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LUCAS HEIGHTS

A METHOD FOR CALCULATING THE REACTIVITY WORTH OF PARTIALLY INSERTED CONTROL RODS USING TWO-DIMENSIONAL GEOMETRY

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ABSTRACT

The three dimensional problem of a reactor with partially inserted control rods is reduced to a two dimensional problem by a redistribution of control material within the reactor. The transformation is exact when the pitch circle radius of the rods and the depth of insertion of the rods into the reactor are large compared with the control rod pitch. The effect of variations in pitch on the accuracy of the transformation is investigated by calculation.

CONTENTS

1. INTRODUCTION

A method is described which simplifies the calculation of the multiplication constant of a cylindrical reactor containing a number of control rods all of which are partially inserted to the same depth and are located on a pitch circle centred at the reactor axis.

The cylindrical co-ordinates R, Z, and Θ must all be used to specify the reactor with partially inserted control rods. To calculate the multiplication constant of such a reactor, using the finite difference approach, a mesh must be imposed in the direction of all three co-ordinates. The resulting large number of mesh points leads to large, if not excessive, computing times.

The abovementioned three dimensional problem is called an RZΘ problem. An RZ reactor problem is one in which the reactor composition at any position RZ is independent of Θ. A similar definition is given to an RΘ problem. To calculate an RZ problem using a finite difference approach, a mesh is needed only in the R and the Z directions. Computing times are an order of magnitude less than for RZΘ problems.

The RZΘ problem is shown in Figure 1. A number of radially thin control rods are inserted to depth d in a cylindrical reactor of height H and diameter D. The rods are located at radius a, have a thickness t, and are spaced at a circumferential pitch $a\beta$. Each rod has a width $a\alpha$.

Reference is made to the control zone. This is the smallest cylindrical annulus which completely encloses the control rods.

2. THE METHOD

A redistribution of control material, within the control zone of the reactor, is required which transforms the RZΘ problem to an RZ problem. In any RZ problem the control material must be in the form of rings centred at the reactor axis. We require a shape and distribution of such rings that will reduce the multiplication constant of the reactor by an amount acceptably close to the reduction by the original rods.

In the limiting case when the control zone depth and radius are large compared with both the control rod pitch and the neutron mean free path, an exact transformation from RZΘ to RZ geometry is possible. The rules which apply in this limiting transformation are also applied when control zone depth and radius are not so large compared with the control rod pitch. The effect of large neutron mean free path is not examined since this is seldom the case in practical reactor systems.

The limiting problem, shown in Figure 2, is describable using the rectangular co-ordinates x and y. In Figure 2 the control rod pattern repeats to infinity in the direction of the y co-ordinate, which, for the original reactor, is the direction of the control rod pitch circle. By regarding the y co-ordinate as the Z co-ordinate of the transformed reactor, the identity between the original and transformed reactors is established.

Thus, when the control zone depth and radius are large compared with the control rod pitch, a successful transformation from RZΘ to RZ geometry is made simply by equating the thickness, axial pitch, and height of the control rings in the transformed reactor to the thickness, circumferential pitch, and width respectively of the control rods in the original reactor.

It is noted that, for infinite control zone depth, the above rules imply equal total control rod surface areas in the original and transformed reactors. For finite control zone depth the control rod surface area in the transformed reactor will depend on the positioning of the array of rings with respect to the Z co-ordinate. An example is given in Figure 5 where the control zone depth is taken equal to 4.5 times the pitch. To maintain total control rod surface area, a half ring must be located at the top or at the bottom of the control zone.

To summarize, the transformation rules are as follows:

1. Equate the thickness, axial pitch, and height of the control rings in the transformed reactor to the thickness, circumferential pitch, and width respectively of the control rods in the original reactor.

2. Maintain the same total control rod surface area by determining the number of control rings from:

$$n = \frac{\text{control zone depth}}{\text{control rod pitch}} .$$

3. If n is not integral, position the fractional portion of a ring at the top or at the bottom of the control zone.

For finite control zone depth and radius it remains to examine by calculation any errors which may be produced by applying the above transformation rules.

3. REACTIVITY WORTH CALCULATIONS

If the control rods are fully inserted in a reactor of uniform axial composition, the original RZΘ problem is independent of Z and becomes an RΘ problem. By calculating the reactivity worths of the RΘ problem and its transformed RZ problem, the accuracy of the transformation can be examined using a two-dimensional computer programme.

The three-group core and control material data of Table 1 were used for all calculations discussed in this report. The materials themselves need not be specified but it is noted that the control material data are derived from extrapolation distances obtained in the manner suggested by Thompson (1963) from transport theory calculations. The removal cross sections consist of absorption plus group-to-group scatter.

TABLE 1

DATA FOR REACTIVITY WORTH CALCULATIONS

Group		1	2	3
Core Data	D	1.122	0.8741	0.8628
	Σ removal	0.009867	0.01364	0.01880
	$\nu\Sigma_f$	0.002094	0.001709	0.02470
	$\Sigma^{g\to g+1}_{scatter}$	0.008004	0.01022	0
Control Material Data	D	0.6410	0.4820	0.5000
	Σremoval	0.07280	57.40	3.61×10^9
	$\nu\Sigma_f$	0	0	0
	$\Sigma^{g\to g+1}_{scatter}$	0	0	0

In Section 3.1 the effect of control zone radius is isolated by considering the control rods fully inserted in a reactor of infinite depth. In Section 3.2 a finite depth, fully inserted control zone is considered. Finally, in Section 3.3, the partially inserted control rod bank is examined.

3.1 Rods Fully Inserted in an Infinite Cylinder

By varying the number of rods in the control zone of an infinite cylindrical reactor we can examine the effect of varying β, the ratio of control rod circumferential pitch to control zone radius.

The original problem corresponds to d = H = ∞ in Figure 1. The transformed problem is shown in Figure 3. Calculations were made using the computer programme Cram by A. Hassitt (1963) for D = 220 cm, a = 55 cm, t = 2 cm, and $\alpha = \pi/8$. Calculations were repeated for 8 rods, 4 rods, and 1 rod corresponding respectively to $\beta = \pi/4$, $\pi/2$, and 2π.

Results are given in Table 2. The controlled reactivity figures are based on the multiplication constant 1.01391 for the reactor without control rods.

TABLE 2

REACTIVITY FIGURES FOR RODS FULLY INSERTED IN AN

INFINITE CYLINDER

β	Multiplication Constant		Controlled Reactivity	
	RΘ	RZ	RΘ	RZ
$\pi/4$	0.85580	0.85571	15.594%	15.603%
$\pi/2$	0.93005	0.93023	8.271%	8.253%
2π	0.9994	1.0007	1.43%	1.30%

The accuracy in estimating the controlled reactivity decreases, as expected, when β increases. However, the controlled reactivity is estimated to 0.2 per cent. of the correct figure even for the case of $\beta = \pi/2$, that is, for four rods.

In Figure 4 the axial distribution of the thermal neutron flux in the transformed reactor is compared with the circumferential distribution of the thermal flux in the original reactor. The comparison is made for the case $\beta = \pi/2$. The distributions are taken at 53 cm radius, that is 1 cm inside the surface of the control zone. The flux is normalised so that the average value at the reactor axis for the transformed reactor is equal to the value at the axis for the original reactor.

We see that the flux rises to slightly too high a level between the control rings of the transformed reactor, thereby allowing too many neutrons to escape capture by passing between the rings. Thus the reactivity worth of the control rods is underestimated by the transformation.

3.2 Rods Fully Inserted in a Finite Cylinder

The infinite cylinder problems were considered initially to reduce computing time. RΘ to RZ comparisons are possible for fully inserted rods in finite cylinders, but the finite RZ problem requires more Z mesh points than the infinite RZ problem.

One finite cylindrical reactor was considered. The original reactor, $\alpha = \pi/8$, $\beta = \pi/4$, and d = H = $4.5a\beta$, transforms to the RZ model of Figure 5b.

Results of RΘ and RZ calculations using Cram are given in Table 3. Reactivities are based on the multiplication constant 0.9655 for the reactor without control rods.

TABLE 3

REACTIVITY FIGURES FOR RODS FULLY INSERTED IN A

FINITE CYLINDER

Multiplication Constant		Controlled Reactivity	
RΘ	RZ	RΘ	RZ
0.818	0.820	15.28%	15.07%

The error in the controlled reactivity, though acceptable, is appreciably greater than in the infinite case. Unfortunately, a rather coarse Z mesh had to be used in the finite RZ calculation and therefore the increased error may not be due entirely to the finite control zone depth.

In Figure 6, a comparison is given of the axial thermal flux distributions at 53 cm radius for the original and transformed reactors. The axial flux distribution in the original reactor varies with the Θ co-ordinate, being at a maximum for Θ midway between adjacent rods and at a minimum when Θ coincides with the centre of a control rod. Both these maximum and minimum axial flux distributions are given. All fluxes are normalised to the same value at the reactor centre.

We see that the flux has risen to too high a level between the control rings of the transformed reactor causing the reactivity to be underestimated by the transformation.

3.3 Rods Partially Inserted in a Finite Cylinder

In the third transformation rule of Section 2 the fractional portion of a control ring could be positioned at the top or at the bottom of the control zone. Consider the reactor of Figure 1 with $\alpha = \pi/8$, $\beta = \pi/4$, $H = 4.5\,a\beta$, and $d = 2.25\,a\beta$. The two possible RZ models are shown in Figure 7.

The reactivities of both models, calculated using Cram, are given in Table 4. The difference is small.

TABLE 4

REACTIVITY FIGURES FOR RODS PARTIALLY INSERTED

IN A FINITE CYLINDER

RZ Model	Multiplication Constant	Controlled Reactivity
Figure 7a	0.9074	6.02%
Figure 7b	0.9071	6.05%

It would be interesting to compare the results of Table 4 with the accurate three-dimensional figure. However a three-dimensional computer programme was not available at the time of writing.

4. DISCUSSION

By following the rules of Section 2, the reactivity worth of a bank of control rods, all partially inserted to the same depth, can be found by considering a two-dimensional RZ reactor problem.

The validity of the method is proven only for the case where control rod pitch circle radius and control rod insertion depth are large compared with the control rod circumferential pitch. However calculations indicate that even for the case of four control rods, where the ratio of rod circumferential pitch to pitch circle radius is $\pi/2$, the transformation is accurate.

The effect of finite control rod insertion depth has been investigated in one case, that is for a depth to circumferential pitch ratio of 4.5. The errors introduced by the transformation were found to be acceptable.

If the insertion depth is large compared with the circumferential pitch, a large number of Z mesh points are required near the control zone. For computer programmes like Cure, by E.L. Wachspress (1957), where the number of Z mesh points can vary with radius, this is no great problem. For programmes like Cram, where the number of Z mesh points is constant with radius, the RZ computing time may be no less than for RZΘ.

The three-dimensional neutron flux shape in the original reactor can be estimated from the RZ flux shape in the transformed reactor provided, of course, that the control rod circumferential pitch is small. For example, at a particular radius the maximum and minimum axial flux shapes are approximated by the envelopes of the axial flux distribution at the same radius in the transformed reactor. This is apparent from Figure 6. It is unlikely however that the deduced flux shape would be as accurate as the multiplication constant of the transformed reactor.

All comparisons in this report were made for bare uniform reactors. If the reactor is not uniform but consists of a number of axial zones, as in Figure 8, then the control rod circumferential pitch should be small compared with the depth of each axial zone through which the control rod passes. This is especially true if the flux level in one zone is likely to be greatly different from the level in the adjacent zone. With this proviso, the transformation should be valid for reflected reactors.

The extension of the method to control rods of any shape is not difficult. Circular rods would be expected to transform to rings of circular cross section, and so on. If any doubt exists, infinite cylinder calculations would assist in verifying the transformation.

Following the original draft of this report, in October 1963, it was reported by B. Micheelsen (1964) that the same methods have found extensive use in calculations associated with the Dragon project.

5. REFERENCES

Hassitt, A. (1962). – A computer program to solve the multigroup diffusion equations TRG Report 229 (R).

Micheelsen, B. (1964). – Control rod studies for the Zenith reactor. Proceedings of November 1963 Symposium on Physics and Material Problems of Reactor Control Rods. I.A.E.A., Vienna.

Thompson, J.J. (1963). – The inclusion of grey slabs in the finite difference neutron diffusion approximation. AAEC/TM186.

Wachspress, E.L. (1957). – Cure: A generalised two-space dimension multigroup coding for the IBM 704. KAPL 1724.

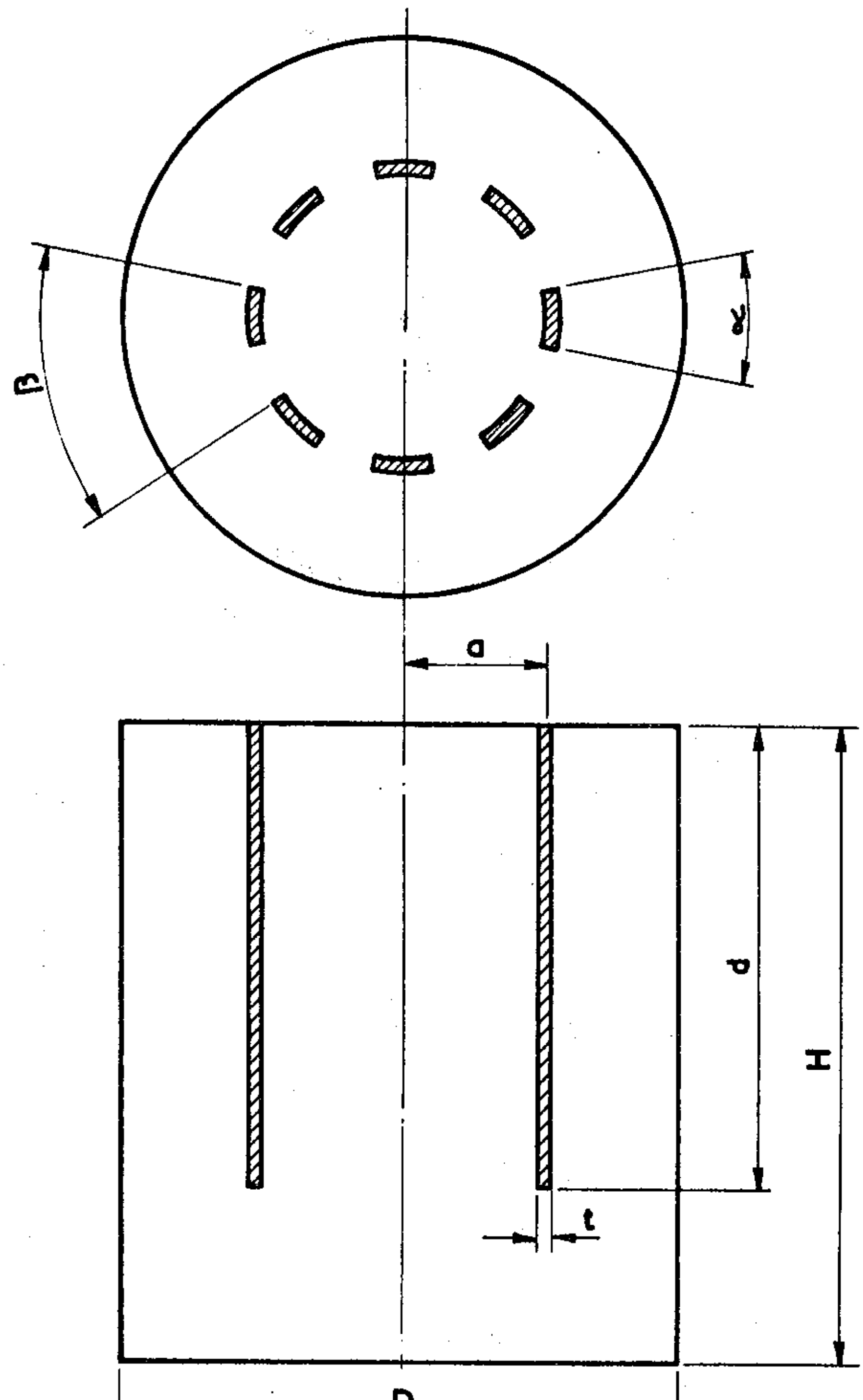

FIGURE 1. THE THREE DIMENSIONAL PROBLEM

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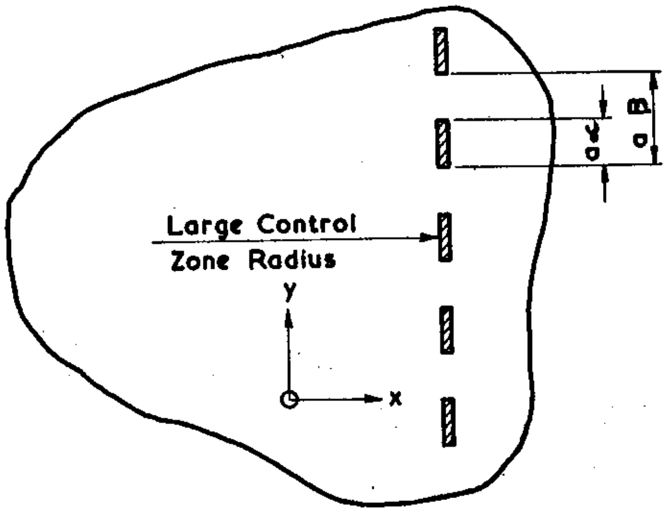

FIGURE 2. THE PLANE CONTROL ZONE

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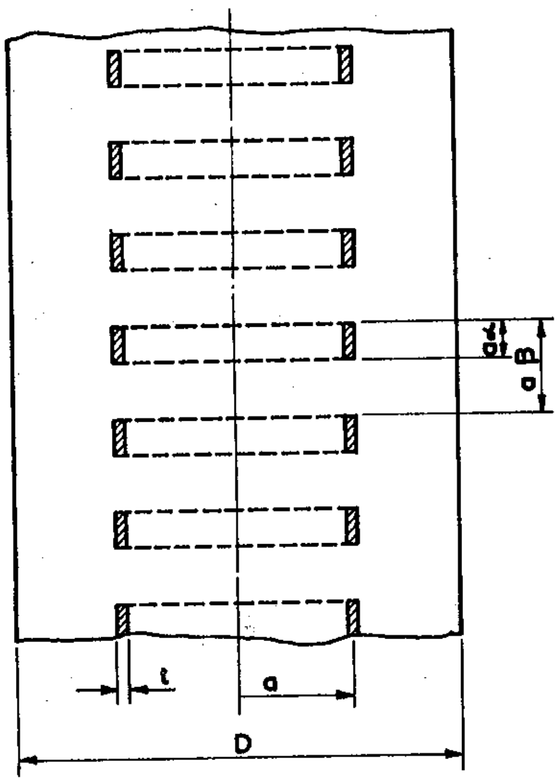

FIGURE 3. THE TRANSFORMED INFINITE PROBLEM

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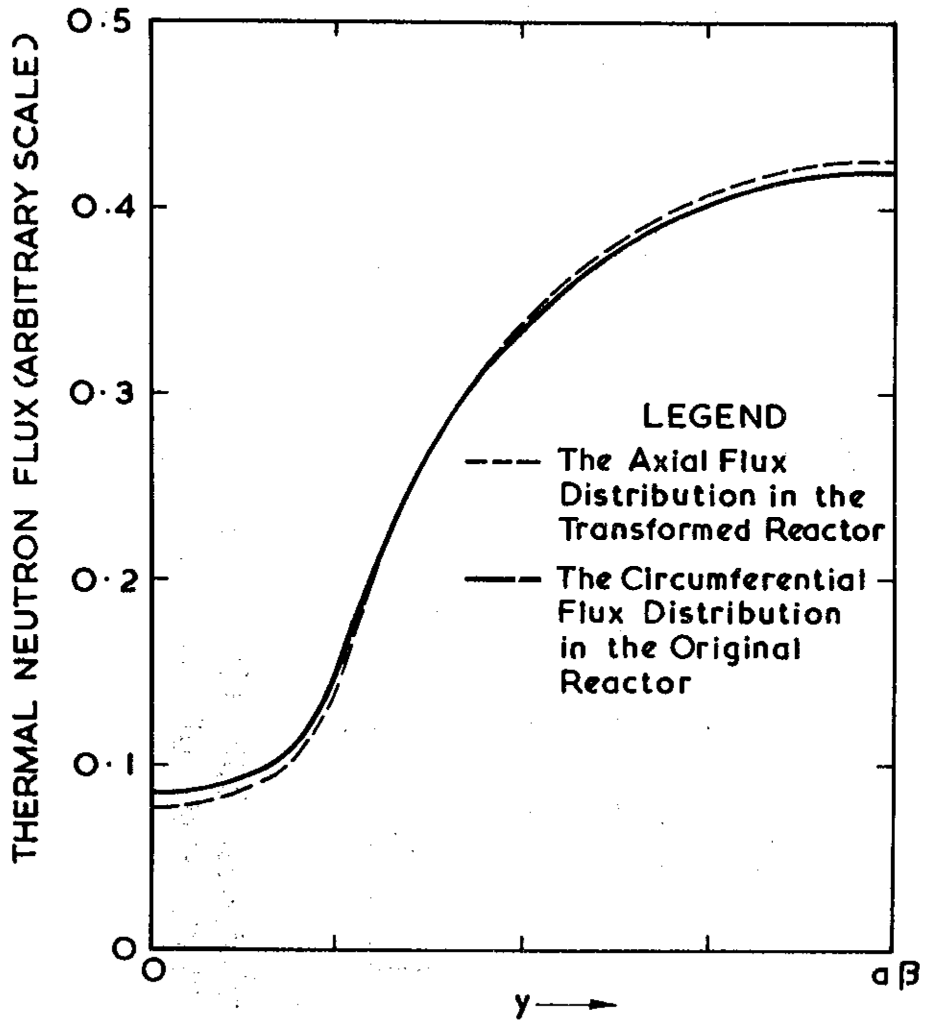

FIGURE 4. A COMPARISON OF THERMAL FLUX DISTRIBUTIONS NEAR THE CONTROL ZONE OF AN INFINITE CYLINDRICAL REACTOR CONTAINING FOUR FULLY INSERTED CONTROL RODS.

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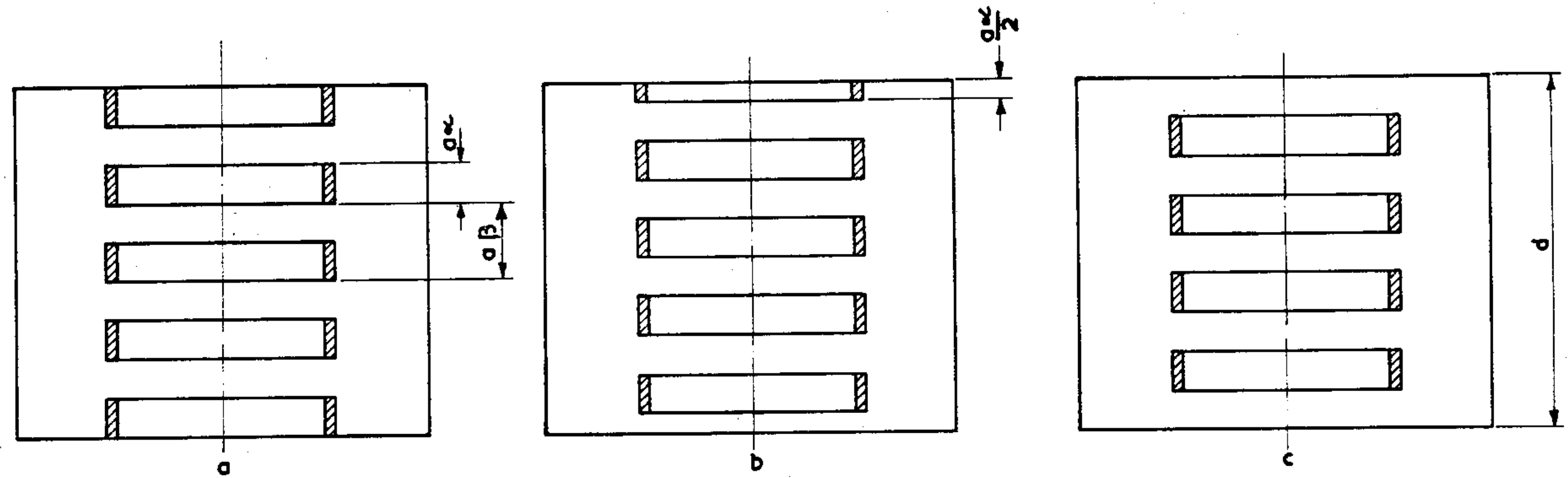

FIGURE 5. THE TRANSFORMED CONTROL ZONE FOR FINITE CONTROL ZONE DEPTH $d = 4 \cdot 5\ a\beta$

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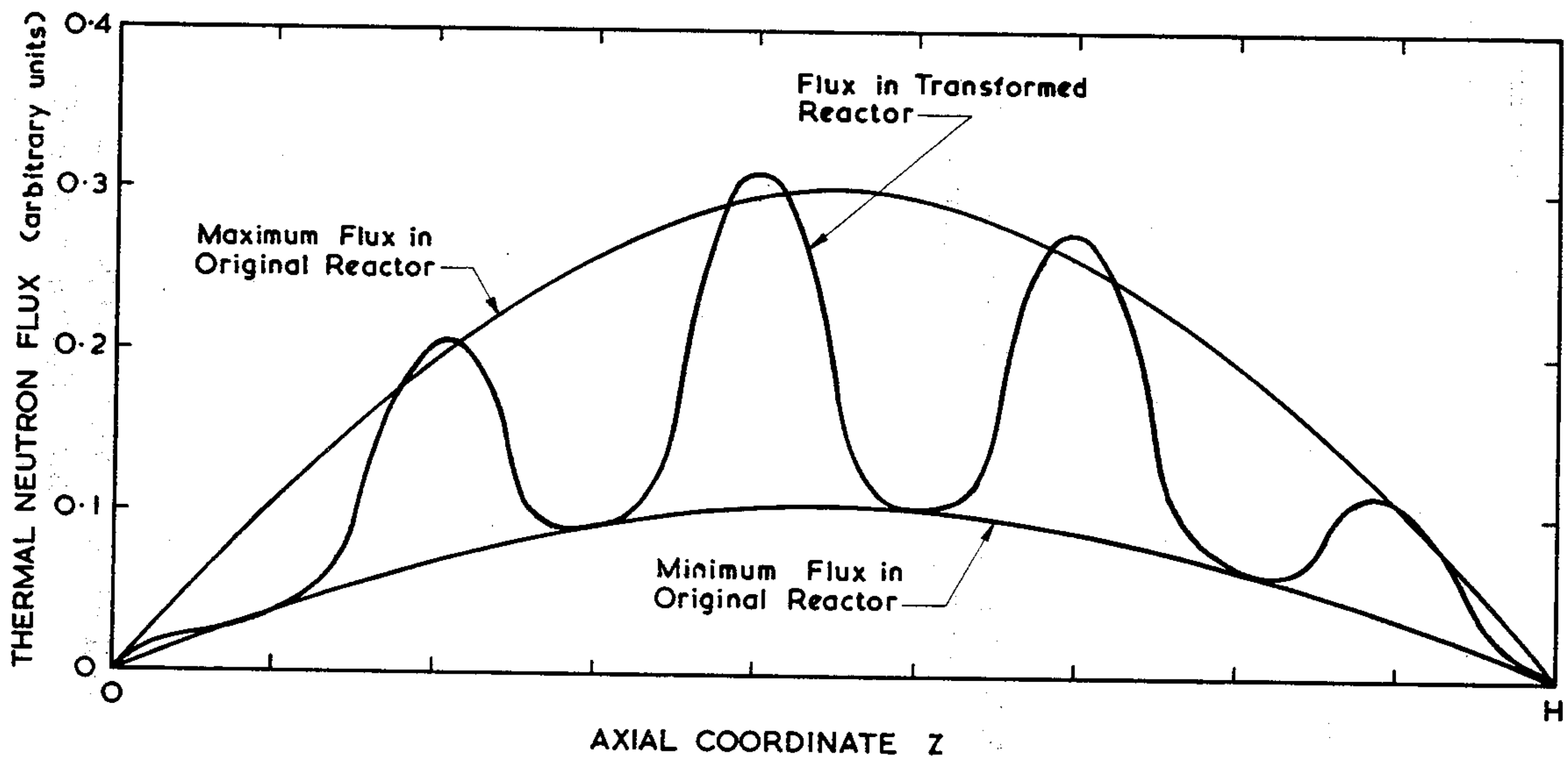

FIGURE 6. A COMPARISON OF THE AXIAL FLUX DISTRIBUTIONS NEAR THE CONTROL ZONE OF A FINITE CYLINDRICAL REACTOR WITH EIGHT FULLY INSERTED CONTROL RODS.

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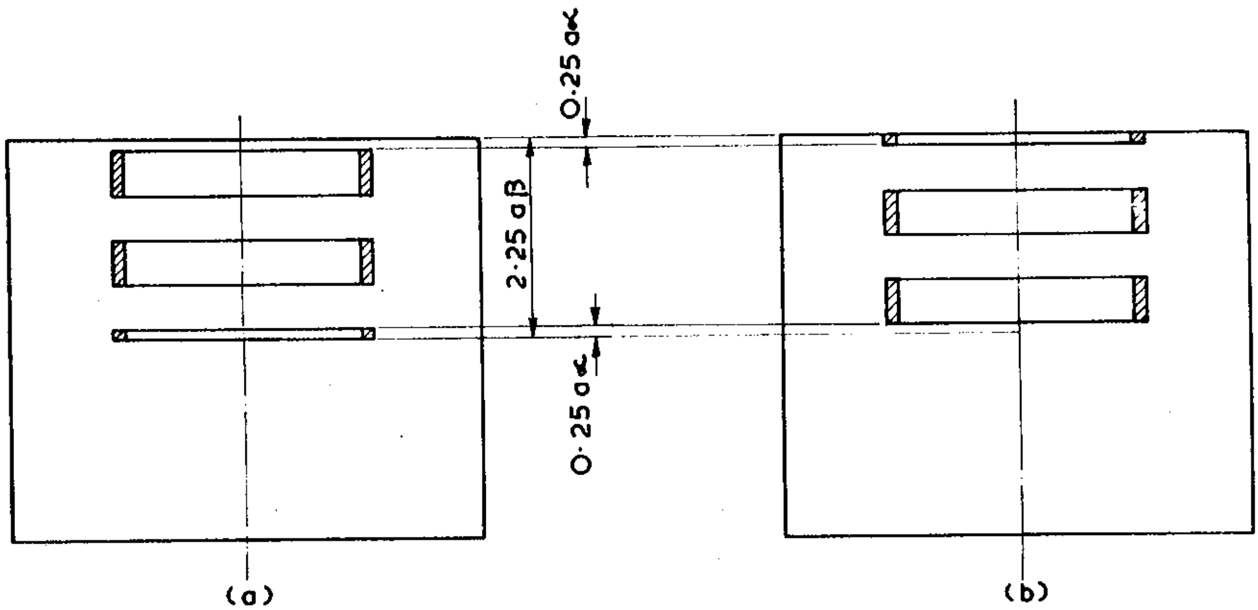

FIGURE 7. POSSIBLE MODELS FOR THE PROBLEM OF PARTIALLY INSERTED CONTROL RODS

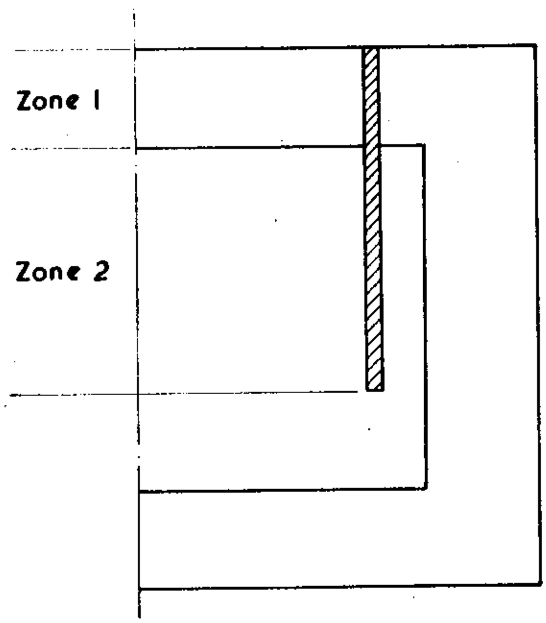

FIGURE 8. AXIAL ZONES IN A NON – UNIFORM REACTOR

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