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ABSTRACT

Angle-dependent spectra of neutrons, emitted by a thick lithium target when bombarded with protons in the energy range 1.881 to 3 MeV, have been evaluated from experimental and theoretical values of the angular distribution of neutrons emitted in the $^7Li(p,n)^7Be$ reaction. The variation of $d^2N/dE_n d\Omega$ with proton energy at different angles of emission, and with angle of emission for different neutron energies, is presented for the ground and excited state reactions. The angle-integrated neutron spectrum is given for different incident proton energies.

CONTENTS

1. INTRODUCTION

The $^{7}Li(p,n)^{7}Be$ reaction is a relatively prolific source of neutrons for proton bombarding energies in the region of 3 MeV. It is a useful source of neutrons for fast reactor experiments, since the spectrum of neutrons emitted from a thick target bombarded with 3 MeV protons covers the range 0 to 1.3 MeV, with the bulk of neutrons in the range 50 keV to 1.0 MeV. It has the added advantage compared, say, with the $^{9}Be(d,n)^{10}B$ reaction, that the neutron energy spectrum is closely controlled by the energy of the incident protons.

Although it is a relatively straightforward process to use experimentally measured differential cross sections ($d^2\sigma/d\Omega dE_p$) to present the number of neutrons emitted at various angles as a function of proton bombarding energy [Theobald, Migneco & Cervini 1971], the information is not in a form suitable for reactor calculations. Such calculations require the angular distribution of neutrons emitted from the source as a function of neutron energy. Usually, this information is required in the form of Legendre moments of the neutron energy spectrum, the commonest requirement being the zeroth moment, which is just the angle-averaged or total spectrum.

In the present paper, both experimental and theoretical data for the ground and excited state $^{7}Li(p,n)^{7}Be$ reactions have been used to present the number of neutrons emitted:

(i) as a function of proton energy for various angles of neutron emission, and

(ii) as a function of angle for various neutron energies, for protons in the energy range 1.881 to 3 MeV.

The data have been further processed to give the total neutron spectrum for a thick target at different proton bombarding energies. Some indication is also given of the extent to which the total neutron spectrum can be tailored by varying the target thickness.

2. FORMALISM FOR CALCULATING THE NEUTRON ENERGY SPECTRUM

The number of neutrons emitted per second into solid angle dΩ by an element dx of a thick target bombarded by a proton beam of i μA, is

$$dN = igD \frac{d\sigma(E_p)}{d\Omega} \, dx \, d\Omega \quad ,$$

where E_p = the proton energy at depth x from the surface of the target,

g = number of protons per μA,

D = atomic density of ^{7}Li, and

$\frac{d\sigma}{d\Omega}$ = the differential cross section in the laboratory frame.

Since there is a well defined relationship between the proton energy and depth of penetration into the target, and between the proton energy E_p and the energy E_n of the neutron emitted into the solid angle Ω, we can express dx as

$$dx = (dx/dE_p)\,|dE_p/dE_n|\,dE_n \quad .$$

The number of neutrons emitted can be usefully expressed in terms of the differential cross section referred to the centre of mass frame, since measurements and theoretical expressions for the differential cross section are most frequently quoted in this frame of reference. The differential neutron energy spectrum then takes the form

$$\frac{d^2N}{dE_n d\Omega} = igD \frac{d\sigma}{d\Omega'} \frac{d\Omega'}{d\Omega} \left(\frac{dE_p}{dx}\right)^{-1} \frac{dE_p}{dE_n} \quad . \qquad \ldots(1)$$

We can consider separately the contributions of the ground and excited state reactions to the total neutron energy spectrum. For each, there are analytical expressions for $d\Omega'/d\Omega$ and dE_p/dE_n, which are most succinctly expressed in the form [Winter & Schmid 1968]

$$\frac{d\Omega'}{d\Omega} = \frac{\gamma}{\xi} (\mu \pm \xi)^2 \qquad \ldots(2)$$

$$\frac{dE_p}{dE_n} = \frac{(m_y + m_n)^2}{m_p\, m_n} [\mu \pm \xi]^{-1} \left[\mu \pm \xi \pm \frac{m_y(m_y + m_n - m_p)}{m_p\, m_n} \frac{E_{th}}{\xi E_p}\right]^{-1} \qquad \ldots(3)$$

$$\gamma^2 = \frac{m_p\, m_n}{(m_y + m_n - m_p)m_y} \frac{E_p}{(E_p - E_{th})} \quad , \qquad \ldots(4)$$

$$\xi^2 = 1/\gamma^2 - 1 + \mu^2 \quad , \qquad \ldots(5)$$

where μ = the cosine of the angle of neutron emission in the laboratory frame,

m_n, m_p, m_y = mass number of the neutron, proton and ^7Li, and

E_{th} = the threshold energy = 1.881 MeV for the ground state reaction
= 2.378 MeV for the excited state reaction.

In the same notation the relationship between E_p, E_n and μ takes the form

$$E_n = \frac{m_p m_n}{(m_y + m_n)^2} [\mu \pm \xi]^2 E_p \qquad \ldots(6)$$

When $1 \leqslant \gamma \leqslant \infty$, the neutron energy is a double-valued function of the proton energy, the two values being given by the positive and negative values of ξ. Similarly, $d\Omega'/d\Omega$ and dE_p/dE_n are double-valued functions of proton energy. In equations (2) and (3), the positive value of ξ is used to evaluate $d\Omega'/d\Omega$ and dE_p/dE_n, associated with the larger value of E_n in equation (6).

The variation of $d\sigma/d\Omega'$ with E_p is not, in general, a simple function of E_p and has to be evaluated from experimental results. On the other hand, an analytical expression [Livingston & Bethe 1937] does exist for the slowing down power dE_p/dx, characterised by I, the mean excitation potential. The choice of I and the agreement between theoretical and experimental values of the proton energy loss are discussed below.

3. DISCUSSION OF DATA USED IN EVALUATING $d\sigma/d\Omega'$ AND dE_p/dx

There is surprisingly little information on the neutron angular distribution from the $^7Li(p,n)^7Be$ reaction as a function of proton energy in the range 1.88 to 3 MeV. Bergström *et al.* [1967] have carried out angular distribution measurements in the range 1.928 to 2.361 MeV, while Buccino, Hollandsworth & Bevington [1964] have measured angular distribution of neutrons from both the ground and excited states in the range 2.48 to 2.977 MeV. Bevington, Rolland & Lewis [1961] have also measured the angular cross section for the ground and excited states at a limited number of points in the present energy range. In the region near threshold, the ground state reaction is isotropic in the centre of mass system, and the energy dependence can be described by a single resonance Breit-Wigner formulation [Macklin & Gibbons 1958].

Macklin & Gibbons have shown that the energy level near threshold is broad and that the Breit-Wigner formula reduces to

$$\frac{d\sigma}{d\Omega'} = \frac{1}{k_p^2} \frac{x}{(1 + x)^2} \qquad \ldots(7)$$

where $x = \Gamma_n/\Gamma_p$ = the ratio of the neutron to proton widths

$$= \frac{k_n}{k_p} \frac{\gamma_n^2}{\gamma_p^2} (F_o^2 + G_o^2)$$

$$= \frac{\gamma_n^2}{\gamma_p^2} (F_o^2 + G_o^2) \left[\frac{E_n - E_{th}}{E_p}\right]^{\frac{1}{2}}$$

and F_o and G_o are the regular and irregular Coulomb functions. Now $(F_o^2 + G_o^2)$

varies only slowly over the limited energy range of interest and a good fit (Figure 1a) to the total cross section is obtained by using

$$x = 4.72 \left[\frac{E_p - E_{th}}{E_p}\right]^{\frac{1}{2}} \qquad \ldots(8)$$

This fit is better than that obtained using $\gamma_n^2/\gamma_p^2 = 5.2$ as derived by Macklin & Gibbons (see Figure 1a) and suggests that a better value for γ_n^2/γ_p^2 would be 3.5. The differential cross section was evaluated in the range 1.881 to 1.928 MeV using equation (8) in equation (7).

The angular distribution data of Bergström *et al.* [1967] were integrated over solid angle and compared with the total cross section (Figure 1b) given in the review paper of Gibbons & Newson [1960]. The agreement is good, but it should be noted that it is necessary to correct two typographical errors that appear in the paper by Bergström *et al.* - at $E_p = 2.161$, $\Delta\sigma$ should be 35 rather than 30 and, at $E_p = 2.231$, $\Delta\sigma$ should be 0 rather than 10.

The angular distribution data of Buccino, Hollandsworth & Bevington [1964] for the excited state indicate that, near threshold, the angular distribution in the centre of mass is nearly isotropic and the cross section can again be described by a single Breit-Wigner resonance. It is also clear from the data that the angular distribution becomes increasingly anisotropic as E_p increases, but in the present calculations, $d\sigma/d\Omega'$ for the excited state has been evaluated over the whole range from threshold at 2.378 MeV to 3 MeV, using the single resonance expression and the parameters given by Buccino, Hollandsworth & Bevington. The angular distribution predicted by the simple expression is in reasonably good agreement with the results of Bevington, Rolland & Lewis except at the higher energies where the maximum deviation is about 30 per cent. Since the excited state cross section is only about 10 per cent of that of the ground state, the error in the neutron energy spectrum introduced by using this approximation is expected to be small. The single resonance approximation was used rather than the experimental values of Buccino *et al.* since these data, when integrated over solid angle, gave poor agreement (see Figure 1c) with the total cross section [Bevington, Rolland & Lewis 1961, Batchelor & Morrison 1955], indicating some uncertainty in the method used by Buccino *et al.* to normalise their angular distributions to the total cross section.

The angular distribution data of Buccino, Hollandsworth & Bevington [1964] for the ground state reaction were also integrated over angle and compared with the total cross section of Gibbons & Newson [1960] from which the excited state contribution had been subtracted. The Buccino *et al.*

cross section has the same energy dependence, but gives better agreement with the Gibbons & Newson curve when reduced by 6 per cent. This correction was applied to the Buccino, Hollandsworth & Bevington data in calculating the neutron energy spectra.

The total cross section curve of Gibbons & Newson has a relative accuracy of about ± 1 per cent and an absolute accuracy of about ± 4 per cent. Bergström *et al.* quote accuracies of 10 per cent for their angular distribution data, and Buccino, Hollandsworth & Bevington quote ± 10 per cent up to 2.7 MeV and ± 15 per cent above 2.7 MeV. It would therefore seem reasonable to expect the present derived angular neutron spectra to be accurate to about 10 to 15 per cent.

The expression for the stopping power derived by Livingston & Bethe [1937] take the form

$$\frac{dE_n}{dx} = \frac{2\pi e^4 z^2 DZ}{E_p} \frac{m_p}{m_e} \left[\log \frac{4 E_p m_e}{I m_p} - \log(1 - \beta^2) - \beta^2 \right] .$$

Since in the present energy regime, β, the ratio of the proton velocity to the velocity of light is very small, the stopping power can be characterised by one parameter, I, the mean excitation potential. Aron, Hoffman & Williams [1949] have used I = 34.5 eV, while Whaling [1958] has assigned I = 60 eV on the basis of some accurate energy loss measurements in the range $0.1 < E_p \leqslant 1.0$ MeV. These two expressions give absolute values of the angular neutron spectra differing by about 12 per cent, but relative variations change by only 1 to 2 per cent. Scott [1971], who found excellent agreement between measured thick target total yields and yields calculated using Whaling's value for the slowing down power, estimated that in the energy range 3 to 10 MeV the Whaling value had a systematic error of less than 6.5 per cent.

Whaling's value for I has been used in the present calculations and the slowing down power for lithium is taken as

$$\varepsilon = D^{-1} \frac{dE_p}{dx} = 7.18 \times 10^{-16} \, [4.69 - \log_e(3/E_p)] E_p^{-1} \text{ eV cm}^2 \quad ,$$

where E_p is in MeV. Based on the accuracy of the experimental data and extrapolating Scott's findings below 3 MeV, the calculated angular spectra should have a relative accuracy of 10 to 15 per cent, with an additional uncertainty of ± 7 per cent in the absolute yields.

4. CALCULATED ANGULAR AND TOTAL NEUTRON SPECTRA

The neutron yield at various angles and at the proton energies at which angular distributions were measured, was evaluated by direct substitution of the experimental values of $d\sigma/d\Omega'$ in equation (1). The neutron yields from the ground state reaction near threshold and from the excited state were calculated using the single Breit-Wigner resonance expression for $d\sigma/d\Omega'$ and the parameters for the resonances indicated above. Figures 2 and 3 show the yields for the ground and excited states respectively.

The double-valued behaviour of the yield at angles less than 90° and for proton energies less than 1.92 MeV in Figure 2 corresponds to the double-valued behaviour of the neutron energy as a function of proton energy in this range. The lower part of the curve corresponds to the lower neutron energies, all of which lie below 30 keV. For ease of presentation, the lower energy branch has been omitted from the neutron energy scales of Figures 2 and 3, and the double-valued portion of the yield curve has been omitted from Figure 3. In all cases the yields correspond to the isotopic abundance of ^{7}Li in natural lithium.

It is a slightly more involved process to calculate $d^2N/dE_n d\Omega$ as a function of μ for a given E_n. Particular values of μ and E_n define a particular value of E_p according to the expression

$$E_p = E_n\left(\frac{m_y + m_n}{m_y - m_p} + 2\mu^2 \frac{m_p m_n}{(m_y - m_p)^2}\right) + \frac{(m_y + m_n - m_p)m_y}{(m_y + m_n)(m_y - m_p)} E_{th} -$$
$$- \frac{2\mu}{(m_y - m_p)^2}\left\{E_n^2[(m_y - m_p)(m_y + m_n)m_n m_p + m_n^2 m_p^2 \mu^2] + \right.$$
$$\left. + E_n E_{th} \frac{m_n m_p m_y (m_y + m_n - m_p)(m_y - m_p)}{(m_y + m_n)}\right\}^{1/2} .$$

For resulting values of E_p close to threshold for the ground state reaction and for all values of E_p for the excited state reaction, the angular cross section as a function of μ can be found by straightforward substitution into the analytical expressions for $d\sigma/d\Omega'$. For resulting values of E_p lying within the range covered by the experiments of Bergström *et al.* [1967] and Buccino, Hollandsworth & Bevington [1964], the value of $d\sigma/d\Omega'$ at the required value of E_p was found by interpolation. The interpolation technique used in practically all cases was to fit a third order polynomial through the angular distribution measurements at the two energy points immediately below, and the two immediately above the one required. When the E_p value lay between the lowest two points used by Bergström *et al.* (1.928 and 2.031 MeV), the angular distribution corresponding to E_p = 1.9 MeV was evaluated using the Breit-Wigner resonance expression, and the four point interpolation method used as before.

For E_p values lying between the highest and second highest of the values used by Buccino *et al.* a simple linear interpolation method was used.

The variation of $d^2N/dE_n d\Omega$ with μ for various E_n is shown in Figures 4 and 5 for the ground and excited states respectively. It should be noted that the backward angle cut-off in these figures corresponds to a proton energy of 2.977 MeV, the highest energy used by Buccino, Hollandsworth & Bevington.

The energy spectrum of neutrons emitted into all possible angles can be derived by integrating the differential cross section over μ. This was done numerically using a simple trapezoidal rule. The lower limit, μ_L, on the integration for a chosen neutron energy and given maximum proton energy, E_{pmax}, incident on the thick target, is given by

$$\mu_L = \frac{1}{2(m_n m_p E_n E_p)^{\frac{1}{2}}} [E_n(m_y + m_n) + E_{th}\frac{m_y(m_y + m_n - m_p)}{(m_y + m_n)} -$$

$$- E_{pmax}(m_y - m_p)] \quad .$$

As E_n increases, μ_L approaches one for a given E_{pmax}. Physically, this follows from the fact that the maximum neutron energy is achieved by protons of the maximum energy interacting with lithium at the very surface of the target and emitting neutrons in the forward direction. Neutrons of slightly lower energies can be produced either at the target surface, but emitted at some angle greater than zero, or at some point below the surface and emitted in the forward direction. Neutrons of even lower energy can be produced over a wider angular range until, at sufficiently low neutron energies, neutrons are produced at all angles.

The neutron energy spectra corresponding to a thick target and maximum incident proton energies of 2.1, 2.4 and 2.8 MeV are shown in Figure 6. Table 1 shows the contribution of the ground state and excited state reactions to the total neutron energy spectra for the two incident proton energies above the excited state threshold.

It is also possible, by changing the target thickness and by judicious choice of the proton bombarding energy, to produce angle integrated spectra which peak at different neutron energies in the range 50 to 700 keV, a range of considerable interest in fast reactor neutronics. The extent to which this is possible can be judged from Figure 7. The maximum proton energy determines the number of high energy neutrons, while the yield of low energy neutrons can be reduced by choosing a thin target which eliminates the reactions involving low energy protons from which come the bulk of low energy neutrons. Choosing a thin target, in fact, introduces a low energy proton limit, E_{pmin}, which in turn introduces an upper limit to the integration over angle for some neutron energies. This corresponds physically to the fact that when E_p is greater than the value for which the neutron energy is a double-valued function of proton energy, there is a minimum possible neutron energy for a given E_{pmin}, and this occurs for neutrons emitted backwards ($\mu = -1$) in the laboratory frame of reference. Figure 7 shows the angle integrated neutron spectra for incident proton energies of 2.1, 2.4 and 2.8 MeV when the target thickness is 220, 200 and 500 keV, respectively. For the two higher proton energies, the low energy neutron tail is due to the excited state reaction.

5. ACKNOWLEDGEMENTS

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TABLE 1

ANGLE-INTEGRATED THICK TARGET NEUTRON SPECTRUM FROM GROUND AND EXCITED STATE AT PROTON ENERGIES OF 2.4 AND 2.8 MeV

Neutron Yield [n/(eV μA s)$^{-1}$]

Neutron Energy (keV)	E_{pmax} = 2.4 MeV		E_{pmax} = 2.8 MeV	
	Ground State (x 10^{-2})	Excited State (x 10^{-2})	Ground State (x 10^{-2})	Excited State (x 10^{-2})
20	9.670	0.025	9.670	0.219
50	14.586	0.029	14.586	0.372
75	16.411	0.019	16.411	0.484
100	17.326	0.004	17.326	0.594
200	14.357	-	14.576	0.654
300	12.436		15.342	0.535
400	13.454		19.235	0.390
500	12.036		19.430	0.217
600	5.583		15.416	0.014
700	-		11.377	-
800			8.256	
900			5.548	
1000			2.781	

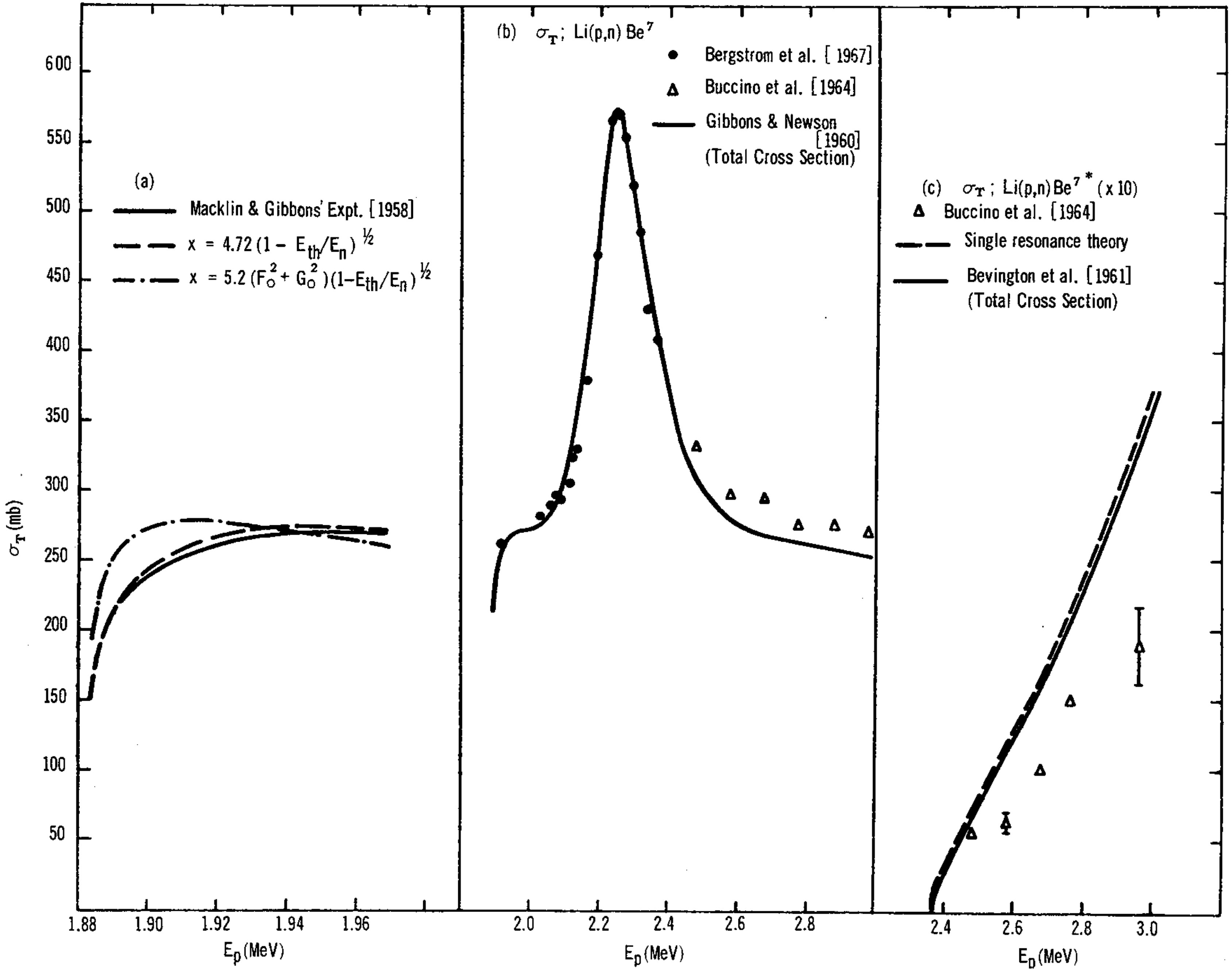

FIGURE 1 COMPARISON OF ANGLE–INTEGRATED DIFFERENTIAL CROSS SECTION DATA WITH TOTAL CROSS SECTIONS

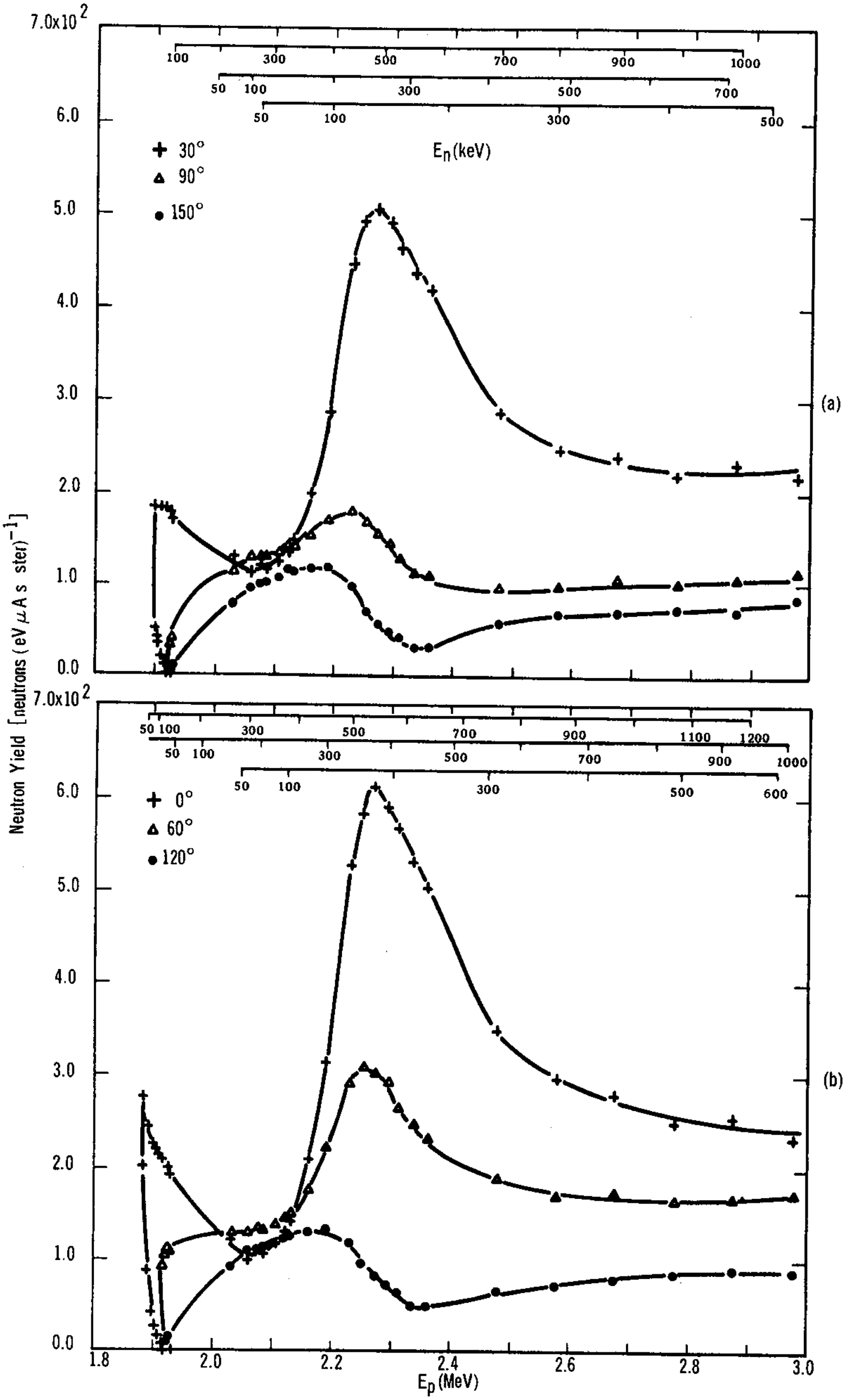

FIGURE 2 THICK TARGET YIELD FOR THE GROUND STATE AS A FUNCTION OF PROTON ENERGY FOR VARIOUS ANGLES

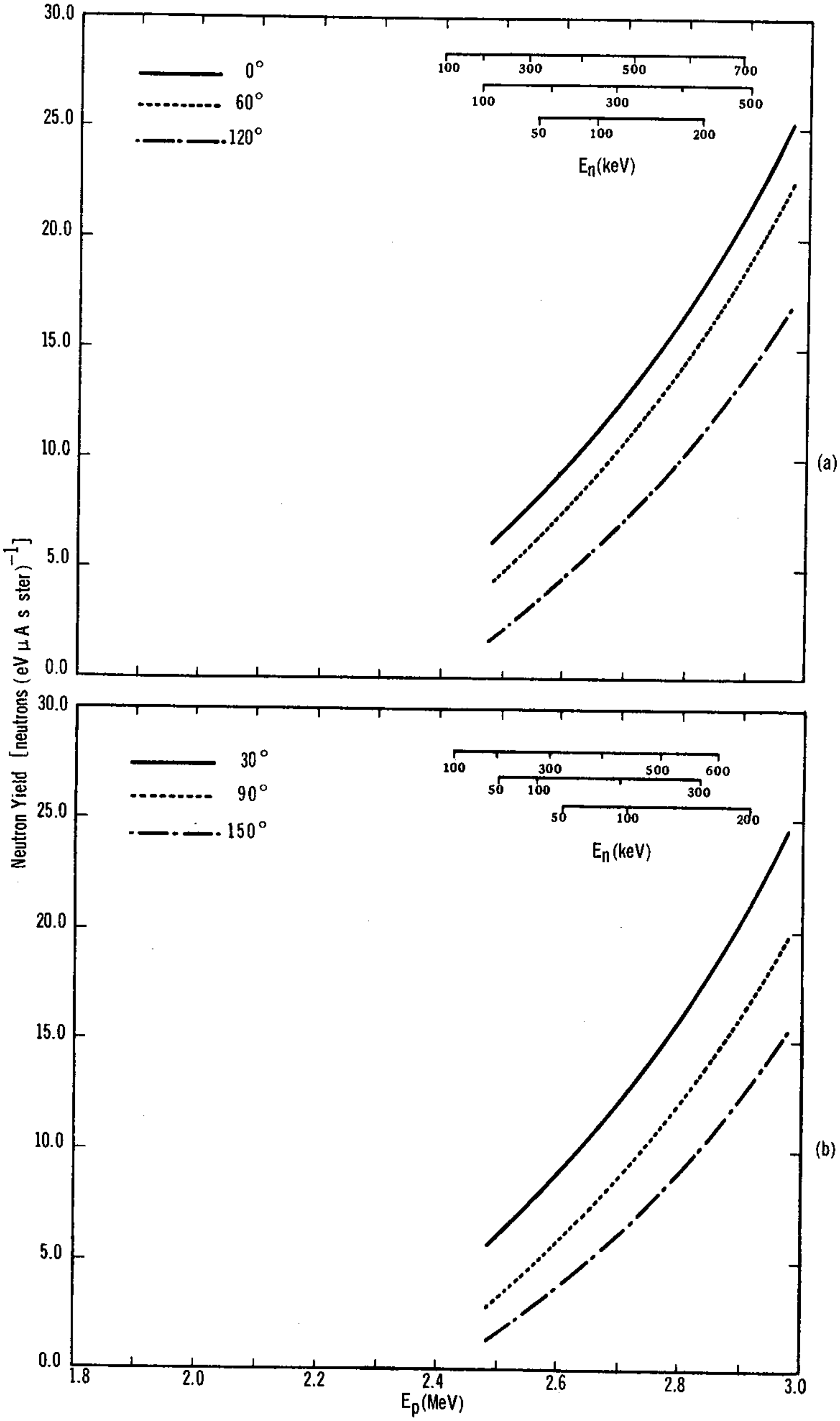

FIGURE 3 THICK TARGET YIELD FOR THE EXCITED STATE AS A FUNCTION OF ENERGY FOR VARIOUS ANGLES

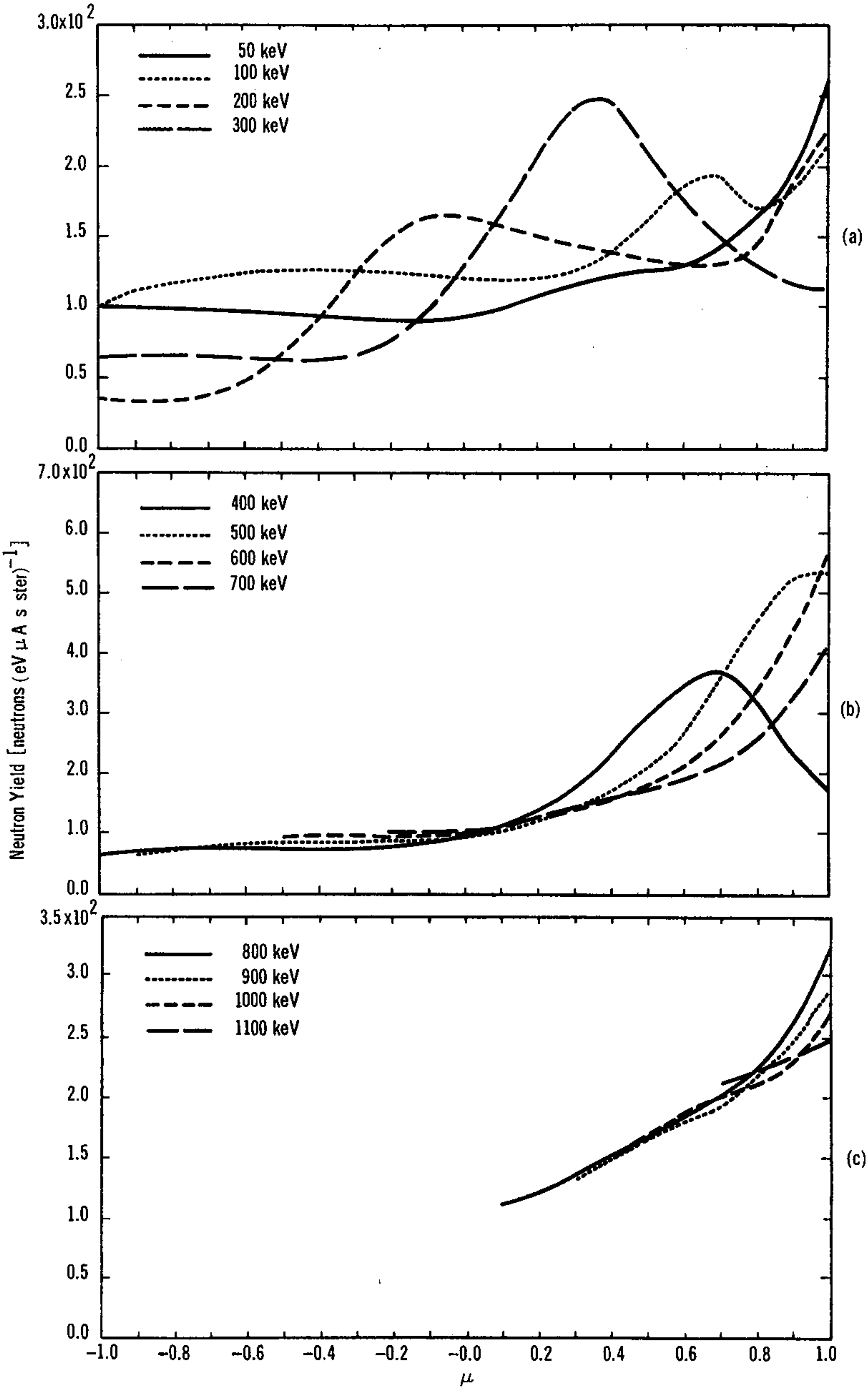

FIGURE 4 THICK TARGET YIELD FOR THE GROUND STATE AS A FUNCTION OF ANGLE FOR VARIOUS NEUTRON ENERGIES

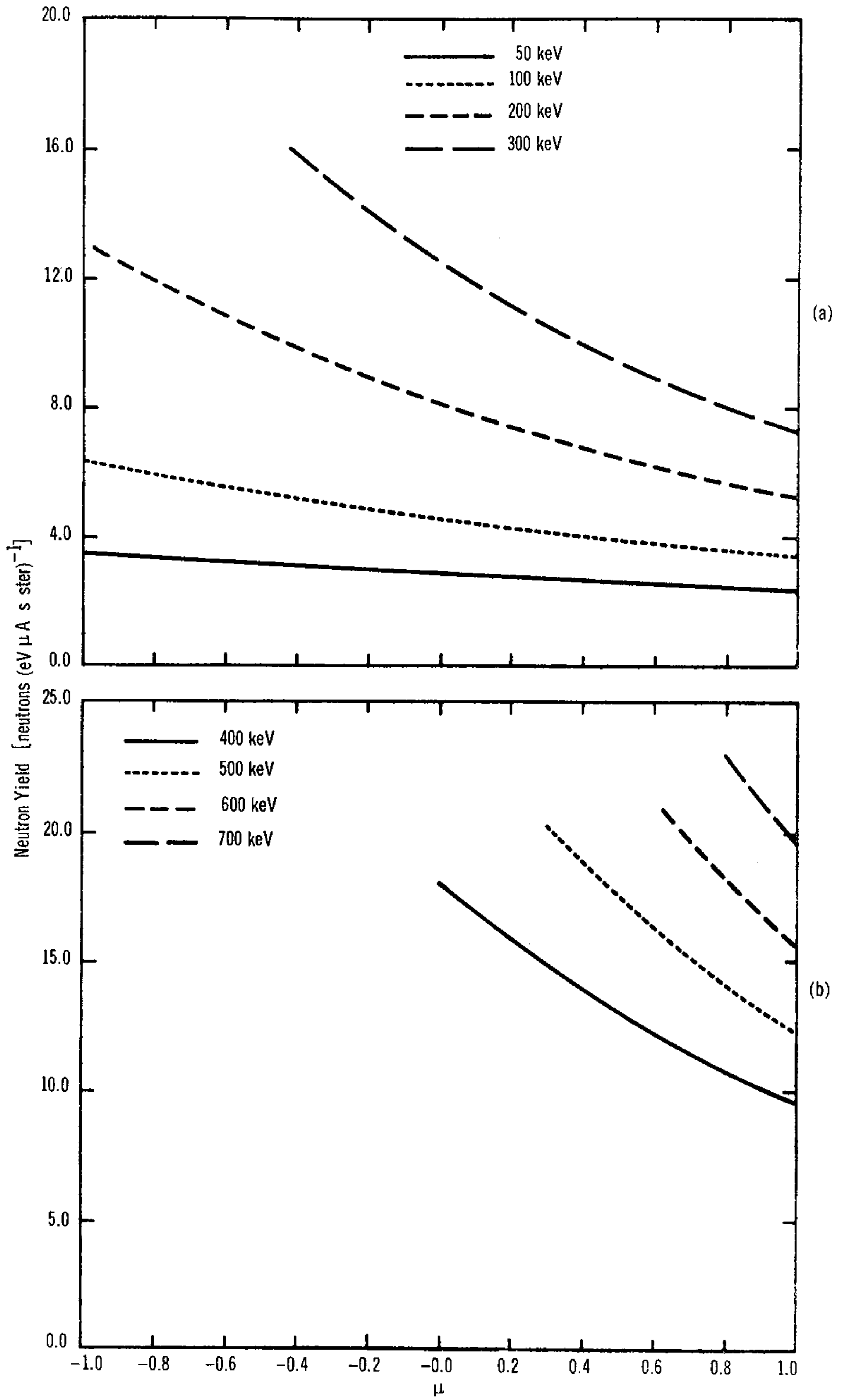

FIGURE 5 THICK TARGET YIELD FOR THE EXCITED STATE AS A FUNCTION OF ANGLE FOR VARIOUS NEUTRON ENERGIES

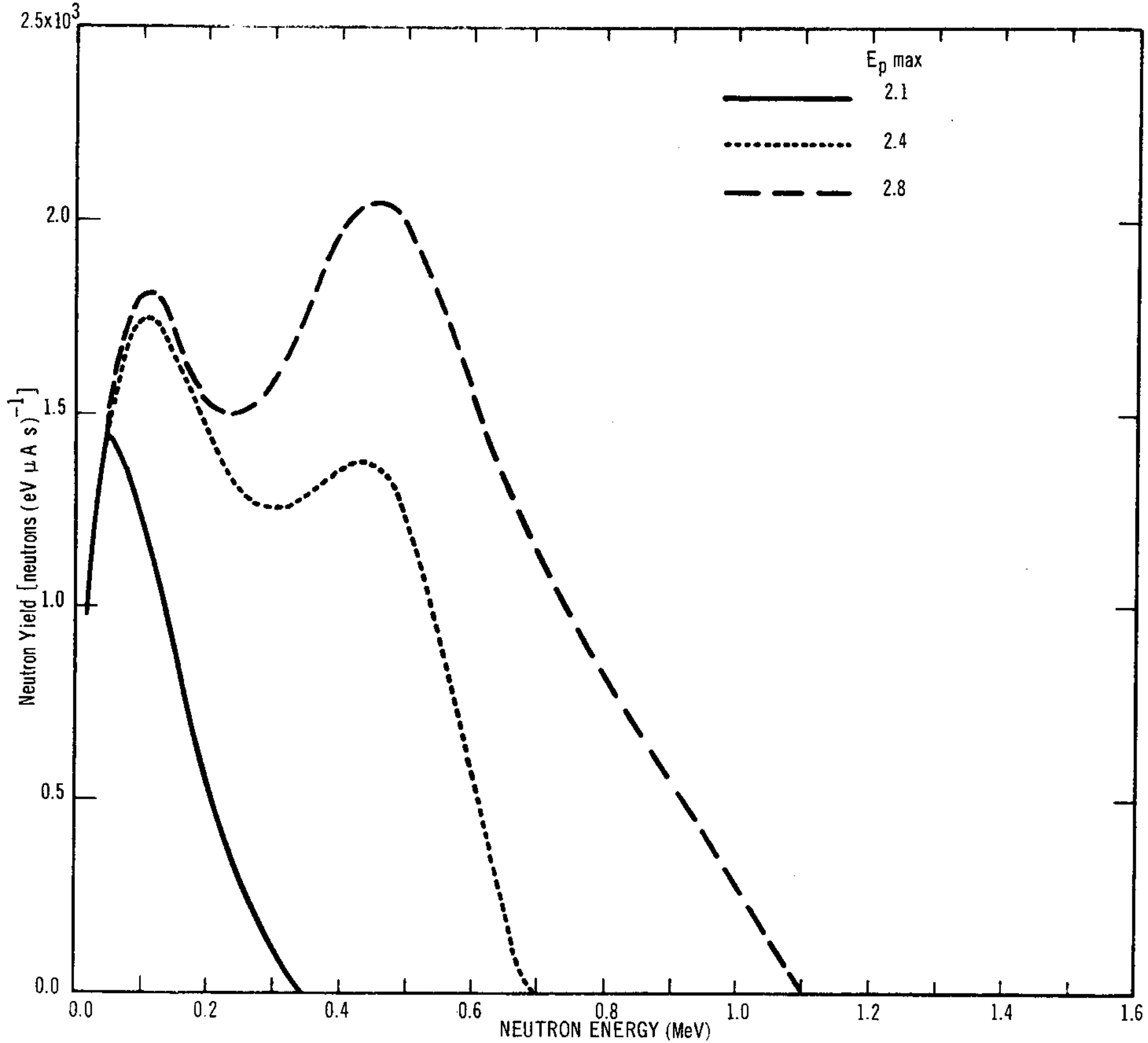

FIGURE 6 ANGLE-INTEGRATED NEUTRON SPECTRA FOR THICK TARGETS AND DIFFERENT INCIDENT PROTON ENERGIES

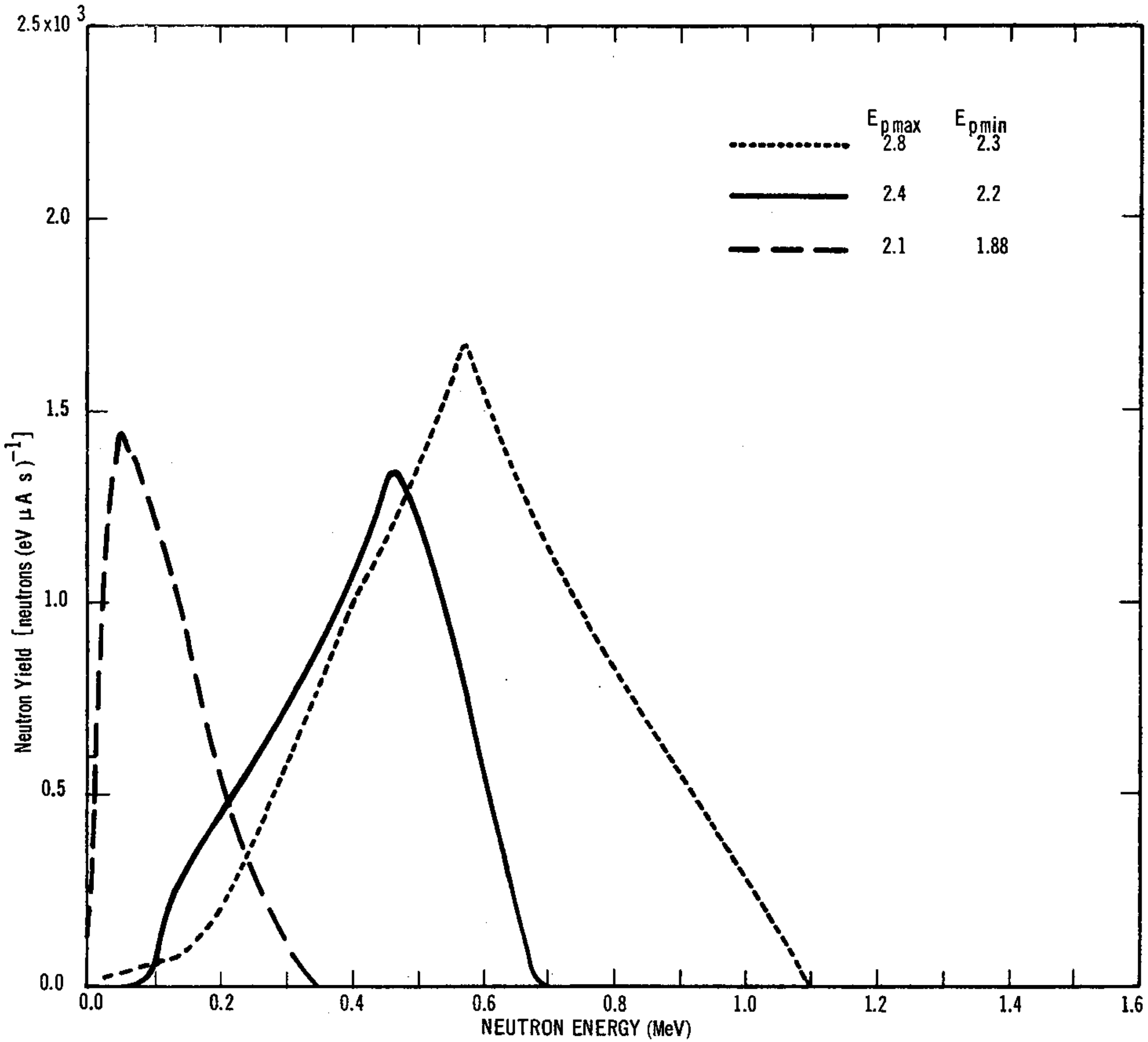

FIGURE 7 ANGLE–INTEGRATED NEUTRON SPECTRA FOR DIFFERENT TARGET THICKNESS AND DIFFERENT INCIDENT PROTON ENERGIES