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RESEARCH ESTABLISHMENT

LUCAS HEIGHTS

A REVIEW OF FLUID FLOW DISTRIBUTION THROUGH RANDOMLY PACKED BEDS

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ABSTRACT

The literature on fluid flow distribution through randomly packed beds is reviewed in relation to a feasibility study of a pebble bed reactor. The methods of measurement used are briefly discussed and existing theoretical solutions are appraised.

The experimental data of different observers are in poor agreement. Existing theoretical derivations are found to be inadequate for general application.

An alternative theory is proposed which relies upon experiment to specify a boundary condition. The existing superficial velocity profiles can be reasonably predicted in terms of the centre line velocity provided this boundary condition is allowed to vary.

CONTENTS

1. INTRODUCTION

Thermal evaluation of the high temperature, gas cooled, pebble bed reactor concept investigated by the A.A.E.C. Research Establishment at Lucas Heights requires a knowledge of the fluid flow distribution through the core.

This report reviews the available literature on fluid flow distribution through randomly packed beds, compares the experimental data and methods of measurement, and examines the theoretical aspects.

The notation used in the text is defined in Appendix 1.

2. LITERATURE REVIEW AND DISCUSSION OF EXPERIMENTAL DATA

The methods of measurement used to determine the fluid velocity distribution through randomly packed beds can be divided, broadly, into two categories; those in which the measurements were made in the open tube at exit from the bed face, giving superficial velocity profiles based upon empty tube areas, and those in which direct measurements were made of the velocities inside the bed. The former category can be further subdivided to separate the methods employing hot wire anemometers in the open tube at exit from the bed, (for example, Morales, Spinn, and Smith 1951, Schwartz and Smith 1953, Dorweiler and Fahien 1959, and O.R.N.L. 1962-1964) and those using flow separators between the bed exit face and the point of measurement (for example, Arthur, Linnet, Raynor, and Sington 1950, and Collins 1958). In the second category measurements made inside the bed have been reported by Hirai (1954), Akehata and Sato (1958) and Cairns and Prausnitz (1959).

With the exception of the O.R.N.L. tests, which were carried out upon $1\frac{1}{2}$ inch diameter pellets inside a 30 inch diameter vessel and utilised specific geometries, the reported data refer to right cylindrical beds of diameter 4 inches or less, vessel to pellet diameter ratios in the range 5 to 32 and Reynolds numbers from 3 to 500. Air was the most common fluid used.

The measurements made inside the bed contribute little to a knowledge of the flow distribution. The results of both Hirai and Akehata and Sato are open to the criticism that the electrodes present in the bed could significantly influence their measurements. This criticism does not apply to Cairns and Prausnitz who measured mean axial velocities over a length of bed. A curve, typical of their results, is shown in Figure 1. Superficial velocities have been plotted assuming that the voidage fraction was constant over the central core of their bed (that is, superficial velocity = e × axial velocity inside the bed). No data were obtained near the walls which is a region of major interest. The dotted portion of the curve was based solely upon considerations of an overall mass balance.

Typical superficial velocity profiles, obtained from measurements made at exit from the bed, are also shown in Figure 1. The data all refer to a vessel to pellet diameter ratio of 16 : 1. Direct comparison may only be made with the solid portion of the curve of Cairns and Prausnitz.

There is disagreement as to the effect of bed length upon the velocity profile. Morales et al. show an apparent variation with bed length whereas both Schwartz and Smith, and Cairns and Prausnitz claim no measurable effect. The latter conclusion is preferred as the trend reported by Morales is not consistent, and would hardly be significant when compared with their spread of data from repeat tests upon beds repacked under nominally similar conditions.

The effect of overall flowrate upon the velocity profile has been considered in most of the references cited and there is general agreement that the normalised velocity profile is independent of flowrate, though some reservations were noted in the O.R.N.L. data at high Reynolds numbers.

It is obvious from the comparison of data in Figure 1 that there is marked disagreement between different investigators for beds having a vessel to pellet diameter ratio of 16; further Schwartz and Smith state that no significant differences were observed between spherical and cylindrical pellets. Similar disagreement is found at other values of vessel to pellet diameter ratio where comparison may be made.

The differences have two main causes, the first concerned with the packing arrangements of the pellets within the bed and the second with the experimental techniques used. Both Morales and Schwartz report that slightly different packing methods significantly influenced their measured superficial velocity profiles and, in Morales' case, even repacking in a consistent manner. The different packing techniques used by the investigators would therefore be expected to exaggerate this effect. Hardly any

data on voidage fraction are given for the beds tested.

The presence of mass balance errors of the order ± 5 per cent. may be demonstrated by evaluating the integral $2/R^2 \int_0^R (U/\bar{V})\, r\, dr$ for the different curves in Figure 1. This comment does not apply to the curve of Cairns and Prausnitz who used this criterion to obtain the dotted portion of their curve. It is possible, however, that more fundamental errors exist. For example the siting requirements are conflicting for circular hot wires placed in the empty tube at exit from the bed to measure the superficial velocity profile. This is discussed at some length in the literature. To minimise changes in the superficial velocity profile which occur in the tube between the bed face and the point of measurement, the anemometer wire should be placed as close as possible to the exit face of the bed but measurements made in this region can be in error because of severe circumferential velocity variations (Morales et al.), non-axial components of velocity, and high fluctuating velocities (Mickley et al. 1965). Errors in the magnitude and position of the maximum velocity could also have occurred because of the limited number of wires covering a region where steep velocity gradients exist. The technique adopted by Collins avoids these problems by using a honeycomb section between the bed exit face and the point of measurement but care is needed to prevent flow redistribution, and consideration has to be given to the problems of high fluctuating velocities. Confining the measurements to only two perpendicular diameters (Collins) needs justification.

3. THEORETICAL SOLUTIONS FOR THE VELOCITY PROFILE THROUGH PACKED BEDS

A velocity profile through a packed bed can be evaluated from a knowledge of the local voidage fraction and a pressure loss correlation derived for whole beds (Blake 1922, Carman 1937, Leva 1947, and Ergun 1952) :

$$\left[\frac{\Delta p\ d}{L\ \rho\ \bar{V}^2}\right] \left[\frac{e^3}{1-e}\right] = f = \phi \left[\frac{N_R}{1-e}\right] ,$$

if this is assumed to apply locally. A velocity distribution can then be calculated for any given voidage distribution. A typical voidage distribution, as measured by Benenati and Brosilow (1962), is shown in Figure 2. In the wall region the predicted profile would depend upon the widths of the annuli assumed; frictional effects at the wall would need to be taken into consideration otherwise the fluid velocity would approach infinity as the voidage fraction approached unity. The method is at best approximate since it assumes that the effective fluid phase shear stresses may be ignored in comparison with the axial pressure gradient. Local application of the general pressure loss correlation to the wall regions could also be in error.

3.1 Theoretical Development of Schwartz and Smith (1953)

A review of the literature shows that in only two references (Schwartz and Smith 1953, Collins 1958) has a more rigorous theoretical treatment been attempted. The derivation was the same in both cases and was developed by Schwartz and Smith.

Figure 3 shows a comparison of their theoretical predictions with some of the experimental data from Figure 1. The theory, which requires the use of empirical coefficients, predicts the form of the velocity profile in terms of $U_o/\overline{V}$ and is therefore coincident with experiment at the axis of the bed. The predictions agree well with the data of Collins (not shown), but differ significantly from the data of Morales and Dorweiler at radii greater than r/R equal to 0.6. Even the data of Schwartz and Smith are not in good agreement with their predictions in this region. The comparison is restricted by the theory to within two pellet diameters from the wall (r/R = 0.75). At this radius the predicted and experimental curves diverge in all the cases shown. The theory does not indicate how the normalised centre line velocity $U_o/\overline{V}$ can be evaluated or offer any physical explanation for the observed differences; its application to beds of D/d outside the range covered by the experiments of Schwartz and Smith is not justified.

Two of the basic assumptions used in their derivation are invalid and a further boundary condition has to be specified. The reasons for this are as follows:

(i) Collins reports, from consideration of his experimental data, that the assumption of a fluid phase shear stress which is a constant fraction of the pressure loss is erroneous.

(ii) The use of the Prandtl mixing length definition of the eddy diffusivity of momentum is unjustified in packed beds. The work of Mickley et al. (1965) clearly indicates that momentum transfer inside packed bed is a function of the gross properties of the bed, depending upon the sidestepping

of the fluid stream as it passes between pellets rather than the actual turbulent structure of the flow, to which the Prandtl mixing length applies.

(iii) For a central core of constant properties, as assumed in the model, increased velocities in the outer regions of this core, which are apparent from Figure 1, can only stem from higher velocities at the outer edge of the core since the axial pressure gradient is assumed to be constant across the bed face. The fluid velocity at the extremities of this central core must therefore be specified as a boundary condition. In their derivation Schwartz and Smith did not need an outer boundary condition but their Equation 6 would seem to require justification.

3.2 Proposed Theory

The theory proposed below is based upon the following assumptions in common with Schwartz and Smith:

(a) It is restricted to right cylindrical geometry and isothermal, fully developed flow conditions.

(b) A differential model may be formulated and the analysis made in terms of the mean circumferential velocities.

(c) The analysis is confined to a central core of the bed in which the voidage fraction is assumed to be constant.

(d) The general pressure loss correlation is assumed to apply locally.

It differs from the theory of Schwartz and Smith mainly in that:

(i) The concept of a pressure defect is dispensed with, thereby eliminating the need to express the fluid phase shear stress as a constant fraction of the pressure loss.

(ii) The eddy diffusivity is taken proportional to the fluid velocity rather than the velocity gradient.

(iii) The velocity profile in the central core is related to an outer boundary condition.

The nature of the flow within a packed bed is interpreted as follows. In the central core of a randomly packed bed it may be assumed that the voids are evenly distributed throughout so that the fractional free area at any of the fluid stream as it passes between pellets rather than the actual turbulent structure of the flow, to which the Prandtl mixing length applies.

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cross section is constant and equal to the voidage fraction. The whole system of voids is interconnected so that where the section of one void is decreasing in the direction of flow the velocity does not have to increase markedly but rather the excess of fluid escapes to a neighbouring void whose section is enlarging in the direction of the flow. The flow path is therefore sinuous, and this explains the high effective eddy diffusivities compared with normal pipe flow.

If a region of high velocity exists near the walls of the containing vessel, the intermixing of the fluid streams as they negotiate their paths will result, effectively, in a transfer of momentum from the region of high velocity near the walls into the central core of the bed. Superimposed upon this gross flow structure will be the variations of velocity within any individual void where, as shown by Mickley et al., the local profiles will depend upon the individual void shape and the shear stress between solid and fluid. This analysis is concerned with the gross radial velocity profile rather than the profiles within the voids.

The basic fluid phase shear equation, written in terms of the local void velocity inside the bed, u, is :

$$\frac{\tau}{\rho} = (E + \nu)\frac{du}{dr} ,$$

and neglecting the kinematic viscosity, ν,

$$\frac{\tau}{\rho} = E\frac{du}{dr} .$$

The analogies of heat, mass, and momentum transfer indicate that, within the governing assumptions, the eddy diffusivities are the same for all three processes. The analogy should also apply to packed beds and may be utilised if the eddy diffusivities of heat or mass can be evaluated. Consideration of the mass transfer tests of Dorweiler and Fahien (1959) and the heat transfer data reported by McAdams (1954), shows that for Reynolds numbers greater than approximately 10^3,

$$N_{PE} = \frac{\overline{V}\,d}{E_H} = \frac{\overline{V}\,d}{E_M} = \text{constant} .$$

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$$N_{PE} = \frac{\overline{V}\,d}{E_H} = \frac{\overline{V}\,d}{E_M} = \text{constant} .$$

Assuming that this relationship may be applied locally:

$$N_{PE} = \frac{Ud}{E'} = \frac{(u\,\delta)\,d}{E'} = \frac{ud}{E'/\delta} = \frac{ud}{E} = \text{constant},$$

hence

$$E = \frac{ud}{N_{PE}} = bdu . \qquad \ldots\ldots (1)$$

The expression for the fluid phase shear stress then becomes :

$$\frac{\tau}{\rho} = bdu\frac{du}{dr} . \qquad \ldots\ldots (2)$$

Consider now a cylindrical section in the packed bed, of unit length and radius r. A force balance upon the fluid may be written :

$$2\pi r\tau + \pi r^2\frac{dp}{dx} = F , \qquad \ldots\ldots (3)$$

where F is the force resisting motion per unit length of bed and arises from the interactions between the solid packing and the fluid. This force is expressed in terms of the fluid velocity by consideration of the general pressure loss correlation, assumed to apply locally. For Reynolds numbers greater than approximately 10^3 the pressure loss is proportional to the square of the velocity. Hence for regions of uniform velocity :

$$F = \text{area}\cdot\frac{dp}{dx} = \text{area}\cdot cu^2 .$$

Since the velocity varies with radius across the cylindrical section considered, the total resistive force acting upon the fluid within the cylinder is :

$$F = \int_0^r 2\pi r\,dr\,cu^2 . \qquad \ldots\ldots (4)$$

Substituting this value of F into Equation 3, differentiating with respect to r and re-arranging gives:

$$\frac{d\tau}{dr} + \frac{\tau}{r} - \left(cu^2 - \frac{dp}{dx}\right) = 0 . \qquad \ldots\ldots (5)$$

The expression for the shear stress, Equation 2, is now substituted into (5) which after re-arrangement becomes :

$$\frac{d}{dr}\left[2u\frac{du}{dr}\right] + \frac{1}{r}\left[2u\frac{du}{dr}\right] - \frac{2c}{\rho bd}\left[u^2 - \frac{1}{c}\frac{dp}{dx}\right] = 0 \text{ .}$$

For the central core the velocity within the bed, u, may be related to the superficial velocity in the empty tube at exit from the bed, U, by the expression :

$$U = \text{constant} \cdot u = \delta u$$

Rewriting the above equation in terms of U, putting $2c/\rho bd$ equal to B^2 and making the substitution :

$$\phi = \left[U^2 - \frac{\delta^2}{c}\frac{dp}{dx}\right] ,$$

results in :

$$\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr} - B^2\phi = 0 \text{ .} \qquad \text{..... (6)}$$

This is a Bessel equation, with a solution of the form :

$$\phi = N\, I_o\,(Br) + M\, K_o\,(Br) ,$$

where N and M are constants and I_o, K_o, modified Bessel functions of the first and second kind respectively. At $r = 0$, $du/dr = d\phi/dr = 0$ and $M = 0$ since $I'(0) = 0$ and $K'(0) = \infty$.

Also $I_o(0) = 1$, $U = U_o$, hence $\phi_o = N = \left[U_o^2 - \frac{\delta^2}{c}\frac{dp}{dx}\right]$.

Thus generally :

$$\phi = \left[U_o^2 - \frac{\delta^2}{c}\frac{dp}{dx}\right] I_o\,(Br) , \qquad \text{..... (7)}$$

A further boundary condition has to be specified. So far the analysis has been restricted to the central core of the bed where the voidage fraction and b in Equation 1 have been assumed constant. The other boundary condition therefore relates to the outer edge of this central core where the increased velocities provide the driving force for momentum transfer into the central core of the bed. These high velocities would arise primarily from the relatively high voidage fraction near the walls, although the reduced radial mixing caused by the presence of the walls, which results in a marked reduction

in the value of b in this region (Dorweiler and Fahien 1959), may also contribute because the fluid would follow a less tortuous path than in the central core. To specify such a condition without recourse to experiment would be extremely difficult and for this analysis the existing data are used.

Equation 7 predicts a superficial velocity profile whose gradient increases with radius, hence the central core is assumed to extend to a radius $\hat{r}$ where the gradient of the velocity profile is no longer increasing. The velocity at this radius is denoted by $\hat{U}$.

Substituting these boundary values into Equation 7 gives :

$$\left[\hat{U}^2 - \frac{\delta^2}{c}\frac{dp}{dx}\right] = \left[U_o^2 - \frac{\delta^2}{c}\frac{dp}{dx}\right] I_o\,(B\hat{r}) \text{ .}$$

Solving for $\frac{\delta^2}{c}\frac{dp}{dx}$ in terms of $\hat{U}$, U_o, and $\hat{r}$; substituting back into Equation 7, and re-arranging gives :

$$U^2\left[I_o\,(B\hat{r}) - 1\right] = U_o^2\left\{\left[\left(\frac{\hat{U}}{U_o}\right)^2 - 1\right] I_o\,(Br) + I_o(B\hat{r}) - \left(\frac{\hat{U}}{U_o}\right)^2\right\} \text{ .}$$

This equation can be normalised with respect to $\overline{V}$:

$$\left(\frac{U}{\overline{V}}\right)^2\left[I_o(B\hat{r}) - 1\right] = \left(\frac{U_o}{\overline{V}}\right)^2\left\{\left[\left(\frac{\hat{U}/\overline{V}}{U_o/\overline{V}}\right)^2 - 1\right] I_o(Br) + I_o(B\hat{r}) - \left(\frac{U/\overline{V}}{U_o/\overline{V}}\right)^2\right\} ,$$

$$\text{or } \left(\frac{U}{\overline{V}}\right)^2\left[I_o(B\hat{r}) - 1\right] = \left(\frac{U_o}{\overline{V}}\right)^2\left[(\alpha^2 - 1)\, I_o(Br) + I_o(B\hat{r}) - \alpha^2\right] , \qquad \text{..... (8)}$$

where

$$\alpha = \frac{U/\overline{V}}{U_o/\overline{V}} = \frac{\hat{U}}{U_o} \text{ .}$$

The above equation predicts the velocity profile in terms of $U_o/\overline{V}$, as in the case of Schwartz and Smith. The resulting profile is independent of flowrate provided that α is independent of flowrate. The reported data show this to be true at least for Reynolds numbers up to 500.

4. COMPARISON OF PROPOSED THEORY WITH EXISTING EXPERIMENTAL DATA

The position of $\hat{r}$ is difficult to specify precisely. The characteristic of increasing velocity gradient is generally exhibited by the experimental data to within close proximity of the radius of the maximum velocity but the latter cannot be coincident with $\hat{r}$ since it corresponds to a surface of zero shear. The theory can only be compared directly with the measured superficial velocity profiles as the data of Cairns and Prausnitz do not permit evaluation of the outer boundary condition. The comparison is made by assuming that the fluid velocity at the outer edge of the central core is equal to the maximum velocity and that $\hat{r}$ is approximately $\frac{1}{8}$ pellet diameter less than the radius at which this maximum velocity occurs.

Figure 4b shows the profiles predicted for values of B of 4/d, 2/d and 1/d respectively taking in each case values of α and $\hat{r}$ from Collins' experimental data for a vessel to pellet diameter ratio of 16. The value of B equal to 1/d gives the best agreement with experiment. The forms of the profiles were then predicted for Collins' other experimental data using in each case the experimentally determined value of α and $\hat{r}$ with B equal to 1/d. The comparisons are shown on Figures 4a and 4c. The same method was then used for the data of Dorweiler and Fahien, and Schwartz and Smith, the resulting comparisons being shown in Figures 5 and 6. In all cases agreement is reasonable using the single value of B equal to 1/d, though it could no doubt be improved in individual cases by modifying the value slightly. Although the radius of maximum velocity is consistently from one to two pellet diameters from the vessel wall, the value of α varies markedly, both with the vessel to pellet diameter ratio for one investigator, and between different investigators.

The variations in the position of $\hat{r}$ may be genuine or may be due to there being too few measuring stations in a region where the profile changes rapidly. The variations in α could only be accounted for by variations in packing structure. Tentative support for this view is obtained from consideration of packing tests upon loosely poured and vibrated beds. The mean change in voidage fraction in the outer annulus, one pellet diameter in width, was a factor of two to three higher than that in the central core. Changes in packing structure in the outer annulus adjacent to the vessel wall would affect $\hat{U}$; changes in the central core would affect U_o and B, though the effect upon B would not be as great as that upon U_o.

The use of Equation 8 to predict the velocity profile requires a knowledge of α, $\hat{r}$, and B. From the above comparison, B could reasonably be taken as 1/d but the values of α and $\hat{r}$ would have to be determined experimentally by measuring the superficial velocity profiles for the particular packing structure

under consideration, over the Reynolds number range of interest. Such tests would also provide a relationship expressing the velocities in terms of the pressure gradient. The effect of different vessel to pellet diameter ratios could be calculated provided the packing geometries near the vessel walls and in the central core did not change.

The above analysis is confined to the central core of the bed, the velocity profile in the wall region remaining undefined. Consideration of this region raises doubts on the validity of the experimental data used in the comparison. The position of the maximum velocity corresponds, approximately, to the voidage fraction peak one pellet diameter from the walls (see Figures 1 and 2). However, the region within a half pellet diameter of the wall has, on average, a significantly higher voidage fraction and its flowpath would be relatively less tortuous. These simple considerations indicate that the maximum velocity would occur within a half pellet diameter from the wall. This view is suggested by Hart, Lawther, and Szomanski (1965) and supported by their experimental data. Measurements of superficial velocity profiles made at various distances behind the bed exit face showed that close to the bed, (0.8d), the maximum velocity was approximately one quarter pellet diameter away from the vessel wall. As the plane of measurement receded from the bed face the position of the maximum velocity moved inwards towards the axis of the bed, approaching one pellet diameter from the wall at a measuring plane 4.8d behind the bed face. These observations point to the possibility that the measurements of superficial velocity made in the open tube could be substantially in error.

There is other evidence to support this view. The value of B has been estimated, taking b equal to 1/11 and using the experimental results of Denton et al (1963) to evaluate c. The estimate of B is 4/d. From Figure 4b it can be seen that with B equal to 4/d, the profile falls away rapidly from the maximum velocity and that over the central core of the bed the velocity profile is uniform. Such a profile would be supported by the limited data of Cairns and Prausnitz (see Figure 1) but is substantially different to the superficial velocity profile measured outside the bed. It is noted that the theoretical analysis assumes a constant friction factor implying Reynolds numbers of the order 10^3 to 10^4, whereas the data used in this comparison were obtained at Reynolds numbers of less than 500. As the Reynolds number decreases, the friction factor increases, hence the above estimate of B, equal to 4/d, would probably be low.

The existing experimental data are insufficient to resolve these anomalies. The proposed theoretical approach cannot proceed until reliable velocity profiles are measured and associated with the packing structure of the bed.

5. CONCLUSIONS

A literature survey of the existing experimental data on fluid flow distribution through randomly packed beds shows inconsistencies in the data which, it is believed, are due partly to the different packing methods employed and partly to the methods of measurement.

The theoretical solution for predicting the flow distribution through packed beds which was proposed by Schwartz and Smith (1953) lacks general application, and some aspects of the derivation are open to criticism.

An alternative theory which has potential general application is compared with existing experimental data, measured at exit from the bed face. These data differ mainly in the boundary conditions. The differences may be due to different packing structures within the bed but there is evidence that the experimental data used in the comparison may be substantially in error owing to momentum transfer between the exit face of the bed and the plane of measurement.

The inconsistencies in the experimental data and the lack of an established theory do not permit confident prediction of the fluid flow distribution through right cylindrical, randomly packed beds. Predictions for a design study of an H.T.G.C. pebble bed reactor would be far more difficult owing to relatively complex geometry and non-isothermal flow. A programme of experimental work is required to resolve the anomalies.

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<u>APPENDIX 1</u>

<u>NOTATION</u>

b	defined in text by Equation 1
B	defined in text as $2c/\rho bd$
c	defined in text as the constant relating to the resistive force F to the velocity u
d	pellet diameter
D	vessel diameter
e	voidage fraction
E	eddy diffusivity (local) based upon u
E'	eddy diffusivity (local) based upon U
E_H	eddy diffusivity of heat, based upon $\overline{V}$
E_M	eddy diffusivity of mass, based upon $\overline{V}$
f	friction factor in the general pressure loss correlation, defined in text
F	resistive force to fluid motion per unit length of bed
I_o	modified Bessel function of the first kind
I_o'	first differential of I_o
K_o	modified Bessel function of the second kind
K_o'	first differential of K_o
L	bed length
M	a constant defined in text
N	a constant defined in text, see Equation 7
N_R	Reynolds number $= \rho \overline{V} d/\mu$
N_{PE}	Peclet number $= (\overline{V} d/E_H) = (\overline{V} d/E_M)$
p	fluid pressure
Δp	pressure loss across packed bed
$\hat{r}$	radius of outer edge of the central core
R	radius of containing vessel
u	local fluid velocity inside the packed bed
U	local superficial fluid velocity (that is, referred to empty tube at exit from the bed).

U_o superficial fluid velocity on the bed centreline

$\hat{U}$ superficial fluid velocity at edge of the central core

$\overline{V}$ average superficial fluid velocity

$\phi = U^2 - \frac{\delta^2}{c}\frac{dp}{dx}$

α ratio $\hat{U}/U_o$

δ Carman constant connecting the fluid velocity inside the central core of the bed with the superficial velocity $= \frac{e}{\sqrt{2}}$

ρ fluid density

τ effective fluid phase shear stress

μ fluid viscosity

ν fluid kinematic viscosity $= \mu/\rho$

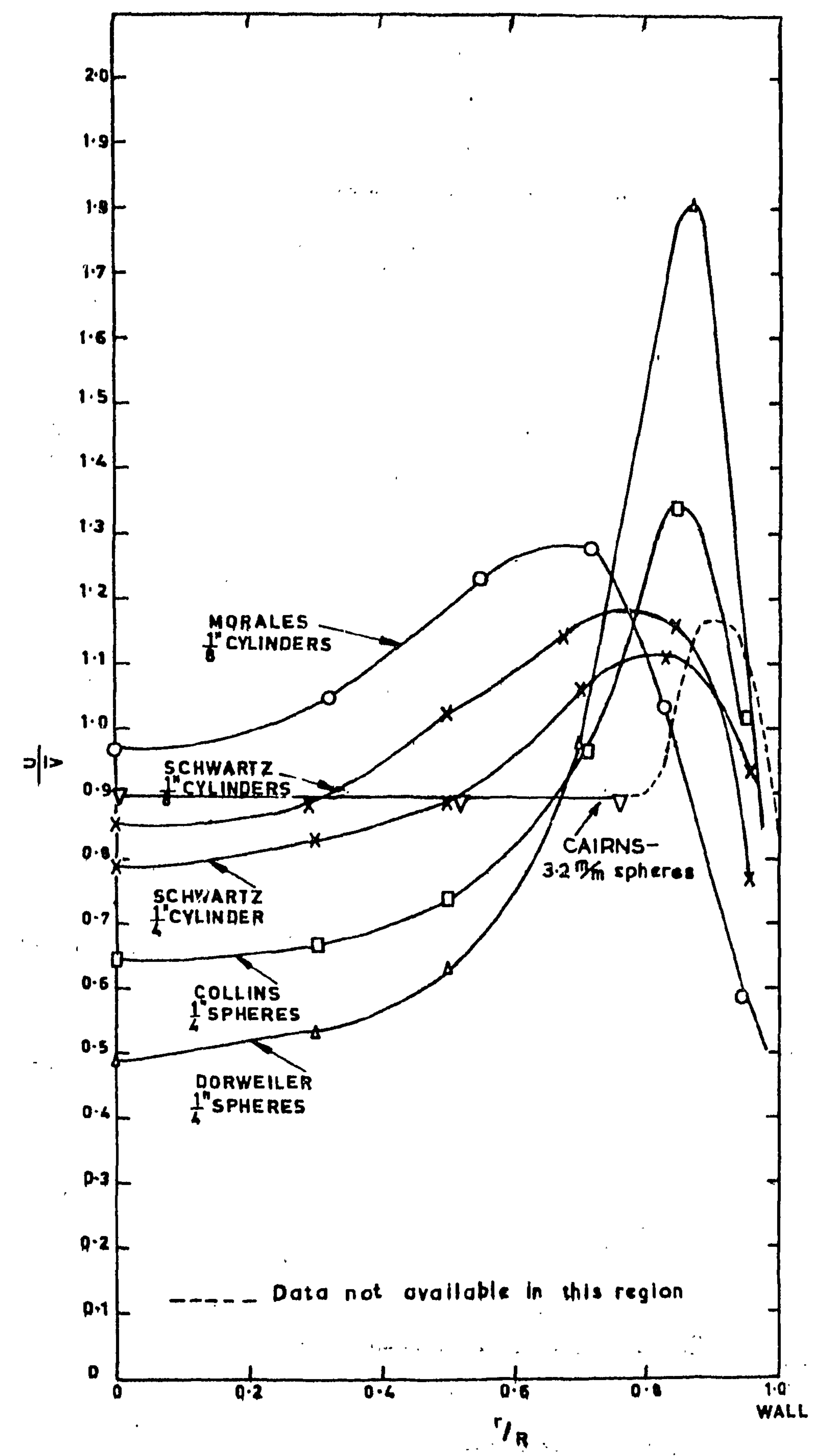

FIGURE 1. MEASURED VELOCITY PROFILES FOR RANDOMLY PACKED BEDS – $D/d = 16$

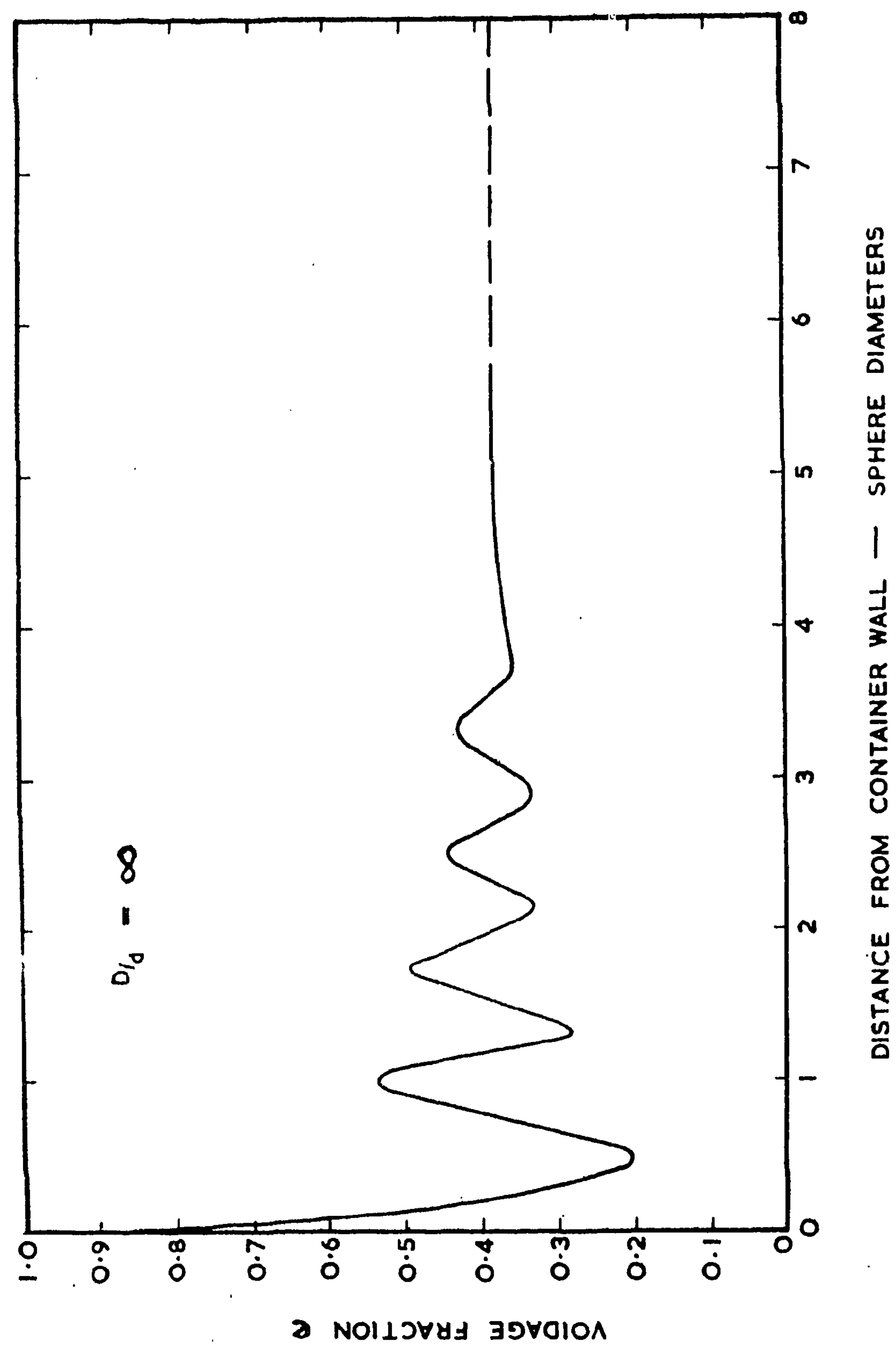

FIGURE 2. VOIDAGE FRACTION IN RANDOMLY PACKED BEDS ACCORDING TO BENENATI AND BROSILOW

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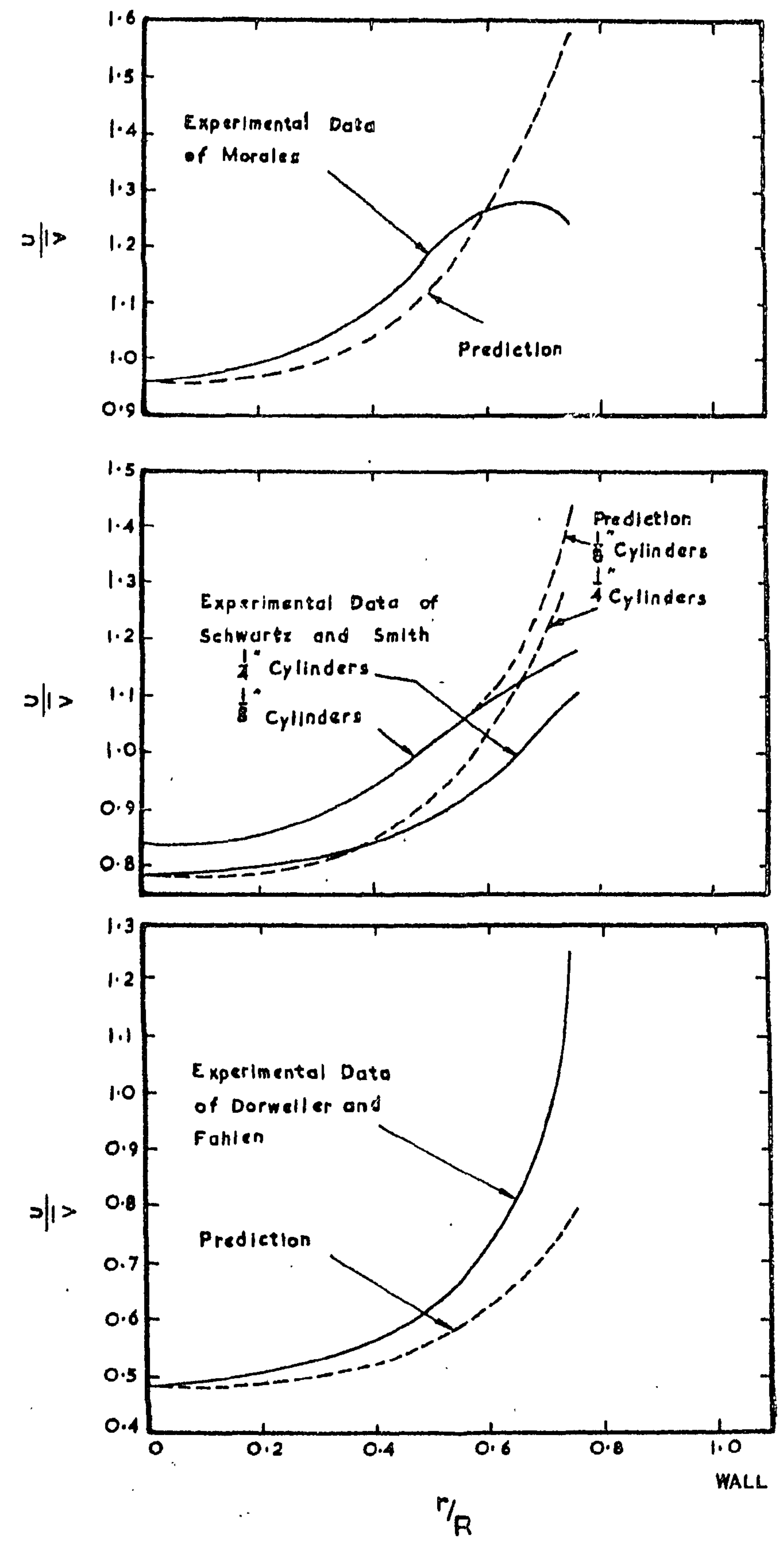

FIGURE 3. COMPARISON OF MEASURED VELOCITY PROFILES WITH THE PREDICTIONS OF SCHWARTZ AND SMITH — D/d = 16

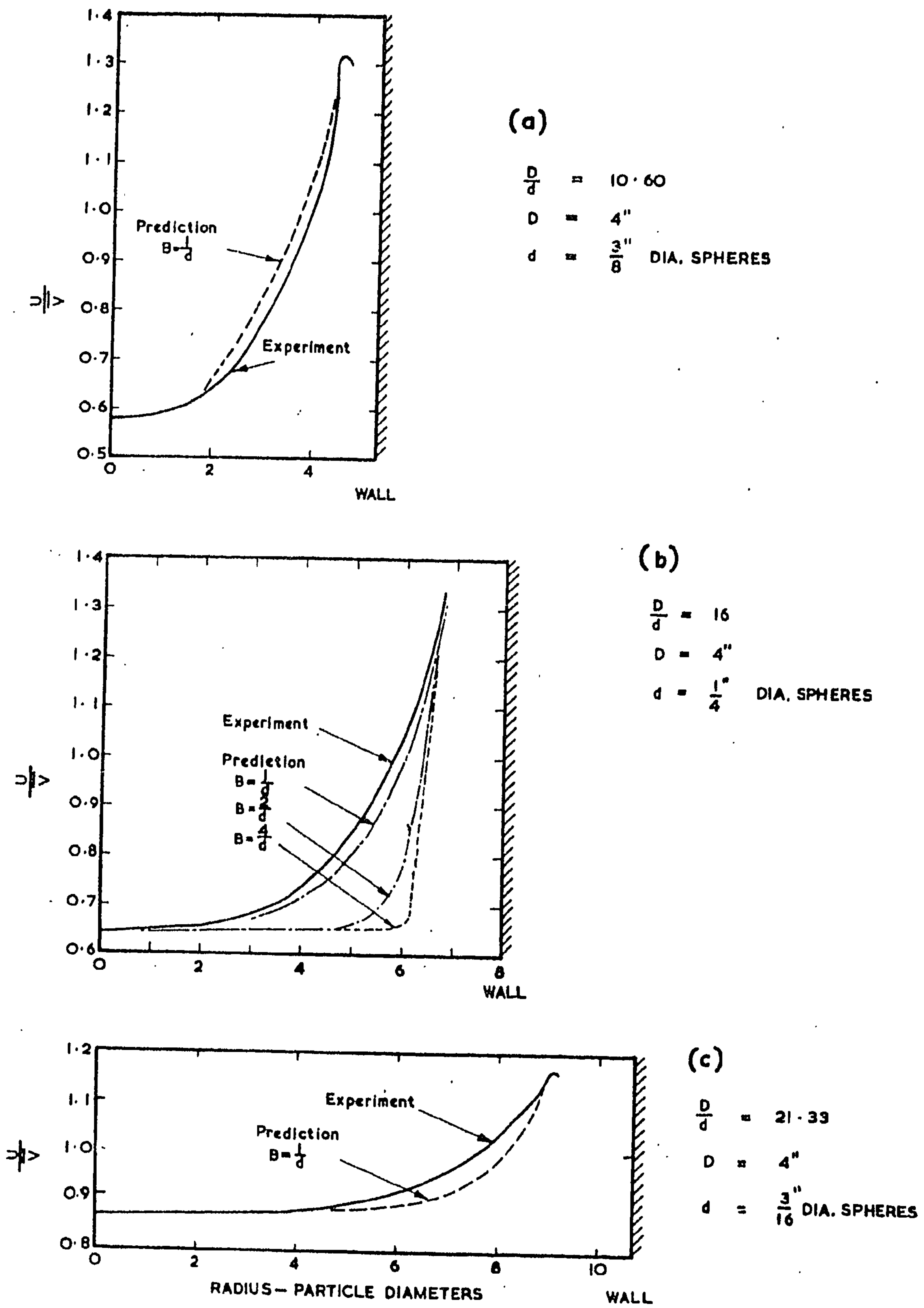

FIGURE 4. COMPARISON OF THEORY AND EXPERIMENT — COLLINS' DATA

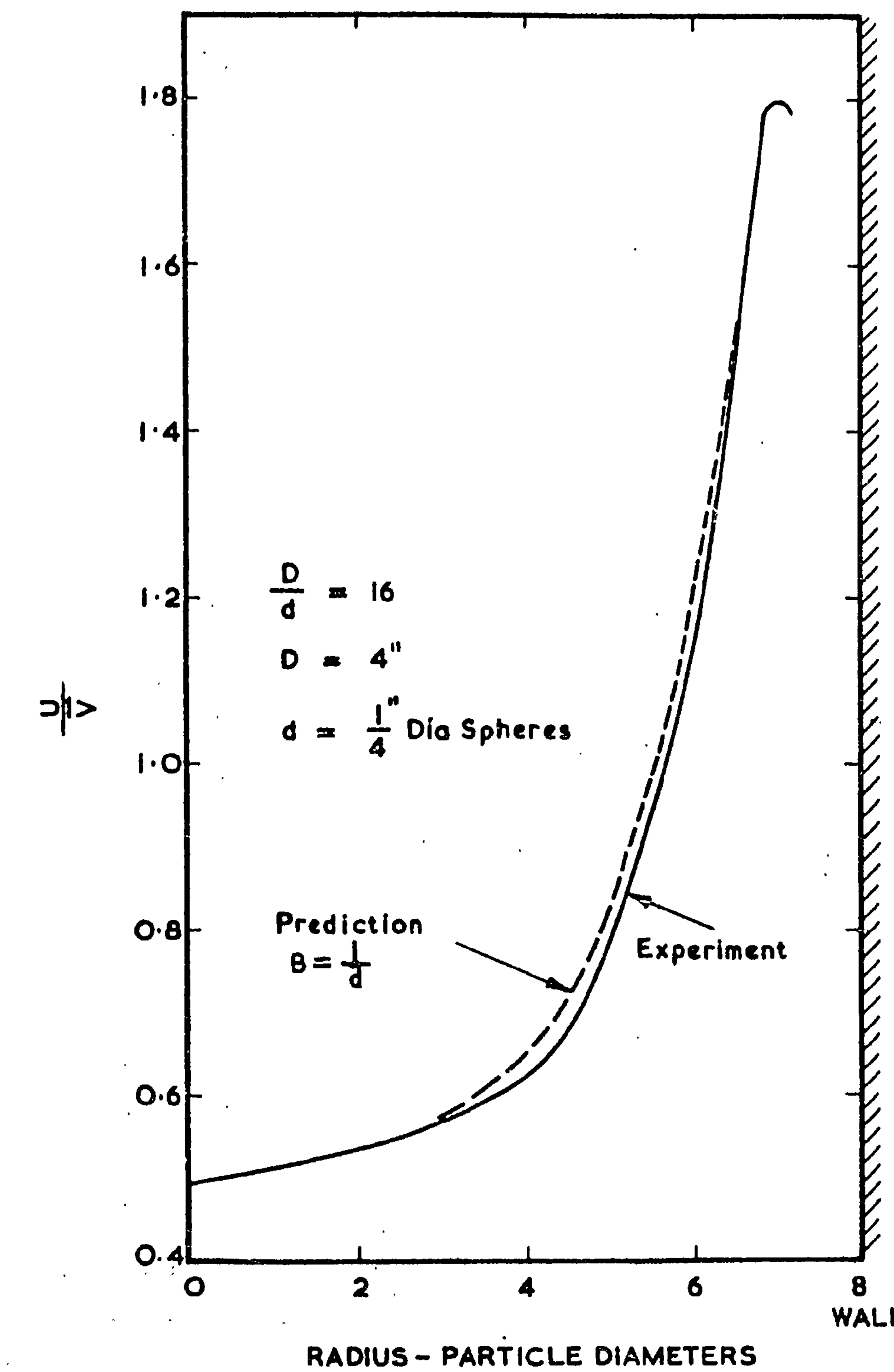

FIGURE 5. COMPARISON OF THEORY AND EXPERIMENT — DORWEILER'S DATA

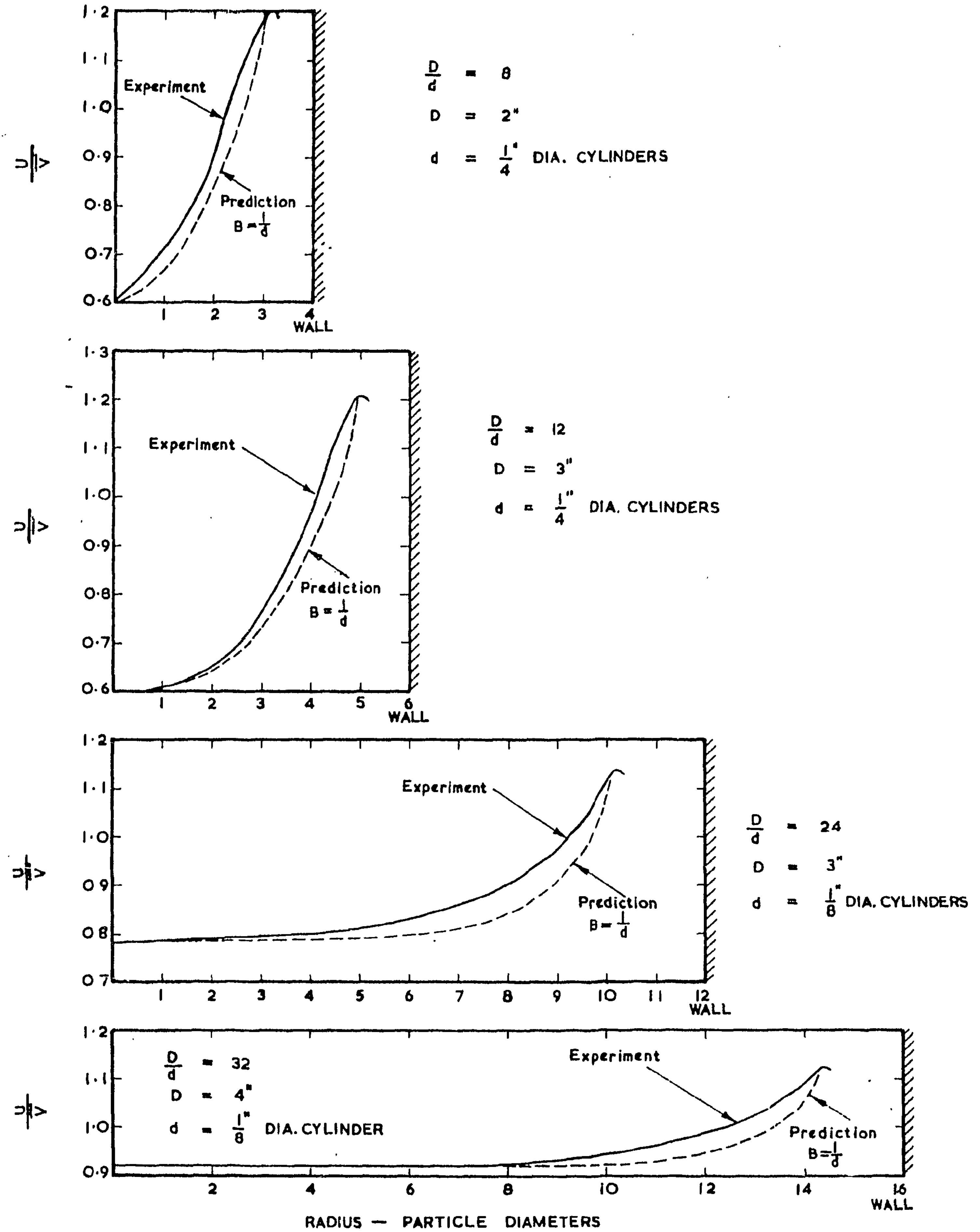

FIGURE 6. COMPARISON OF THEORY AND EXPERIMENT — SCHWARTZ' DATA

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