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A TWO-GROUP ANALYSIS OF A FINITE FULLY REFLECTED CYLINDRICAL REACTOR

by

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A TWO-GROUP ANALYSIS OF A FINITE FULLY REFLECTED CYLINDRICAL REACTOR

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Abstract

A method is developed for the calculation of the critical size or effective multiplication constant of a fully reflected cylindrical reactor with a uniform core and uniform reflectors, using two-group theory. Solutions of the basic differential equations are superimposed, and boundary conditions satisfied by the use of orthogonal functions. The method appears suitable for a small digital computer.

CONTENTS

1. INTRODUCTION

Fully reflected cylinders are of particular interest as being reasonable models of practical reactor configurations. The solution of the diffusion equations for such systems usually involves finite difference approximation and the application of the source iteration technique using digital computers. Machines with large storage capacity are required to store the constants for each group relating to each mesh point. Non–uniform cores represent no significant increase in time, as non–uniform mesh lengths are universally employed.

Without ready access to the large machines required, there is some incentive to develop a method which would necessarily be restricted to a few uniform regions and a few groups, but which would enable these restricted solutions to be obtained on a much smaller capacity computer. Such a method should be capable of yielding a good estimate of the flux distributions as well as the eigenvalue.

There are two approaches that might be adopted. The first is to take a series of functions that satisfy the differential equations but involve a number of arbitrary constants which must be adjusted to satisfy the boundary conditions to the required degree of accuracy. The other is to choose functions that satisfy the boundary conditions and force them to approximate to the solution of the differential equations. In this paper the first method was chosen. The theory is developed in matrix form for a two–group analysis. To avoid complications, both fluxes are made to vanish at the outer reflector boundary.

2. THEORY

A cylindrical core of radius R and height 2H is surrounded by a reflector of thickness T in the radial direction and Z in the axial direction. The system is symmetrical about the r axis through the centre of the cylinder. The overall height is $2L = 2(H + Z)$. The core, region C, occupies the space $0 \leq r < R$, $-H < z < H$, the top reflector, region A, occupies the space $0 \leq r < R$, $H < z < L$ and the side reflector region B, occupies the space $R < r < R+T$, $-L < z < L$. Regions A, B and C are of uniform composition and nuclear properties but A can be different from B. In the following development it will be assumed that the diffusion coefficients in each region are the same in both radial and axial directions, but the extension to include anisotropy is obvious.

The reflector B, and the end reflected core A + C will be considered separately. The final condition for criticality will be obtained from the conditions of continuity of neutron fluxes and neutron currents over the surface $r = R$. A set of orthogonal functions in the range $-L < z < L$ which will be used is

$$\cos\left(\frac{n\pi z}{L}\right), \quad 2n = 1, 3, 5 \ldots\ldots$$

$$\text{i.e.} \quad 2\int_0^1 \cos\left(\frac{n\pi z}{L}\right) \cos\left(\frac{m\pi z}{L}\right) d\left(\frac{z}{L}\right) = \delta_{mn}$$

An affix n, subscript or superscript, will be used to indicate quantities which are functions of n.

In region B, the diffusion equations are

$$\nabla^2 \phi_{1B} - k_{1B}^2 \phi_{1B} = 0 \qquad (1a)$$

$$\nabla^2 \phi_{2B} - k_{2B}^2 \phi_{2B} + \left(\frac{\Sigma_s}{D_2}\right)_B \phi_{1B} = 0 \qquad (1b)$$

where Σ_s is the cross–section for transfer from group 1 to 2.

A solution satisfying the boundary conditions of vanishing flux at $r = R + T$ and $z = \pm L$ is therefore

$$\phi_B^n(r, z) = \cos\left(\frac{n\pi z}{L}\right) S_B \Phi_{Bn}(r) A_B^n$$

i.e.
$$\begin{Bmatrix} \phi_{B1} \\ \phi_{B2} \end{Bmatrix}^n = \cos\left(\frac{n\pi z}{L}\right) \begin{bmatrix} 1 & 0 \\ S(B) & 1 \end{bmatrix} \begin{bmatrix} (\Phi_{Bn}(r))_1 & 0 \\ 0 & (\Phi_{Bn}(r))_2 \end{bmatrix} \begin{Bmatrix} A_{B1} \\ A_{B2} \end{Bmatrix}^n$$

where $S(B) = (\Sigma_s/D_2)_B \; / \; (k_{2B}^2 - k_{1B}^2)$

$$(\Phi_{Bn}(r))_1 = I_o(\beta_{1n} r) - \kappa_o(\beta_{1n} r) \left\{ \frac{I_o(\beta_{1n}(R+T))}{\kappa_o(\beta_{1n}(R+T))} \right\}$$

with a similar expression for $(\Phi_{Bn}(r))_2$ with β_{1n} replaced by β_{2n} and

$$\beta_{1n}^2 = k_{1B}^2 + \left(\frac{n\pi}{L}\right)^2$$

$$\beta_{2n}^2 = k_{2B}^2 + \left(\frac{n\pi}{L}\right)^2$$

The neutron current vector in the +r direction, directed into region B at r = R is therefore

$$J_B^n(R) = -\cos\left(\frac{n\pi z}{L}\right) D_B S_B \Phi_{Bn}^1(R) A_B^n$$

where D_B is the diagonal matrix of diffusion coefficients and dashes will always denote differentiation (not transposition).

A flux matrix $\overline{\lambda}^n$ can now be defined as

$$\overline{\lambda}^n = -D_B S_B \Phi_{Bn}^1(R) \Phi_{Bn}^{-1}(R) S_B^{-1}$$

This matrix relates current and flux at r = R. Thus, if in the general case of a fully reflected reactor, the flux distribution along z at r = R can be expressed as the Fourier series

$$\phi(R) = \sum_n \cos\left(\frac{n\pi z}{L}\right) G_n$$

then the neutron current distribution will be

$$J(R) = \sum_n \cos\left(\frac{n\pi z}{L}\right) \overline{\lambda}^n G_n$$

The matrix $\overline{\lambda}^n$ when evaluated is quite simple. It is

$$\overline{\lambda}^n = \begin{bmatrix} D_{1B} & 0 \\ S(B)D_{2B} & D_{2B} \end{bmatrix} \begin{bmatrix} X_{1n} & 0 \\ -S(B)X_{2n} & X_{2n} \end{bmatrix}$$

where

$$X_{1n} = \beta_{1n} \left\{ \frac{\kappa_1(\beta_{1n}R)\, I_o(\beta_{1n}(R+T)) + I_1(\beta_{1n}R)\, \kappa_o(\beta_{1n}(R+T))}{\kappa_o(\beta_{1n}R)\, I_o(\beta_{1n}(R+T)) - I_o(\beta_{1n}R)\, \kappa_o(\beta_{1n}(R+T))} \right\} \qquad \text{etc.}$$

It should be noted that as n and hence β_{1n} increases

$$X_{1n} \rightarrow \beta_{1n}\kappa_1(\beta_{1n}R) / \kappa_o(\beta_{1n}R) \qquad \text{etc.}$$

In region A, the diffusion equations are obtained from 1a and 1b by replacing the suffix B by A.

A solution can be obtained as

$$\phi_{An}(r,z) = \cos\left(\frac{n\pi z}{L}\right) S_A\, \Phi_{An}(r)\, A_A^n$$

where

$$\Phi_{An} = \begin{bmatrix} I_o(\alpha_{1n}r) & 0 \\ 0 & I_o(\alpha_{2n}r) \end{bmatrix}$$

$$\alpha_{1n}^2 = k_{1A}^2 + \left(\frac{n\pi}{L}\right)^2 \qquad \text{etc.}$$

In the core, region C, the two-group equations reduce to

$$(\nabla^2 + \kappa^2)(\nabla^2 - \mu^2)\,\phi = 0$$

The relevant solutions involving $\cos\left(\frac{n\pi z}{L}\right)$ are

$$\phi_{Cn}(r,z) = \cos\left(\frac{n\pi z}{L}\right) \begin{bmatrix} S_\kappa & -S_\mu \\ 1 & 1 \end{bmatrix} \begin{bmatrix} J_o(\kappa_n r) & 0 \\ 0 & I_o(\mu_n r) \end{bmatrix} \begin{Bmatrix} A_{C1} \\ A_{C2} \end{Bmatrix}^n$$

i.e.

$$\phi_{Cn}(r,z) = \cos\left(\frac{n\pi z}{L}\right) S_C\, \Phi_{Cn}(r)\, A_C^n$$

where

$$\kappa_n^2 = \kappa^2 - \frac{n^2\pi^2}{L^2}$$

$$\mu_n^2 = \mu^2 + \frac{n^2\pi^2}{L^2}$$

If $\frac{n^2\pi^2}{L^2} > \kappa^2$ the κ_n function is $I_o\left[\left(\frac{n^2\pi^2}{L^2} - \kappa^2\right)^{\frac{1}{2}} r\right]$

To obtain a continuous cosine flux distribution at r = R

$$S_C\ \Phi_{Cn}(R)\ A_C^n = S_A\ \Phi_{An}(R)\ A_A^n$$

$$\therefore\ A_A^n = \Phi_{An}^{-1}(R)\ S_A^{-1}\ S_C\ \Phi_{Cn}(R)\ A_C^n = P_n A_C^n$$

where $P_n = \begin{bmatrix} \frac{S_\kappa\ J_o(\kappa_n R)}{I_o(\alpha_{1n} R)} & \frac{-S_\mu I_o(\mu_n R)}{I_o(\alpha_{1n} R)} \\ \frac{(1 - S_\kappa S(A))\ J_o(\kappa_n R)}{I_o(\alpha_{2n} R)} & \frac{(1 + S_\mu S(A))\ I_o(\mu_n R)}{I_o(\alpha_{2n} R)} \end{bmatrix}$

Now the solutions ϕ_{An} and ϕ_{Cn} will not give continuity of flux (except at r = R) or current over the interface between A and C. Additional solutions of the relevant diffusion equations must therefore be added to satisfy these conditions. The solutions can be chosen to involve radial functions orthogonal over the range $0 \leqslant r < R$. Suitable functions are

$$J_o(\omega_m r/R) \quad m = 1, 2, 3, \ldots$$

where $\quad J_o(\omega_m) = 0$

The values of ω_m are 2.405, 5.520, 8.654, 11.792, 14.931, etc. The orthogonal relations that will be required are

$$\int_o^1 \frac{r}{R} J_o\left(\omega_m \frac{r}{R}\right) J_o\left(\omega_n \frac{r}{R}\right) d\left(\frac{r}{R}\right) = \delta_{mn}\ J_1^2(\omega_m)/2$$

In region A, the solution in terms of these functions is

$$\phi^*_{An}(r, z) = \sum_m J_o\left(\frac{\omega_m r}{R}\right) S_A\ \psi_{Am}(z)\ A_A^{mn}$$

$$\psi_{Am}(z) = \begin{bmatrix} \sinh(\sigma_{1m}(L - z)) & 0 \\ 0 & \sinh(\sigma_{2m}(L - z)) \end{bmatrix}$$

$$\sigma_{1m}^2 = \left(\frac{\omega_m}{R}\right)^2 + k_{1A}^2 \qquad \text{etc.}$$

In region C, the solution is

$$\phi^*_{Cn}(r, z) = \sum_m J_o\left(\frac{\omega_m r}{R}\right) S_C\ \psi_{Cm}(z)\ A_C^{mn}$$

$$\psi_{Cm}(z) = \begin{bmatrix} \cosh(\kappa_m z) & 0 \\ 0 & \cosh(\mu_m z) \end{bmatrix}$$

$$\kappa_m^2 = (\frac{\omega_m}{R})^2 - \kappa^2$$

$$\mu_m^2 = (\frac{\omega_m}{R})^2 + \mu^2$$

If $\frac{\omega_m}{R} < \kappa$ then the κ_m function becomes $\cos\left((\kappa^2-(\frac{\omega_m}{R})^2)^{1/2} z\right)$ but the modifications to the following analysis are obvious.

At this stage, the flux distributions in A and C are as follows,

$$\phi_A^n = \phi_{An} + \phi_{An}^*$$

$$= \cos(\frac{n\pi z}{L})\, S_A \Phi_{An}(r)\, P_n\, A_C^n + \sum_m J_o(\frac{\omega_m r}{R})\, S_A\, \psi_{Am}(z)\, A_A^{mn}$$

$$\phi_C^n = \phi_{Cn} + \phi_{Cn}^*$$

$$= \cos(\frac{n\pi z}{L})\, S_C\, \Phi_{Cn}(r)\, A_C^n + \sum_m J_o(\frac{\omega_m r}{R})\, S_C\, \psi_{Cm}(z)\, A_C^{mn}$$

The functions Φ_{An} and Φ_{Cn} can be expanded in terms of the Bessel functions, i.e., with Fourier – Bessel coefficients U^{mn},

$$\Phi_{An}(r) = \sum_m J_o(\frac{\omega_m r}{R})\, U_A^{mn}$$

$$\Phi_{Cn}(r) = \sum_m J_o(\frac{\omega_m r}{R})\, U_C^{mn}$$

Using the result

$$\int_o^1 \frac{r}{R} J_o(\frac{\omega_m r}{R})\, I_o(a\frac{r}{R})\, d(\frac{r}{R}) = \omega_m\, J_1(\omega_m)\, \frac{I_o(a)}{\omega_m^2 + a^2}$$

and the similar integral for $J_o(\frac{ar}{R})$, it follows that

$$U_A^{mn} = \frac{2\omega_m}{J_1(\omega_m)} \begin{bmatrix} I_o(\alpha_{1n} R)/(\omega_m^2 + \alpha_{1n}^2 R^2) & 0 \\ 0 & I_o(\alpha_{2n} R)/(\omega_m^2 + \alpha_{2n}^2 R^2) \end{bmatrix}$$

$$U_C^{mn} = \frac{2\omega_m}{J_1(\omega_m)} \begin{bmatrix} J_o(\kappa_n R)/(\omega_m^2 - \kappa_n^2 R^2) & 0 \\ 0 & I_o(\mu_n R)/(\omega_m^2 + \mu_n^2 R^2) \end{bmatrix}$$

The continuity of neutron flux and current at z = H requires that for each value of m,

$$\cos\left(\frac{n\pi H}{L}\right) S_A U_A^{mn} P_n A_C^n + S_A \psi_{Am}(H) A_A^{mn}$$

$$= \cos\left(\frac{n\pi H}{L}\right) S_C U_C^{mn} A_C^n + S_C \psi_{cm}(H) A_C^{mn}$$

and

$$-\left(\frac{n\pi}{L}\right) \sin\left(\frac{n\pi H}{L}\right) D_A S_A U_A^{mn} P_n A_C^n + D_A S_A \psi_{Am}^1(H) A_A^{mn}$$

$$= -\left(\frac{n\pi}{L}\right) \sin\left(\frac{n\pi H}{L}\right) D_C S_C U_C^{mn} A_C^n + D_C S_C \psi_{Cm}^1(H) A_C^{mn}$$

To solve these equations, let

$$D_A S_A \psi_{Am}^1(H) = -\lambda_{Am} S_A \psi_{Am}(H)$$

$$D_C S_C \psi_{Cm}^1(H) = \lambda_{Cm} S_C \psi_{Cm}(H)$$

i.e.

$$\lambda_{Am} = -D_A S_A \psi_{Am}^1(H) \psi_{Am}^{-1}(H) S_A^{-1}$$

$$\lambda_{Cm} = D_C S_C \psi_{Cm}^1(H) \psi_{Cm}^{-1}(H) S_C^{-1}$$

These matrices are again comparatively simple

$$\lambda_{Am} = -\begin{bmatrix} D_{1A} & 0 \\ S(A) D_{2A} & D_{2A} \end{bmatrix} \begin{bmatrix} -\sigma_{1m} \coth(\sigma_{1m} Z) & 0 \\ S(A)\, \sigma_{2m} \coth(\sigma_{2m} Z) & -\sigma_{2m} \coth(\sigma_{2m} Z) \end{bmatrix}$$

$$\lambda_{Cm} = \begin{bmatrix} D_{1C} S_\kappa & -D_{1C} S_\mu \\ D_{2C} & D_{2C} \end{bmatrix} \begin{bmatrix} \kappa_m \tanh(\kappa_m H) & S_\mu \kappa_m \tanh(\kappa_m H) \\ -\mu_m \tanh(\mu_m H) & S_\kappa \mu_m \tanh(\mu_m H) \end{bmatrix} \frac{1}{S_\kappa + S_\mu}$$

The two boundary conditions now reduce to

$$(\lambda_{Am} + \lambda_{Cm}) S_A \psi_{Am}(H) A_A^{mn} = Q_A^{mn} A_C^n$$

$$(\lambda_{Am} + \lambda_{Cm}) S_C \psi_{Cm}(H) A_C^{mn} = Q_C^{mn} A_C^n$$

where

$$Q_A^{mn} = -\{\lambda_{Cm}\cos(\frac{n\pi H}{L}) + D_A(\frac{n\pi}{L})\sin(\frac{n\pi H}{L})\}\, S_A U_A^{mn} P_n$$
$$+\{\lambda_{Cm}\cos(\frac{n\pi H}{L}) + D_C(\frac{n\pi}{L})\sin(\frac{n\pi H}{L})\}\, S_C U_C^{mn}$$

$$Q_C^{mn} = \{\lambda_{Am}\cos(\frac{n\pi H}{L}) - D_A(\frac{n\pi}{L})\sin(\frac{n\pi H}{L})\}\, S_A U_A^{mn} P_n$$
$$-\{\lambda_{Am}\cos(\frac{n\pi H}{L}) - D_C(\frac{n\pi}{L})\sin(\frac{n\pi H}{L})\}\, S_C U_C^{mn}$$

The solution is therefore

$$A_A^{mn} = \psi_{Am}^{-1}(H)\, S_A^{-1}(\lambda_{Am} + \lambda_{Cm})^{-1} Q_A^{mn} A_C^n = X_A^{mn} A_C^n$$
$$A_C^{mn} = \psi_{Cm}^{-1}(H)\, S_C^{-1}(\lambda_{Am} + \lambda_{Cm})^{-1} Q_C^{mn} A_C^n = X_C^{mn} A_C^n$$

The complete solution for the end reflected core A + C when the fluxes along $r = R$ are distributed as $\cos(\frac{n\pi z}{L})$ is now as follows,

$$\phi_A^n(r,z) = \{\cos(\frac{n\pi z}{L})\, S_A\, \Phi_{An}(r)\, P_n + \sum_m J_o(\frac{\omega_m r}{R})\, S_A\, \psi_{Am}(z)\, X_A^{mn}\}\; A_C^n$$

$$\phi_C^n(r,z) = \{\cos(\frac{n\pi z}{L})\, S_C\, \Phi_{Cn}(r) + \sum_m J_o(\frac{\omega_m r}{R})\, S_C\, \psi_{Cm}(z)\, X_C^{mn}\}\; A_C^n$$

To match up the neutron current at $r = R$ with that in the reflector, it is necessary to have the neutron current distribution at $r = R$ in the A + C system. This current will be taken as positive if directed into the material, i.e., in the $-r$ direction.

For $O < z < H$

$$\mathcal{J}_C^n(R,z) = D_C\, \{\cos(\frac{n\pi z}{L})\, S_C\, \Phi_C^1(R) - \sum_m \frac{\omega_m}{R} J_1(\omega_m)\, S_C\, \psi_{Cm}(z)\, X_C^{mn}\}\; A_C^n$$

and for $H < z < L$

$$\mathcal{J}_A^n(R,z) = D_A\{\cos(\frac{n\pi z}{L})\, S_A\, \Phi_{An}^1(R)\, P_n - \sum_m \frac{\omega_m}{R} J_1(\omega_m)\, S_A\, \psi_{Am}(z)\, X_A^{mn}\}\; A_C^n$$

This distribution can be expanded as

$$\mathcal{J}^n(R,z) = \sum_S \cos\left(\frac{S\pi z}{L}\right) \theta^{Sn} A_C^n \quad , 2S = 1,3,5\ldots\ldots \quad , O \leqslant z \leqslant L$$

where

$$\theta^{Sn} A_C^n = 2\int_o^{\frac{H}{L}} \mathcal{J}_C^n (R,z) \cos\left(\frac{S\pi z}{L}\right) d\left(\frac{z}{L}\right) + 2\int_{\frac{H}{L}}^{1} \mathcal{J}_A^n (R,z) \cos\left(\frac{S\pi z}{L}\right) d\left(\frac{z}{L}\right)$$

The first integral produces $2\theta(S,n)$ from $\cos\left(\frac{n\pi z}{L}\right)$

and $2\theta_C(S,m)$ from $\psi_{Cm}(z)$

The second integral produces $\delta_{Sn} - 2\theta(S,n)$ from $\cos\left(\frac{n\pi z}{L}\right)$

and $2\theta_A(S,m)$ from $\psi_{Am}(z)$

The integrations present no difficulty and the results are

$$2\theta(S,n) = \frac{H}{L} + \frac{1}{n\pi} \sin\left(\frac{n\pi H}{L}\right) \cos\left(\frac{n\pi H}{L}\right) \quad , \quad S = n$$

$$= \frac{2S \sin\left(\frac{S\pi H}{L}\right) \cos\left(\frac{n\pi H}{L}\right) - 2n \cos\left(\frac{S\pi H}{L}\right) \sin\left(\frac{n\pi H}{L}\right)}{(S^2 - n^2)\pi} \quad , \quad S \neq n$$

The matrices $2\theta_C(S,m)$ and $2\theta_A(S,m)$ are diagonal. The leading element of $2\theta_C(S,m)$ is

$$\frac{2}{S^2\pi^2 + L^2\kappa_m^2} \left\{ \kappa_m L \cos\left(\frac{S\pi H}{L}\right) \sinh(\kappa_m H) + S\pi \sin\left(\frac{S\pi H}{L}\right) \cosh(\kappa_m H) \right\}$$

with a similar expression for the other element in μ_m. Similarly the leading element of $2\theta_A(S,m)$ is

$$\frac{2}{S^2\pi^2 + L^2\sigma_{1m}^2} \left\{ \sigma_{1m} L \cos\left(\frac{S\pi H}{L}\right) \cosh(\sigma_{1m} Z) - S\pi \sin\left(\frac{S\pi H}{L}\right) \sinh(\sigma_{1m} Z) \right\}$$

with a similar expression for the other element in σ_{2m}.

The complete expression for θ^{Sn} is therefore

$$\theta^{Sn} = D_C S_C \left\{ 2\theta(S,n)\, \Phi_{Cn}^1(R) - \sum_m \left(\frac{\omega_m}{R}\right) J_1(\omega_m)\, 2\theta_C(S,m)\, X_C^{mn} \right\}$$

$$+ D_A S_A \left\{ (\delta_{Sn} - 2\theta(S,n))\, \Phi_{An}^1(R)\, P_n - \sum_m \left(\frac{\omega_m}{R}\right) J_1(\omega_m)\, 2\theta_A(S,m)\, X_A^{mn} \right\}$$

Thus, finally, if the flux distribution along r = R in the A + C system is

$$\phi^{n}(R) = \cos\left(\frac{n\pi z}{L}\right) S_C \, \Phi_{Cn}(R) \, A_C^{n}$$

$$= \cos\left(\frac{n\pi z}{L}\right) \Delta^{n} \, A_C^{n}$$

then the neutron current distribution is

$$\mathcal{J}^{n}(R) = \sum_{S} \cos\left(\frac{S\pi z}{L}\right) \theta^{Sn} A_C^{n}$$

If, in the general case of a fully reflected system, A + B + C

$$\phi(R) = \sum_{n} \cos\left(\frac{n\pi z}{L}\right) G_n$$

then the neutron current distribution due to A + C is

$$\mathcal{J}(R) = \sum_{n}\sum_{S} \cos\left(\frac{S\pi z}{L}\right) \theta^{Sn} (\Delta^{n})^{-1} G_n$$

$$= \sum_{n}\sum_{S} \cos\left(\frac{S\pi z}{L}\right) \lambda^{Sn} G_n$$

In order therefore to satisfy the boundary conditions between A + C and B

$$-\sum_{n} \cos\left(\frac{n\pi z}{L}\right) \bar{\lambda}^{n} G_n = \sum_{n}\sum_{S} \cos\left(\frac{S\pi z}{L}\right) \lambda^{Sn} G_n$$

$$= \sum_{n}\sum_{p} \cos\left(\frac{n\pi z}{L}\right) \lambda^{np} G_p$$

This condition can be satisfied if

$$\begin{vmatrix} (\bar{\lambda}^{1} + \lambda^{11}) & \lambda^{12} & \lambda^{13} & \cdot & \cdot & \cdot \\ \lambda^{21} & (\bar{\lambda}^{2} + \lambda^{22}) & \lambda^{23} & \cdot & \cdot & \cdot \\ \lambda^{31} & \lambda^{32} & (\bar{\lambda}^{3} + \lambda^{33}) & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix} = 0$$

In practice it will be necessary to truncate this determinant. Some very limited experience has indicated that the vanishing of a 6th order determinant is sufficient to establish a critical radius with reasonable accuracy. If, however, the flux distributions are required, then it will be necessary to take account of the higher orders.

A similar method to that outlined above can be applied if the separation into two sub–systems is made by means of a plane cut at $z = \pm H$. A further extension, which will involve doubling the size of the determinant at least, would be to develop the analysis for a system unsymmetrical about the r axis, i.e. with different top and bottom reflectors.

In searching for critical systems it is possible that a much smaller determinant would be satisfactory if the theory were developed in terms of neutron importance. Neutron importance is a much smoother function than flux and fewer terms would be required in the expansion along the boundary between sub–systems.

3. CONCLUSION

The method developed involves the manipulation of second order matrices only. Matrix inversion is therefore not a problem. In addition, many of the matrices involved are diagonal. It is suggested that a small computer could handle the calculation in two or three stages. In the first stage the elements of the λ matrices would be computed for different values of the independent parameter, e.g. critical radius or effective multiplication constant. This output would be used as input for the second stage in which the determinants are evaluated. The third stage would be the computation of neutron flux distributions.

4. NOTATION

A	Constants appearing in solutions of the diffusion equations.
D	Diagonal matrix of diffusion coefficients.
H	Half height of cylinder core.
$\mathfrak{J}$	Neutron current vector.
k^2	Inverse diffusion area in reflector.
L	Half overall height of core plus reflectors.
R	Radius of core.
S	Matrix of coupling coefficients in two group diffusion equation solutions.
S_κ	Ratio of fast to thermal flux derived from $(\nabla^2 + \kappa^2)\,\phi = 0$.
S_μ	,, ,, ,, ,, ,, ,, ,, ,, $(\nabla^2 - \mu^2)\phi = 0$.
T	Thickness of radial reflector.
Z	Thickness of end reflectors.
δ_{mn}	Kronecker delta.
$\phi(r, z)$	Flux vector.
$\Phi(r)$	Diagonal matrix of radial functions associated with a cosine axial distribution.

$\psi(z)$	Diagonal matrix of axial functions associated with a J_o Bessel function radial distribution.
ω_m	m'th root of $J_o(\omega) = 0$.
Suffix A,B,C	Refer to top reflector, radial reflector and core regions respectively.
Suffix 1, 2	Refer to fast and thermal groups respectively.
Affix n	Refers to functions associated with $\cos\left(\frac{n\pi z}{L}\right)$.
Affix m	Refers to functions associated with $J_o(\omega_m r/R)$.