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LUCAS HEIGHTS

VELOCITY DISTRIBUTION AND PRESSURE LOSSES FOR RANDOMLY PACKED BEDS OF SPHERES

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ABSTRACT

Measurements were made of fluid velocity distributions for a number of randomly packed beds of spheres using a flow separator between the exit face of the bed and the plane of measurement. The results for right cylindrical beds show that the departures from uniformity were small, except for a region within one half sphere diameter from the walls of the containing vessel where a marked increase in velocity occurred. The distribution was found to be independent of Reynolds number ($1470 < N_R < 4350$) and bed length ($9 < L/d < 36$). It was only slightly affected by the method of loading the bed, the density and tolerance of the spheres, and sphere diameter ($12 < D/d < 48$). A marked change was found for a bed which was peaked at exit.

Measurements of the pressure losses across the beds were also made. The resulting friction factors for the right cylindrical beds were correlated using the general pressure loss correlation in its normal form and also in a modified form which took the measured velocity distributions into consideration. Both catered adequately for different packing methods and sphere diameter, but not for different bed lengths. Results are presented for the additional pressure losses caused by a peak at exit.

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1. INTRODUCTION

A review of the published literature on the fluid flow distribution through randomly packed beds has shown that marked discrepancies exist in the data for nominally similar beds and has raised doubts on the validity of some of the experimental techniques used (Price 1966). This view was prompted by theoretical considerations and is supported by recent experiments on a randomly packed bed of glass spheres (Price and Stevens 1966).

The present series of tests was designed to provide reliable experimental data over a range of bed parameters which would resolve the anomalies in the published literature. It was also hoped to establish a sound theoretical approach as a base for calculations relating to a design study of a high temperature gas-cooled pebble bed reactor.

A study of the pressure losses through packed beds was also made to appraise the best-known pressure loss correlation in the light of the measured velocity distributions.

The notation used in this report is given in Appendix 1.

2. APPARATUS

The apparatus is shown schematically in Figure 1. The tests were made in a vertical wind tunnel which exhausted to atmosphere. The 12 in. diameter test bed was contained between 5-mesh screens and a honeycomb was located on its exit face. A pitot static tube, which could be traversed over all sections of the bed, was used in conjunction with a micro-manometer to measure the air velocities. Overall flowrate was measured by means of an orifice plate upstream from the 90° bend. The bend was fitted with turning vanes and a 60-mesh screen at outlet to ensure a uniform velocity profile at inlet to the bed. The apparatus is described in detail by Price and Stevens (1966).

Spheres used were ¼ in., ½in. or 1 in. diameter, and the bed length was varied from 4½ in. to 18 in.

The honeycomb used for the right cylindrical beds of ½ in. diameter spheres was that developed by Price and Stevens in their test series III. The cross sectional area of its passages converged in the direction of flow such that the ratio of inlet area to outlet area was approximately 2.1 for all but the outer ring of the honeycomb where the ratio was reduced to 1.6. These contractions greatly facilitated measurement without causing flow redistribution within the bed.

In the tests on the right cylindrical beds of ¼ in. or 1 in. diameter spheres the contraction area ratios were modified slightly in the outer ring and the one adjacent to it. These modifications were based on measurements made with a honeycomb whose flow passages were of constant cross section along the length (Price and Stevens test series I). This same honeycomb was used in the tests on the peaked bed but required the addition of a section contoured to suit the peak angle of the bed.

3. METHOD OF MEASUREMENT

3.1 Velocity Distribution

Local air velocities were measured by a pitot static tube traversed across a plane which was about 1/16 inch downstream from the exit face of the honeycomb. Impact pressures were recorded on a micro-manometer. A number of readings were taken for each compartment of the honeycomb ranging from six for the largest to two for the smallest. At each flow rate, approximately 350 readings were taken with the honeycomb whose flow passages converged and 1000 to 1200 with the one having uniform flow passages.

3.2 Pressure Loss

The honeycomb was first removed from the test section and static pressure differences between the atmosphere and each of three piezometer rings were measured; one ring was placed upstream of the bed and the other two at stations along the length of the bed (see Section 5 for a discussion on the suitability of these positions). Each piezometer ring contained three pressure tappings spaced at 120°.

Readings were taken for a range of flow rates, the upper limit being set by the start of levitation. Care was necessary in defining this upper limit, particularly for loosely packed beds, as significant increases in friction factor were noted at flows well below the theoretical levitation flow. These were usually associated with a slight settling of the bed. Since the levitation flow decreases with bed weight the upper limit of the flow range was decreased as the sphere density decreased. The tendency to settle was particularly noticeable in the tests on peaked beds, the peak angle tending to flatten as the flow was increased.

The friction factors reported here all refer to beds whose packing structure was unaltered by the action of the air flow. In a number of cases this necessitated repeat testing.

3.3 Voidage Fraction

The average voidage fraction was determined from the volume of spheres in the cylindrical test section, the lower end of which was closed by a taut 5-mesh wire screen. In all cases the top surface of the bed was arranged so that no spheres were allowed to project beyond the test bed length but all available spaces on the top surface were filled if they could be filled by whole spheres. This left vacancies which could have been filled by part spheres.

The number of spheres in each bed was hand-counted and weight-checked; their volume was determined by water displacement tests and diameter measurement.

4. VELOCITY DISTRIBUTIONS FOR RIGHT CYLINDRICAL BEDS

4.1 Test Programme

The following parameters were investigated in the tests made upon right cylindrical beds:

Flow rate or Reynolds Number

Bed Length

Packing Method

Sphere Density and Tolerance

Sphere Diameter or Vessel/Sphere Diameter Ratio D/d.

The tests on flow rate, bed length and D/d ratio provide a direct check of the corresponding data in the literature. Packing method and sphere material were included as both could influence the velocity profile by changing the packing arrangement of the spheres inside the bed. They may have contributed to the discrepancies in the literature (Price 1966).

4.2 Evaluation and Presentation of Velocity Distributions

The method of calculation of the velocity distribution differed slightly for the two types of honeycombs, though for both the mean air velocity v at exit from a honeycomb compartment was first evaluated from the pitot static tube measurements.

For the honeycomb with contracting flow passages the local volume flow was obtained as the product of this mean exit velocity and the measured compartment exit area A_0. The superficial velocity, corresponding to the section of the bed monitored by that particular honeycomb compartment U_L, was evaluated by dividing this local volume flow by the catchment area at inlet to the compartment, A_1. Assuming a constant air density, this relationship is written:

$$U_L = \frac{\text{local volume flow}}{\text{corresponding empty tube area}} = \frac{v A_0}{A_1} .$$

The mean superficial velocity at each radial station U was obtained as the average of the eight values of U_L for any one ring of compartments. Summation of the volume flows over the whole cross section of the bed gave the overall flow rate from which the mean superficial exit velocity $\overline{V}$ (based on the cross sectional area of the empty tube) was determined. The distributions were plotted in normalised form as $U/\bar{V}$ versus the radius in sphere diameters, each bar representing the mean value of $U/\bar{V}$ over the width of that particular ring of the honeycomb.

The values of $\bar{V}$ derived by summation were higher than the corresponding values from the orifice plate. The differences arose from the effects of friction as this reduced the velocities near the walls of each compartment. The amount varied between 9 per cent. and 16 per cent. depending on the flow rate, being smaller for the higher flow rates.

For the honeycomb with flow passages of constant cross section, the mean velocities measured at exit from the honeycomb compartments corresponded directly to the mean velocities on the exit face of the bed after a small correction was made for the blockage area of the honeycomb. The differences between the values of $\bar{V}$ from summation and those from the orifice plate were slightly greater in this case than those given above, owing to increased turbulence intensities and less uniform flow within compartments (Price and Stevens 1966).

4.3 The Effect of Flow Rate

The tests for the effect of flow rate were made on a loosely packed bed of ½ in. diameter bright steel spheres (diametral tolerance ± 0.0005 in.). The 9 in. long test bed was prepared by hand-pouring the spheres from a 600 ml beaker and this gave an average voidage fraction of 0.402. The drop height from the beaker to the surface of the bed was kept small.

The velocity distribution was measured at four different flow rates. The mean superficial velocities ranged from approximately 7 ft. sec^{-1} to 21 ft. sec^{-1}, the corresponding Reynolds numbers (based on sphere diameter) having values from 1470 to 4350. The bed was not repacked between tests.

The results are plotted in Figure 2. They show that the normalised velocity distribution is independent of flow rate over the range tested. This confirms the less conclusive data in the literature, though the distributions differ substantially (Price 1966, see also Figure 9).

4.4 Repeatability with Repacking

To assess the significance of any parameter changes requiring repacking of the bed between tests, it was necessary to determine the variations associated with repeat tests on beds repacked as identically as possible.

Four tests were made using the ½ in. diameter steel spheres in a bed 9 in. long. Between tests the bed was repacked as identically as possible by hand-pouring the spheres from the beaker. The numbers of spheres loaded into the bed in successive tests were 9351, 9397, 9373 and 9379, the first figure corresponding to an average voidage fraction of 0.402.

The results of these tests are shown in Figure 3. The scatter is markedly less than that obtained by Morales et al. (1951) in a similar series of experiments.

The values of $U/\bar{V}$ plotted at each radial station (that is, for each ring of the honeycomb) are the mean circumferential values. Each was obtained by averaging the values of U_L from the eight equally spaced compartments around the circumference with the exception of the value from the single central compartment at the axis of the bed. The standard deviation of the mean circumferential values about the station means obtained from the four repeat tests is shown in Figure 3. The central value may be a poor estimate as it is based on four results only, compared with thirty-two at all other stations.

The local volume flow through any honeycomb compartment is the product of the void area of the corresponding surface of the bed and the fluid velocity in the voids. In the absence of dense or loosely packed clusters of spheres on the surface (and no evidence of these was found) it seems reasonable to assume that the void velocities in one packed bed would be similar to those in an identically packed bed. On the other hand the void areas corresponding to a particular honeycomb compartment could change markedly, depending on the position relative to the honeycomb of the spheres in the last row and this could cause the observed variations. The trend shown in Figure 3, variations increasing with decreasing radius, is consistent with this view.

The mean velocity distribution from the four tests shows that a region of high velocity exists within a half sphere diameter from the wall and that over the rest of the bed there is little variation. This was observed in all subsequent tests and is the major finding of the work.

4.5 The Effect of Bed Length

To test the effect of bed length, two additional tests were made using the ½ in. diameter steel spheres in bed lengths of 4½ in. ($L/d = 9$) and 18 in. ($L/d = 36$). Both beds were prepared by hand-pouring the spheres from a beaker as before, the average voidage fractions being respectively 0.409 and 0.400.

The normalised velocity distributions are shown in Figure 4 and are compared with those from the repeat tests described in Section 4.4 on the 9 in. long bed ($L/d = 18$). The values of $U/\overline{V}$ at each radial station for both the 4½ and 18 in. bed lengths lie within or close to the range of values obtained for the 9 in. bed. The normalised velocity distribution is therefore considered to be independent of the bed length over the range $9 < L/d < 36$.

This result differs from that of Morales et al. (1951) but is in agreement with that of Schwartz and Smith (1953) and Cairns and Prausnitz (1959).

4.6 The Effect of Packing Method

Tests were made to assess the effects of bed packing methods on the velocity distributions by comparing measurements for loosely and tightly packed beds. For these tests ½ in. diameter spheres were employed in a 9 in. long bed. Extreme methods of forming randomly packed beds were chosen. The loose packings were prepared as before by hand-pouring the spheres from a beaker. The tight packings were also loaded by beaker but the bed was vibrated during loading, and then for a further two hours, subsidiary tests having shown that little further consolidation took place after this period. The vibrator produced 50 c/s transverse oscillations of small amplitude and these were perpendicular to the bed axis.

In this investigation glass spheres, as well as the steel spheres used previously were employed. The diametral tolerance of the available glass spheres was approximately ±0.020 inches (compared with ±0.0005 for the steel spheres). The average voidage fraction for the loosely packed bed of steel spheres was 0.402 and for the tightly packed bed 0.382, while for the loosely packed glass spheres it was 0.41 and for the tightly packed 0.373. Each bed was tested at two flow rates, repeatability similar to that shown in Figure 2 being obtained in each case. Measurements were also made on loosely packed and tightly packed beds of wooden spheres (tolerance ±0.010 inches). However, problems of bed movement even at low flows, invalidated the results for the loosely packed bed, and may have slightly influenced those for the tightly packed bed.

The normalised velocity distributions for the steel and glass spheres are shown in Figure 5, and there are only small differences between the compared curves. The significance of the differences between circumferential means at each radial station was assessed, the standard error of the difference being based on the variances calculated from the repacking tests. Differences which appeared to be significant were noted at a number of radial stations, but only those which were consistent in direction and which reached at least the 25 per cent. level for both the steel and glass spheres were selected for further consideration. These were confined to regions near the wall, as shown in column (1) of Table 1 below. Column (2) gives the observed differences between the circumferential means for loosely and tightly packed beds of steel spheres; the significance level of these differences is given in column (3). Columns (4) and (5) give the results for the beds of glass spheres.

The percentage difference between circumferential means is based on $\overline{V}$.

Once significance is established for any areas of the bed, real differences must exist in the flows through the remaining areas to compensate. The net result of the change from loose to tight packing was to decrease slightly the flow near the walls. The compensating increase in flow through the remainder of the bed averages approximately 1 per cent. and this is too small to be detected.

TABLE 1

EFFECT OF PACKING METHOD ON BEDS OF STEEL AND GLASS SPHERES

(1) Region of the bed to which the result applies – Annulus Width	Steel Spheres		Glass Spheres	
	(2) Difference Between Circumferential Means Loose Value – Tight Value	(3) Significance Level	(4) Difference Between Circumferential Means Loose Value – Tight Value	(5) Significance Level
Containing wall to one half sphere from the wall	+ 6.5%	25%	+5.5%	25%
A half sphere to one sphere from the wall	+10.2%	5%	+12%	5%
One sphere to one and a half spheres from the wall	–7%	25%	– 8.5%	25%

The significance levels noted above are not conclusive, except perhaps for the region from a half to one sphere diameter from the wall (where the low significance level indicates strong correlation between the effects), but there is qualitative support for the measured trend. It is known that as loosely packed beds are consolidated by vibration, the spheres immediately adjacent to the wall densify more than those in the central region of the bed (Tingate 1966). The fluid velocities near the wall relative to those in the centre of the bed would therefore be expected to decrease. It is also known that the number of spheres not touching the wall but projecting into this outer layer decreases with consolidation. This could account for the relative increase in velocity for a vibrated bed in the region from one to one and a half sphere diameters from the wall.

For each of the four distributions the departures from uniformity over the region of the bed not affected by packing method can be attributed to the variations arising from random sphere position. Large variations in the central value are to be expected as it is based on measurements in one compartment only.

4.7 The Effect of Sphere Density and Tolerance

The combined effect of sphere density and tolerance, each of which could influence the distribution by causing variation in the packing structure of the bed, was investigated by comparing the velocity results for the steel spheres with those for the glass spheres. Comparison was made for both the loosely packed and the tightly packed beds (Figure 6), the analysis being similar to that used for the packing method. For both the loosely packed and the tightly packed comparisons consistent differences were found at the axis of the bed and in the region extending one sphere diameter from the wall into the bed.

The differences observed at the axis of the bed are ignored for the purpose of this comparison. Subsidiary tests showed that the central readings were markedly influenced by changes in the position of a single sphere at the centre of the bed, confirming that the standard deviation referred to in Section 4.4 was low. Further, an earlier test on a bed of glass spheres did not show a high central value (Price and Stevens 1966).

Significance at the 10 per cent. level was found for the differences within one sphere diameter from the wall. The normalised velocity was about 7 per cent. lower for the glass spheres indicating only a small influence of sphere density/tolerance on the packing arrangements of the spheres. The results for the wooden spheres in a tightly packed bed were similar to those for steel. Although the effects of sphere density and tolerance cannot be separated, the combinations tested indicate that the distribution is relatively insensitive to both.

4.8 The Effect of Sphere Diameter

The effect of sphere diameter, or D/d ratio, was investigated by testing ¼ and 1 in. diameter spheres separately in the 12 in. diameter test bed. The bed length was 9 in. in both cases. The ¼ in. diameter spheres available from stock were steel spheres, precision machined to a tolerance of ± 0.0002 inches, with surfaces which had been coarsened slightly through treatment in an anti-corrosion bath; the 1 in. spheres available were aluminium spheres with a tolerance of ± 0.002 inches. The beds were loaded by hand-pouring from beakers of 250 ml capacity for the ¼ in. and 600 ml capacity for the 1 in. diameter spheres; the average voidage fractions obtained were 0.423 and 0.430 respectively.

The results of these tests are shown in Figure 7. For the 1 in. diameter spheres, where resolution was to a ¼ sphere diameter near the wall, major departures from uniformity were confined to the region extending one half sphere diameter from the wall just as for the ½ in. diameter spheres. There were slightly greater departures from uniformity over the remainder of the bed than there were for the ¼ in. diameter spheres. This would be expected if they arose from random sphere position (in relation to honeycomb compartment) on the exit face of the bed. It was found that a single sphere could virtually block the central compartment and hence the central value was averaged with those from the adjacent ring. For the ¼ in. diameter spheres the high velocity region adjacent to the walls could not be resolved to fractions of a sphere diameter owing to the spacing of the honeycomb.

To provide a direct comparison, the results for the ½ in. and 1 in. diameter spheres were replotted to a base of one sphere diameter and the abcissa changed so that the curves were coincident at the wall (Figure 8; point rather than bar representation is used for clarity). The similarity of the distributions is apparent. They are essentially uniform to within one sphere diameter from the wall, where in all cases the velocity increases but to different extents. The ratio of the mean velocity through the outer annulus, one sphere diameter in width, to the mean velocity over the rest of the bed gives an indication of the effect due to D/d ratio. The velocity ratio was evaluated from the results and is given in Table 2 below.

TABLE 2

MEAN VELOCITIES THROUGH THE CENTRAL CORE AND OUTER ANNULUS

Sphere Diameter (in.)	D/d Ratio	Packing Method	Sphere Material	Mean Velocity through outer annulus, one sphere diameter wide	Mean Velocity through central region (diameter = D − 2d)	Velocity Ratio
1	12	Loose	Aluminium	1.18 $\bar{V}$	0.92 $\bar{V}$	1.28
½	24	Loose	Steel	1.33 $\bar{V}$	0.94 $\bar{V}$	1.42
½	24	Tight	Steel	1.25 $\bar{V}$	0.95 $\bar{V}$	1.31
½	24	Loose	Glass	1.26 $\bar{V}$	0.95 $\bar{V}$	1.32
½	24	Tight	Glass	1.18 $\bar{V}$	0.96 $\bar{V}$	1.22
¼	48	Loose	Steel	1.50 $\bar{V}$	0.96 $\bar{V}$	1.57

If the velocity ratio for the ¼ in. spheres is compared with that for the ½ inch loosely packed steel spheres (similar packing methods, sphere density and tolerance) it is seen to increase slightly with increasing D/d ratio. Comparison of the ratio for the 1 in. diameter aluminium spheres with those for the ½ in. diameter loosely packed steel and glass spheres (similar packing methods, different sphere density and tolerance) confirms the trend though here the increase is smaller.

4.9 Discussion of Velocity Distributions

The measurements presented in the previous sections show that a region of high velocity exists within one half sphere diameter from the container walls and that over the rest of the bed departures from uniformity are small.

These results are supported by theoretical considerations (Price 1966). The region of high velocity is attributed to the local increase in voidage fraction that occurs close to a bounding surface and to the straighter flowpath taken by the fluid. The equation derived in the reference predicts a velocity distribution that is essentially uniform for a central core of constant voidage fraction; only in the immediate vicinity of the boundary adjacent to the high velocity region are small increases in velocity predicted to occur due to momentum transfer into the central core. The assumption of a constant voidage fraction for the bulk of the central core is well founded. However it is a little surprising that the cyclic variations in voidage fraction that are known to exist in the region from one half to two sphere diameters from the wall (Benenati and Brosilow 1962) do not cause greater variations of velocity than those reported here. It is possible that these variations were not detected owing to the spacing of the honeycomb.

The nature of the results greatly simplifies prediction of the flow distribution through right cylindrical beds. For most practical purposes it is adequately defined by specifying the mean flows through two zones; a central zone, and a wall zone which extends one sphere diameter from the wall into the bed. For a given flow rate and bed geometry this requires a knowledge of the flow through the wall region only.

The flows through the wall region for the different beds are tabulated in the previous section. The differences due to packing method are small though the packing methods approach the extremes likely to be found in non-fluidised random packings. The differences noted between steel and glass spheres were also small and measurements on the tightly packed bed of wooden spheres suggest that this result will apply to a range of material and sphere tolerances. The differences due to sphere diameter or D/d ratio, probably arise from changes in the ratio between the mean voidage fraction in the wall region and that in the central core (e_w/e_c). Benenati and Brosilow (1962) showed that small changes in e_w/e_c are to be expected for different D/d ratios but the variations are not as consistent as the changes in velocity ratio suggest.

The normalised velocity distribution was found to be independent of both the flow rate, or Reynolds number ($1470 < N_R < 4350$) and bed length ($9 < L/d < 36$) and this further simplifies prediction of the flow distribution. The literature suggests that the former result may reasonably be applied over the range $100 < N_R < 6000$ and the latter to bed lengths greater than 36.

Velocity distributions from the literature are shown in Figure 9 plotted as $U/\bar{V}$ versus r/R. For comparison the smoothed result for the ½ in. diameter loosely packed steel spheres is shown dotted. Only the limited data of Cairns and Prausnitz (1959) which extend to r/R = 0.75 show agreement with the dotted curve.

Price (1966) concluded that the differences between investigators arose either from the experimental techniques employed or the different packing methods and sphere materials used. The relatively weak influences of packing method and sphere density/tolerance reported here indicate that the differences did arise mainly from the experimental techniques used. Reasons for this are given by Price and Stevens (1966).

5. PRESSURE LOSSES FOR RIGHT CYLINDRICAL BEDS

These measurements were not intended to provide a comprehensive investigation of the generally accepted pressure loss correlation; rather the aim was to appraise it in the light of the measured velocity distributions. For this reason the tests were confined mainly to those beds on which the velocity distributions were measured.

5.1 Evaluation of Friction Factors

Friction factors, defined by the equation:

$$f = \frac{\Delta p g d \bar{\rho}}{L 2 G^2}$$

were calculated from the pressure loss measurements.

The term $\Delta p/L$ was evaluated from the pressure tappings placed immediately upstream from the bed. Their greater reliability was demonstrated by the fact that the pressures at the three individual tapping points in the plane upstream from the bed agreed closely, while those in the planes along the bed length did not, owing to the location of the spheres with respect to the tappings. Also the pressures measured upstream from the bed showed better repeatability for identically packed beds than those along the length of the bed. Some improvement was noted when these latter tappings were arranged in the form of piezometer rings.

5.2 Correlation of Results and Discussion

In Figure 10 the range of friction factors obtained in these tests is superimposed on the survey plot of Denton (1963) which covers a wide range of Reynolds number (note ordinate equals 2f).

In Figure 11a the friction factors are shown in more detail. The substantial variations reflect the changes in voidage fraction caused in turn by changes in packing method, sphere material/tolerance and diameter. Also an increase in friction factor of approximately 5 per cent. was observed as the bed length was halved, though the same packing method was used for both bed lengths. The effect was consistent for all three sphere diameters.

The most widely accepted pressure loss correlation for packed beds (Blake 1922, Leva 1947) may be written:

$$\left[\frac{\Delta p g d \bar{\rho}}{L 2 G^2}\right]\left[\frac{\bar{e}^3}{1-\bar{e}}\right] = f\left[\frac{\bar{e}^3}{1-\bar{e}}\right] = \phi\left[\frac{N_R}{1-\bar{e}}\right].$$

The data are plotted according to this equation in Figure 11b. The spread, though much reduced, suggests that the correlation could be further improved.

In this form the correlation makes no provision for variations of velocity distribution in and between different beds. It is placed on a more rigorous basis by referring to the central core where the velocity is uniform, rather than the whole bed. Replacing $\bar{e}$ by e_c and $\bar{V}$ by $q\bar{V}$ gives:

$$\left[\frac{\Delta p g d \bar{\rho}}{L 2 G^2}\right]\left[\frac{e_c^3}{1-e_c}\right]\left[\frac{1}{q^2}\right] = f\left[\frac{e_c^3}{1-e_c}\right]\left[\frac{1}{q^2}\right] = \phi\left[\frac{N_R q}{1-e_c}\right].$$

The data are plotted according to this equation in Figure 12a. The values of q were taken from the table in Section 4.8. The values of e_c, which depend on the ratios e_w/e_c and D/d, were estimated from the measured average values assuming a constant ratio e_w/e_c equal to 1.10 (Benenati and Brosilow 1962). For the loosely packed beds this assumption slightly exaggerates the correction due to the change from $\bar{e}$ to e_c. However, this correction does not materially influence data correlation, as may be seen from Figure 12b in which correlation is based on $\bar{e}$ but includes the q terms. Allowance could have been made for a reduction in the value of e_w/e_c for the tightly packed beds but supporting tests in 5 and 12 in. diameter glass vessels (Tingate 1966) showed that this reduction was less than 4 per cent. and hence produced a negligible change on the correlation of the data.

Comparison of Figures 11b and 12a shows that the two correlations proposed above give a similar spread of results. The relative positions of the points change slightly. The data tend to form three distinct levels corresponding to the three different L/d ratios.

The results for the beds with an L/d ratio of 18 correlate well and demonstrate the adequacy of both forms of the correlation to cater for different packing methods and sphere diameter. The results for the two beds with an L/d ratio of 9 agree well with each other but lie above the general level of the beds with an L/d ratio of 18; those for the bed with an L/d ratio of 36 lie below.

The difference in level arises partly from the change in friction factor with bed length and partly from the voidage correction term ($e^3/1-e$). The average voidage fraction was found to decrease as the bed length increased for all three sphere diameters as shown in the table below; the decrease is attributed to the diminishing effect on voidage fraction of the missing part spheres at the end of the bed as the bed length was increased (Section 3.3).

TABLE 3

VARIATION OF VOIDAGE FRACTION WITH BED LENGTH

Sphere Diameter	Average Voidage Fraction $\bar{e}$		
	L/d = 9	L/d = 18	L/d = 36
¼ inch		0.428	0.423
½ inch	0.409	0.402	
1 inch	0.43	0.423	

The increase in friction factor with decreasing bed length can be explained qualitatively by postulating entry and exit losses which outweigh the reduction caused by the absence of part spheres in the top layer of the bed. Quantitatively the results do not quite conform as the difference associated with the change from an L/d ratio of 18 to one of 9 should be greater than that from 36 to 18.

6. PEAKED BEDS

In application many packed beds retain the natural peak, or 'fill' cone, which is formed on the top surface of the bed during the loading process. In such cases the superficial velocity distribution and the pressure loss across the bed will be affected by the peak. The tests made to establish the magnitude of these effects are reported below.

6.1 Velocity Distribution for a Peaked Bed

Measurements of velocity distribution were taken on a single 12 in. diameter bed prepared by hand-pouring ½ in. diameter steel balls from a beaker as before. The cylindrical portion of the bed was 9 in. long and the angle of repose approximately 22°.

The honeycomb with passages of constant cross section was used for these tests as the velocity distribution was expected to vary across the bed. To accommodate the conical surface of the peaked bed a transition section was fitted between the bed exit face and the honeycomb, as shown inset in Figure 13. This was, in effect, a replica of the honeycomb with which it was used except that its bottom surface was contoured to suit the repose angle of the bed. Measurements of the velocity distribution were made both with and without the peak.

The results are shown in Figure 13. There is a marked change in the velocity distribution due to the peak. It is no longer uniform over the central core but varies continuously from a value of approximately 0.5 $\bar{V}$ on the central axis to 2.1 $\bar{V}$ near the wall. This is caused by distribution of flow within the bed which has increased the velocities near the wall and decreased those towards the centre of the bed.

6.2 Pressure Loss for Peaked Beds

Whereas only one bed was tested in the peaked bed velocity distribution measurements, a number of beds were tested to assess the effect of the peak on pressure loss. All were prepared by hand-pouring the spheres from a beaker and their dimensions, including approximate measurements of the angle of repose, are given in Table 4.

Pressure loss measurements were first made on the bed complete with its conical peak, over a range of flow rates, using the piezometer ring upstream of the bed. The peak was then removed, so that the remaining cylindrical portion conformed to the definition of bed length given in Section 3.3, and the measurements were repeated over the same range of flow rates. The maximum flow was limited by the tendency of the peaks to flatten; this was caused by loosening of the packing by aerodynamic forces, especially noticeable in the tests on the ¼ in. diameter spheres.

The effect of the peak was expressed as an equivalent length of cylindrical bed: the tests on the cylindrical portion were used to evaluate a friction factor which was, in turn, used to evaluate the equivalent cylindrical length of the whole peaked bed. The difference between the latter and the actual length of the cylindrical section (9 in. or 4½ in.) is quoted as the equivalent length of the peak. This was found to be independent of the flow rate over the range tested.

TABLE 4

DIMENSIONS OF BEDS TESTED AND EQUIVALENT LENGTH OF PEAK

Length of Cylindrical Portion of Bed (in.)	Sphere Diameter (in.)	L/d Ratio of Cylindrical Portion	D/d Ratio	Approx. Angle of Repose ± 2°	Equivalent Length of Peak (in.)	Reynolds Number Range of Tests
4½	½	9	24	22°	.45	1000–2500
9	½	18	24	22°	.41	1000–2500
4½	¼	18	48	24°	.46	500–750
9	1	9	12	24°	.40	2000–4500

The equivalent lengths tabulated above show reasonable agreement. The nature of the tests was such that the differences could be ascribed to experimental error and a mean value of 0.43 inches is recommended. This indicates that, to a first approximation, the equivalent length is independent of bed length ($L/d > 9$) and sphere diameter.

The vessel diameter was constant throughout these tests and the peak height approximately so. The results may also be applicable to geometrically similar beds of different vessel diameter. For this purpose the equivalent length is expressed as 0.036D.

7. CONCLUSIONS

The velocities measured at exit from right cylindrical randomly packed beds of spheres show little variation over the bed to within a distance of one half sphere diameter from the vessel walls. In this region the velocity increases due to the local increase in voidage fraction that occurs near a bounding surface. These measurements are in accord with predictions but differ markedly from those of previous investigators who measured at exit from the bed.

The distribution is found to be independent of Reynolds number ($1470 < N_R < 4350$) and bed length ($9 < L/d < 36$). It is only slightly influenced by the method of loading the bed and by the use of spheres of different density, tolerance, and diameter ($12 < D/d < 48$). This indicates that the large discrepancies that exist in the literature for similar beds are probably due to the shortcomings in the various experimental techniques used.

The most generally accepted correlation for friction factors of right cylindrical randomly packed beds, based upon average conditions through the bed, adequately correlates for packing method and sphere diameter. Correlation is not significantly improved by the use of velocities and voidage fractions applicable only to the central core of the bed. Neither method of correlation appears to cater adequately for different bed lengths, owing to the difficulty of allowing for end effects on beds of finite length.

The velocity profile is markedly affected by the presence of a peak at exit from the bed; fluid is diverted away from the centre of the bed towards the walls. To a reasonable approximation the additional pressure loss due to the peak is independent of bed length (L/d > 9) and sphere diameter, for a given vessel diameter.

8. ACKNOWLEDGEMENTS

The author specially acknowledges the assistance of Mr. J. R. Stevens who assisted throughout this project, participating in the design and manufacture of equipment, the carrying out of experiments and initial data analysis.

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APPENDIX 1

NOTATION

A_1 catchment area of bed monitored by each honeycomb compartment

A_O exit area of honeycomb compartment

d sphere diameter

D vessel diameter

e voidage fraction

$\bar{e}$ mean voidage fraction of whole bed

e_c mean voidage fraction of central core, diameter = D−2d

e_w mean voidage fraction of annulus adjacent to the wall, width d.

f friction factor defined in text

G mass velocity through the bed, based on empty tube area = $\frac{W}{\frac{\pi}{4} D^2}$

L bed length

N_R Reynolds number = $\frac{\rho \bar{V} d}{\mu}$

q ratio of the mean superficial velocity through the central core, (diameter = D−2d), to $\bar{V}$

R radius of containing vessel

U_L superficial velocity for each honeycomb compartment, based on A_1

U superficial velocity at radius, r, average for each honeycomb ring

$\bar{V}$ mean superficial velocity at exit from the bed, based upon empty tube area

v mean velocity at exit from each honeycomb compartment

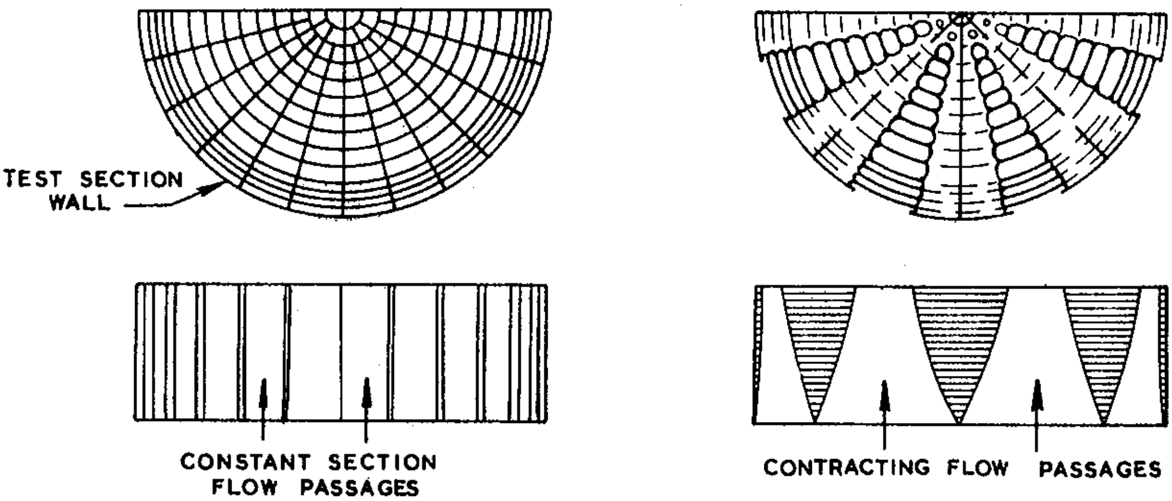
W mass flow through the bed

Δp pressure loss across the packed bed

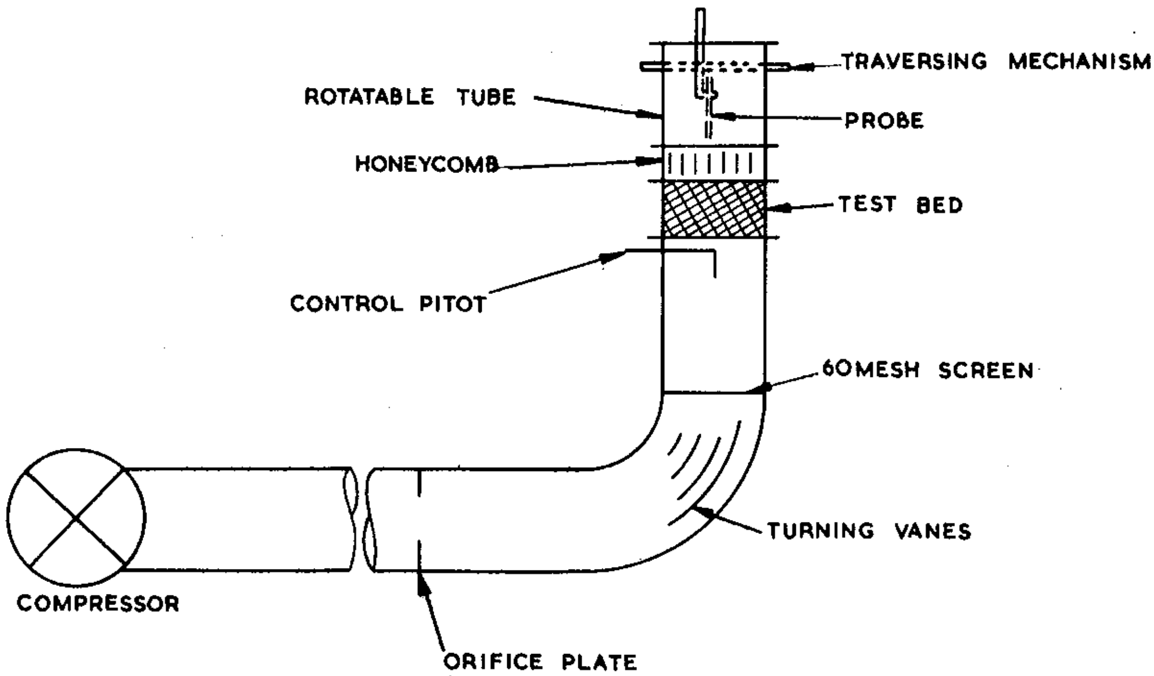
ρ fluid density

$\bar{\rho}$ mean fluid density through the bed

μ fluid viscosity



HONEYCOMB DESIGNS



EXPERIMENTAL ARRANGEMENT

FIGURE 1. HONEYCOMB DESIGNS AND EXPERIMENTAL ARRANGEMENT FOR TESTS ON RANDOMLY PACKED BEDS

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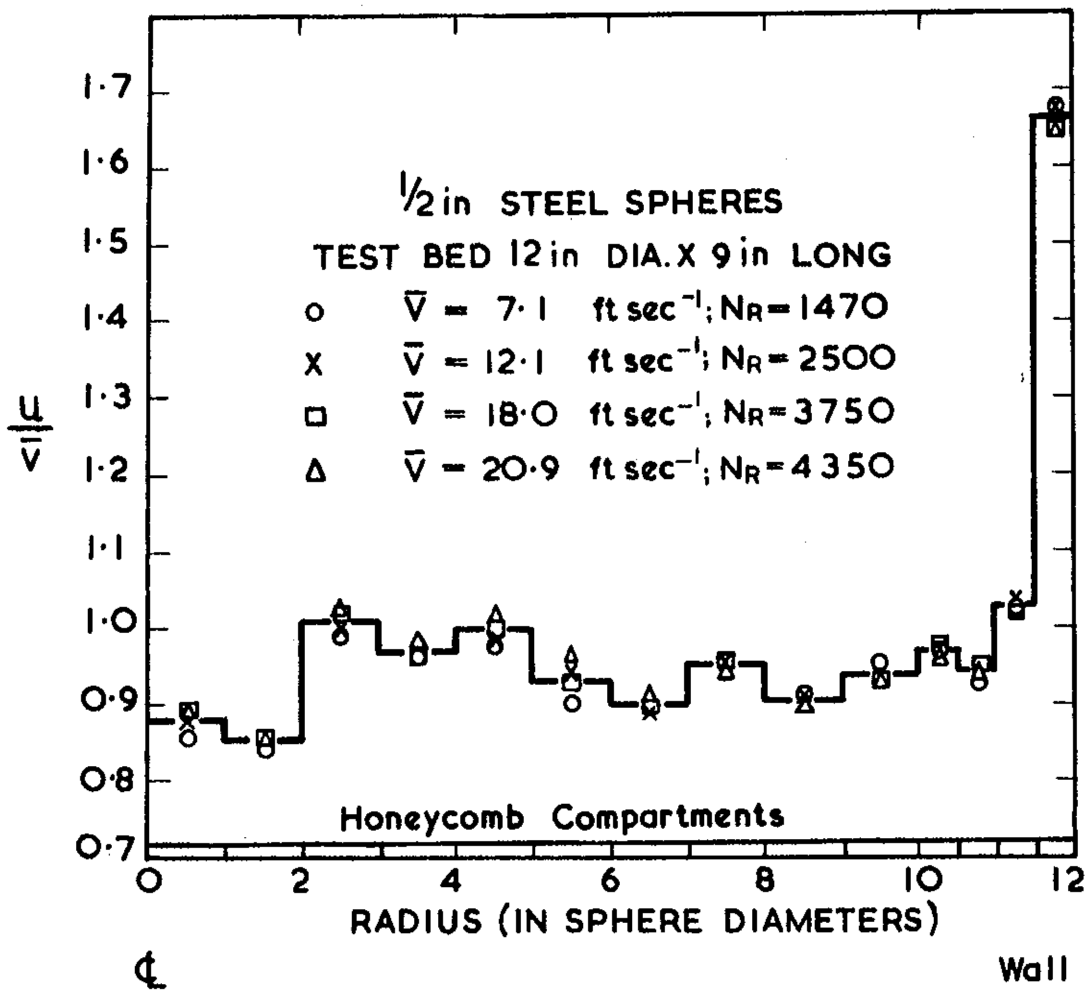

FIGURE 2. THE EFFECT OF FLOW RATE ON THE VELOCITY DISTRIBUTION OF A RIGHT CYLINDRICAL RANDOMLY PACKED BED

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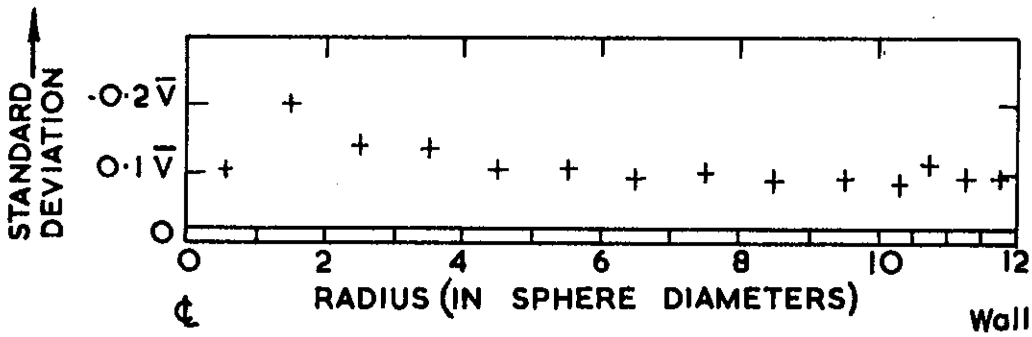
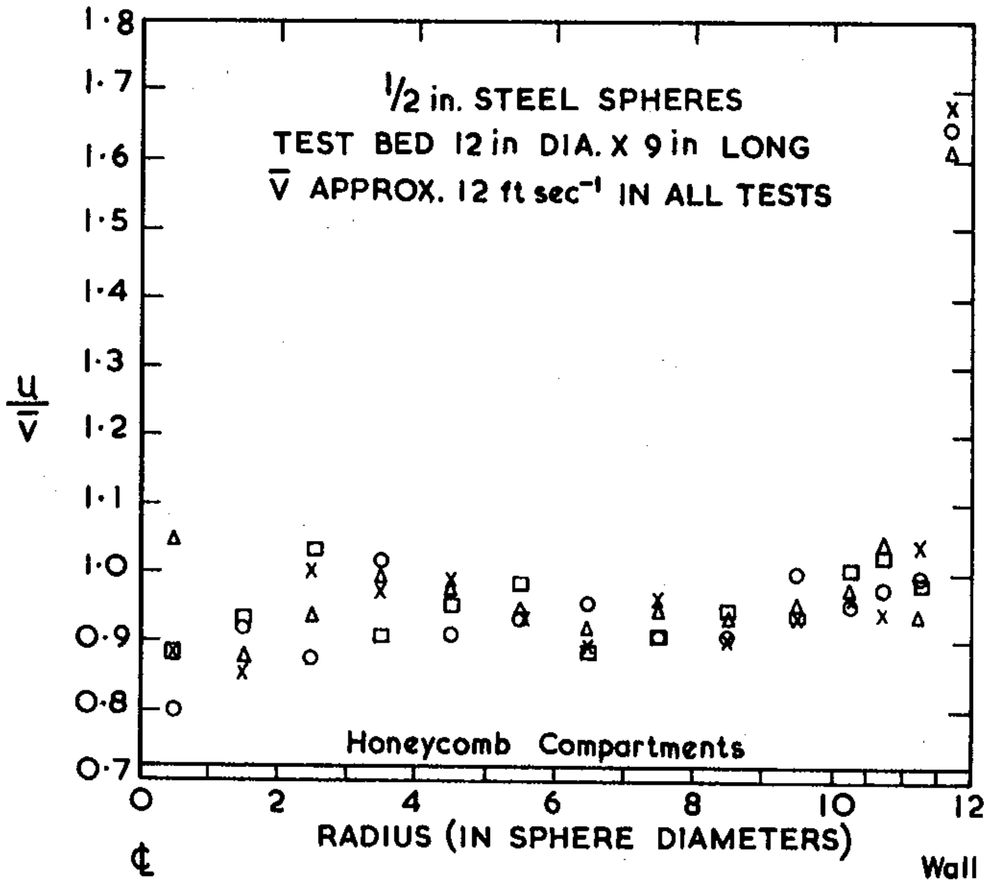


STANDARD DEVIATIONS OF THE LOCAL VALUES OF $\frac{u}{\bar{V}}$ ABOUT THE GRAND STATION MEANS FROM THE REPEAT TESTS



MEAN VELOCITY DISTRIBUTIONS FOR THE FOUR REPEAT TESTS

FIGURE 3. THE VARIATION OF THE VELOCITY DISTRIBUTION OF A RIGHT CYLINDRICAL, RANDOMLY PACKED BED WITH REPEAT TESTING

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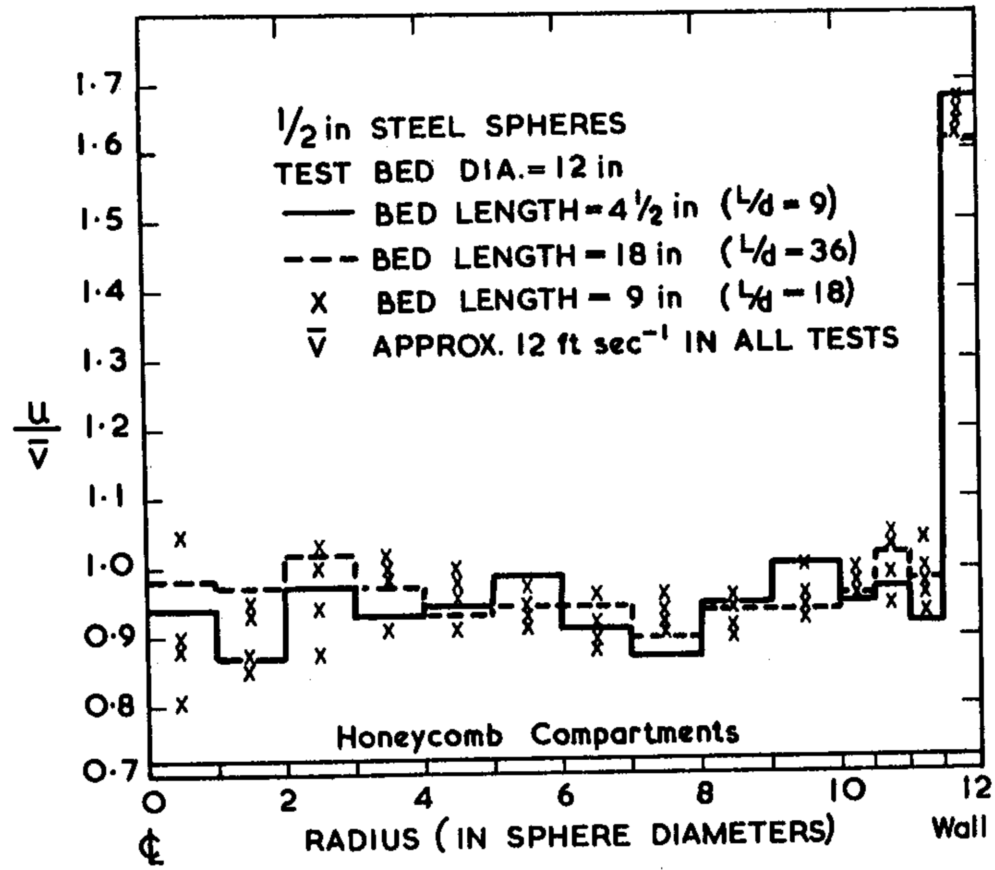

FIGURE 4. THE EFFECT OF BED LENGTH ON THE VELOCITY DISTRIBUTION OF A RIGHT CYLINDRICAL, RANDOMLY PACKED BED

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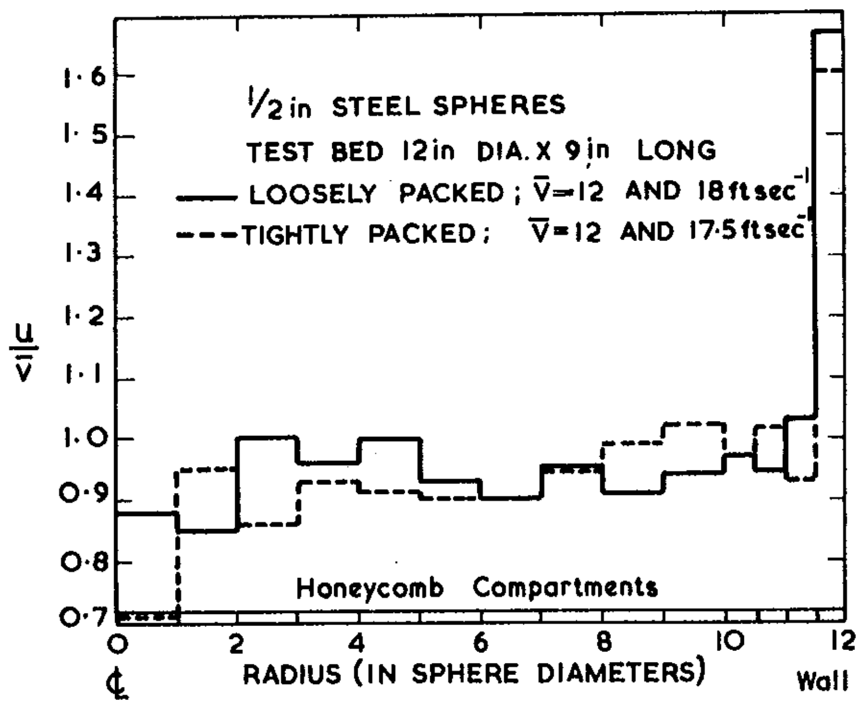
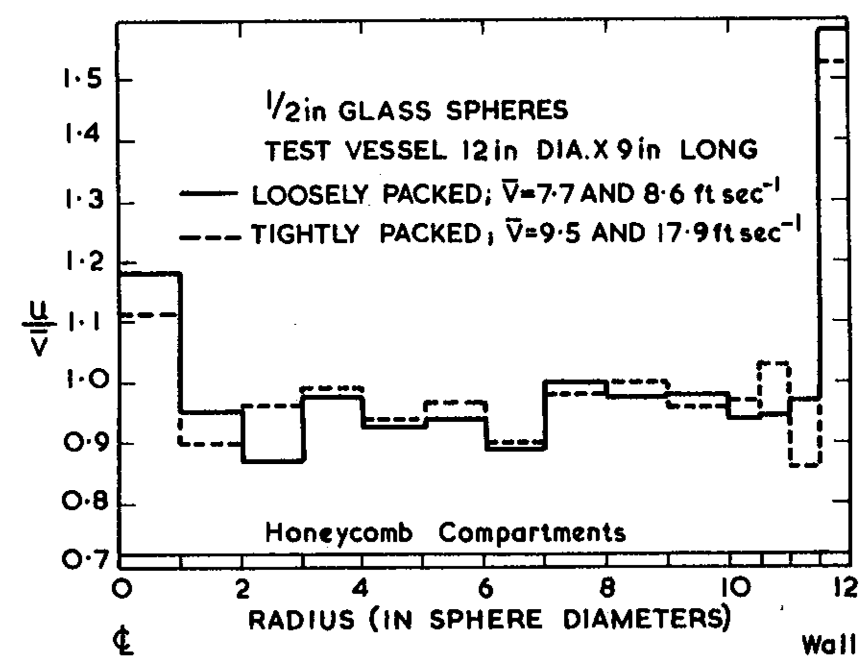


RESULTS FOR LOOSE AND TIGHT STEEL PACKINGS



RESULTS FOR LOOSE AND TIGHT GLASS PACKINGS

FIGURE 5. THE EFFECT OF PACKING METHOD ON THE VELOCITY DISTRIBUTION OF RIGHT CYLINDRICAL BEDS OF STEEL AND GLASS SPHERES

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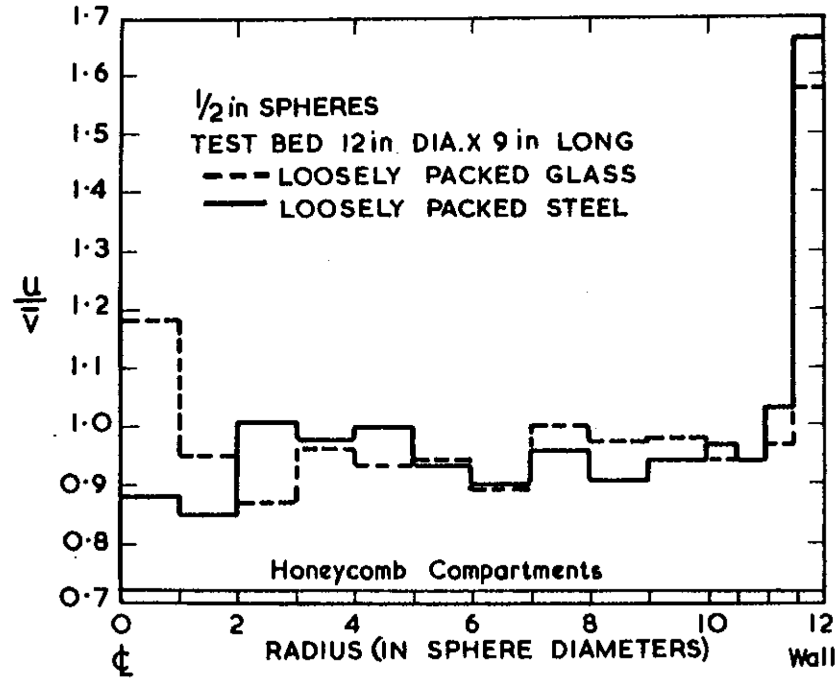
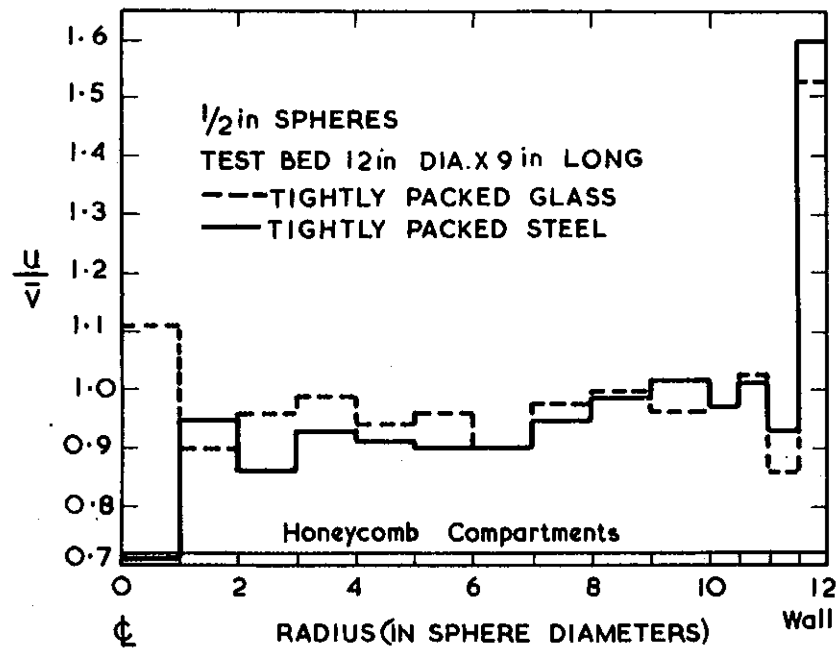


RESULTS FOR LOOSELY PACKED GLASS AND STEEL BEDS



RESULTS FOR TIGHTLY PACKED GLASS AND STEEL BEDS

FIGURE 6. THE EFFECT OF SPHERE DENSITY/TOLERANCE ON THE VELOCITY DISTRIBUTION OF LOOSELY PACKED AND TIGHTLY PACKED, RIGHT CYLINDRICAL BEDS OF SPHERES

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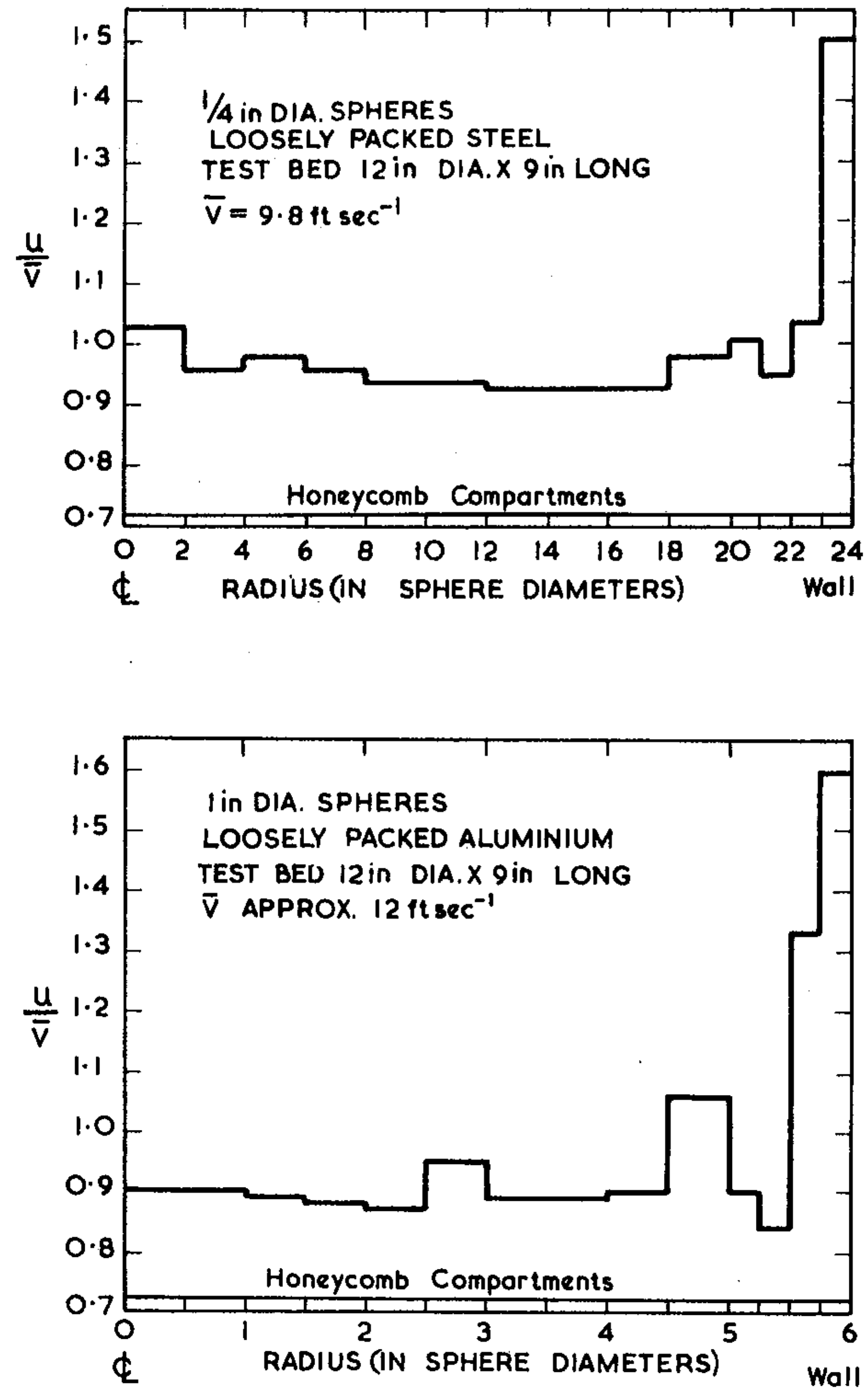

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FIGURE 7. THE VELOCITY DISTRIBUTION OF RIGHT CYLINDRICAL, RANDOMLY PACKED BEDS OF 1/4 in. AND 1 in. DIAMETER SPHERES

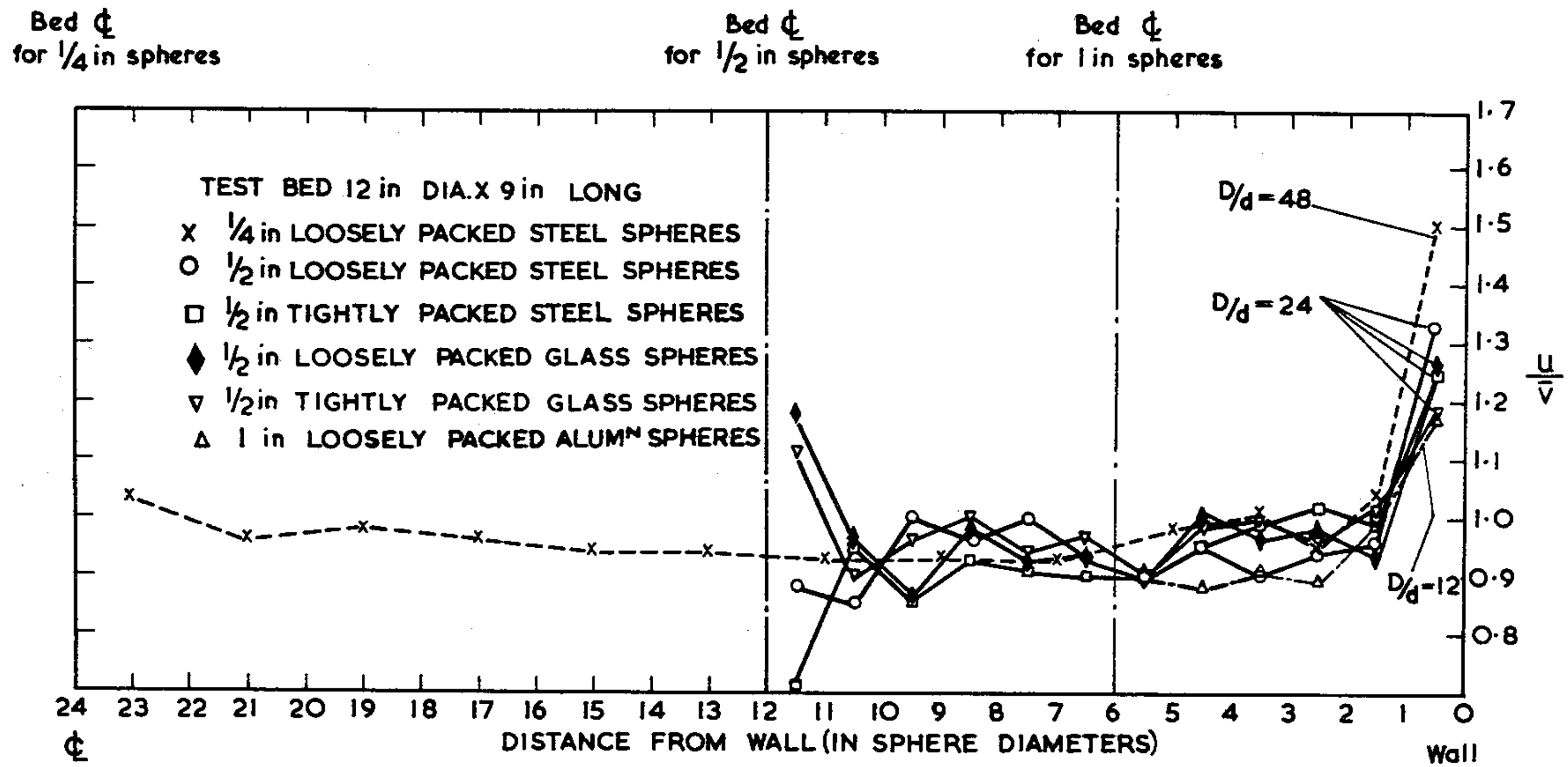

FIGURE 8. COMBINED PLOT OF VELOCITY DISTRIBUTIONS FOR ALL THE BEDS TESTED

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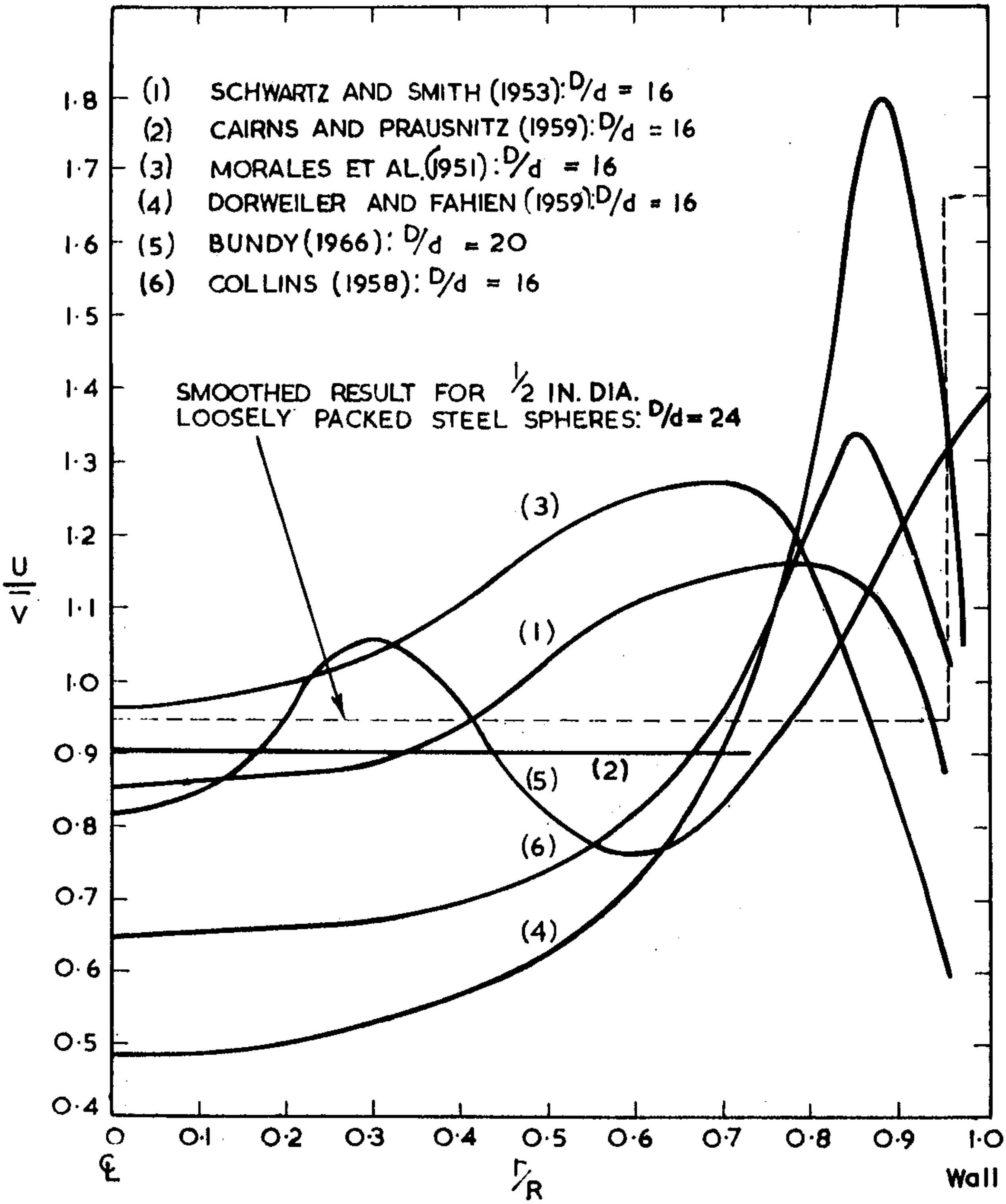

FIGURE 9. COMPARISON OF THE VELOCITY DISTRIBUTIONS FOR RANDOMLY PACKED BEDS

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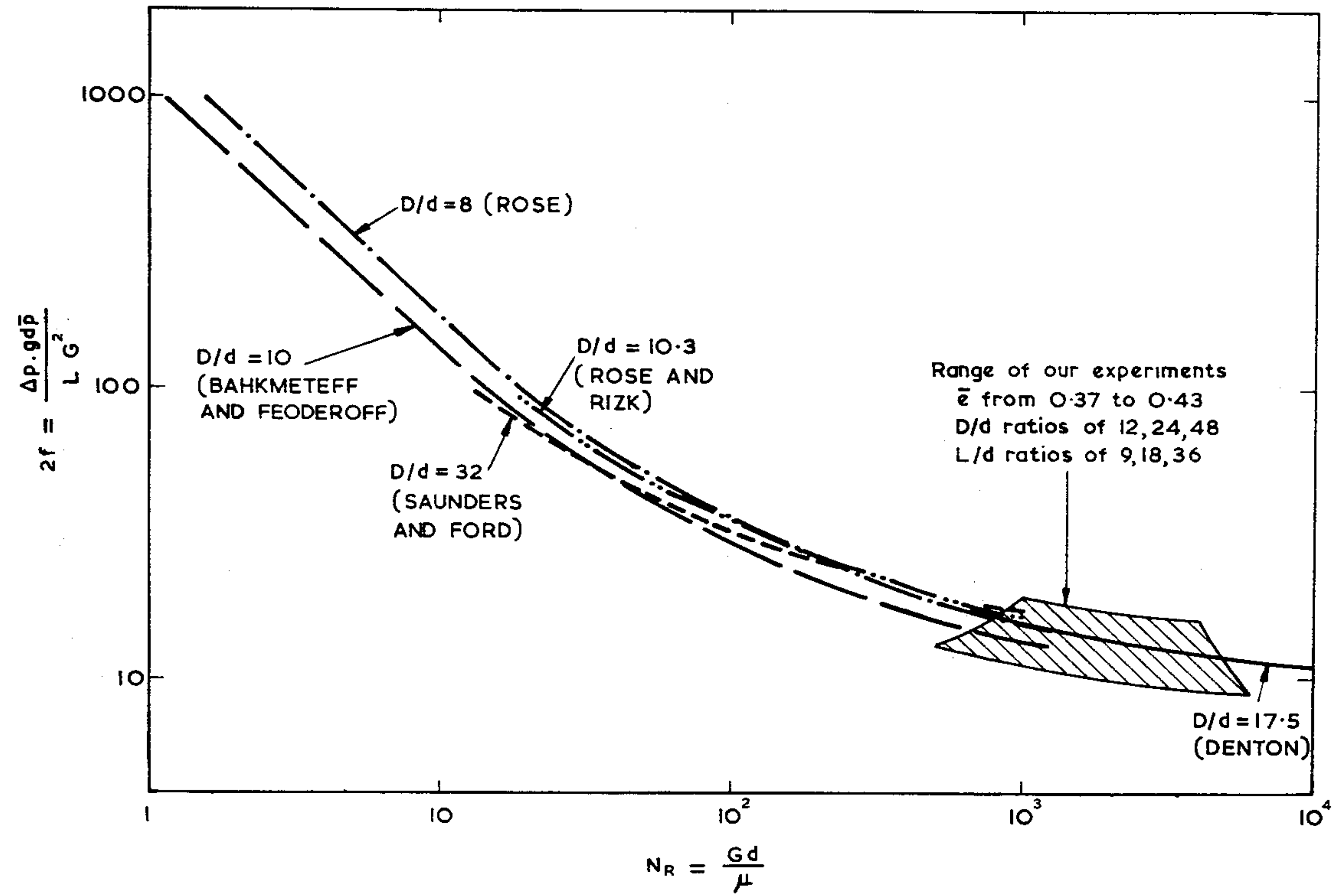

FIGURE 10. COMPARISON OF THE FRICTION FACTORS FROM OUR EXPERIMENTS WITH THOSE FROM THE LITERATURE (TAKEN FROM DENTON 1963)

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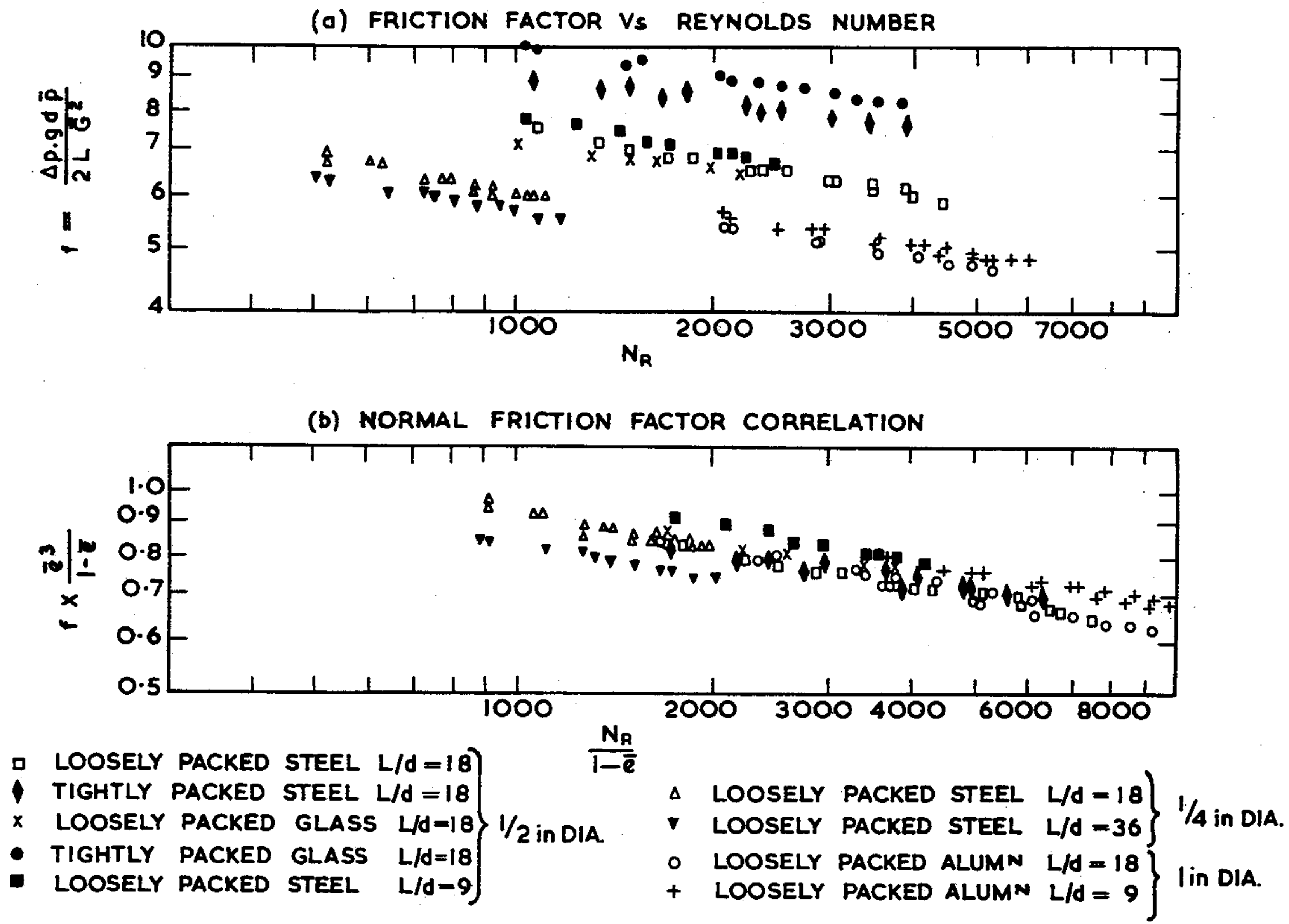

FIGURE 11. FRICTION FACTORS AND NORMAL CORRELATION

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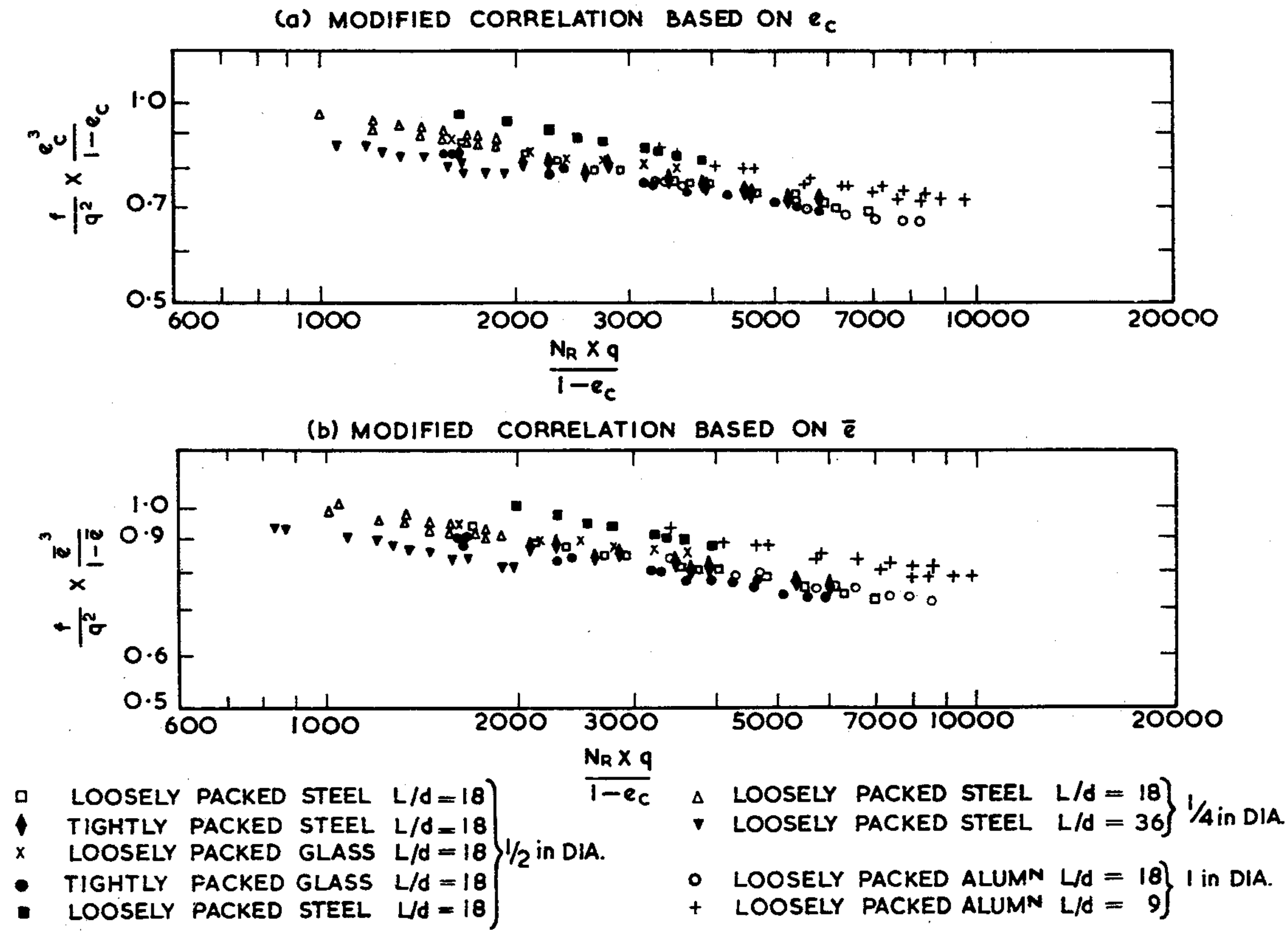

FIGURE 12. MODIFIED CORRELATION OF FRICTION FACTORS BASED ON CONDITIONS IN THE CENTRAL CORE

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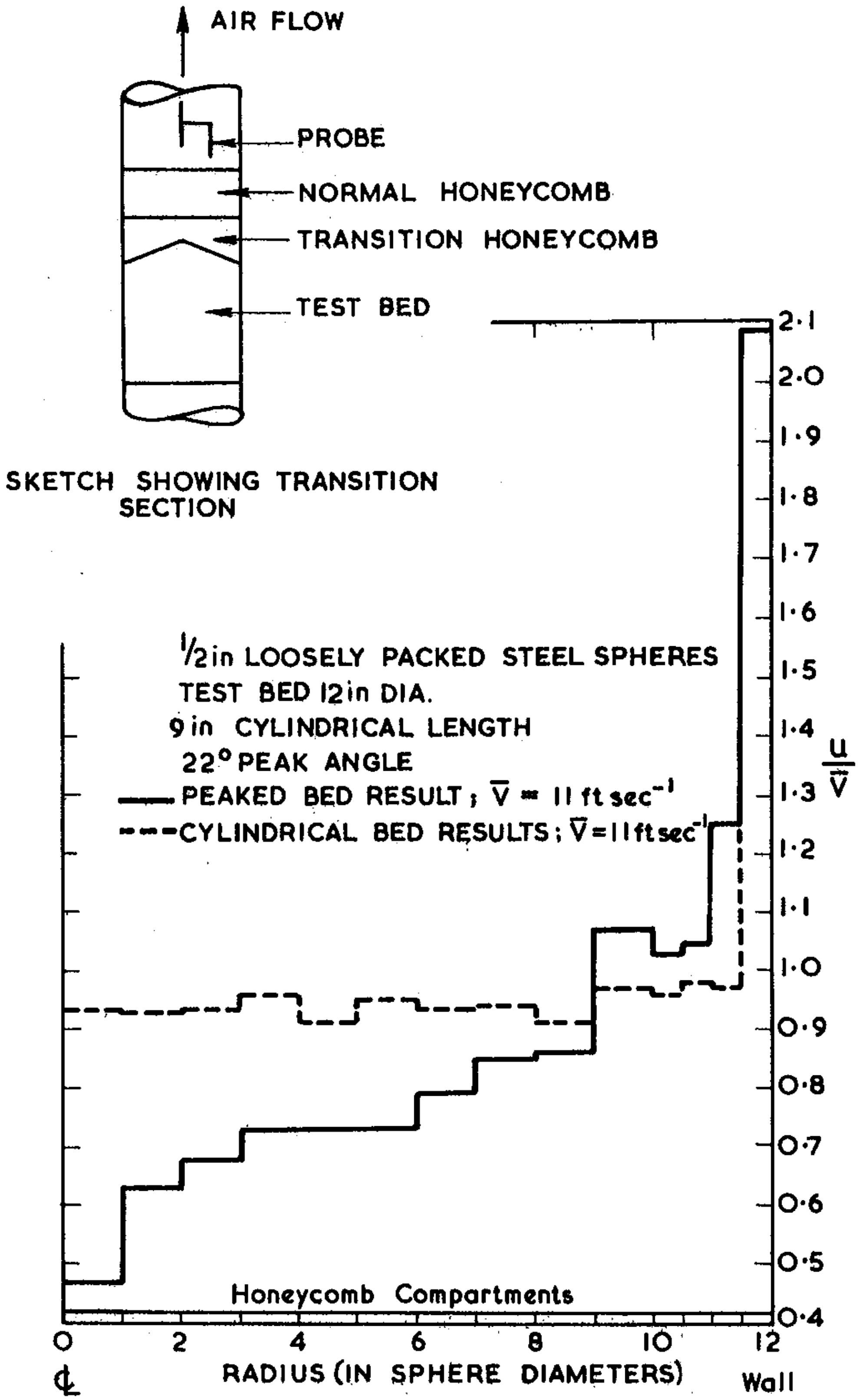

FIGURE 13. THE EFFECT OF A PEAK ON THE VELOCITY DISTRIBUTION OF A RANDOMLY PACKED BED

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