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AUSTRALIAN ATOMIC ENERGY COMMISSION

RESEARCH ESTABLISHMENT
LUCAS HEIGHTS

FORTRAN IV PROGRAMMES FOR COMPUTATION OF TEMPERATURE AND THERMOELASTIC STRESS IN A HOMOGENEOUS SPHERICAL FUEL ELEMENT DUE TO AXISYMMETRIC HEAT TRANSFER VARIATION OVER THE SURFACE

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ABSTRACT

Computer programmes together with a brief outline of the theory are presented which enable computation of temperature and thermoelastic stress fields in homogeneous spherical fuel elements due to axisymmetric heat transfer variation over the surface. Uniform heat generation in the fuel element is assumed.

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1. INTRODUCTION

The spherical fuel elements in a pebble bed reactor core are subject to non-uniform heat transfer over their surface, which causes temperature and stress increases compared with those based on average heat transfer. While the solutions for temperature and stresses due to arbitrary heat transfer variation over the surface have been discussed by Thompson (1964 and 1965), practical application of the theory for computation is not available because of the extreme complexity of the problem. A considerable simplification can be achieved if an axisymmetric heat transfer variation over the surface is assumed. This has been analysed (Holy 1967) and results obtained for a number of experimentally derived and artificially created distributions.

The determination of temperature and stresses for an axisymmetric heat transfer variation should be of considerable practical importance, as the results can be used as a first approach in a design analysis of the spherical fuel elements, which as a rule are stress limited. Another application is the temperature calibration of the instrumented spheres used for the determination of the heat transfer coefficient in a bed or array of spheres. The instrumented sphere, which is internally heated, is calibrated in a stream of coolant, giving rise to axisymmetric heat transfer distribution over the surface.

In this report programmes written in Fortran IV language are presented which can be used on any computer having a storage equivalent to or larger than an IBM 7040.

The computation is split into three separate stages. The first expands an arbitrary axisymmetric heat transfer distribution in terms of Legendre polynomials. Four different approaches are used, depending on the type of distribution analysed, each one programmed separately. The second stage consists of a programme which uses the results from the first stage to determine the expansion coefficients, which are used as input data in the third stage programme to give the estimates of the temperature and stress fields through the sphere.

A brief outline of theory involved in each computational stage is now presented.

2. REVIEW OF THE THEORY

For an axisymmetric heat transfer distribution over the surface of a heat producing sphere, such as is shown in Figure 1, the heat transfer coefficient variation is a function of angle θ only. Solutions for the temperature and stresses can be expressed in terms of a truncated series of Legendre polynomials. As a first step in obtaining the solution it is necessary to expand the heat transfer distribution in terms of Legendre polynomials.

2.1 Expansion of a Heat Transfer Distribution in Terms of Legendre Polynomials

Let the distribution (Figure 2) be re-plotted as a function of an alternative variable $\mu = \cos\theta$. The heat transfer coefficient H (Figure 3) can then be written.

$$H = H_o \left\{ 1 + \frac{h(\mu)}{H_o} \right\} = H_o \left\{ 1 + \psi\,(\mu) \right\} .$$

The expansion required is that of $\psi\,(\mu)$, the dimensionless heat transfer coefficient variation about the mean H_o.

Thus:

$$\psi\,(\mu) = \sum_n \psi_n P_n\,(\mu) ,$$

where ψ_n are the expansion coefficients with $\psi_o = 0$, and $P_n(\mu)$ are the Legendre polynomials of the first kind and the n^{th} degree. The ψ_n coefficients are then used as input data for the next programme stage.

Two distinct methods are used in the programme to obtain the required expansion coefficients:

(1) Methods based on $\psi_n = \frac{2n+1}{2} \int_{-1}^{+1} \psi(\mu) P_n(\mu) d\mu$,

with integral being evaluated by suitable numerical quadratures.

(2) Least Squares method.

2.1.1 Numerical quadrature methods

To evaluate the integral $\int_{-1}^{+1} \psi(\mu)\; P_n(\mu) d\mu$ the following are used:

(a) Simpson's "One-third rule",

(b) Linear interpolation with Gaussian quadrature,

(c) Lagrange four point interpolation with Gaussian quadrature.

The abscissae and weight factors used here for the 80th order Gaussian quadrature are those quoted by Davis and Rabinowitz (1958), and the Lagrange four point interpolation formula can be found, for instance, in Abramowitz and Stegun (1964).

2.1.2 Least Squares Method

The principle of Least Squares states that the sum of the squares of the deviations should be a minimum. Thus from the expansion:

$$\psi(\mu) = \sum_n \psi_n P_n(\mu) ,$$

$$I = \sum_i \left\{ \psi(\mu_i) - \sum_n \psi_n P_n(\mu_i) \right\}^2$$

is formed, where i refers to the i^{th} point of the given heat transfer distribution.

For a minimum:

$$\frac{\partial I}{\partial \psi_m} = -2 \sum_i \left[\left\{ \psi(\mu_i) - \sum_n \psi_n P_n(\mu_i) \right\} P_m(\mu_i) \right] = 0 ,$$

where m = 0, 1, 2, 3 . . . .

Therefore:

$$\sum_n \psi_n \sum_i \left\{ P_n(\mu_i)\; P_m(\mu_i) \right\} = \sum_i \psi(\mu_i)\; P_m(\mu_i) .$$

This can be written concisely in matrix form as:

$$M\psi = G ,$$

where M is a symmetric matrix with elements $M_{n,m} = \sum_i P_n(\mu_i)\; P_m(\mu_i)$, and vectors ψ and G are:

$$\psi = \begin{Bmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix} , \qquad G = \begin{Bmatrix} \sum_i \psi(\mu_i)\; P_0(\mu_i) \\ \sum_i \psi(\mu_i)\; P_1(\mu_i) \\ \sum_i \psi(\mu_i)\; P_2(\mu_i) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix} .$$

The solution is then simply:

$$\psi = M^{-1}G .$$

Individual Legendre polynomials are generated by a recurrence relation quoted by Kizner (1966) which is well suited for computation:

$$P_n(\mu) = w_0 ,$$

with $\quad w_n = 1, \quad w_{n-1} = b_{n-1}\mu w_n ,$

$$w_j = b_j \mu w_{j+1} - c_j w_{j+2} ,$$

for $j = n-2, n-3, \ldots\ldots 0;$

$$b_i = \frac{2i+1}{i+1} , \quad c_i = \frac{i+1}{i+2} ,$$

for $i = 0, 1, \ldots\ldots$.

The programmes are designed to give the coefficients ψ_n and also the average value H_0, the input data being the heat transfer coefficient H associated with the discrete values of μ .

2.2. Coefficients of the Temperature and Stress Series

The next programme stage is concerned with the calculation of the temperature and stress series coefficients, using the results from the stage one programme.

Consider a homogeneous sphere of radius a, with uniform internal heat generation per unit volume $Q(r) = Q$ and axisymmetric heat transfer variation over the surface as shown in Figure 1. Under steady state conditions and measuring temperature T relative to coolant, the problem can be formulated, after introducing the dimensionless parameter $\rho = \frac{r}{a}$, as:

$$\nabla_\rho^2 T + \frac{Qa^2}{k} = 0 ,$$

with $$\frac{\partial T}{\partial \rho}\Big|_1 + \frac{H_0 a}{k}\left(1 + \psi(\mu)\right) T(1) = 0 , \text{ at } \rho = 1.$$

Let $T = T_0 + X$; then the formulation may be separated as follows:

$$\nabla_\rho^2 T_0 + S = 0 , \qquad \ldots\ldots(1)$$

$$\frac{\partial T_0}{\partial \rho}\Big|_1 + \beta T_0(1) = 0, \qquad \ldots\ldots(2)$$

and $$\nabla_\rho^2 X = 0 , \qquad \ldots\ldots(3)$$

$$\frac{\partial X}{\partial \rho}\Big|_1 + \beta\left(1 + \psi(\mu)\right) X(1) = - \beta\psi(\mu) T_0(1) , \qquad \ldots\ldots(4)$$

where Biot No. $\beta = \frac{H_0 a}{k}$ and $S = \frac{Qa^2}{k}$ is the heat source term.

The solution to (1) and (2) which gives the temperature associated with the mean value of the heat transfer coefficient H_0 , is simply:

$$T_0 = \frac{S}{6}(1 - \rho^2) + \frac{S}{3\beta} .$$

Because of the spherical symmetry involved, the solution to (3) for the perturbation temperature is:

$$X = \sum_n \lambda_n \rho^n P_n(\mu) . \qquad \ldots\ldots(5)$$

Let $\psi(\mu)$, the dimensionless heat transfer coefficient distribution about the mean, be expanded in terms of Legendre polynomials.

Thus:

$$\psi(\mu) = \sum_n \psi_n P_n(\mu) . \qquad \ldots\ldots(6)$$

Substituting (5) and (6) into (4) and applying the orthogonal properties of Legendre polynomials, there results a set of infinite equations for the coefficients λ_n. In matrix formulation this can be written:

$$\left[N + \beta \left\{ I + \sum_n \sum_r (-1)^r \frac{(2n+2r)!\, \psi_{n+2r}}{2^{n+2r} n!(n+r)!r!} \tilde{\Omega}^n \right\} \right] \lambda = - \frac{S}{3} \psi ,$$

where N is a diagonal matrix with $N_{ii} = (i-1)$, I is a unit matrix, and vectors λ and ψ are defined as:

$$\lambda = \begin{Bmatrix} \lambda_0 \\ \lambda_1 \\ \lambda_2 \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix} , \quad \psi = \begin{Bmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix}$$

and $\tilde{\Omega}$ denotes transpose of the matrix Ω defined by:

$$\Omega = \begin{bmatrix} 0 & 1 & & & & \\ \frac{1}{3} & 0 & \frac{2}{3} & & & \\ & \ddots & \ddots & \ddots & & \\ & & \frac{n}{2n+1} & 0 & \frac{n+1}{2n+1} & \\ & & & \ddots & \ddots & \ddots \\ & & & & & 0 \end{bmatrix}$$

Truncation of this set of infinite equations and their solution yields the coefficients λ_n, which enable the computation of estimated temperatures and stresses.

The coefficients obtained from the programme are normalised in the sense that they are computed for $\frac{S}{6} = \frac{Qa^2}{6k} = 1$. The same normalisation is also applied in the third stage programme when computing the dimensionless temperatures and stresses.

Suitable subroutines are used to handle the necessary matrix operations. The matrix inversion is done by a [illegible].

2.3 Evaluation of Temperature and Stresses

The third stage programme is concerned with the calculation of the temperature and stresses at any point r,θ of the sphere, based on the truncated coefficients λ_n from the second stage.

The temperature at a point r,θ is given as:

$$T(r,\theta) = \sum_n \lambda_n \left(\frac{r}{a}\right)^n P_n(\cos\theta) + T_c(r) .$$

The perturbation stresses involved are shown in Figure 4 and are obtained using a formulation by Nowacki (1962) as.

$$\sigma_{rr} = \sum_n \lambda_n (\sigma_{rr})_n , \quad \sigma_{\theta\theta} = \sum_n \lambda_n (\sigma_{\theta\theta})_n ,$$

$$\sigma_{\Phi\Phi} = \sum_n \lambda_n (\sigma_{\Phi\Phi})_n , \quad \text{and } \sigma_{r\theta} = \sum_n \lambda_n (\sigma_{r\theta})_n .$$

The stress components associated with the individual λ_n are:

$$(\sigma_{rr})_n = -\delta_n n(n-1)\left(1-\left(\frac{a}{r}\right)^2\right)\left(\frac{r}{a}\right)^n P_n(\mu) ,$$

$$(\sigma_{\theta\theta})_n = \delta_n \left[n\left(n+2-n\left(\frac{a}{r}\right)^2\right)P_n(\mu) - \left(3-\left(\frac{a}{r}\right)^2\right)\mu P_n'(\mu)\right]\left(\frac{r}{a}\right)^n ,$$

$$(\sigma_{\Phi\Phi})_n = -\delta_n \left[n\left(2n+1-\left(\frac{a}{r}\right)^2\right)P_n(\mu) - \left(3-\left(\frac{a}{r}\right)^2\right)\mu P_n'(\mu)\right]\left(\frac{r}{a}\right)^n ,$$

$$(\sigma_{r\theta})_n = \delta_n (n-1)\left(1-\left(\frac{a}{r}\right)^2\right)\left(\frac{r}{a}\right)^n \hat{\mu} P_n'(\mu) ,$$

where:

$$\mu = \cos\theta, \quad \hat{\mu} = \sin\theta, \quad P_n' = \frac{dP_n(\mu)}{d\mu} ,$$

$$\delta_n = \frac{\omega(1-\nu)}{n^2+n+1+(2n+1)\nu} , \quad \omega = \frac{E\alpha}{2(1-\nu)} .$$

In the centre of the sphere $\left(\frac{r}{a}\right) = 0$ and the perturbation stresses are then:

$$\sigma_{rr} = 2\lambda_2\delta_2 P_2(\mu) , \quad \sigma_{\theta\theta} = \lambda_2\delta_2\left[-4P_2(\mu)+\mu P_2'(\mu)\right] ,$$

$$\sigma_{\Phi\Phi} = \lambda_2\delta_2\left[2P_2(\mu) - \mu P_2'(\mu)\right] \quad \text{and } \sigma_{r\theta} = -\lambda_2\delta_2\hat{\mu}P_2'(\mu) .$$

The total stresses are obtained by adding the components due to uniform distribution of the mean heat transfer H_0 over the sphere surface.

Thus:

$$(\sigma_{rr})_t = \sigma_{rr} + (\sigma_{rr})_0 , \quad (\sigma_{\theta\theta})_t = \sigma_{\theta\theta} + (\sigma)_0 ,$$

$$(\sigma_{\Phi\Phi})_t = \sigma_{\Phi\Phi} + (\sigma)_0 \quad \text{and } (\sigma_{r\theta})_t = \sigma_{r\theta} ,$$

where:

$$(\sigma_{rr})_0 = \frac{2E\alpha\Delta T_0}{5(1-\nu)}\left\{\left(\frac{r}{a}\right)^2 - 1\right\} ,$$

$$(\sigma_{\theta\theta,\Phi\Phi})_0 = (\sigma)_0 = \frac{2E\alpha\Delta T_0}{5(1-\nu)}\left\{2\left(\frac{r}{a}\right)^2 - 1\right\} ,$$

and ΔT_0 is the temperature difference between the surface and the centre of the sphere.

It is also useful to combine the effects of all the stress field components by considering an equivalent stress derived from some type of failure criterion. Hencky-von Mise's yield criterion of maximum shear strain energy (Finnie and Heller 1959) is used here, for which the equivalent stress is defined as:

$$\sigma^* = \frac{1}{\sqrt{2}}\left[\left\{(\sigma_{\theta\theta})_t - (\sigma_{\Phi\Phi})_t\right\}^2 + \left\{(\sigma_{\Phi\Phi})_t - (\sigma_{rr})_t\right\}^2 + \left\{(\sigma_{rr})_t - (\sigma_{\theta\theta})_t\right\}^2 + 6(\sigma_{r\theta})_t^2\right]^{\frac{1}{2}} .$$

The first derivative $P_n'(\mu)$ required in the calculation of the perturbation stresses is obtained from:

$$P_n'(\mu) = (2n-1)P_{n-1}(\mu) + (2n-5)P_{n-3}(\mu) + (2n-9)P_{n-5}(\mu) + \ldots\ldots ,$$

which results simply from the relation (Morse and Feshbach 1953):

$$P_{n+1}'(\mu) - P_{n-1}'(\mu) = (2n+1)P_n(\mu) .$$

In the programme the temperatures and stresses are calculated both in dimensionless form and as actual magnitude values. The dimensionless form is for a normalised heat source of $\frac{S}{6} - 1$, the temperatures being expressed as fractions of the surface temperature T_0, while the stresses are given as $\frac{\sigma}{E\alpha\Delta T_0}$ where ΔT_0 is the surface to centre temperature difference. This form is useful when comparing various heat transfer distributions. The actual magnitudes of temperatures and stresses are then calculated for a particular power density which is read in as input data.

3. DESCRIPTION OF THE PROGRAMMES

The Fortran IV programmes and subroutines are listed in the Appendix. In this section their application and the input and output arrangements are discussed. The input for the first stage programmes depends on the method used to obtain the required expansion coefficient of the heat transfer distribution, while the output is the same for all the programmes. It is assumed that the distribution is given as a function of $\mu = \cos\theta$ and also that the programme variable $X \equiv \mu$ takes on values from 0 to 2 which correspond to $\mu = -1$ to $+1$. The expansion of the distribution in terms of Legendre polynomials is calculated for a specified number of terms and then tested by synthesising a distribution for a varied number of terms and comparing this with the original distribution. Depending on the accuracy required, the number of expansion coefficients is then selected to represent the distribution in the next programme stage.

The programmes based on the four different methods of obtaining the expansion are now detailed.

3.1 Simpson's "One-third Rule"

This programme is used when the distribution is a moderately fast varying function, values of which are given for a medium to large number (say 150 to 220) of uniformly spaced values of the argument μ.

Input:

N, L, M, MN, LN, DX

- N = Number of values of the distribution
- L = Number of coefficients required in the Legendre expansion
- M = The initial number of the expansion coefficients used in generating the synthetic distribution for comparison purposes

- MN = Number by which the synthetic distribution coefficients are incremented
- LN = The final number of the synthetic distribution coefficients
- DX = Step lengths between the values of μ

Y(I),I = 1,N

Values of the distribution at the relevant values of μ, starting with the value at $\mu = -1$ for I = 1.

3.2 Linear Interpolation with Gaussian Quadrature

If the distribution is a moderately fast varying function with values given for a large number (say 220) of uniformly or non-uniformly spaced values of the argument μ, linear interpolation with Gaussian quadrature can be used. This programme can also be used for a slowly varying function given by a small number of values (say 80).

Input:

C(I),B(I),I = 1,40

The abscissae and weight factors for the 80th order Gaussian quadrature. (Values are listed in the Appendix).

N, L, M, MN, LN

The same as in Section 3.1.

X(I),I = 1,N

Values of the variable X are given as $X = 1 + \mu$, that is, the variable X takes on values from 0 to 2, which correspond to $\mu = -1$ to $+1$.

Y(I),I = 1,N

The same as Section 3.1.

3.3 Lagrange Four Point Interpolation with Gaussian Quadrature

This programme should be used mainly for a distribution which is a fast varying function given by a medium to large number of values (say 150 to 220). The values of the argument can be spaced either uniformly or non-uniformly. This can also be used for slowly varying functions for any number of values and its accuracy is superior to the methods used in Sections 3.1 and 3.2.

Input:

Identical to that of Section 3.2.

3.4 Least Squares Method

This is most useful for distributions which are slow to medium fast varying functions with uniformly or non-uniformly spaced values of the argument μ. If a small number (say 60) of values is used, the accuracy is satisfactory only if the argument values are uniformly spaced.

Input:

N, L, M, MN, LN
X(I),I = 1,N
Y(I),I = 1,N

Identical to that in Section 3.2. There is no quadrature.

3.5 Output Form for Stage One Programmes

This is the same for all the stage one programmes and is given as follows:

INPUT SPECIFICATION

N= , L= , M= , MN= , LN=

VALUE OF FUNCTION

X(I), Y(I), I = 1,N

Values of the original distribution are printed out.

ACTUAL LEG COF

A(I),I = 1,L

This gives the actual values of the expansion coefficients.

REDUCED LEG COF

G(I),I = 1,L

Normalised coefficients obtained by dividing the actual values by the value of the first coefficient.

AVERAGE VALUE OF HEAT TRANSFER = A(1)

This is the value of H_o.

EXPANSION COEFFICIENTS

Q,G(I), I = 2,L

This lists all the coefficients which may eventually be used as input data for the stage two programme. The number to be used will depend on the accuracy of the synthetic distribution.

COMPARISON OF SYNTHETIC FUNCTION
POLYNOMIAL DEGREE = M - 1

Highest degree of the polynomial used in synthesising the distribution for comparison purposes.

EXPANSION COEFFICIENTS NOW

Q,G(I),I = 2,M

The values and the number of the expansion coefficients, which, subject to the accuracy test, are used as input data for the stage two programme.

X COORD	ACT FUNCT	SYNT FUNCT
EX	FX	TR

For values of the coordinate X(I) this gives Y(I) and the corresponding values of the synthetic distribution. The accuracy of the print-out is then visually compared and, if satisfactory, the number and values of the expansion coefficient selected for the next programme stage.

3.6 Coefficients of the Temperature and Stress Series

This programme enables the computation of the normalised temperature and stress series coefficients, which are subsequently used in the stage three programme to obtain the estimates of temperature and stresses. The normalisation is effected by taking the heat source term as $\frac{S}{6} = 1$. The data used as input are obtained from the results of the stage one programmes. Examination of the coefficients in the output, after discarding any with magnitude smaller than 10^{-4}, determines the input data for the stage three programme.

Input:

IDE

This serves as identification for a particular calculation.

N, NN, BIO

N = order of matrices required to ensure a satisfactory convergence of the truncated temperature and stress series coefficients. As a rule $N \geqq 40$.

NN = number of expansion coefficients from the stage one programmes which give satisfactory accuracy in representing the original distribution. $NN \equiv M$.

BIO = Biot No. = $\frac{H_c a}{k}$, where H is the average value of the heat transfer coefficient, which is again obtained from the stage one programmes.

FE(I),I = 1,N

Values of the expansion coefficients from stage one programme with FE(1) = 0.0, also FE(I) = 0.0 for $NN < I \leq N$

Output:

IDENTIFICATION IDE= , N= , NN= , BIO=

INPUT LEGENDRE COEFFICIENTS

FE(I),I = 1,N

EXPANSION COEFFICIENTS

A(I),I = 1,N

These coefficients are used as input for the stage three programme after discarding any smaller than 10^{-4}.

3.7 Temperature and Thermal Stress in a Sphere

The last programme enables the temperature and stresses to be obtained through the whole body of the sphere by varying radius r and angle θ, or simply for a fixed radius and variable θ only. In order to be able to compare the effects of various heat transfer distributions, the output is expressed both in dimensionless form and as actual magnitude values. The dimensionless temperatures are given as fractions of the surface temperature T_o associated with the average value of the heat transfer coefficient H_o. The dimensionless stresses are expressed as $\frac{\sigma}{E\alpha\Delta T_o}$, with ΔT_o being the centre to surface temperature drop associated with H_o. Further, the dimensionless forms are calculated for the same normalised heat source of $\frac{S}{6} = 1$ as used in the stage two programme. The actual magnitudes of the temperature and stresses are calculated for a particular uniform heat generation Q, the input being in the form $\frac{Qa}{H_o}$.

Input:

BIO, U, E, ALF, M

BIO = Biot No.

U = ν, Poisson's ratio

E = Young's modulus

ALF = α, coefficient of thermal expansion

M = number of temperature and stress series coefficients from the stage two programme.

A(I),I = 1,M

Values of the temperature and stress series coefficients from the stage two programme.

R, Q, DRX, RDX, KB, KC, KD

R = ρ, dimensionless radius, for the surface R = 1.0

Q = $\frac{Qa}{H_o}$, uniform heat generation term

DRX = increments in angle θ in degrees, usually 10°

RDX = decrements in the radius R, usually 0.1

KB = 1, temperature and stress calculation is carried through the whole or part of the sphere, depending on the value of R, in decrements of RDX to R = 0.0

KB = 2, calculation one for a fixed value of R

KC = 1, calculation carried on for all values of θ from 0^o to 180^o with increments of DRX. If R = 1, that is, on the surface, the increments are automatically taken as DRX/2.

KC = 2, this applies for a distribution symmetric about the equator, that is, θ varies only from 0^o to 90^o

KD = 1, only the dimensionless form is calculated

KD = 2, both the dimensionless form and the actual magnitudes are calculated.

Output:

IDENTIFICATION

BIO= , U= , E= , ALF= , M=

COEFFICIENTS A

A(I),I = 1,M

The coefficients A read in as input are printed out as identification.

RAD=R, ANGLE=DG, Q = Q

This serves as a heading to identify the point r,θ of the sphere at which the following temperatures and stresses are calculated. Q is the uniform heat generation term. The notation used in describing the temperatures and stresses is that detailed in Section 2.

NORMALISED AND DIMENSIONLESS TEMPERATURES

These temperatures are calculated for the normalised heat source term of $\frac{S}{6} = 1$.

PERTUBATION T=T, DUE HO TU=T_0, TOTAL TOT=$T+T_0$, SURF HO TW=$T_0(1)$

RATIOS

This denotes temperatures expressed as fractions of the surface temperature T_0 due to H_0.

PERTUBATION TH=$T/T_0(1)$, DUE HO TUD=$T_0/T_0(1)$ TOTAL TX=$(T+T_0)/T_0(1)$

ACTUAL TEMPERATURES

Actual magnitudes of temperatures due to uniform heat generation term Q.

PERTUBATION TP=T, DUE HO TUT=T_0, TOTAL TE=$T+T_0$, SURF HO TUS=$T_0(1)$

NORMALISED DIMENSIONLESS STRESSES

These stresses are calculated for the normalised heat source term of $\frac{S}{6} = 1$, and also expressed in the dimensionless form $\frac{\sigma}{E\alpha\Delta T_0}$.

PERTUBATION SA = $\frac{\sigma_{rr}}{E\alpha\Delta T_0}$, SB = $\frac{\sigma_{\theta\theta}}{E\alpha\Delta T_0}$, SE = $\frac{\sigma_{\Phi\Phi}}{E\alpha\Delta T_0}$, SD = $\frac{\sigma_{r\theta}}{E\alpha\Delta T_0}$

DUE UNIFORM HO SRU = $\frac{(\sigma_{rr})_0}{E\alpha\Delta T_0}$, STU $\frac{(\sigma)_0}{E\alpha\Delta T_0}$

TOTAL ZSA = $\frac{(\sigma_{rr})_t}{E\alpha\Delta T_0}$, ZSB = $\frac{(\sigma_{\theta\theta})_t}{E\alpha\Delta T_0}$, ZSC = $\frac{(\sigma_{\Phi\Phi})_t}{E\alpha\Delta T_0}$, ZSD = $\frac{(\sigma_{r\theta})_t}{E\alpha\Delta T_0}$

EQUIVALENT SIG = $\frac{\sigma^*}{E\alpha\Delta T_0}$

PERTB HO RATIOS XRR = $\frac{\sigma_{rr}}{(\sigma)_0}$, XTT = $\frac{\sigma_{\theta\theta}}{(\sigma)_0}$, XFF = $\frac{\sigma_{\Phi\Phi}}{(\sigma)_0}$, XRT = $\frac{\sigma_{r\theta}}{(\sigma)_0}$

This gives the ratios of the perturbation stresses to the surface stresses caused by H_0. This is of use when plotting the results.

ACTUAL MAGNITUDE OF STRESSES

The actual magnitude of stresses caused by the uniform heat generation Q in the sphere.

PERTUBATION ZA = σ_{rr}, ZB = $\sigma_{\theta\theta}$, ZC = $\sigma_{\Phi\Phi}$, ZD = $\sigma_{r\theta}$

DUE UNIFORM HO SRA = $(\sigma_{rr})_0$, STA = $(\sigma)_0$

TOTAL ZAA = $(\sigma_{rr})_t$, ZAB = $(\sigma_{\theta\theta})_t$, ZAC = $(\sigma_{\Phi\Phi})_t$, ZAD = $(\sigma_{r\theta})_t$

EQUIVALENT SAC = σ^* .

3.8 Sample of Typical Output

IDENTIFICATION BIO = 2.000U = 0.310E = 0.41800E + 08ALF = 0.10000E-04M=8

COEFFICIENTS A 0.32495E + 00 -0.97483E + 00 0.34324E + 00 - 0.85341E - 01 0.16668E - 01 - 0.26944E - 02 0.36940E - 03 - 0.44170E - 04

RAD = 1.000 ANGLE = 0. Q = 75.00

NORMALISED AND DIMENSIONLESS TEMPERATURES

PERTUBATION T = - 0.37768E + 00DUE HO TU = 0.10000E + 01

TOTAL TOT = 0.62232E + 00SURF HO TW = 0.10000E + 01

RATIOS

PERTUBATION TH = - 0.37768E + 00DUE HO TUD = 0.10000E + 01

TOTAL TX = 0.62232E + 00

ACTUAL TEMPERATURES

PERTUBATION TP = - 0.94421E + 01DUE HO TUT = 0.25000E + 02

TOTAL TE = 0.15558E + 02SURF HO TUS = 0.25000E + 02

NORMALISED DIMENSIONLESS STRESSES

PERTUBATION SA = -0. SB = -0.26792E - 01SE = -0.26792E - 01SD = 0.

DUE UNIFORM HO SRU = 0. STU = 0.57971E + 00

TOTAL ZSA = -0. ZSB = 0.55292E + 00ZSC = 0.55292E + 00ZSD = 0.

EQUIVALENT SIG = 0.55292E + 00

PERTB HO RATIOSXRR = -0. XTT = -0.46216E - 01XFF = -0.46216E -01XRT = 0.

ACTUAL MAGNITUDE OF STRESSES

PERTUBATION ZA = -0. ZB = -0.27997E + 03ZC = -0.27997E + 03ZD = 0.

DUE UNIFORM HO SRA = 0. STA = 0.60580E + 04

TOTAL ZAA = -0. ZAB = 0.57780E + 04ZAC = 0.57780E + 04ZAD = 0.

EQUIVALENT SAC = 057780E + 04

4. ACKNOWLEDGEMENTS

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6. NOTATION

a	radius of sphere
b	coefficient in recurrence relation for $P_n(\mu)$
c	coefficient in recurrence relation for $P_n(\mu)$
E	Young's modulus
G	vector in Least Squares Method
$h(\mu)$	heat transfer coefficient variation about mean H_o
H	heat transfer coefficient distribution
H_o	mean value of heat transfer coefficient
I	unit matrix; also denotes sum of squares of derivations
k	thermal conductivity
M	symmetric matrix in Least Squares Method
n	positive integer
N	diagonal matrix used in solution
$P_n(\mu)$	n^{th} degree Legendre polynomial of the first kind
Q	internal heat generation per unit volume
r	radius vector; also denotes positive integer
S	heat source term $\frac{Qa^2}{k}$
T	temperature measured relative to coolant
T_o	temperature component due to mean value of heat transfer H_o
x, y, z	coordinate axes
X	temperature component due to $h(\mu)$
w	variable in recurrence relation for $P_n(\mu)$
α	coefficient of thermal expansion
β	Biot number $\frac{H_o a}{k}$
δ_n	stress coefficient associated with state n
θ, Φ	angles of spherical coordinate system
λ	vector used in solution
λ_n	expansion coefficient of truncated series associated with $P_n(\mu)$
μ	alternative independent variable, $\mu = \cos\theta$
$\hat{\mu}$	variable in stress formulation, $\hat{\mu} = \sin\theta$
ν	Poisson's ratio
ρ	dimensionless radius vector, $\rho = \frac{r}{a}$
σ	stress components as defined in Figure 4

σ^*	equivalent total stress
ψ	vector used in solution
$\psi(\mu)$	normalised heat transfer coefficient variation $\psi(\mu) = \frac{h(\mu)}{H_o}$
ψ_n	coefficient in expansion of $\psi(\mu)$ associated with $P_n(\mu)$
ω	thermoelastic coefficient, $\omega = \frac{E\alpha}{2(1-\nu)}$
Ω	matrix used in solution

Subscripts

O	due to H_o
n	associated with λ_n
t	total
$\theta\theta, \Phi\Phi, rr, r\theta$	as defined in Figure 4 for stress components

APPENDIX

LISTING OF FORTRAN IV PROGRAMMES AND SUBROUTINES

APPENDIX 1 LISTING OF FORTRAN IV PROGRAMMES AND SUBROUTINES

```
      EXPANSION OF HEAT TRANSFER IN TERMS OF LEGENDRE POLYNOMIALS
      SIMPSONS ONE THIRD RULE
      DIMENSION Y(220),A(50),G(50),XU(220)
   42 READ(5,1)N,L,M,MN,LN,DX
    1 FORMAT(5I4,F7.3)
      READ(5,2)(Y(I),I=1,N)
    2 FORMAT(10F7.3)
      K=(N-1)/2
      DO 10 I=1,L
      BN=I-1
      SUM=0.0
      X=1.0
      CALL POL(XMN,BN,X)
      SUM=SUM+XMN*Y(N)
      X=-1.0
      CALL POL(XMN,BN,X)
      SUM=SUM+XMN*Y(1)
      DO 11 J=2,K,2
      Z=K-J+1
      X=Z*DX
      MA=N-J+1
   25 CALL POL(XMN,BN,X)
      SUM=SUM+4.*XMN*Y(MA)
      IF(X.LT.0.0)GO TO 11
      X=-X
      MA=J
      GO TO 25
   11 CONTINUE
      KN=K-1
      DO 12 J=3,KN,2
      Z=K-J+1
      X=Z*DX
      MA=N-J+1
   26 CALL POL(XMN,BN,X)
      SUM = SUM+2.*XMN*Y(MA)
      IF(X.LT.0.0)GO TO 12
      X=-X
      MA=J
      GO TO 26
   12 CONTINUE
      X=0.0
      CALL POL(XMN,BN,X)
      KM=K+1
      SUM=SUM+2.*XMN*Y(KM)
   10 A(I)=(2.*BN+1.0)*SUM*DX/6.0
      DO 70 I=1,L
   70 G(I)=A(I)/A(1)
      DO 35 I=1,N
      TU=I-1
   35 XU(I)=TU*DX
      WRITE(6,110)
  110 FORMAT(1X,19HINPUT SPECIFICATION)
      WRITE(6,111)N,L,M,MN,LN,DX
  111 FORMAT(1X,2HN=I4,2HL=I4,2HM=I4,3HMN=I4,3HLN=I4,3HDX=F7.3)
      WRITE(6,66)(XU(I),Y(I),I=1,N)
```

```
 66 FORMAT(1X,17HVALUE OF FUNCTION/(1X,10E12.4))
    WRITE(6,45)(A(I),I=1,L)
 45 FORMAT(1X,14HACTUAL LEG COF/(1X,10E13.5))
    WRITE(6,72)(G(I),I=1,L)
 72 FORMAT(1X,15HREDUCED LEG COF/(1X,10E13.5))
    WRITE(6,205)A(1)
205 FORMAT(1X,31HAVERAGE VALUE OF HEAT TRANSFER=E13.5)
    WRITE(6,206)
206 FORMAT(1X,22HESPANSION COEFFICIENTS)
    Q=0.0
    WRITE(6,207)Q,UG(I),I=2,L)
207 FORMAT(1X,10E13.5/(1X,10E13.5))
    WRITE(6,82)
 82 FORMAT(1X,32HCOMPARISON OF SYNTHETIC FUNCTION)
150 IJ=M-1
    WRITE(6,151)IJ
151 FORMAT(1X,18HPOLYNOMIAL DEGREE=I4)
    WRITE(6,208)
208 FORMAT(1X,26HESPANSION COEFFICIENTS NOW)
    WRITE(6,209)Q,(G(I),I=2,M)
209 FORMAT(1X,10E13.5/(1X,10E13.5))
    WRITE(6,152)
152 FORMAT(1X,13H   XCOORD    ,13H  ACT FUNCT  ,13H  SYNT FUNCT )
    K=1
156 EX=XU(K)
    XY = EX-1.0
    TR=0.0
    DO 153 I=1,M
    BN=I-1
    CALL POL(HA,BN,XY)
153 TR=TR+HA*A(I)
    FX=Y(K)
    WRITE(6,154)EX,FX,TR
154 FORMAT(1X,3E13.5)
    IF(K.EQ.N)GO TO 155
    K=K+1
    GO TO 156
155 IF(M.GE.LN)GO TO 42
    M=M+MN
    GO TO 150
    END

    INCLUDING THE FOLLOWING SUBROUTINES
    SUBROUTINE POL
```



```
    EXPANSION OF HEAT TRANSFER IN TERMS OF LEGENDRE POLYNOMIALS
    LINEAR INTERPOLATION WITH GAUSSIAN QUADRATURE
    DIMENSION B(80D,C(80),X(220),Y(220),D(80),A(50),G(50),Z(220)
    READ(5,11)(C(I),B(I),I=1,40)
 11 FORMAT(4F10.7)
 42 READ(5,1)N,L,M,MN,LN
  1 FORMAT(5I4)
    READ(5,2)(X(I)TI=1,N)
    READ(5,2)(Y(I),I=1,N)
  2 FORMAT(10F7.3)
    DO 12 I=1,40
    K=81-I
    C(K)=C(I)
    C(I)=-C(I)
 12 B(K)=B(I)
    DO 3 I=1,N
  3 Z(I)=X(I)-1.0
    I=1
 10 J=1
  6 U=Z(J)-C(I)
    IF(U.EQ.0.0)GO TO 4
    IF(U.GT.0.0)GO TO 5
    J=J+1
    GO TO 6
  4 D(I)=Y(J)
    GO TO8
  5 V=Z(J)-Z(J-1)
    R=Y(J)-Y(J-1)
    D(I)=Y(J-1)+(V-U)*R/V
  8 IF(I.EQ.80)GO TO 9
    I=I+1
    GO TO10
  9 DO 21 I=1,L
    BN=I-1
    SU=0.0
    DO 22 J=1,80
    XX=C(J)
    CALL POL(XMN,BN,XX)
 22 SU=SU+XMN*D(J)*B(J)
 21 A(I)=(2.*BN+1.0)*SU/2.0
    DO 70 I=1,L
 70 G(I)=A(I)/A(1)
    WRITE(6,110)
110 FORMAT(1X,19HINPUT SPECIFICATION)
    WRITE(6,111)N,L,M,MN,LN
111 FORMAT(1X,2HN=I4,2HL=I4,2HM=I4,3HMN=I4,3HLN=I4)
    WRITE(6,66)(X(I),Y(I),I=1,N)
 66 FORMAT(1X,17HVALUE OF FUNCTION/(1X,10E12.4))
    WRITE(6,45)(A(I),I=1,L)
 45 FORMAT(1X,14HACTUAL LEG COF/(1X,10E13.5))
    WRITE(6,72)(G(I),I=1,L)
 72 FORMAT(1X,15HREDUCED LEG COF/(1X,10E13.5))
    WRITE(6,205)A(1)
205 FORMAT(1X,31HAOERAGE VALUE OF HEAT TRANSFER=E13.5)
    WRITE(6,206)
206 FORMAT(1X,22HEXPANSION COEFFICIENTS)
```

```
      Q=0.0
      WRITE(6,207)Q,UG(I),I=2,L)
  207 FORMAT(1X,10E13.5/(1X,10E13.5))
      WRITE(6,82)
   82 FORMAT(1X,32HCOMPARISON OF SYNTHETIC FUNCTION)
  150 IJ=M-1
      WRITE(6,151)IJ
  151 FORMAT(1X,18HPOLYNOMIAL DEGREE=I4)
      WRITE(6,208)
  208 FORMAT(1X,26HEXPANSION COEFFICIENTS NOW)
      WRITE(6,209)Q,(G(I),I=2,M)
  209 FORMAT(1X,10E13.5/(1X,10E13.5))
      WRITE(6,152)
  152 FORMAT(1X,13H   XCOORD    ,13H  ACT FUNCT  ,13H  SYNT FUNCT )
      K=1
  156 EX=X(K)
      XY = EX-1.0
      TR=0.0
      DO 153 I=1,M
      BN=I-1
      CALL POL(HA,BN,XY)
  153 TR=TR+HA*A(I)
      FX=Y(K)
      WRITE(6,154)EX,FX,TR
  154 FORMAT(1X,3E13.5)
      IF(K.EQ.N)GO TO 155
      K=K+1
      GO TO 156
  155 IF(M.GE.LN)GO TO 42
      M=M+MN
      GO TO 150
      END

      INCLUDING THE FOLLOWING SUBROUTINES
      SUBROUTINE POL
```



```
      EXPANSION OF HEAT TRANSFER IN TERMS OF LEGENDRE POLYNOMIALS
      LAGRANGE FOUR POINT INTERPOLATION WITH GAUSSIAN QUADRATURE
      DIMENSION B(80),C(80),X(220),Y(220),Z(220),A(50),G(50),D(80)
      READ(5,11)(C(I),B(I),I=1,40)
   11 FORMAT(4F10.7)
   42 READ(5,1)N,L,M,MN,LN
    1 FORMAT(5I4)
      READ(5,2)(X(I),I=1,N)
      READ(5,2)(Y(I),I=1,N)
    2 FORMAT(10F7.3)
      DO 12 I=1,40
      K=81-I
      C(K)=C(I)
      C(I)=-C(I)
   12 B(K)=B(I)
      DO 3 I=1,N
    3 Z(I)=X(I)-1.0
      I=1
   10 J=1
    6 U=Z(J)-C(I)
      IF(U.EQ.0.0)GO TO 4
      IF(U.GT.0.0)GO TO 5
      J=J+1
      GO TO 6
    4 D(I)=Y(J)
      GO TO 8
    5 IF((J-2).LE.0)GO TO 201
      IF((J+1).GT.N)GO TO 202
      AO=Y(J-2)
      A3=Y(J+1)
      XO=Z(J-2)
      X3=Z(J+1)
      GO TO 255
  201 AO=Y(J)
      XO=Z(J-1)+Z(J-1)-Z(J)
      A3=Y(J+1)
      X3=Z(J+1)
      GO TO 255
  202 A3=Y(J-1)
      X3=Z(J)+Z(J)-Z(J-1)
      AO=Y(J-2)
      XO=Z(J-2)
  255 A1=Y(J-1)
      A2=Y(J)
      X1=Z(J-1)
      X2=Z(J)
      T=C(I)
      V=(T-X1)*(T-X2)*(T-X3)*AO/((XO-X1)*(XO-X2)*(XO-X3))
      V=V+(T-XO)*(T-X2)*(T-X3)*A1/((X1-XO)*(X1-X2)*(X1-X3))
      V=V+(T-XO)*(T-X1)*(T-X3)*A2/((X2-XO)*(X2-X1)*(X2-X3))
      V=V+(T-XO)*(T-X1)*(T-X2)*A3/((X3-XO)*(X3-X1)*(X3-X2))
      D(I)=V
    8 IF(I.EQ.80)GO TO 9
      I=I+1
      GO TO 10
    9 DO 21 I=1,L
```

```
      BN=I-1
      SU=0.0
      DO 22 J=1,80
      XX=C(J)
      CALL POL(XMN,BN,XX)
   22 SU=SU+XMN*D(J)*B(J)
   21 A(I)=(2.*BN+1.0)*SU/2.0
      DO 70 I=1,L
   70 G(I)=A(I)/A(1)
      WRITE(6,110)
  110 FORMAT(1X,19HINPUT SPECIFICATION)
      WRITE(6,111)N,L,M,MN,LN
  111 FORMAT(1X,2HN=I4,2HL=I4,2HM=I4,3HMN=I4,3HLN=I4)
      WRITE(6,66)(X(I),Y(I),I=1,N)
   66 FORMAT(1X,17HVALUE OF FUNCTION/(1X,10E12.4))
      WRITE(6,45)(A(I),I=1,L)
   45 FORMAT(1X,14HACTUAL LEG COF/(1X,10E13.5))
      WRITE(6,72)(G(I),I=1,L)
   72 FORMAT(1X,15HREDUCED LEG COF/(1X,10E13.5))
      WRITE(6,205)A(1)
  205 FORMAT(1X,31HAVERAGE VALUE OF HEAT TRANSFER=E13.5)
      WRITE(6,206)
  206 FORMAT(1X,22HEXPANSION COEFFICIENTS)
      Q=0.0
      WRITE(6,207)Q,(G(I),I=2,L)
  207 FORMAT(1X,10E13.5/(1X,10E13.5))
      WRITE(6,82)
   82 FORMAT(1X,32HCKMPARISON OF SYNTHETIC FUNCTION)
  150 IJ=M-1
      WRITE(6,151)IJ
  151 FORMAT(1X,18HPOLYNOMIAL DEGREE=I4)
      WRITE(6,208)
  208 FORMAT(1X,26HEXPANSION COEFFICIENTS NOW;
      WRITE(6,209)Q,(G(I),I=2,M)
  209 FORMAT(1X,10E13.5/(1X,10E13.5))
      WRITE(6,152)
  152 FORMAT(1X,13H    XCOORD     ,13H  ACT FUNCT   ,13H  SYNT FUNCT )
      K=1
  156 EX=X(K)
      XY = EX-1.0
      TR=0.0
      DO 153 I=1,M
      BN=I-1
      CALL POL(HA,BNTXY)
  153 TR=TR+HA*A(I)
      FX=Y(K)
      WRITE(6,154)EX,FX,TR
  154 FORMAT(1X,3E13.5)
      IF(K.EQ.N)GO TO 155
      K=K+1
      GO TO 156
  155 IF(M.GE.LN)GO TO 42
      M=M+MN
      GO TO 150
      END
```


INCLUDING THE FOLLOWING SUBROUTINES
SUBROUTINE POL

ABSCISSAS AND WEIGHTS OF GAUSSIAN QUADRATURE C(I),B(I),I=1,40

0.[illegible]95539	0.0011449	0.9976499	0.0026635
0.9942275	0.0041803	0.9892913	0.0056909
0.9828485	0.0071929	0.9749091	0.0086839
0.9654850	0.0101617	0.9545907	0.0116241
0.9422427	0.0130687	0.9284598	0.0144935
0.9132631	0.0158961	0.8966755	0.0172746
0.8787225	0.0186268	0.8594314	0.0199506
0.8388314	0.0212440	0.8169541	0.0225050
0.7938327	0.0237318	0.7695024	0.0249225
0.7440002	0.0260752	0.7173651	0.0271882
0.6896376	0.0282598	0.6608598	0.0292883
0.6310757	0.0302723	0.6003306	0.0312101
0.5686712	0.0321004	0.5361459	0.0329419
0.5028041	0.0337332	0.4686966	0.034473[illegible]
0.4338753	0.0351605	0.3983934	0.035794[illegible]
0.3623047	0.0363737	0.3256643	0.036897[illegible]
0.2885280	0.0373654	0.2509523	0.0377763
0.2129945	0.0381297	0.1747122	0.0384249
0.1361640	0.0386617	0.0974083	0.0388396
0.0585044	0.0389583	0.0195113	0.0390178

```
      EXPANSION OF HEAT TRANSFER IN TERMS OF LEGENDRE POLYNOMIALS
      LEAST SQUARES METHOD
      DIMENSION P(50T220),X(220),Y(220),D(50),Z1(50,50),Z(50,50),
     1A(50),G(50)
   42 READ(5,1)N,L,MTMN,LN
    1 FORMAT(5I4)
      READ(5,2)(X(I),I=1,N)
      READ(5,2)(Y(I),I=1,N)
    2 FORMAT(10F7.3)
      DO 84 I=1,L
      BN=I-1
      DO 85 J=1,N
      XY=X(J)-1.0
      CALL POL(XM,BN,XY)
   85 P(I,J)=XM
   84 CONTINUE
      DO 86 I=1,L
      D(I)=0.0
      DO 87 K=1,N
   87 D(I)=D(I)+Y(K)*P(I,K)
      DO 89 J=I,L
      Z(I,J)=0.0
      DO 88 K=1,N
   88 Z(I,J)=Z(I,J)+P(I,K)*P(J,K)
   89 Z(J,I)=Z(I,J)
   86 CONTINUE
      CALL BORD(Z1,ZTL)
      CALL COL(A,Z1,D,L)
      DO 70 I=1,L
   70 G(I)=A(I)/A(1)
      WRITE(6,110)
  110 FORMAT(1X,19HINPUT SPECIFICATION)
      WRITE(6,111)N,L,M,MN,LN
  111 FORMAT(1X,2HN=I4,2HL=I4,2HM=I4,3HMN=I4,3HLN=I4)
      WRITE(6,66)(X(I),Y(I),I=1,N)
   66 FORMAT(1X,17HVALUE OF FUNCTION/(1X,10E12.4))
      WRITE(6,45)(A(I),I=1,L)
   45 FORMAT(1X,14HACTUAL LEG COF/(1X,10E13.5))
      WRITE(6,72)(G(I),I=1,L)
   72 FORMAT(1X,15HREDUCED LEG COF/(1X,10E13.5))
      WRITE(6,205)A(1)
  205 FORMAT(1X,31HAVERAGE VALUE OF HEAT TRANSFER=E13.5)
      WRITE(6,206)
  206 FORMAT(1X,22HEXPANSION COEFFICIENTS)
      Q=0.0
      WRITE(6,207)Q,(G(I),I=2,L)
  207 FORMAT(1X,10E13.5/(1X,10E13.5))
      WRITE(6,82)
   82 FORMAT(1X,32HCOMPARISON OF SYNTHETIC FUNCTION)
  150 IJ=M-1
      WRITE(6,151)IJ
  151 FORMAT(1X,18HPOLYNOMIAL DEGREE=I4)
      WRITE(6,208)
  208 FORMAT(1X,26HEXPANSION COEFFICIENTS NOW)
      WRITE(6,209)Q,UG(I),I=2,M)
  209 FORMAT(1X,10E13.5/(1X,10E13.5))
```



```
      WRITE(6,152)
  152 FORMAT(1X,13H   XCOORD    ,13H   ACT FUNCT  ,13H   SYNT FUNCT )
      K=1
  156 EX=X(K)
      XY = EX-1.0
      TR=0.0
      DO 153 I=1,M
      BN=I-1
      CALL POL(HA,BNTXY)
  153 TR=TR+HA*A(I)
      FX=Y(K)
      WRITE(6,154)EX,FX,TR
  154 FORMAT(1X,3E13.5)
      IF(K.EQ.N)GO TO 155
      K=K+1
      GO TO 156
  155 IF(M.GE.LN)GO TO 42
      M=M+MN
      GO TO 150
      END

      INCLUDING THE FOLLOWING SUBROUTINES
      SUBROUTINE POL
      SUBROUTINE BORD
      SUBROUTINE COL
```

```
      COEFFICIENTS OF THE TEMPERATURE AND STRESS SERIES
      DIMENSION FE(50),WR(50),VG(50,50),ZZ(50,50),WG(50,50),
     1TW(50,50),A(50)
   25 READ(5,4)IDE
    4 FORMAT(I4)
      READ(5,1)N,NN,BIO
    1 FORMAT(2I4,F6.3)
      READ(5,2)(FE(I),I=1,N)
    2 FORMAT(5E12.4)
      DO 105 I=1,NN
      J=1
      II=I-1
      WR(I)=0.0
  102 JJ=J-1
      K=II+2*JJ+1
      IF(K.GT.NN)GO TO 105
      CALL WA(AW,II,JJ)
      WR(I)=WR(I)+AW*FE(K)
      IF(K.EQ.NN)GO TO 105
      J=J+1
      GO TO 102
  105 CONTINUE
      L=1
      I=1
      WRR=WR(I)
      CALL UN(ZZ,N,L)
      CALL EQW(TW,ZZ,N)
      CALL SC(WG,ZZ,WRR,N)
      CALL EQW(ZZ,WG,N)
      IF(I.EQ.NN)GO TO 107
      I=2
  106 WRR=WR(I)
      CALL OMG(ZZ,N)
      CALL TRP(VG,ZZ,N)
      CALL PWR(ZZ,VG,TW,N)
      CALL EQW(TW,ZZ,N)
      CALL SC(VG,ZZ,SRR,N)
      CALL ADD(ZZ,VG,WG,N)
      IF(I.EQ.NN)GO TO 107
      CALL EQW(WG,ZZ,N)
      I=I+1
      GO TO 106
  107 CALL UN(VG,N,L)
      CALL ADD(TW,VG,ZZ,N)
      CALL SC(ZZ,TW,BIO,N)
      L=2
      CALL UN(WG,N,L)
      CALL ADD(VG,WG,ZZ,N)
      CALL BORD(ZZ,V+,N)
      SA=-2.
      CALL SC(VG,ZZ,SA,N)
      CALL COL(A,VG,FE,N)
      WRITE(6,35)IDE,N,NN,BIO
   35 FORMAT(1X,15HIDENTIFICATION=I4,2HN=I4,3HNN=I4,4HBIO=F5.2)
      WRITE(6,250)
  250 FORMAT(1X,27HINPUT LEGENDRE COEFFICIENTS)
```



```
      WRITE(6,8)(FE(I),I=1,N)
    8 FORMAT(1X,5E12.4/(1X,5E12.4))
      WRITE(6,251)
  251 FORMAT(1X,22HEXPANSION COEFFICIENTS/(1X,5E13.5))
      WRITE(6,20)(A(I),I=1,N)
   20 FORMAT(1X,5E13.5/(1X,5E13.5))
      GO TO 25
      END

      INCLUDING THE FOLLOWING SUBROUTINES
      SUBROUTINE TRP
      SUBROUTINE SC
      SUBROUTINE ADD
      SUBROUTINE OMG
      SUBROUTINE EQW
      SUBROUTINE PWR
      SUBROUTINE WA
      SUBROUTINE UN
      SUBROUTINE BORD
      SUBROUTINE COL
```

```
      TEMPERATURE AND THERMAL STRESS IN SPHERE
      DIMENSION A(50),SRR(50),STT(50),SFF(50),SRT(50)
   22 READ(5,1)BIO,UTE,ALF,M
    1 FORMAT(2F6.3,2E13.5,I4)
      READ(5,2)(A(I),I=1,M)
    2 FORMAT(5E13.5)
      READ(5,6)R,Q,DRX,RDX,KB,KC,KD
    6 FORMAT(4F8.3,3I3)
      WRITE(6,150)
  150 FORMAT(1X,14HIDENTIFICATION)
      WRITE(6,81)BIO,U,E,ALF,M
   81 FORMAT(1X,4HBIO=F6.3,2HU=F6.3,2HE=E13.5,4HALF=E13.5,2HM=I4)
      WRITE(6,82)(A(I),I=1,M)
   82 FORMAT(1X,14HCOEFFICIENTS A/(5E13.5))
      PFE=3.14159265
      QU=Q*BIO/6.
      OM=E*ALF/(2.*(1.-U))
      G=OM*(1.-U)
      TW=2./BIO
   55 X=1.0
      DG=0.0
      IF((R-0.01).LE.0.0)GO TO 53
      RK=(1./R)**2
      RB=1.-RK
      RH=3.-RK
   53 T=0.0
      RA=1.0
      DO 7 I=1,M
      BN=I-1
      CALL POL(XMN,BN,X)
      T=T+A(I)*RA*XMN
    7 RA=RA*R
      TU=1.0+2./BIO-R**2
      TOT=TU+T
      TX=TOT/TW
      TUD=TU/TW
      TH=T/TW
      WRITE(6,8)R,DG,Q
    8 FORMAT(1X,4HRAD=F5.3,6HANGLE=F5.1,2HQ=F6.2)
      WRITE(6,270)
  270 FORMAT(1X,41HNORMALISED AND DIMENSIONLESS TEMPERATURES)
      WRITE(6,61)T,TU,TOT,TW
   61 FORMAT(1X,15HPERTUBATION  T=E13.5,12HDUE HO   TU=E13.5/
     11X,15HTOTAL        TOT=E13.5,12HSURF HO   TW=E13.5)
      WRITE(6,63)
   63 FORMAT(1X,6HRATIOS)
      WRITE(6,64)TH,TUD,TX
   64 FORMAT(1X,15HPERTUBATION TH=E13.5,12HDUE HO  TUD=E13.5/
     11X,15HTOTAL         TX=E13.5)
      IF(KD.EQ.1)GO TO 260
      TUT=QU*TU
      TE=QU*TOT
      TUS=QU*TW
      TP=QU*T
      WRITE(6,271)
  271 FORMAT(1X,19HACTUAL TEMPERATURES)
```



```
      WRITE(6,272)TP,TUT,TE,TUS
  272 FORMAT(1X,15HPERTUBATION TP=E13.5,12HDUE HO  TUT=E13.5/
     11X,15HTOTAL         TE=E13.5,12HSURF HO TUS=E13.5)
  260 XD=(1.-X**2)**0.5
      IF((R-0.01).LE.0.0)GO TO50
      DO 11 I=1,M
      AB=I-1
      AZ=AB**2+AB+1.+(2.*AB+1.)*U
      DEN=G/AZ
      CALL POL(XA,AB,X)
      RD=R**AB
      SRR(I)=-DEN*AB*(AB-1.)*RB*RD*XA
      CALL DER(AB,X,KD)
      ZU=RH*X*PD
      STT(I)=DEN*RD*(AB*(AB+2.-AB*RK)*XA-ZU)
      SFF(I)=-DEN*RD*(AB*(2.*AB+RB)*XA-ZU)
   11 SRT(I)=DEN*(AB-1.)*RB*RD*XD*PD
      SRRT=0.0
      STTT=0.0
      SFFT=0.0
      SRTT=0.0
      DO 13 I=1,M
      SRRT=SRRT+A(I)*SRR(I)
      STTT=STTT+A(I)*STT(I)
      SFFT=SFFT+A(I)*SFF(I)
   13 SRTT=SRTT+A(I)*SRT(I)
      GO TO 51
   50 AB=2.0
      AZ=7.0+5.0*U
      DEN=G/AZ
      CALL POL(XA,AB,X)
      CALL DER(AB,X,PD)
      SRRT=2.*DEN*XA*A(3)
      STTT=DEN*(X*PD-4.*XA)*A(3)
      SFFT = DEN*(2.*XA-X*PD)*A(3)
      SRTT=-DEN*XD*PD*A(3)
   51 W=E*ALF
      SA=SRRT/W
      SB=STTT/W
      SE=SFFT/W
      SD=SRTT/W
      WRITE(6,273)
  273 FORMAT(1X,33HNORMALISED DIMENSIONLESS STRESSES)
      WRITE(6,72)SA,SB,SE,SD
   72 FORMAT(1X,15HPERTUBATION    4HSA =E13.5,4HSB =E13.5,4HSE =E13.5,
     14HSD =E13.5)
      RP=R**2
      SW=2./(5.*(1.-U))
      SV=(2.*E*ALF)/(5.*(1.-U))
      SR=SV*(RP-1.)
      ST=SV*(2.*RP-1.)
      SRU=SR/W
      STU=ST/W
      WRITE(6,70)SRU,STU
   70 FORMAT(1X,15HDUE UNIFORM HO 4HSRU=E13.5,4HSTU=E13.5)
      ZRRT=SRRT+SR
```

```
      ZTTT=STTT+ST
      ZFFT=SFFT+ST
      ZSA=ZRRT/W
      ZSB=ZTTT/W
      ZSC=ZFFT/W
      ZSD=SD
      WRITE(6,40)ZSA,ZSB,ZSC,ZSD
   40 FORMAT(1X,15HTOTAL          4HZSA=E13.5,4HZSB=E13.5,4HZSC=E13.5,
     14HZSD=E13.5)
      SUM=(ZRRT+ZTTT+ZFFT)/3.
      DRR=ZRRT-SUM
      DTT=ZTTT-SUM
      DFF=ZFFT-SUM
      DRT=SRTT
      SIGM=(DRR**2+DTT**2+DFF**2)*3./2.
      SIGM=(SIGM+3.*DRT**2)**0.5
      SIG=SIGM/W
      WRITE(6,135)SIG
  135 FORMAT(1X,15HEQUIVALENT     4HSIG=E13.5)
      XRR=SA/SW
      XTT=SB/SW
      XFF=SE/SW
      XRT=SD/SW
      WRITE(6,89)XRR,XTT,XFF,XRT
   89 FORMAT(1X,15HPERTB HO RATIOS4HXRR=E13.5,4HXTT=E13.5,4HXFF=E13.5,
     14HXRT=E13.5)
      IF(KD.EQ.1)GO TO 261
      WRITE(6,275)
  275 FORMAT(1X,28HACTUAL MAGNITUDE OF STRESSES)
      ZA=SRRT*QU
      ZB=STTT*QU
      ZC=SFFT*QU
      ZD=SRTT*QU
      WRITE(6,280)ZA,ZB,ZC,ZD
  280 FORMAT(1X,15HPERTUBATION    4HZA =E13.5,4HZB =E13.5,4HZC =E13.5,
     14HZD =E13.5)
      SRA=SR*QU
      STA=ST*QU
      WRITE(6,281)SRA,STA
  281 FORMAT(1X,15HDUE UNIFORM HO 4HSRA=E13.5,4HSTA=E13.5)
      ZAA=ZRRT*QU
      ZAB=ZTTT*QU
      ZAC=ZFFT*QU
      ZAD=SRTT*QU
      WRITE(6,282)ZAA,ZAB,ZAC,ZAD
  282 FORMAT(1X,15HTOTAL          4HZAA=E13.5,4HZAB=E13.5,4HZAC=E13.5,
     14HZAD=E13.5)
      SAC=SIGM*QU
      WRITE(6,283)SAC
  283 FORMAT(1X,15HEQUIVALENT     4HSAC=E13.5)
  261 IF((X+1.0).EQ.0.0)GO TO 54
      DU=DRX
      IF(R.EQ.1.0)DU3DRX/2.0
      DG=DG+DU
      IF(DG.GT.180.0)GO TO 54
      ARC=PFE*DG/180.
      X=COS(ARC)
```



```
      IF(DG.GE.90.5)GO TO 32
      GO TO 53
   32 IF(KC.EQ.2)GO TO 54
      IF(DG.GE.179.999)X=-1.0
      GO TO 53
   54 IF(KB.EQ.2)GO TO 22
      R=R-RDX
      IF((R+0.05).LE.0.0)GO TO 22
      GO TO 55
      END
```

INCLUDING THE FOLLOWING SUBROUTINES
SUBROUTINE POL
SUBROUTINE DER

LISTING OF SUBROUTINES

```
      SUBROUTINE POL(Q,BN,X)
      IF(BN.GE.2.0)GO TO 4
      IF(BN.EQ.0.0)GO TO 3
      IF(BN.EQ.1.0)W3=X
      GO TO 2
    4 N=BN-1.0
      W1=1.0
      AN=N
      B=(2.*AN+1.0)/(AN+1.0)
      W2=B*X*W1
    1 N=N-1
      AN=N
      B=(2.*AN+1.0)/(AN+1.0)
      C=(AN+1.0)/(AN+2.0)
      W3=B*X*W2-C*W1
      IF(N.EQ.0)GO TO 2
      D=W2
      W2=W3
      W1=D
      GO TO 1
    3 W3=1.0
    2 Q=W3
      RETURN
      END

      SUBROUTINE DER(AN,X,RE)
      IF(AN.GT.1.01)GO TO 1
      IF(AN.LE.1.01) RE=1.0
      IF(AN.LE.0.01) RE=0.0
      GO TO 4
    1 RE=0.0
      AD=2.*AN-1.0
      AE=AN-1.0
    3 CALL POL(XMN,AE,X)
      RE=RE+AD*XMN
      IF(AE.LE.1.01)GO TO 4
      AD=AD-4.0
      AE=AE-2.0
      GO TO 3
    4 RETURN
      END

      SUBROUTINE TRP(HA,CH,N)
      DIMENSION HA(50,50),CH(50,50)
      DO 1 I=1,N
      DO 2 J=1,N
    2 HA(I,J)=CH(J,I)
    1 CONTINUE
      RETURN
      END

      SUBROUTINE SC(FH,GZ,BO,N)
      DIMENSION GH(50,50),GZ(50,50)
      DO 1 I=1,N
```



```
      DO 2 J=1,N
    2 GH(I,J)=BO*GZ(I,J)
    1 CONTINUE
      RETURN
      END

      SUBROUTINE ADD(HD,HE,HG,N)
      DIMENSION HD(50,50),HE(50,50),HG(50,50)
      DO 1 I=1,N
      DO 2 J=1,N
    2 HD(I,J)=HE(I,J)+HG(I,J)
    1 CONTINUE
      RETURN
      END

      SUBROUTINE OMGUYR,N)
      DIMENSION YR(50,50)
      DO 1 I=1,N
      DO 2 J=1,N
    2 YR(I,J)=0.0
    1 CONTINUE
      M=N-1
      DO 3 I=1,M
      AN=I-1
      AM=I
      YR(I,I+1) = (AN+1.0)/(2.*AN+1.0)
    3 YR(I+1,I)=AM/(2.*AM+1.0)
      RETURN
      END

      SUBROUTINE EQW(AX,ATA,N)
      DIMENSION AX(50,50),ATA(50,50)
      DO 1 I=1,N
      DO 2 J=1,N
    2 AX(I,J)=ATA(I,J)
    1 CONTINUE
      RETURN
      END

      SUBROUTINE PWRUF,G,H,N)
      DIMENSION F(50,50),G(50,50),H(50,50)
      K=1
    4 DO 1 I=1,N
      F(K,I)=0.0
      DO 2 J=1,N
    2 F(K,I)=F(K,I)++(K,J)*H(J,I)
    1 CONTINUE
      IF(K.EQ.N)GO TO 3
      K=K+1
      GO TO 4
    3 RETURN
      END

      SUBROUTINE WA(W,NN,NR)
      ZN=NN
      ZR=NR
```

```
      L=1
    4 IF(L.EQ.1)BN=ZN
      IF(L.EQ.2)BN=ZR
      IF(L.EQ.3)BN=ZN+ZR
      IF(L.EQ.4)BN=2.*ZN+2.*ZR
      IF((BN 1.01).GT.0.0)GO TO 1
      IF((BN-1.01).LE.0.0)FN=1.0
      GO TO 3
    1 FN=BN
      CN=BN
    2 CN=CN 1.0
      FN=[illegible]N/ [illegible]
      IF((CN 1.01).LE.0.0)GO TO 3
      GO TO 2
    3 IF(L.EQ.1)Q1=FN
      IF(L.EQ.2)Q2=FN
      IF(L.EQ.3)Q3=FN
      IF(L.EQ.4)Q4=FN
      L=L+1
      IF(L.LT.5)GO TO 4
      M=NN+2*NR
      IF(M.GT.0) GO TO 7
      Q5=1.0
      GO TO 8
    7 Q5=2**M
    8 W=Q4/(Q1*Q2*Q3MQ5)
      NA=(ZR/2.+0.1)
      CX=NA*2
      CX=CX+0.1
      IF((ZR-CX).LE.0.0)GO TO 5
      W=-W
    5 RETURN
      END

      SUBROUTINE UN(UZ,N,L)
      DIMENSION UZ(50,50)
      DO 1 I=1,N
      DO 2 J=1,N
    2 UZ(I,J)=0.0
    1 CONTINUE
      IF(L.EQ.1)GO TO 4
      DO 3 I=1,N
    3 UZ(I,I)=I-1
      GO TO 6
    4 DO 5 I=1,N
    5 UZ(I,I)=1.0
    6 RETURN
      END

      SUBROUTINE COLUDA,HB,CE,N)
      DIMENSION DA(50),HB(50,50),CE(50)
      DO 1 I=1,N
      DA(I)=0.0
      DO 2 J=1,N
    2 DA(I)=DA(I)+HBUI,J)*CE(J)
    1 CONTINUE
```



```
      RETURN
      END

      SUBROUTINE BORD(H,W,M)
      DIMENSION W(50,50),H(50,50),R(50,50),S(50,50)
      X=W(2,1)/W(2,2)
      H(1,1)=1.0/(W(1,1)-W(1,2)*X)
      H(2,1)=-X*H(1,1)
      X=W(1,2)/W(1,1)
      H(2,2)=1.0/(W(2,2)-W(2,1)*X)
      H(1,2)=-X*H(2,2)
      N=2
    1 K=N
      N=N+1
      DO 2 I=1,K
      R(N,I)=0.0
      S(I,N)=0.0
      DO 2 J=1,K
      R(N,I)=R(N,I)-W(N,J)*H(J,I)
    2 S(I,N)=S(I,N)-+(I,J)*W(J,N)
      ALN=0.0
      DO 3 I=1,K
    3 ALN=ALN+W(I,N)*R(N,I)
      ALN=ALN+W(N,N)
      X=1.0/ALN
      DO 4 I=1,K
      H(I,N)=S(I,N)*X
      H(N,I)=R(N,I)*X
      DO 4 J=1,K
    4 H(I,J)=H(I,J)+S(I,N)*R(N,J)*X
      H(N,N)=X
      IF(N.LT.M)GO T- 1
      RETURN
      END
```

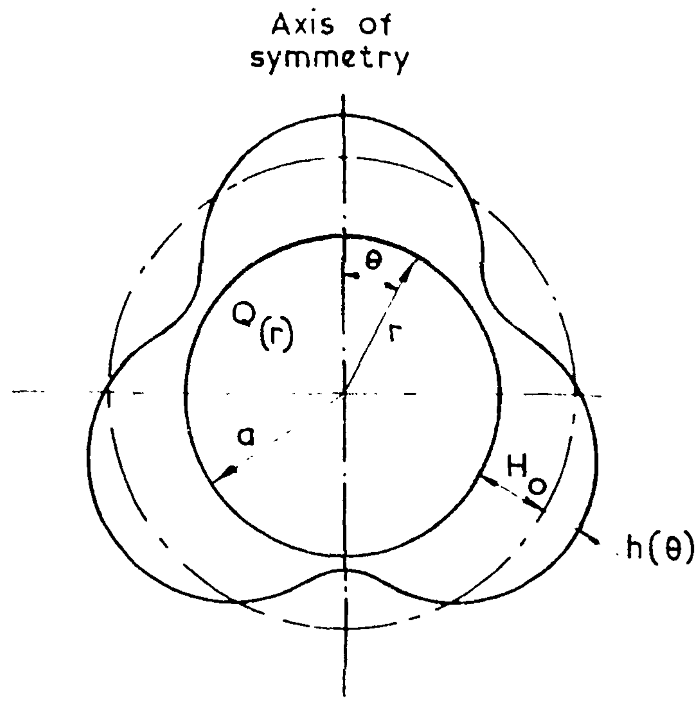

FIGURE 1. AXIALLY SYMMETRIC HEAT TRANSFER VARIATION OVER SURFACE OF A SPHERE

P1007/1

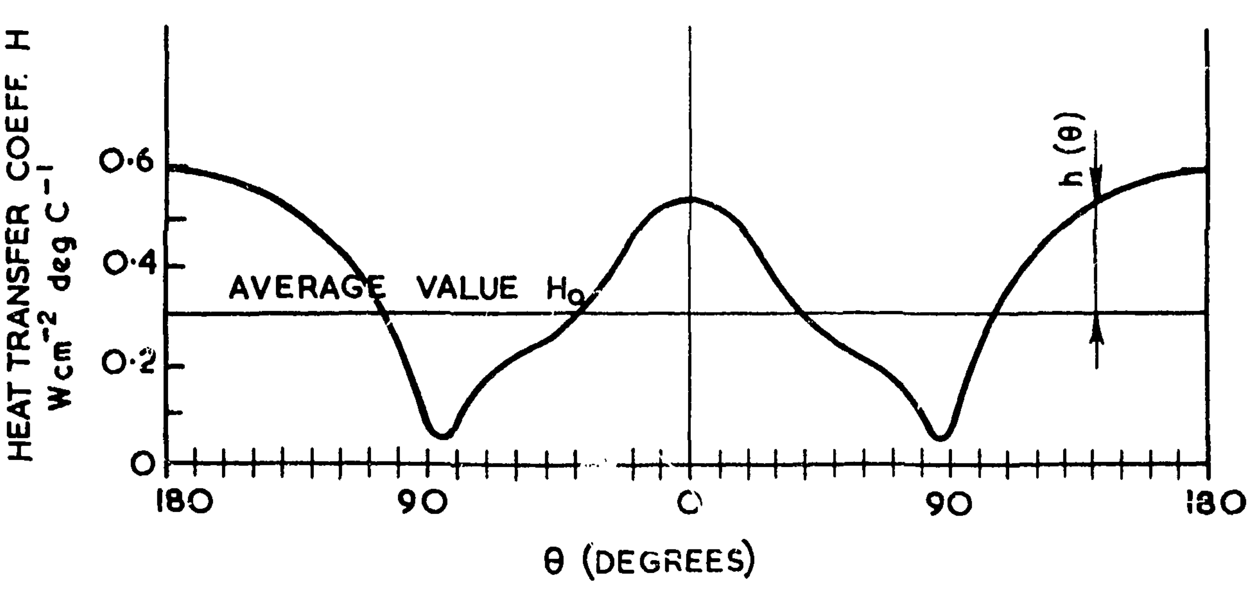

FIGURE 2. TYPICAL HEAT TRANSFER DISTRIBUTION AS A FUNCTION OF θ

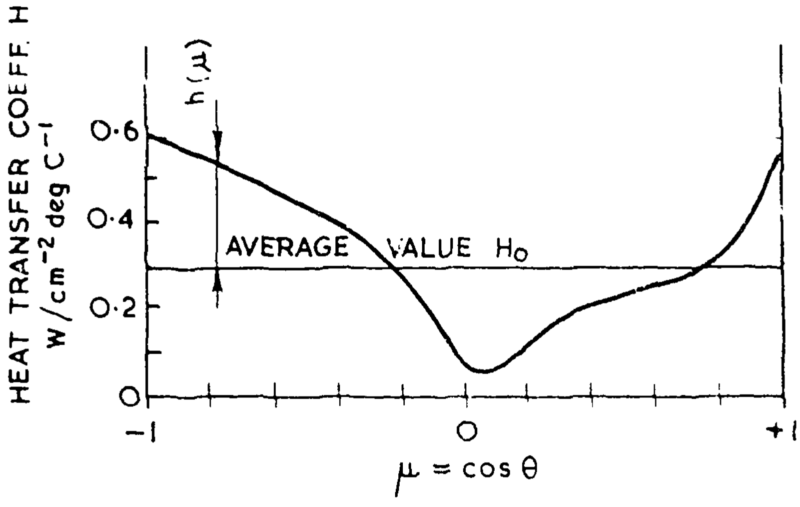

FIGURE 3. HEAT TRANSFER DISTRIBUTION RE-PLOTTED AS A FUNCTION OF $\mu = \cos\theta$

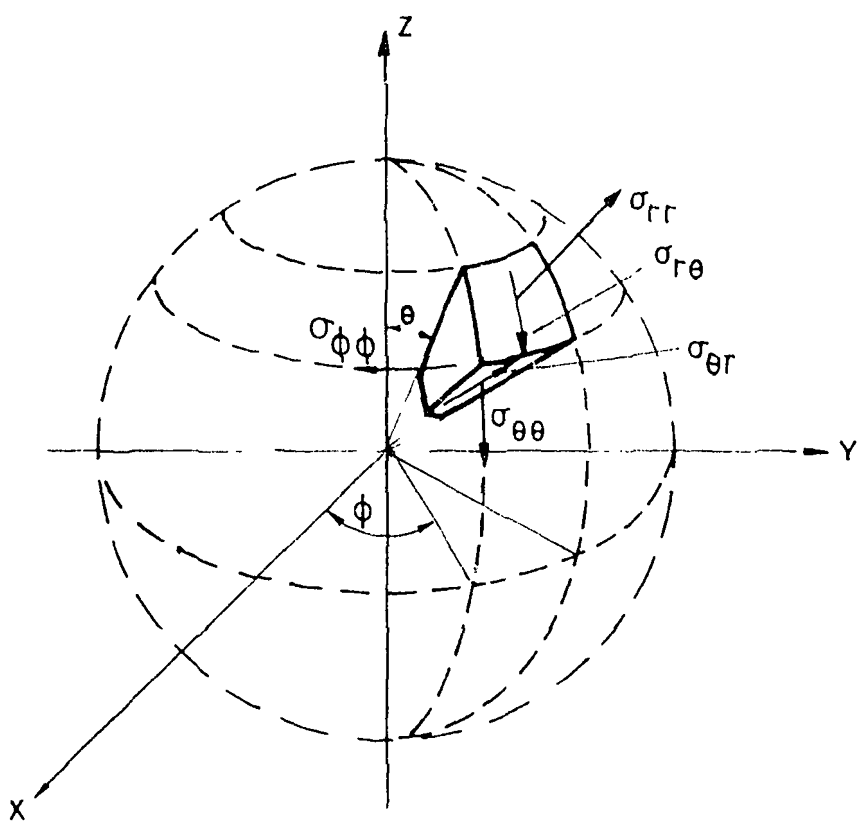

FIGURE 4. THERMOELASTIC STRESS COMPONENTS FOR A SPHERE