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CALCULATIONS OF THERMAL STRESSES IN HIFAR REACTOR SHELL STRUCTURE

by

D. R. Ebeling

and

N. D. Holt

Sydney, 1st May 1958.

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Summary

Thermal stresses will be induced in the circular crane beam by expansion of the reactor shell. Calculations assuming 100°F temperature difference show that thermal stresses of about 11000 p.s.i are probable and that the combined stresses due to expansion and crane loading, are close to the maximum allowable.

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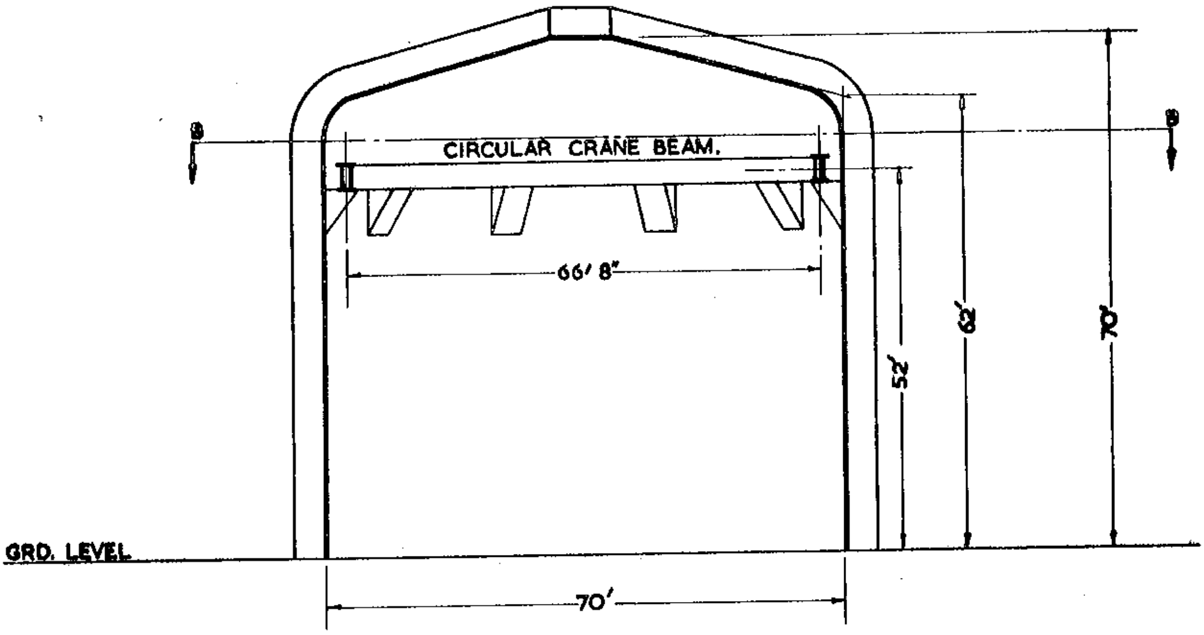
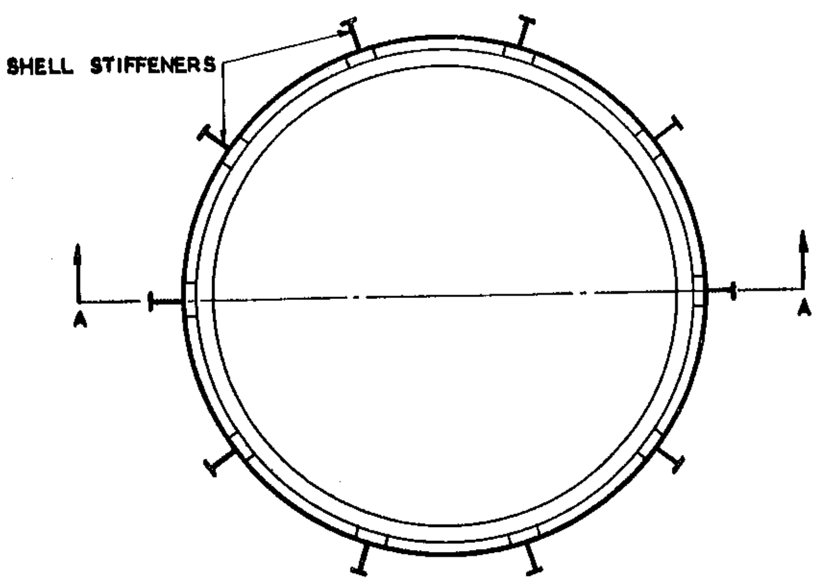


SECTION A-A



SECTION B-B

REACTOR SHELL & CRANE BEAM STRUCTURE.

DIAGRAMMATIC ONLY — STIFFENERS NOT TO SCALE.

1. INTRODUCTION

Early in 1957 it was suggested that due to the large range in temperatures in N.S.W., thermal stresses in parts of the reactor structure could be excessive. A survey showed that the most serious stresses would be induced in the circular crane beam.

The circular crane beam is located inside the shell and will be substantially maintained at the inside temperature of the reactor hall by the air conditioning. It is supported in ten places with the load being transferred to the stiffeners running down the outside of the reactor shell. If the shell were free to expand, it would increase in diameter on hot days. However, the shell is rigidly attached to the crane beam and the thermal expansion will be restrained by the crane beam giving rise to thermal stresses.

2. OUTLINE OF CALCULATIONS

2.1 Assumed temperature difference.

A temperature difference of 100°F was used in the calculations. This represents a mean shell temperature of 160°F.

2.2 Various other assumptions.

As the reactor shell is rather complex to analyse, two different assumptions were made to obtain high and low limits for the stresses. When these calculations indicated that a high stress was possible, a further calculation was done to try to obtain a more accurate result. The various assumptions are set out below.

(a) The stiffeners alone were assumed to take the load; the stiffener was assumed to be of I beam form which included a width of 12 inches of the shell plates. The remainder of the shell plates and the inclination of the roof were neglected and the stiffeners were assumed to be portal frames, pin jointed at the ground. This assumption should give low values for the stresses.

(b) The roof was considered to be infinitely rigid but free to expand thermally. The stiffeners were considered as beams running from the roof to the ground and built in at each end. This assumption should give high values for the stresses.

(c) The vertical stiffeners were considered as beams on an elastic foundation to take account of the wall plates. As the curved plate between the walls and the roof forms a circular beam of comparable moment to the crane beam, and since this is further stiffened by the roof stiffeners and plates the assumption was made that there is no radial deflection of the vertical stiffeners from the freely expanded position at the junction with the roof. The moment at this point was considered to be resisted by bending of the roof stiffeners. Twisting of the circular crane beam was also taken into account.

3. RESULTS

The maximum thermal stress in the circular crane beam was a tension in the outer edges of the flanges. The values found by the three calculations were:-

(a) 4400 psi (lowest value)

(b) 15500 psi (highest value)

(c) 10800 psi (Most Probable Value)

The results of calculation (c) were combined with the stresses due to the crane loading and the maximum stresses in the crane beam and the stiffeners were calculated.

Crane beam:-

Due to combined thermal and crane loading.

Tension at top outside corner over supports	17500 psi
Compression at bottom inside corner over supports	18100 psi

Stiffener:-

Due to combined thermal and crane loading

Tension in flange below the crane beam	2800 psi
Compression in flange above the crane beam	8700 psi
Shear in web above the crane beam	3000 psi

The maximum allowable stresses due to SAA Int 351

Basic bending stress	22400 psi	
Factor for plate girders	0.95	
Factor for 100% live load	0.84	
Maximum allowable bending stress		18000 psi
Basic shear stress	14600 psi	
Reduction Factor	0.84	
Maximum allowable shear stress		12000 psi

4. CONCLUSIONS

Significant stresses will be induced in the circular crane beam by thermal expansion of the reactor shell. The combined stresses due to the crane loading and thermal stresses may reach the maximum allowable values. However this maximum combination should be a rare occurrence and should not represent a dangerous condition.

The stresses in the stiffener are well within safe limits.

Some uncertainty is introduced in these calculations by the necessary assumptions made. The main ones are:-

The assumption of 100° F temperature difference between the crane beam and the outer shell. Since the air conditioning plant is capable of producing 60°F inside temperatures in hot weather and it does not seem unlikely that 160°F outer skin temperature could be achieved in mid summer, this temperature difference is expected to be high but not excessive.

The assumption of zero deflection of the stiffener from the expanded position at the roof junction would tend to give a slightly high result.

The assumption that only twelve inches of the reactor shell act with the stiffener in bending would tend to give a low result. These effects tend to cancel each other and it is felt that the calculations give an adequate analysis of the structure.

Since such assumptions are necessary for calculation the only method to give a sure result would be to use strain gauges to measure the stresses in the beam although it is doubtful whether an accuracy greater than ± 15% could be achieved.

NOTATION

Symbol	Description	Units
A	Area of section	in^2
a	Height of roof above crane beam	in
	Height of crane beam above ground	in
b	Suffix referring to crane beam	
E	Modulus of elasticity	p.s.i.
I	Moment of Inertia of section	in^4
I_y	Moment about vertical axis	in^4
I_z	Moment about horizontal axis	in^4
k	Radius of gyration of section	in
L	Height of roof above ground	in
M	Applied moments and bending moments	lb. in.
P	Applied forces	lb.
r	Radius	in
S	Suffix referring to stiffener	
T	Tension in member	lb.
t	Temperature difference	^oF
U	Strain energy	lb. in.
α	Coefficient of Thermal Expansion	^oF
Δ	Radial free expansion of the shell	in
δ	radial deflection of crane beam or stiffener due to coupling force	in
θ	Rotations of members	
θ, ϕ	Various angles as indicated in text	

Note: In appendices references to equations by a number alone refer to equations derived in the same appendix. Where reference is made to an equation in another appendix the number will be prefixed by the appropriate letter.

References: Timoshenko – Strength of materials Vol. 2

Roark – Formulas for Stress and Strain

Hartree – Numerical Analysis

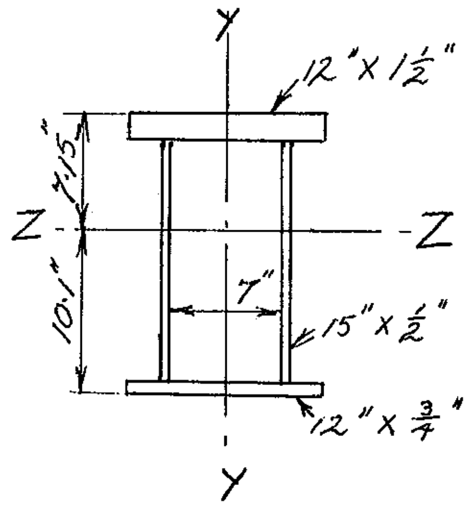
SAA Int. 351

APPENDIX A

Properties of Crane beam and Stiffener Sections.

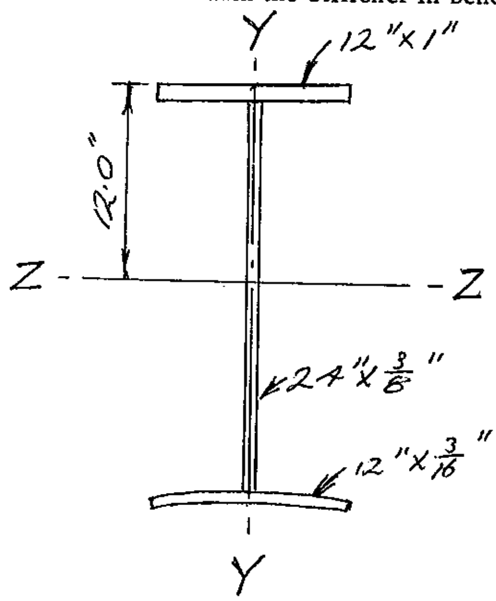
1. Crane beam

A	=	42 in^2
I_z	=	1920 in^4
I_y	=	538 in^4
K_y	=	3.58 in



2. Stiffener. A 12 inch strip of plate from the shell is considered to act with the stiffener in bending.

A	=	30.8 in^2
I_z	=	3780 in^4



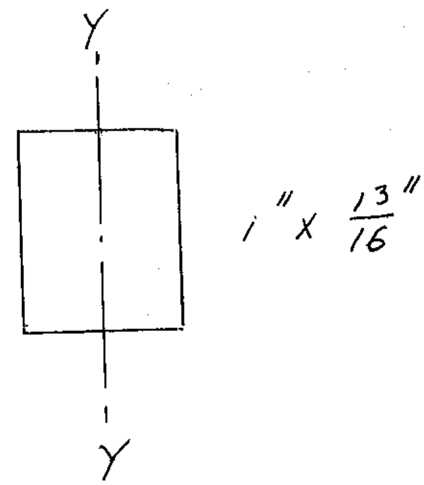
Appendix A (Contd.)

3. One inch strip of shell

$A = 0.812\ in^2$

$I_y = 0.0445\ in^4$

$K_y = 0.234\ in$



APPENDIX B

Calculation of deflection of a ring subject to a number of radial forces as shown in fig. 1.

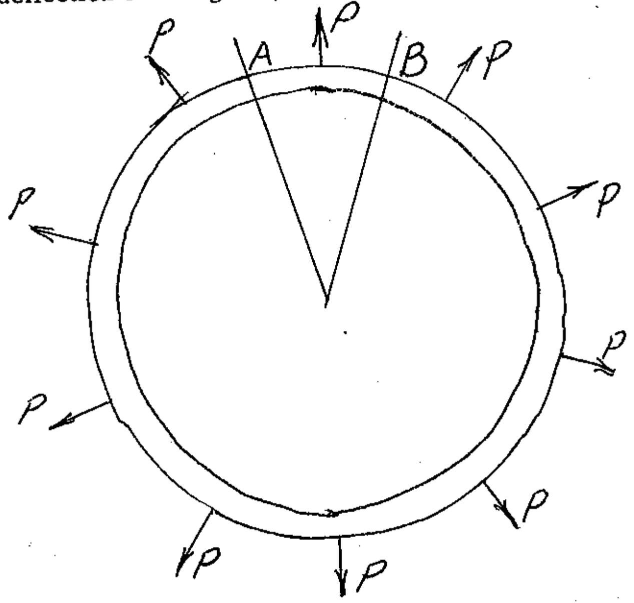


Fig. 1.

Appendix B (Contd.)

Consider the forces on the segment A B where A and B are midway between the forces applied to the ring.

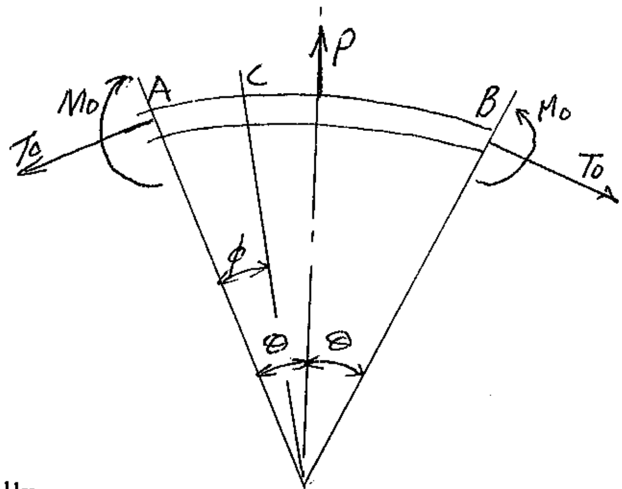


Fig. 2
Forces on segment A. B.

Resolving vertically

$P = 2 \, To \sin \theta$ (1)

Now consider the segment AC defined by the angle ϕ as shown on Fig. 2.

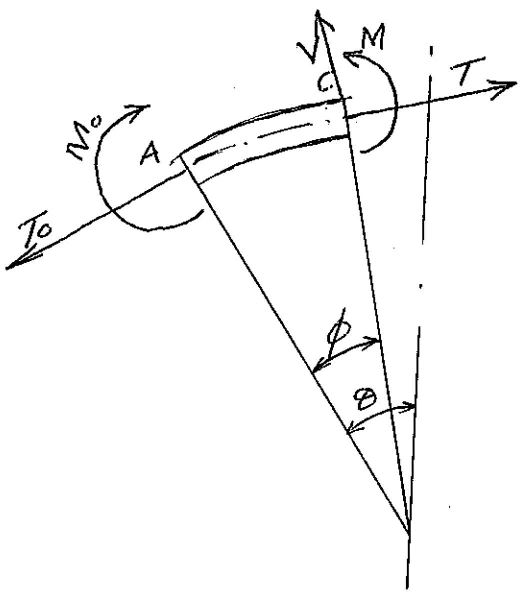


Fig. 3

Resolving forces and moments

$M = M_o - r \, To \, (1 - \cos \phi)$
$T = To \cos \phi$ (2)
$V = To \sin \phi$ (3)

The strain energy in a beam due to bending and compression is defined as

$$U = \frac{1}{2EI}\int_o^1 M^2\, dx + \frac{1}{2EA}\int_o^1 T^2\, dx \qquad (4)$$

$$\therefore \frac{\partial U}{\partial Mo} = \frac{1}{EI}\int_o^1 M \frac{\partial M}{\partial Mo}\, dx + \frac{1}{EA}\int_o^1 T \frac{\partial T}{\partial Mo}\, dx \qquad (5)$$

In this case $dx = r\, d\phi$
and the integration must be carried out between the limits $\phi = O$
and $\phi = \theta$ and multiplied by 2 to give the total strain energy in the segment A. B.

Strain Energy due to the shearing force V is neglected since the deflection due to shear will be small compared to the deflection due to bending.

By the theory of strain energy $\frac{\partial U}{\partial Mo} = O$ since Mo is an internal force and does no work.

∴ Substituting (2) and (3) in (5) and equating to O

$$\frac{1}{EI}\, 2\int_o^\theta (Mo - r\, To\,(1 - \cos\phi))\, r\, d\phi = O$$

$$\therefore Mo\,\theta - r\, To\,(\theta - \sin\theta) = O$$

$$Mo = r\, To\,(1 - \frac{\sin\theta}{\theta}) \qquad (6)$$

Substitute in (2)

$$M = r\, To\,(\cos\phi - \frac{\sin\theta}{\theta}) \qquad (7)$$

Substitute (3) and (7) in (4)

$$U = \frac{1}{EI}\int_o^\theta r^3\, To^2\, (\frac{\sin\theta}{\theta} - \cos\phi)^2\, d\phi + \frac{1}{EA}\int_o^\theta r\, To^2 \cos^2\phi\, d \qquad (8)$$

where U is the strain energy from A to B (Fig. 2)

$$\int_o^\theta (\frac{\sin\theta}{\theta} - \cos\phi)^2\, d\phi = \frac{\theta}{2}\left[1 + \frac{\sin 2\theta}{2\theta} - 2\,(\frac{\sin\theta}{\theta})^2\right]$$

In our case $\theta = \frac{\pi}{10}$ as there are 10 forces

$$\int_o^{\frac{\pi}{10}} (\frac{\sin\theta}{\theta} - \cos\theta)^2\, d\phi = 67.0.10^{-6} \qquad (9)$$

$$\int_0^{\theta} \cos^2 \phi \, d\phi = \frac{\theta}{2} \left(1 + \frac{\sin 2\theta}{2\theta} \right)$$

Substituting $\theta = \frac{\pi}{10} = 18^o$

$$\int_0^{\frac{\pi}{10}} \cos^2 \phi \, d\phi = 0.304 \qquad (10)$$

Substitute (9) and (10) in (8) and putting $I = Ak^2$

$$U = \frac{r\,To^2}{EA} \left(67.0 \left(\frac{r}{k}\right)^2 10^{-6} + 0.304 \right) \qquad (11)$$

By the theory of strain energy $U = \frac{P\delta}{2}$ where δ is the radial deflection of the point of application of P.

From (1) $To = \frac{P}{2 \sin\theta}$

$$\therefore \frac{P\delta}{2} = \frac{r\,P^2}{4EA \sin^2\theta} \left(67.0 \left(\frac{r}{k}\right)^2 10^{-6} + 0.304 \right)$$

where $\theta = 18^o$

$$\therefore \delta = \frac{rP}{EA} \left(3.50 \left(\frac{r}{k}\right)^2 10^{-4} + 1.59 \right) \qquad (12)$$

Substituting (1) in (7) gives

$$M = rP \frac{1}{2 \sin\theta} \left(\cos\phi - \frac{\sin\theta}{\theta} \right)$$

for the point of application of P $\phi = \theta = 18^o$

$$\therefore M = rp \frac{1}{2} \left(\frac{1}{\tan 18^o} - \frac{10}{\pi} \right) \qquad (13)$$

at support $M = 0.0527\, rP$

Substituting (1) in (3)

$$T = \frac{P}{2 \sin\theta} (\cos\phi)$$

$\therefore$ at the support $\phi = \theta = \frac{\pi}{10} = 18^o$

$$T = \frac{P}{2 \tan 18^o}$$

at the support $T = 1.539P \qquad (14)$

Summary. For 10 equal radial forces of magnitude P acting on a ring of radius r the bending moments and axial tensions in the ring are:–

	Bending Moment	Axial Tension
At points of application of forces	–0.0527 rP	1.539 P
Midway between forces	+0.0265 rP	1.618 P

Since the tension remains almost constant the greatest stresses will be at the supports where the bending moment is greatest.

The radial deflection at the points of application of the forces is given by:–

$$\delta = \frac{rP}{EA}\left(3.50\left(\frac{r}{k}\right)^2 10^{-4} + 1.59\right) \qquad (12)$$

where r = mean radius of ring

P = applied force

E = modulus of elasticity

A = section area

k = radius of gyration of section about y y axis

δ = radial deflection

APPENDIX C

Calculation of the stress by the first assumption, i.e. Minimum shell stiffness to give the lower limit of stress.

All the reaction from the crane beam was considered to be taken by the stiffeners which form frames across the reactor. The frames were considered as pin jointed at the ground. To simplify calculation the slope of the roof was neglected as the low slope will have only a small effect.

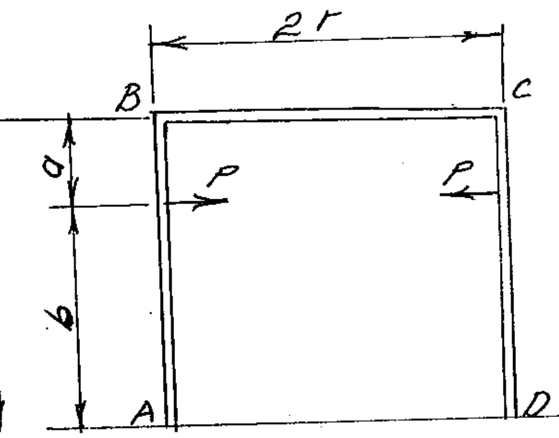


Fig. 1 Assumed Stiffener frame

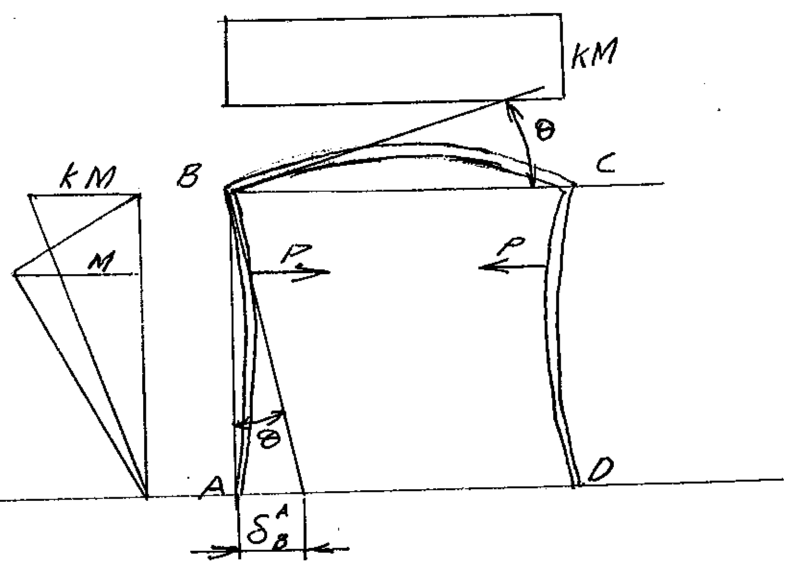


Fig. 2 Mode of deflection and bending moments

Assume that the B.M. in A B due to P and neglecting moment at B is M as shown in figure 2.

$$\therefore M = P \frac{ab}{L} \qquad (1)$$

Assume that the moment at B is kM where k is a constant.

Using the Moment – Area method.

Change of slope of BC between B and C

$$= \frac{1}{EI} \text{ (Area of BM diagram)}$$

$$\therefore -2\theta = \frac{1}{EI} \; 2\,kMr$$

$$\therefore \theta = \frac{-1}{EI} \; kMr \qquad (2)$$

δ_B^A = deflection of A above the tangent at B

$$= \theta L$$

$$= -\frac{1}{EI} \; kMrL \qquad (3)$$

But by area moment method

$$\delta_B^A = \frac{1}{EI} \text{ (Moment of BM diagram about A)}$$

$$\therefore \delta_B^A = \frac{1}{EI} \left(\frac{1}{3} kML^2 + \frac{1}{6} ML \; (L + b) \right) \qquad (3)$$

equating (2) and (3)

$$-kMrL = \frac{1}{3} kML^2 + \frac{1}{6} ML^2 + \frac{1}{6} MLb$$

$$\therefore k = -\frac{L + b}{2(L+3r)} \qquad L = 744 \text{ in}, \; b = 624 \text{ in} \qquad (4)$$

$$= -0.342 \qquad r = 420 \text{ in}$$

Let δ be the deflection at the point E

$$\therefore \delta = \theta a - \delta_B^E$$

$$\delta_B^E = \frac{1}{EI} \text{ (moment of diagram between E and B about E)}$$

$$= \frac{1}{EI} \left(\frac{1}{3} kMa^2 + \frac{1}{6} (1 + \frac{b}{L} k) \; Ma^2 \right) \qquad (5)$$

$$\theta_a = -\frac{1}{EI} kMra \qquad (6)$$

$$\therefore \quad \delta = -\frac{1}{EI}\left(kMra + \frac{1}{3} kMa^2 + \frac{1}{6}Ma^2 + \frac{1}{6}\frac{b}{1} kMa^2 \right)$$

$$\delta = -\frac{Ma^2}{6EI}\left(1 + k\left(6\frac{r}{a} + \frac{b}{L} + 2 \right) \right) \qquad (7)$$

Substituting values

$a = 120$ in

$b = 624$ in

$L = 744$ in

$r = 420$ in

$E = 30.10^6$ p.s.i

$I = 3780$ in^4

$M = P\frac{ab}{L}$

$k = -0.342$

$$\delta_s = 15.2 \, P \, 10^{-6} \qquad (8)$$

Calculation of the stresses in the circular crane beam

Equation B (12)

$$\delta_b = \frac{rP}{EA}\left(3.50\left(\frac{r}{k}\right)^2 + 1.59\right)$$

Substituting values for the crane beam

$r_b = 400$ in

$E = 30.10^6$ p.s.i

$A = 42$ in^2

$k = 3.58$ in

$$\delta_b = 1.89 \, P.10^{-6} \qquad (9)$$

but $\Delta = \delta_b + \delta_s$

$\Delta = r_s \alpha t$ — $r_s = 420$ in, $\alpha = 6.7.10^{-6}$ $^oF^{-1}$, $t = 100$ ^oF

$$\therefore \quad \Delta = 0.28 \text{ in} \qquad (10)$$

$\therefore$ $1.89\ P.10^{-6} + 15.2\ P.10^{-6} = 0.28$

$P = \frac{0.28.\ 10^6}{17.1}$ lb.

$= 16\ 400$ lb.

Substitute in B (13)

$\therefore$ BM at supports $= -0.0527\ r_b P$ $\quad r_b = 400$ in

$= -3.46 \times 10^5$ lb. in

Maximum bending stresses are at inside and outside fibres of the circular beam.

$fb = \pm \frac{Mz}{Iy}$ $\quad$ Z max = 6 in; $\quad$ Iy = 538 in^4

$= \frac{3.46 \times 6 \times 10^5}{538}$

$= 3800$ p.s.i.

From B (14)

Axial tension $= 1.539P$

$\therefore$ $fa = \frac{1.539P}{A}$ $\quad$ $A = 42\ in^2$

$= 600$ p.s.i.

The greatest stress will be in the outside fibres where the two tensile stresses combine

f max = fb + fa

$= 3800 + 600$

$= 4400$ p.s.i.

A compressive stress of 3200 psi is induced in the inner fibres.

APPENDIX D

Calculation of the stresses by the second assumption, i.e. Maximum shell stiffness to give the upper limit of stress.

The roof was considered as rigid and the stiffeners were taken as beams between the roof and the ground and built in at each end.

Appendix D (Contd.)

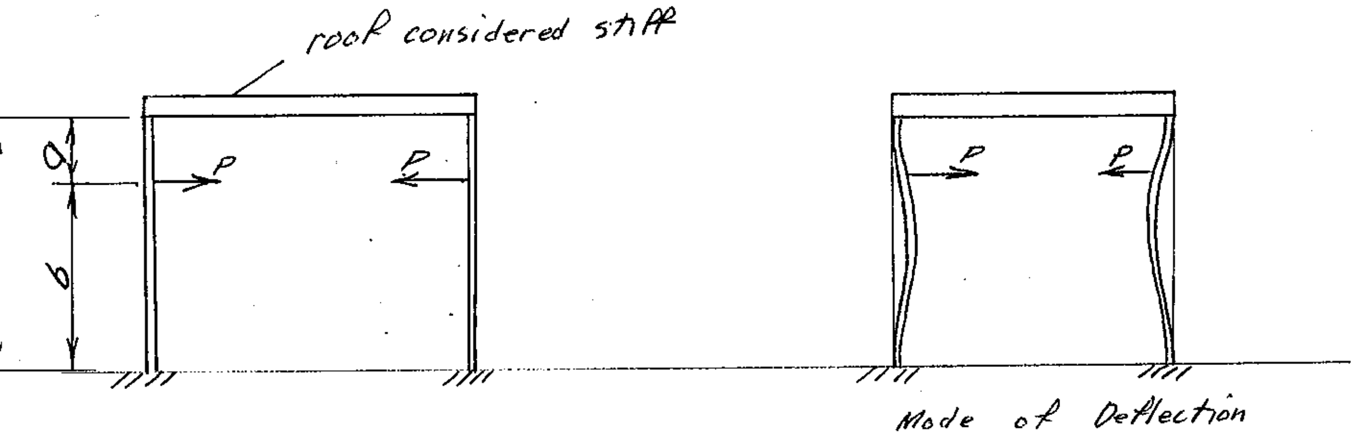


Fig. 1. Second assumption for stiffeners.

For a beam built in at each end and loaded as shown the deflection at C is given by

$$\delta = p \frac{a^3 \ b^3}{3 \ EI \ L^3} \quad (1)$$

a = 120 in
b = 624 in
L = 744 in
E = 30.10^6 p.s.i
I = 3780 in^4

$\therefore \ \delta = 3.00 \ P \ 10^{-6}$ (2)

Combining this with the deflection of the circular crane beam as in appendix C gives

$(1.89 + 3.00) \ P \ 10^{-6} = 0.28$

$P = 57000$ lb.

∴ The maximum stress by comparison with appendix C.

$$f \ max = 4400 \ \frac{57000}{16400}$$

$= 15500$ p.s.i.

APPENDIX E

Calculation of the stress in the circular crane beam by treating the stiffener as a beam on an elastic foundation to take account of the shell plates. Twisting of the circular crane beam and flexibility of the roof beams are considered.

Reference: Timoshenko – Strength of Materials
Volume 2, CH 1

As the stiffeners deflect radially, the shell plates between them will buckle and exert a resisting force on the stiffener.

Calculation of Foundation Modulus

The shell will deflect in a similar way to the circular crane beam but as we are considering bending of a plate we use the modified modulus E^1.

$$E^1 = \frac{E}{1-\mu^2} \quad \text{where } \mu = \text{poisson's ratio}$$

$$\mu = 0.3$$

$$\therefore \quad E^1 = \frac{30.10^6}{0.91} \text{ p.s.i}$$

$$= 33.10^6 \text{ p.s.i}$$

By equation B (12)

$$\delta = \frac{rP}{E'A} \left(3.50 \left(\frac{r}{k}\right)^2 10^{-4} + 1.59\right)$$

$r = 420$ in
$E' = 33.10^6$
$A = 0.812$
$k = 0.234$ in

$$\therefore \quad \delta = 1.77 P\, 10^{-2}$$

$$\text{or } P = 55.5\delta$$

the foundation modulus is defined as

$$k = \frac{P}{\delta}$$

for the remainder of this calculation k will stand for the foundation modulus.

$$\therefore \quad k = 55.5 \text{ p.s.i}$$

For stiffener

$$\beta = \sqrt[4]{\frac{k}{4EI}}$$

$k = 55.5$ p.s.i
$E = 30.10^6$ p.s.i
$I = 3780$ in^4

$$= 0.0033 \text{ in}^{-1}$$

The stiffener is considered as part of an infinite beam on an elastic foundation of foundation modulus k. Forces and moments are applied by the crane beam, roof, and ground. The deflections at these three points due to the applied loads are calculated and equated to the assumed values. Coefficients for deflections and rotations are plotted in Timoshenko P5 Table I. in terms of the parameter β X

a = 120 in		βa = 0.4
b = 624 in		βb = 2.0
L = 744 in		BL = 2.4

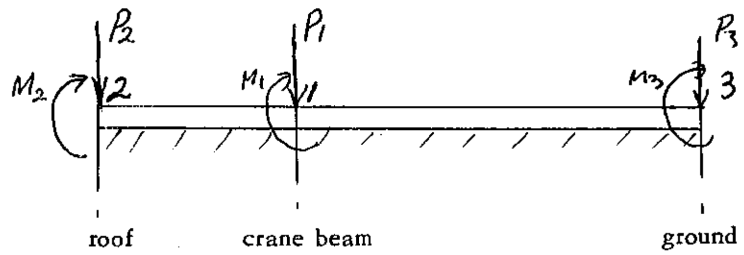


Fig. 1. Forces acting on stiffener

I Equation for deflection at point 1 (crane beam)

Let the deflection at point 1 be δ

then

$$\frac{P_1\beta}{2k}\phi(O) + \frac{M_1\beta^2}{k}S(0) + \frac{P_2\beta}{2k}\phi(0.4) + \frac{M_2\beta^2}{k}S(0.4) + \frac{P_3\beta}{2k}\phi(2.0)$$

$$-\frac{M_3\beta^2}{k}S(2.0) = S$$

Divide by $\frac{\beta}{k}$ and insert values of ϕ and S from table I

$$\therefore\ 0.500\,P_1 + O\,\beta M_1 + 0.489\,P_2 + 0.261\,\beta M_2 + 0.033\,P_3 = 0.123\,\beta M_3 = \frac{k\delta}{\beta}$$

II Equation for rotation at point 1 (crane beam)

Rotation of the stiffener at point 1 is resisted by twisting of the crane beam. Ref. Timoshenko p 177.

M_E = twisting moment per inch of ring

$$M_E = \frac{10\, M_1}{2\pi r}$$

r = 400 in
β = 0.0033 in^{-1}

$$= 0.0040\, M_1$$

$$= 1.20\, \beta M_1$$

Angular twist of the beam

$$\theta = -\frac{M_E\, r^2}{E\, I_x}$$

$$= \frac{10\, M_1\, r}{2\pi E\, I_x}$$

r = 400 in
E = 30.10^6 p.s.i
I_x = 1920 in^4
k = 55.5
β = 0.0033

$$= -M_1 \frac{\beta^3}{k} \frac{10\, k\, M_1\, r}{2\pi \beta^3 E I_x}$$

$$\theta = -16.5\, M_1 \frac{\beta^3}{k}$$

But $\theta = -P_1 \frac{\beta^2}{k} S(O) + M_1 \frac{\beta^3}{k} \psi\ (O) - P_2 \frac{\beta^2}{k} S(0.4) + M_2 \frac{\beta^3}{k} \psi\ (0.4)$

$$+ P_3 \frac{\beta^2}{k} S(2.0) + M_3 \frac{\beta^3}{k} \psi\ (2.0)$$

Equating and dividing by $\frac{\beta^2}{k}$

$$O\, P_1 + 17.5\, \beta M_1 - 0.261\, P_2 + 0.356\, \beta M_2 + 0.123\, P_3 - 0.179\, \beta M_3 = O$$

III Equation for deflection at point 2, (roof.)

Due to the low angle of the roof and the stiffening effect of the roof plates, especially the curved plates at the junction of the walls and roof it is assumed that there is no deflection at point 2. The curved plate between the walls and the roof forms a circular beam of much greater moment than the crane beam. This combined with the roof plates and stiffeners will give a much more rigid structure than the crane beam and hence the errors due to this assumption should not be large. The stress found will be slightly higher than actual.

Developing the equation in a similar way to I gives

$$0.439\, P_1 - 0.261\, \beta M_1 + 0.500\, P_2 + 0\, \beta M_2 - 0.003\, P_3 - 0.061\, \beta M_3 = O$$

IV Equation for rotation at point 2 (roof)

The rotation at point 2 is resisted by bending of the roof beams. Since we are using formulae for an infinite beam an additional moment equal to the bending moment in the infinite beam to the left of point 2 must also be supplied by the roof beam to maintain equivalence.

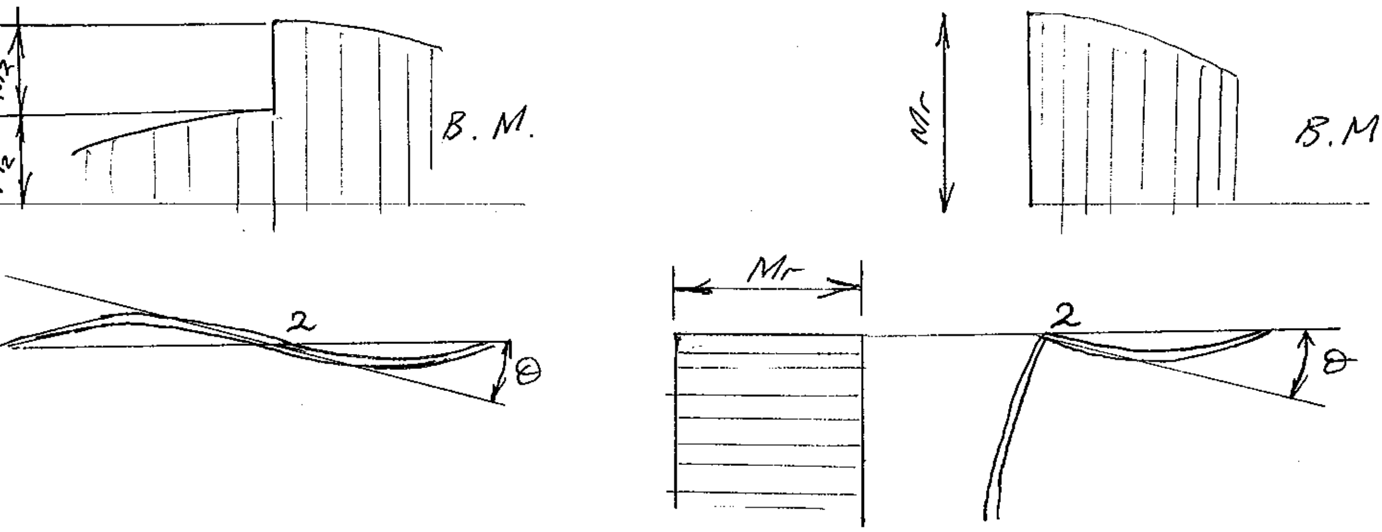


Fig. 2 (a) B.M. Fig. 2 (b)

Fig. 2 (a) Shows the BM in the infinite beam at point 2 with a moment M_2 applied.

Fig. 2 (b) Shows the BM in the stiffener and roof beam at point 2.

From Fig. 2 (b)

The moment in the stiffener to the right of 2

= the BM in the roof beam

= Mr

But the two diagrams must be equivalent to each other on the right of point 2.

$\therefore \quad Mr = M_2{}^1 + M_2$

where $M_2{}^1$ is the BM at 2 to the left. in Fig 2 (a)

$\therefore \quad M_2 = Mr - M_2{}^1$

Mr is found by considering one roof stiffener

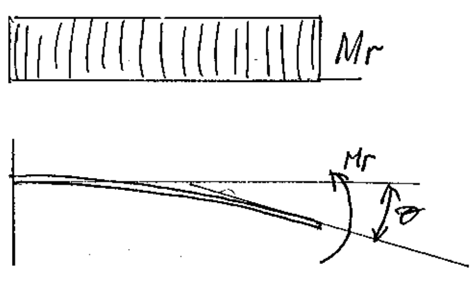


Fig. 3 (d) By the moment – area method

$$\theta = -\frac{1}{EI}\, 420\, Mr$$

$$= -Mr\,\frac{\beta^3}{k}\;\frac{420\,k}{EI\,\beta^3}$$

$$= -5.72\, Mr\,\frac{\beta^3}{k}$$

$\beta = 0.0033$
$k = 55.5$
$E = 30.10^6$
$I = 3780$

$$\therefore\ Mr\,\frac{\beta^3}{k} = -\frac{1}{5.72}\,\theta$$

$M_2{}^1$ is found by considering the semi infinite beam to the left of 2.

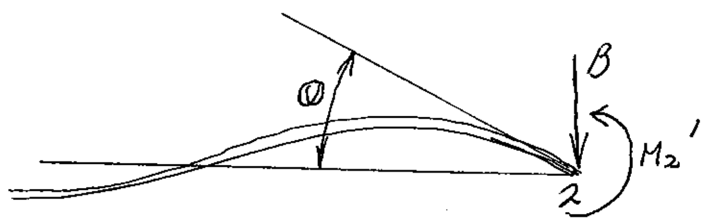


Fig. 3 (b) Semi infinite cylinder to left of 2 for this

$$2\,M_2{}^1\,\frac{\beta^3}{k} = -\theta$$

$$\therefore\ M_2{}^1\,\frac{\beta^3}{k} = -\frac{1}{2}\,\theta$$

but

$$M_2 = M_r - M_2{}^1$$

$$\therefore\ M^2\,\frac{\beta^3}{k} = -\frac{\theta}{5.72} + \frac{\theta}{2}$$

$$= \frac{\theta}{3.08}$$

$$\therefore\ \theta = 3.08\, M_2\,\frac{\beta^3}{k}$$

Developing the equation for θ in a similar manner to II and equating to this result gives

$$0.261\,P_1 + 0.356\,\beta M_1 + O\,P_2 - 2.08\,\beta M_2 + 0.061\,P_3 - 0.128\,\beta M_3 = O$$

VI. Equation for rotation at point 3 (ground.)

There will be no rotation at this point which is considered as built in

$\therefore \quad 0.123\,P_1 - 0.179\,\beta M_1 - 0.061\,P_2 - 0.128\,\beta M_2 + 0\,P_3 + \beta M_3 = 0$

The equations developed in I to VI give a set of 6 linear simultaneous equations.

$$\begin{bmatrix} 0.500 & 0 & 0.489 & 0.261 & 0.033 & -.123 \\ 0 & 17.5 & -.261 & 0.356 & 0.123 & -.179 \\ 0.439 & -.261 & 0.500 & 0 & -.003 & -.061 \\ 0.261 & 0.356 & 0 & -2.08 & 0.061 & -.128 \\ 0.033 & 0.123 & -.003 & 0.061 & 0.500 & 0 \\ -.123 & -.179 & -.061 & -.128 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_1 \\ \beta M_1 \\ P_2 \\ \beta M_2 \\ P_3 \\ \beta M_3 \end{bmatrix} = \frac{k}{\beta} \begin{bmatrix} \delta \\ 0 \\ 0 \\ 0 \\ \Delta \\ 0 \end{bmatrix}$$

The equations were solved for P_1 and M_1 by elimination. Ref. Hartree – Numberical Analysis P157.

Solution of the equations gives

$$0.089\,P_1 = \frac{k\delta}{\beta} - 0.082\,\frac{k\Delta}{\beta} \qquad (7)$$

$$\therefore \quad 17.3\,\beta M_1 = -0.241\,P_1 - 0.262\,\frac{k\Delta}{\beta} \qquad (8)$$

from (7) $\delta_s = \dfrac{0.089\,\beta}{k}\,P_1 + 0.082\,\Delta$

$\beta = 0.0033\ \text{in}^{-1}$
$k = 55.5\ \text{p.s.i}$
$\Delta = 0.28\ \text{in}$

$$\therefore \quad \delta_s = 5.3\,P_1\,.10^{-6} + 0.082\,\Delta \qquad (9)$$

and (9) $\delta_b = 1.89\,P.10^{-6}$

but $\delta_s = \delta_b = \Delta$

$\therefore \quad 7.2\,P.10^{-6} = (1-0.082)\,\Delta$

$\therefore \quad P = \dfrac{0.28 \times 0.918}{7.2}\,.10^{6}$

$$P = 36000\ \text{lb.} \qquad (10)$$

from (8) $17.3\,\beta M_1 = -0.241 \times 36000 - 0.262\,\dfrac{55.5 \times 0.28}{0.0033}$

$$\therefore \quad \beta M_1 = -580\ \text{lb.} \qquad (11)$$

B (13) gives bending moment M = –0.0527 rP

∴ M = –0.0527x400x36000

= $-7.60 \cdot 10^5$ lb. in.

∴ bending stress = $\pm \frac{Mz}{Iy}$

z = 6 in
Iy = 538 in^4

= $\pm \frac{7.60 \times 6}{538} 10^5$

= ± 8500 p.s.i

+ ve on outside fibres

– ve on inside fibres

B (14) gives

Axial tension T = 1.539P

P = 36000 lb.

= 55000 lb.

∴ axial stress = $\frac{T}{A}$

A = 42 in^2

= $\frac{55000}{42}$

= + 1300 p.s.i

Stresses due to twisting of the crane beam

M_t = 1.20 βM_1 see II

βM_1 = –580 lb.

= –1.20x580

= –700 lb.

f = $-\frac{M_t\, r\, y}{I_z}$ Ref. Timoshenko

for the top fibres y = 7.15 in

r = 400 in
y = 7.15 in
I_z = 1920 in

f = $\frac{700 \times 400 \times 7.15}{1920}$ p.s.i

= + 1000 p.s.i

for bottom fibres y = –10.1 in

f = $-\frac{700 \times 400 \times 10.1}{1920}$

= – 1400 p.s.i

Total combined stresses for the four corners of the beam at the supports as shown on Fig. 3 are tabulated below.

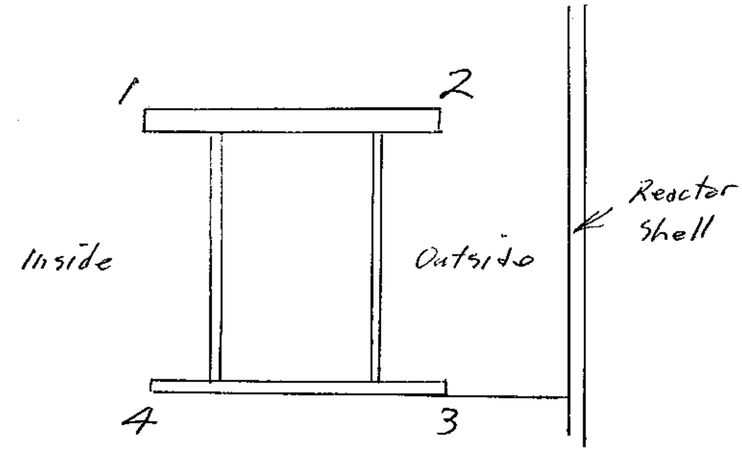


Fig. 3 Crane Beam Section positions of calculated stresses.

Position	Stresses			
	Bending	Tension	Twisting	Total
1	–8500	+1300	+1000	–6200
2	+8500	+1300	+1000	+10800
3	+8500	+1300	–1400	+8400
4	–8500	+1300	–1400	–8600

Calculation of stresses in the stiffener

Complete solution of the 6 equations gives the following values:–

P_1 = 36000 lb

βM_1 = –580 lb

P_2 = –31000 lb

βM_2 = 4400 lb

P_3 = 1800 lb

βM_3 = 700 lb

The maximum bending moment in the stiffener is found to be just above the crane beam level. From Timoshenko

$$M = \frac{P_1}{4\beta}\psi(10) - \frac{M_1}{2}\theta(0) + \frac{P_2}{4\beta}\psi(0.4) + \frac{M_2}{2}\theta(0.4) + \frac{P_3}{4\beta}\psi(2.0) - \frac{M_3}{2}\theta(2.0)$$

Substituting calculated values of P_1 and M_1 etc., and values of ψ and θ from table 1 gives

$\beta M = 7900$ lb. $\qquad\qquad \beta = 0.0033$

$M = 2.4.10^6$ lb.in

Compressive stress in outer flange

$f = \frac{My}{I}$ $\qquad\qquad y = 12$ in

$\qquad\qquad I = 3780$ in^4

$= 7800$ p.s.i

Similarly the maximum shear force is just above 2

$$V = \frac{P_1}{2}\theta(0) - \frac{\beta M_1}{2}\phi(0) - \frac{P_2}{2}\theta(0.4) - \frac{\beta M_2}{2}\phi(0.4) + \frac{P_3}{2}\theta(2.0) - \frac{\beta M_3}{2}\phi(2.0)$$

$V = 26000$ lb

∴ Max shear in web $= \frac{26000}{24x0.375}$ p.s.i

$= 2900$ p.s.i

Summary:

The maximum stresses in the stiffener are:–

Compression in flange 7800 p.s.i

Shear in web 1900 p.s.i

APPENDIX F

Calculation of the stresses in the crane beam due to the crane.
Wheel loading. The maximum wheel loading is given approximately by
Wheel load = 1/4 (weight of crane) + 1/2 (maximum load)

Weight of crane = 28.5 tons
maximum load = 25 tons

∴ Wheel load $= \frac{28.5}{4} + \frac{25}{2}$

$= 19.6$ tons

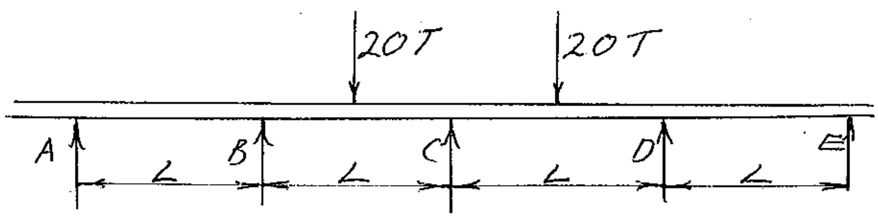
For the calculation assume a load of 20 tons per wheel.

Consider the beam as a straight simply supported continuous beam. The effect of curvative is small and tends to reduce the moments at the supports so that this assumption should give a high estimate.

Span between supports $= \frac{2\pi r}{10}$ $\qquad r = 400$ in

$= 250$ in

The wheel spacing is slightly greater than the span but for convenience assume two loads of 20 tons each in the centres of adjacent spans. Consider a total of four spans simply supported.



$L = 250$ in.

Fig. 1 Assumed loading.

By symmetry it is seen that there will be no rotation at C. We need only consider the two spans A B and B C with the beam built in at C.

The bending moment diagram will be

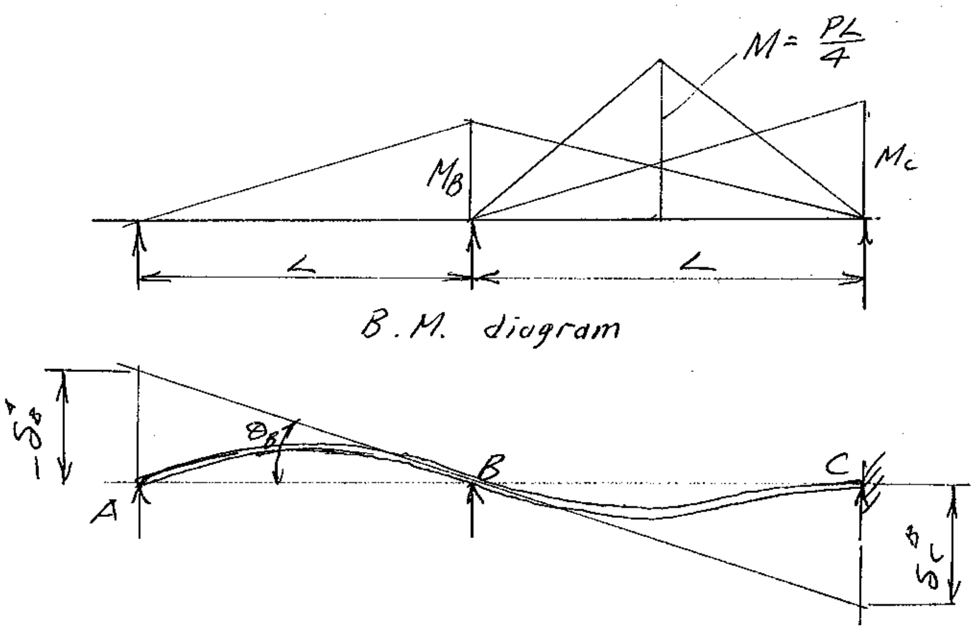


Fig. 2. Mode of deflection.

By the moment – area method.

$$\theta = \frac{1}{EI}\text{ (area of diagram between B and C)}$$

$$= \frac{1}{EI}\left(1/2\,ML + 1/2\ M_bL + 1/2\,M_oL\right)$$

$$= \frac{1}{2EI}\left(M + M_B + M_c\right) \qquad (1)$$

Since the tangent at C is horizontal (by symmetry)

$$\delta_C^B = O$$

but $\delta_C^B = \frac{1}{EI}$ (moment about B of diagram between B and C)

$$\therefore\ O = \frac{1}{EI}\left(\frac{1}{2}ML\frac{1}{2}L + \frac{1}{2}M_BL\frac{1}{3}L + \frac{1}{2}M_cL\frac{2}{3}L\right)$$

$$\therefore\ 3M + 2M_B + 4M_c = O \qquad (2)$$

$$-\delta_B^A = \theta L = +\frac{L^2}{2EI}\left(M + M_B + M_c\right) \qquad (3)$$

but $\delta_B^A = \frac{1}{EI}$ (moment about A of diagram between A and B)

$$\therefore\ = \frac{1}{EI}\left(\frac{1}{3}M_B L^2\right) \qquad (4)$$

Combining (3) and (4)

$$3M + 5M_B + 3M_c = O \qquad (5)$$

Solving (2) and (5) gives

$$M_B = \frac{-3}{14}M$$

$$M_C = \frac{-9}{14}M$$

and $M = \frac{PL}{4} = \frac{20x250}{4} = 1250$ ton in.

$$\therefore\ M_B = \frac{-3}{14}\,1250 = -270 \text{ ton in.} \qquad (6)$$

$$M_C = \frac{-9}{14}\,1250 = -800 \text{ ton in.} \qquad (7)$$

Moment at mid span $= M + \frac{1}{2}(M_B + M_c)$

$= \frac{8}{14}\ 1250$

$= +\ 710$ ton in. (8)

The stress at the support is

$f = M_c \frac{y}{I_z}$

$I_z = 1920\ in^4$
for top fibres $y = 7.15$ in
for bottom fibres $y = 10.1$ in

∴ Tension in top fibres

$fb = +\frac{800 \times 7.15}{1920}$

$= 3.0$ tons in^{-2}

$= +\ 6700$ p.s.i

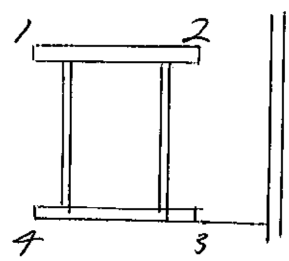
Compression in bottom fibres

$fb = \frac{-800 \times 10.1}{1920}\ 2240$ p.s.i

$= -9500$ p.s.i

Combined stresses in the beam are

Position	Thermal	Stresses Crane loading	Total
1	-6200	+6700	+500
2	+10800	+6700	+17500
3	+8400	-9500	-1100
4	-8600	-9500	-18100



The maximum stresses likely to be encountered are +17500 at the top outside corner and -18100 at the bottom inside corner.

Reaction at C $= P - 2\frac{M_c}{e} - 2\frac{M_B}{e}$

$= P + \frac{2P}{4}\ \frac{9}{14} - \frac{2P}{4}\ \frac{3}{14}$

$= P\ \frac{17}{14}$

$= 24$ tons

Calculation of Stresses in Stiffener due to crane loading.

As an approximation consider one stiffener pin jointed at floor and roof and loaded eccentrically by the crane

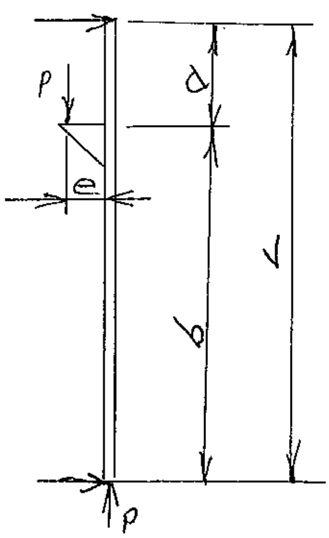
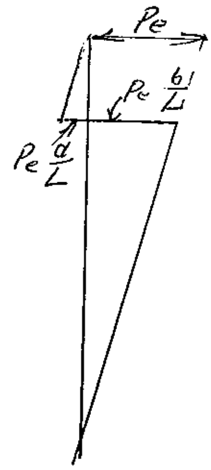


Fig. 3



BM Diagram.

The maximum load on one support is 24 tons from Appendix F (9)

e = (radius of shell) + 12" – (radius of crane rail)

= 420 + 12 – 400 in.

= 32 inches

Max BM = $Pe \frac{b}{L}$ just below the support

= $24.32 \frac{624}{744}$

= 640 in ton

P	=	24 tons
e	=	32 inches
b	=	624 in
I	=	744 in
y	=	12 "
I	=	3780 in^4

Max bending stress = $\frac{My}{I}$

= $\frac{640x2240x12}{3780}$ p.s.i

= 4600 tension in flange

axial compression = $\frac{P}{A}$ A = 30.8 in^2

= $\frac{24x2240}{30.8}$ = 1750 p.s.i

∴ The maximum tension in the flange is 4600 – 1750 p.s.i

= 2850 p.s.i

The maximum compressive force in the flange will be just above the crane support.

$M = P_e \frac{a}{L}$		$P = 24$ tons	
=		$e = 32$ in	
= 125 in ton		$a = 120$ in	
$f = \frac{My}{I}$		$l = 744$ in	
		$y = 12$ in	
$= \frac{125 \times 2240 \times 12}{3780}$		$I = 3780 \text{ in}^4$	
= 900 p.s.i			

∴ This will be additional to the stress of 7800 due to thermal expansion.
Maximum combined stresses in the stiffener flange are 8700 p.s.i compression above the support 2850 p.s.i tension below the support.

This is only an approximate calculation but it indicates reasonably low stresses in the stiffener and a more accurate calculation is not justified.

The shear stress in the stiffener is small, the maximum shear force being $\frac{Pe}{L}$ = 2400 lb. compared to 26000 lb. due to thermal expansion.