) gives

$$D F(u) + g \mathcal{C}_{11} F(u) = 0 ,$$

as $\mathcal{C}_{11}$ is assumed to be a slowly varying function of lethargy;

$$\therefore \; F(u) = \frac{1}{\xi} e^{-\int_{u_T}^{u} \mathcal{C}_{11} \, g \, du'} \tag{28}$$

7. SUMMARY

The analysis presented in this report relates, in general, the collision density of neutrons to a sum of terms which are products of parameters dependent on the slowing down medium and functions related to the fission spectrum (equation 22), with a correction which allows for neutron loss through absorption (equation 21). For single element materials for which neutron absorption is assumed negligible the error in the collision density obtained using the Greuling – Goertzel equation is seen to be in error by at most 13 per cent., and this occurs for deuterium. Other materials have collision densities which are approximated more closely by the Greuling – Goertzel solution as may be seen from Figure 3.

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TABLE 1

FISSION SPECTRUM FUNCTIONS

Note: 9.861(−1) = 9.861 x 10^{-1}, etc.

E in Mev	$\int_{-\infty}^{u} s(u')\,du'$	s(u)	$\frac{d}{du} s(u)$
0.1	9.861 (−1)	2.029 (−2)	−2.909 (−2)
0.2	9.622 (−1)	5.369 (−2)	−7.339 (−2)
0.3	9.333 (−1)	9.227 (−2)	−1.198 (−1)
0.4	9.012 (−1)	1.328 (−1)	−1.633 (−1)
0.5	8.672 (−1)	1.734 (−1)	−2.011 (−1)
0.6	8.321 (−1)	2.129 (−1)	−2.318 (−1)
0.7	7.964 (−1)	2.505 (−1)	−2.547 (−1)
0.8	7.606 (−1)	2.855 (−1)	−2.697 (−1)
0.9	7.251 (−1)	3.178 (−1)	−2.769 (−1)
1.0	6.901 (−1)	3.470 (−1)	−2.767 (−1)
1.1	6.558 (−1)	3.731 (−1)	−2.697 (−1)
1.2	6.223 (−1)	3.961 (−1)	−2.566 (−1)
1.3	5.898 (−1)	4.159 (−1)	−2.379 (−1)
1.4	5.583 (−1)	4.327 (−1)	−2.145 (−1)
1.5	5.280 (−1)	4.466 (−1)	−1.871 (−1)
1.6	4.988 (−1)	4.577 (−1)	−1.563 (−1)
1.7	4.708 (−1)	4.662 (−1)	−1.229 (−1)
1.8	4.439 (−1)	4.722 (−1)	−8.756 (−2)
1.9	4.183 (−1)	4.760 (−1)	−5.077 (−2)
2.0	3.938 (−1)	4.777 (−1)	−1.313 (−2)
2.1	3.705 (−1)	4.774 (−1)	2.487 (−2)
2.2	3.484 (−1)	4.753 (−1)	6.279 (−2)
2.3	3.273 (−1)	4.717 (−1)	1.002 (−1)
2.4	3.073 (−1)	4.667 (−1)	1.368 (−1)
2.5	2.884 (−1)	4.604 (−1)	1.723 (−1)
2.6	2.705 (−1)	4.530 (−1)	2.064 (−1)
2.7	2.536 (−1)	4.446 (−1)	2.389 (−1)
2.8	2.376 (−1)	4.353 (−1)	2.696 (−1)
2.9	2.225 (−1)	4.253 (−1)	2.985 (−1)
3.0	2.082 (−1)	4.148(−1)	3.253 (−1)
3.1	1.948 (−1)	4.037 (−1)	3.501 (−1)
3.2	1.822 (−1)	3.922 (−1)	3.728 (−1)
3.3	1.703 (−1)	3.804 (−1)	3.933 (−1)
3.4	1.591 (−1)	3.684 (−1)	4.117 (−1)
3.5	1.486 (−1)	3.562 (−1)	4.280 (−1)
3.6	1.387 (−1)	3.440 (−1)	4.423 (−1)
3.7	1.295 (−1)	3.317 (−1)	4.545 (−1)
3.8	1.208 (−1)	3.194 (−1)	4.647 (−1)
3.9	1.126 (−1)	3.072 (−1)	4.731 (−1)
4.0	1.050 (−1)	2.952 (−1)	4.796 (−1)
4.1	9.787 (−2)	2.833 (−1)	4.844 (−1)
4.2	9.119 (−2)	2.716 (−1)	4.876 (−1)
4.3	8.494 (−2)	2.601 (−1)	4.892 (−1)

E in Mev	$\int_{-\infty}^{u} s(u')\,du'$	s(u)	$\frac{d}{du}\,s(u)$
4.4	7.909 (−2)	2.488 (−1)	4.894 (−1)
4.5	7.362 (−2)	2.378 (−1)	4.883 (−1)
4.6	6.851 (−2)	2.271 (−1)	4.858 (−1)
4.7	6.374 (−2)	2.167 (−1)	4.823 (−1)
4.8	5.928 (−2)	2.066 (−1)	4.777 (−1)
4.9	5.512 (−2)	1.968 (−1)	4.721 (−1)
5.0	5.124 (−2)	1.873 (−1)	4.656 (−1)
5.1	4.763 (−2)	1.782 (−1)	4.584 (−1)
5.2	4.425 (−2)	1.694 (−1)	4.505 (−1)
5.3	4.111 (−2)	1.609 (−1)	4.419 (−1)
5.4	3.818 (−2)	1.527 (−1)	4.328 (−1)
5.5	3.545 (−2)	1.448 (−1)	4.232 (−1)
5.6	3.291 (−2)	1.373 (−1)	4.132 (−1)
5.7	3.054 (−2)	1.301 (−1)	4.029 (−1)
5.8	2.834 (−2)	1.232 (−1)	3.923 (−1)
5.9	2.629 (−2)	1.165 (−1)	3.815 (−1)
6.0	2.439 (−2)	1.102 (−1)	3.706 (−1)
6.1	2.262 (−2)	1.042 (−1)	3.595 (−1)
6.2	2.097 (−2)	9.843 (−2)	3.483 (−1)
6.3	1.944 (−2)	9.294 (−2)	3.372 (−1)
6.4	1.802 (−2)	8.772 (−2)	3.260 (−1)
6.5	1.670 (−2)	8.275 (−2)	3.149 (−1)
6.6	1.547 (−2)	7.803 (−2)	3.039 (−1)
6.7	1.433 (−2)	7.354 (−2)	2.929 (−1)
6.8	1.327 (−2)	6.928 (−2)	2.821 (−1)
6.9	1.229 (−2)	6.524 (−2)	2.715 (−1)
7.0	1.138 (−2)	6.141 (−2)	2.611 (−1)
7.1	1.053 (−2)	5.778 (−2)	2.508 (−1)
7.2	9.749 (−3)	5.434 (−2)	2.407 (−1)
7.3	9.022 (−3)	5.109 (−2)	2.309 (−1)
7.4	8.348 (−3)	4.802 (−2)	2.213 (−1)
7.5	7.722 (−3)	4.511 (−2)	2.120 (−1)
7.6	7.143 (−3)	4.236 (−2)	2.029 (−1)
7.7	6.606 (−3)	3.977 (−2)	1.940 (−1)
7.8	6.109 (−3)	3.732 (−2)	1.854 (−1)
7.9	5.648 (−3)	3.501 (−2)	1.771 (−1)
8.0	5.221 (−3)	3.283 (−2)	1.691 (−1)
8.1	4.828 (−3)	3.078 (−2)	1.613 (−1)
8.2	4.462 (−3)	2.885 (−2)	1.538 (−1)
8.3	4.123 (−3)	2.703 (−2)	1.465 (−1)
8.4	3.810 (−3)	2.532 (−2)	1.395 (−1)
8.5	3.519 (−3)	2.371 (−2)	1.328 (−1)
8.6	3.251 (−3)	2.219 (−2)	1.263 (−1)
8.7	3.003 (−3)	2.077 (−2)	1.201 (−1)

E in Mev	$\int_{-\infty}^{u} s(u')\,du'$	s(u)	$\frac{d}{du}$ s(u)
8.8	2.773 (−3)	1.943 (−2)	1.141 (−1)
8.9	2.560 (−3)	1.817 (−2)	1.084 (−1)
9.0	2.364 (−3)	1.699 (−2)	1.029 (−1)
9.1	2.183 (−3)	1.588 (−2)	9.764 (−2)
9.2	2.015 (−3)	1.484 (−2)	9.260 (−2)
9.3	1.860 (−3)	1.387 (−2)	8.778 (−2)
9.4	1.717 (−3)	1.295 (−2)	8.318 (−2)
9.5	1.584 (−3)	1.210 (−2)	7.879 (−2)
9.6	1.461 (−3)	1.130 (−2)	7.459 (−2)
9.7	1.348 (−3)	1.054 (−2)	7.059 (−2)
9.8	1.244 (−3)	9.839 (−3)	6.677 (−2)
9.9	1.147 (−3)	9.179 (−3)	6.314 (−2)
10.0	1.058 (−3)	8.562 (−3)	5.968 (−2)

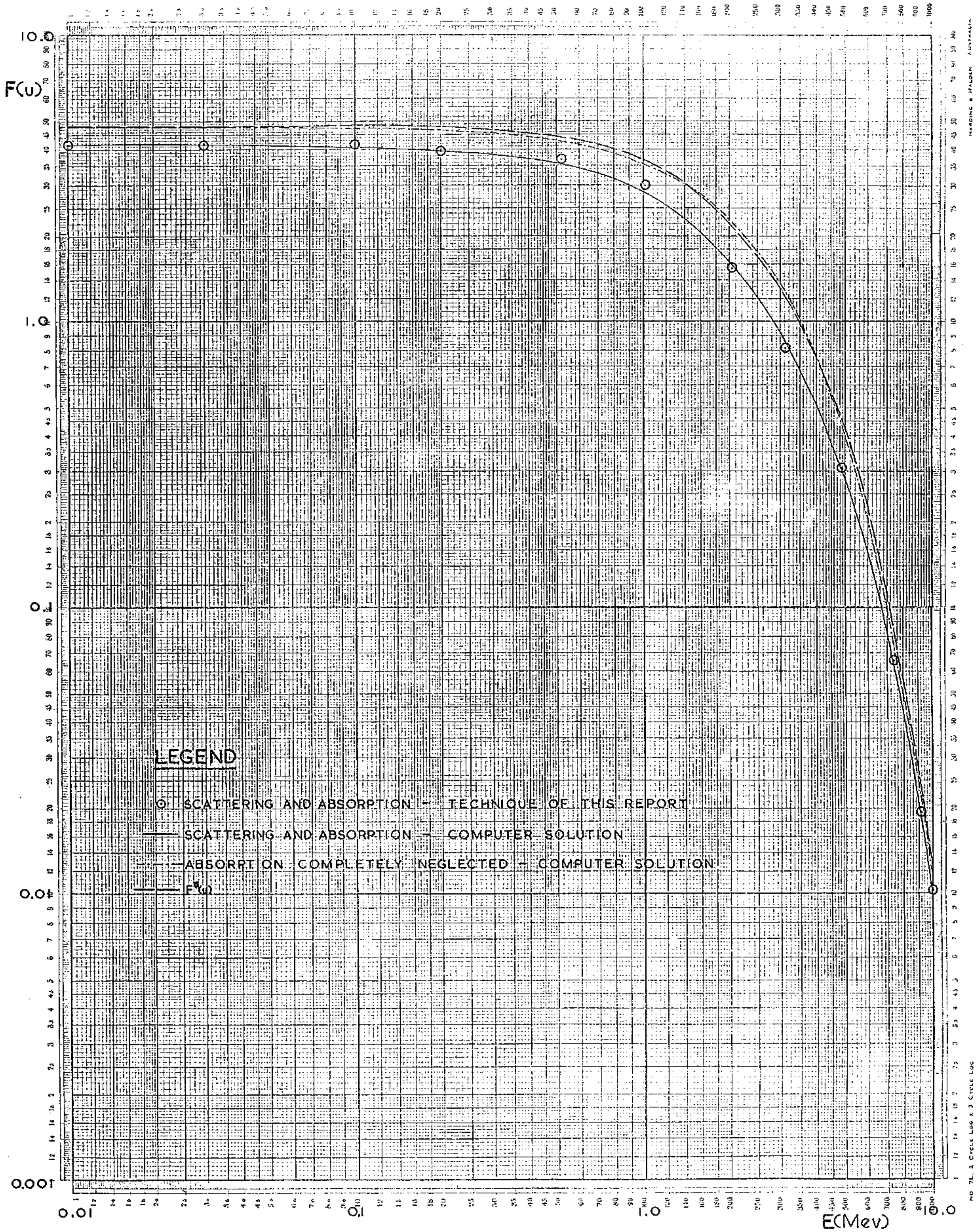

FIGURE 1 SLOWING DOWN SPECTRA OF NEUTRONS IN BERYLLIUM

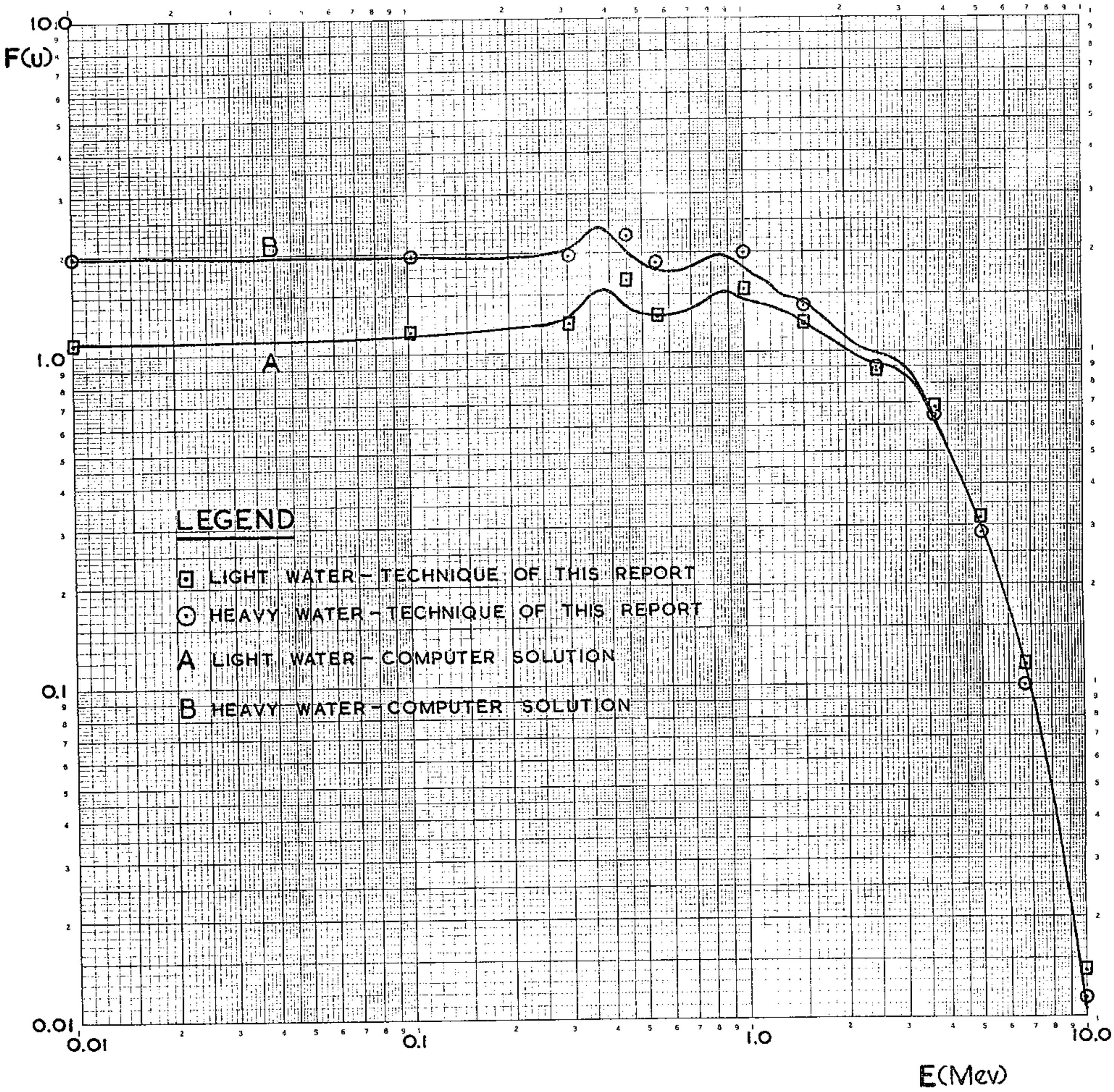

FIGURE 2 SLOWING DOWN SPECTRA OF NEUTRONS IN LIGHT WATER AND HEAVY WATER

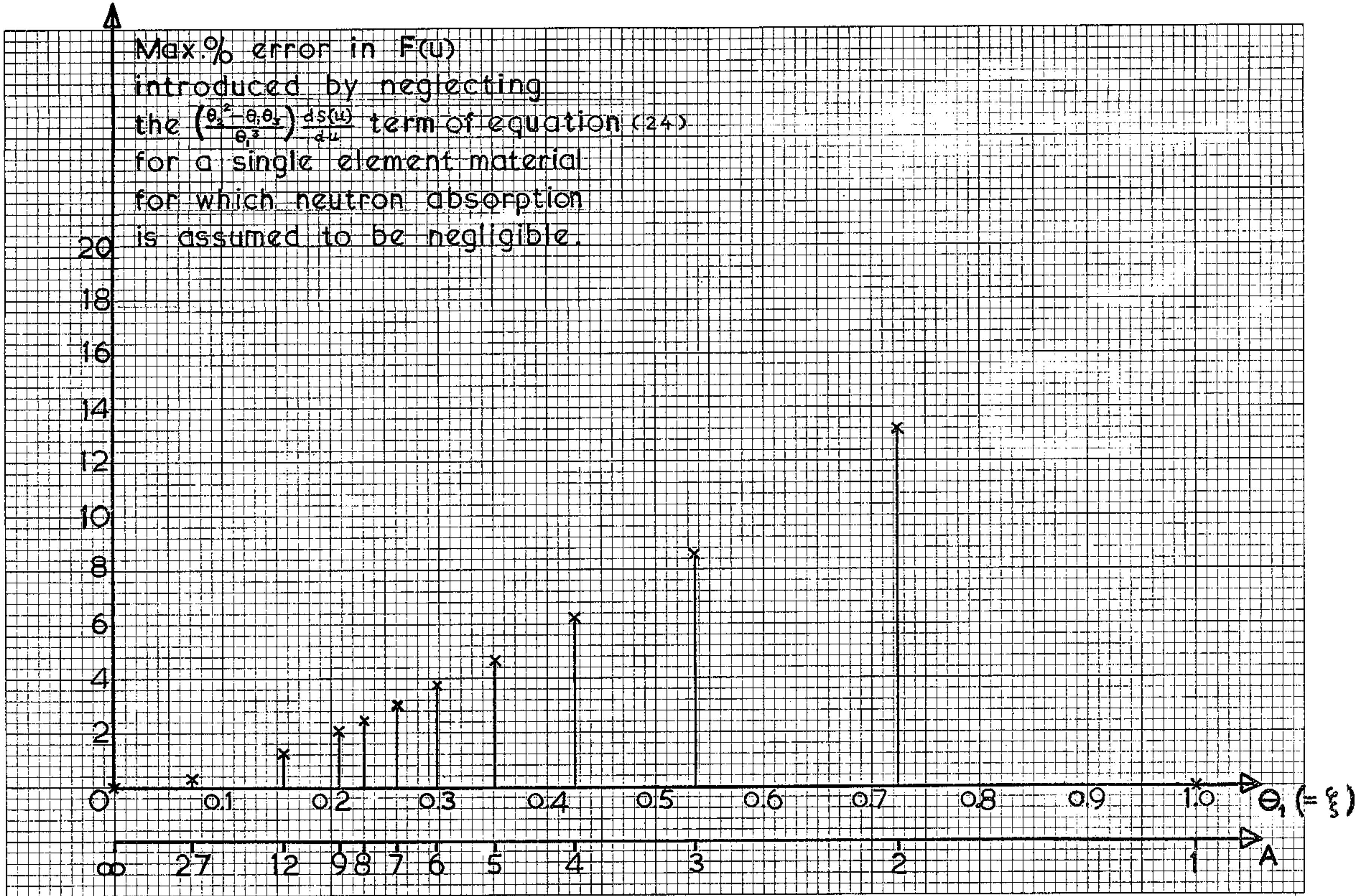

FIGURE 3 MAXIMUM EFFECT OF THIRD TERM OF EQUATION (24)