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ABSTRACT

It is shown that the effect of missing small fission widths in the analysis of a fission width distribution is to give an apparent number of channels open greater than the actual number open. This is demonstrated both by a numerical experiment and by analytical considerations.

A set of resolution probabilities is postulated such that when the apparent distribution is calculated from the true distribution, the effective number of degrees of freedom increases by a specific amount. The theory is applied to the experimental set of fission widths for neutron fission of ^{235}U in both the $J^{\pi} = 3^{-}$ and 4^{-} states.

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URANIUM 235; CROSS SECTIONS; ENERGY LEVELS; LEVEL WIDTHS; FISSION+NEUTRONS

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1. INTRODUCTION

In a recent paper, Boldeman *et al.* [1976] computed the fission cross sections of ^{233}U and ^{235}U from low energies up to 600 keV incident neutron energy, obtaining good agreement with experiment. They used the values for the fission barriers and the energy scheme of low lying fission bands measured by Back *et al.* [1971, 1973], which included the various K bands measured by Dabbs *et al.* [1969] for ^{235}U as reported by Michaudon [1973]. From these data, they calculated the various partial cross sections for each process. For ^{233}U they assumed that there are between two and three channels open in each of the 2^+ and 3^+ states. Previously, the situation with the resolved resonance region fission widths, as summarised by Lynn [1968], was that there were between two and four channels open for the $J^\pi = 2^+$ state, and between one and two channels open for $J^\pi = 3^+$. We have analysed the lumped distribution and found that adequate fits are obtained with two channels open in each state, so there is no real discrepancy between the low and high energy results for this nuclide.

Again for ^{235}U, Boldeman *et al.* used only one channel appreciably open in each of the $J^\pi = 3^-$ and 4^- states at low energies. Lynn's analysis gave between one and two channels open for the 3^- state, and only one channel open for the 4^- state. The behaviour of the fission width distribution for each state was investigated, using recent data, by Cook & Rose [1975] with derived distributions applicable to the case in which channels are open to differing degrees. It was confirmed that there is one dominant channel in each state but, in the 4^- case, the contribution from two other channels is significantly greater than that used by Boldeman *et al.* and Lynn.

One factor which could influence the analysis of Cook & Rose is the possibility that there are an appreciable number of levels which remain undetected because their fission widths are too small. It is the purpose of this paper to examine the influence of missing small widths upon the effective number of channels open. This is done with the ^{235}U data in which there is an apparent anomaly for the 4^- state. Two approaches are adopted. First, a numerical experiment is reported, and then an analytical approach is made using proposed laws for the probability of missing levels.

Preliminary calculations to estimate the average fission widths show that the distributions are very insensitive to these quantities, and it is possible to get acceptable fits in the 4^- state with either all widths equal, or two equal and one zero. In view of this, we employ the effective χ^2_N distributions rather than the intermediate ones reported by Cook & Rose.

2. NUMERICAL METHOD

An initial distribution of fission widths was assumed to be given by a χ^2_N distribution with integer value of N:

$$P_N(\Gamma)\, d\Gamma = \frac{1}{\Gamma(\frac{1}{2}N)} \left(\frac{N}{2<\Gamma>} \right)^{\frac{1}{2}N} \Gamma^{\frac{1}{2}N-1} \exp -\left(\frac{N\Gamma}{2<\Gamma>} \right) d\Gamma \qquad (1)$$

The range for $x = \Gamma/<\Gamma>$ was divided into a number of groups, and the integrals of equation (1) were calculated for each group. Next, the value of the first group from the origin was reduced by a fraction α, representing the fractional number of fission widths detected, and the distribution was normalised to unity. The resulting distribution with modified mean, was fitted by least squares analysis to a χ^2_ν distribution with ν, in general, being non-integer. A set of group boundaries was chosen as follows:

Seven groups for x:

(0.0-0.25), (0.25-0.5),
(0.5-1.0), (1.0-2.0),
(2.0-4.0), (4.0-6.0),
(6.0-∞) .

The initial average fission width was taken to be unity, and the change in this as a function of α was also computed.

In general, it was found that ν as a function of α and $<\Gamma>$ are monotonically decreasing functions, showing that the more the small fission widths are missed, the greater is the effective number of degrees of freedom in the modified distribution, and the greater is the mean fission width. However, the value of ν reaches a maximum of 2.6 when $\alpha = 0$, whereas Rose & Cook [1975] obtained a value of 2.7 ± 0.3 to the distribution for the $J^\pi = 4^-$ state in ^{235}U. This suggests that missing levels are producing the effective shift from the experimental value of $\nu = 1$.

Good empirical fits were obtained in the following cases:

$$N=1 \quad \nu = 1.1475\alpha^2 - 2.799\alpha + 2.6578$$
$$<\Gamma> = 1.51 \times \exp(-0.45\alpha) \qquad (2)$$

$$N=2 \quad \nu = 0.2473\alpha^2 - 1.8708\alpha + 3.6029$$
$$\langle\Gamma\rangle = 1.24 \times \exp(-0.22\alpha) \tag{3}$$

$$N=3 \quad \nu = -0.251\alpha^2 - 1.282\alpha + 4.5082$$
$$\langle\Gamma\rangle = 1.142 \times \exp(-0.134\alpha) \tag{4}$$

All of these equations exhibit a cutoff value for ν when α reaches zero; it is noted that, for the N=1 case mentioned in the introduction, 100 per cent of levels in the interval $0 \leqslant x \leqslant 0.25$ are to be missed if the observed $\nu = 2.7$ of the 4^- state of ^{235}U is to be achieved. The discrepancy between the value of Boldeman *et al.* [1976] and the experimental value for ν cannot be explained by missing levels.

3. ANALYTICAL INVESTIGATION

It is well known that in the single-level approximation, the fission cross section in the resolved resonance range is given by a sum of Breit-Wigner terms:

$$\sigma_f(E) = \sum_{i=1}^{N} \sigma_{oi} \frac{\Gamma_{fi}}{\Gamma_i} \cdot \frac{1}{1 + \left(\frac{2}{\Gamma_i}(E_{oi}-E)\right)^2} + Q(E) \quad , \tag{5}$$

where $\sigma_{oi} = 4\pi\lambda^2\Gamma^0_{ni}\sqrt{E}$,

E = neutron energy,

Γ_{fi} = the i^{th} fission width,

Γ^0_{ni} = the i^{th} reduced neutron width,

$\Gamma_i = \Gamma^0_{ni}\sqrt{E} + \Gamma_{\gamma i} + \Gamma_{fi}$

= the total width of the i^{th} resonance,

$\Gamma_{\gamma i}$ = the i^{th} radiation width, and

$Q(E)$ = experimental background noise.

For very small Γ_{fi}

$$\sigma_f(E) \propto \alpha_i\Gamma_{fi} + Q(E) \quad , \tag{6}$$

where α_i = constant,

and we may conjecture that the probability of seeing a small level is proportional to Γ_{fi}:

$$P_1(\Gamma_f) \propto \Gamma_{fi} \quad . \tag{7}$$

However, for large values of Γ_{fi}, this probability should be unity. Therefore, a probability function (dropping the suffix fi) is conjectured for missing a level

$$P_2(\Gamma) = \exp(-A\Gamma) \quad ,$$

i.e. $$P_1 = 1-P_2 = 1-\exp(-A\Gamma) \quad . \tag{8}$$

Assuming all channels to be open, an effective distribution is obtained from equations (1) and (8):

$$P_{eff}(\Gamma) = \frac{1}{B^N_{eff}} (1-\exp(-A\Gamma)) \ \Gamma^{\frac{1}{2}N-1} \exp\left(-\frac{N\Gamma}{2<\Gamma>}\right) \quad , \tag{9}$$

with $$B^N_{eff} = \Gamma(\tfrac{1}{2}N) \left[\left(\frac{2<\Gamma>}{N}\right)^{\frac{1}{2}N} - \left(\frac{1}{\frac{N}{2<\Gamma>} + A}\right)^{\frac{1}{2}N}\right]$$

and $$\int_0^\infty P(\Gamma)d\Gamma = 1 \quad .$$

For a χ^2_N distribution

$$<\Gamma^2> = (1 + \frac{2}{N}) <\Gamma>^2 \quad , \tag{10}$$

so the effective number of degrees of freedom is defined as

$$\nu_{eff} = \frac{1}{<\Gamma^2>/<\Gamma>^2-1} \quad ; \tag{11}$$

this definition is used for any modified distribution.

The mean of the distribution (equation (9)) is:

$$<\Gamma_{eff}> = \int_0^\infty P_{eff}(\Gamma)d\Gamma$$

$$= <\Gamma> \left\{ \frac{1 - \left(1 + \frac{2A<\Gamma>}{N}\right)^{-\frac{1}{2}N-1}}{1 - \left(1 + \frac{2A<\Gamma>}{N}\right)^{-\frac{1}{2}N}} \right\} \quad . \tag{12}$$

In the range $\infty > A \geqslant 0$

$$<\Gamma> \leqslant <\Gamma_{eff}> \leqslant (1 + \frac{2}{N}) <\Gamma> \quad , \tag{13}$$

so the mean increases monotonically from the value at which no levels are missed, to a maximum asymptotic limit. The variance is

$$<\Gamma^2_{eff}> = (1 + \frac{2}{N}) <\Gamma>^2 \left\{ \frac{1 - \left(1 + \frac{2A<\Gamma>}{N}\right)^{-\frac{1}{2}N-2}}{1 - \left(1 + \frac{2A<\Gamma>}{N}\right)^{-\frac{1}{2}N}} \right\} . \tag{14}$$

For the interval $\infty > A \geqslant 0$

$$(1 + \frac{2}{N}) <\Gamma>^2 \leqslant <\Gamma^2_{eff}> \leqslant (1 + \frac{2}{N}) (1 + \frac{4}{N}) <\Gamma>^2 \tag{15}$$

and $$N \leq \nu_{eff} \leq N+2 \tag{16}$$

The result (equation (16)) is not too surprising because the distribution (9) at low values of Γ behaves as

$$P_{eff}(\Gamma) \sim \frac{A}{B_{eff}} \quad \Gamma^{\frac{1}{2}(N+2)-1} \quad ,$$

and effectively shifts the number of degrees of freedom by two.

4. MORE GENERAL CONSIDERATIONS

With cross sections of the form (5), levels may be missed not only when the signal to noise ratio is low but also when small levels are overshadowed by large ones, either in the same or in a different J state. It is difficult to assess the precise nature of the power law in this case so, to generalise, we postulate a more flexible law for the probability of detecting a level as

$$P_2(\Gamma) = \exp\{-A(\Gamma)^{\mu/2}\} \quad . \tag{17}$$

Substituting into equation (1) and integrating, one finds for $\infty > A > 0$

$$\langle\Gamma\rangle \leq \langle\Gamma_{eff}\rangle \leq \left(\frac{N+\mu}{N}\right) \langle\Gamma\rangle \quad , \tag{18}$$

and $$\langle\Gamma^2\rangle \leq \langle\Gamma^2_{eff}\rangle \leq (N+\mu+2)\ (N+\mu)\langle\Gamma^2\rangle/N \quad , \tag{19}$$

$$N \leq \nu_{eff} \leq N+\mu \quad . \tag{20}$$

The functions (17) effectively shift the number of degrees of freedom uniformly upwards until an asymptotic shift of μ degrees of freedom is attained.

For the special case of μ = 1, the integral representation of Gradshteyn & Ryzhik [1965] is used

$$\int_0^\infty x^{\nu-1}\exp(-\beta x^2-\gamma x)dx = (2\beta)^{-\nu/2}\ \Gamma(\nu)\ e^{-\gamma^2/8\beta}\ D_{-\nu}\left(\frac{\gamma}{\sqrt{2\beta}}\right), \tag{21}$$

where $D_{-\nu}(t)$ is the parabolic cylinder function, to obtain

$$\langle\Gamma_{eff}\rangle = \frac{2}{N}\left\{\frac{\Gamma(\frac{1}{2}N+1)-2^{-N/2}\ \Gamma(N+2)\ e^{A^2/4N} D_{-(N+2)}\left(\frac{A}{\sqrt{N}}\right)}{\Gamma(\frac{1}{2}N)-2^{-(N-1)/2}\ \Gamma(N+1)\ e^{A^2/4N}\ D_{-(N+1)}\left(\frac{A}{\sqrt{N}}\right)}\right\} \tag{22}$$

and $$\langle\Gamma^2_{eff}\rangle = \frac{2}{N}\left\{\frac{\Gamma(\frac{1}{2}N+1)-2^{-N/2}\ \Gamma(N+2)\ e^{A^2/4N}D_{-(N+3)}\left(\frac{A}{\sqrt{N}}\right)}{\Gamma(\frac{1}{2}N)-2^{-(N-1)/2}\ \Gamma(N+1)\ e^{A^2/4N}\ D_{-(N+1)}\left(\frac{A}{\sqrt{N}}\right)}\right\}, \quad (23)$$

from which ν_{eff} as a function of A can be calculated.

5. APPLICATION

From equations (12) and (14) it can be seen that if the mean and variance of the experimental distribution are calculated, then it is possible to solve the equations to find the appropriate A and $\langle\Gamma\rangle$. In addition, it can be seen from equation (9) that the number of levels missed is

$$M_{missed} = M_{exp}\left(\frac{1}{B^N_{exp}} - 1\right) . \quad (24)$$

From Cook & Rose [1975] data

$$\langle\Gamma_{eff}(3^-)\rangle = 0.1122, \quad \langle\Gamma^2_{eff}(3^-)\rangle = 0.03417$$

and, from equation (11),

$$\nu_{eff}(3^-) = 1.167, \ A\langle\Gamma\rangle = 52.58, \ \langle\Gamma(3^-)\rangle = 0.1014,$$

$$M_{exp} = 26 \quad ,$$

$$M_{missed} = 3 \text{ (or 10 per cent) missed.}$$

The $J^\pi = 4^-$ case gives the results

$$\langle\Gamma_{eff}(4^-)\rangle = 0.05723; \quad \langle\Gamma^2_{eff}(4^-)\rangle = 0.005055$$

which yields

$$\nu_{eff}(4^-) = 3.68 \quad .$$

For N = 1, the equations (12) and (13) do not hold since the maximum possible $\nu_{eff}(4^-)$ is 3.0.

With N = 2

$$A\langle\Gamma(4^-)\rangle = 0.835, \quad \langle\Gamma(4^-)\rangle = 0.03704 ,$$

$$M_{exp}(4^-) = 88$$

$$M_{missed}(4^-) = 48 \text{ (or 54.5 per cent) missed.}$$

For N = 3

$$A\langle\Gamma(4^-)\rangle = 4.575, \quad \langle\Gamma(4^-)\rangle = 0.05178 ,$$

$M_{exp}(4^-) = 40,$

$M_{missed}(4^-) = 6$ (or 13 per cent) missed.

The data therefore favour $N(4^-) = 3$, since one would expect about the same fraction of missed levels in the 3^- and 4^- states.

6. CONCLUSION

It has been demonstrated both numerically and analytically that the effect of missing small fission widths, for whatever reason, is to increase the observed number of degrees of freedom of the fission width distribution. It is not likely that this is the origin of the mentioned discrepancy for ^{235}U in the number of channels open.

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