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AUSTRALIAN NUCLEAR SCIENCE & TECHNOLOGY ORGANISATION

LUCAS HEIGHTS RESEARCH LABORATORIES

DERIVATION OF THE DEAD-TIME CORRECTION EQUATION FOR A RADIOACTVITY DETECTOR

by

H. A. WYLLIE

ABSTRACT

When a radioactive source is being counted, a detected disintegration cannot be recorded during a Dead-Time period. In this report experiments are described which show that the graph of the distribution of detected disintegrations during Dead-Time periods is a horizontal line. Based on this observation, an alternative derivation is produced of the widely used Dead-Time correction equation for a radioactivity detector.

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1. GLOSSARY OF SYMBOLS

List of symbols and what they refer to

p1	1st of a pair of disintegrations detected during a β dead-time.
p2	2nd of a pair of disintegrations detected during a β dead-time.
tr1	1st of a trio of disintegrations detected during a β dead-time.
tr3	3rd of a trio of disintegrations detected during a β dead-time.
tet1	1st of a tetrad of disintegrations detected during a β dead-time.
tet4	4th of a tetrad of disintegrations detected during a β dead-time.
ADC	Analogue-to-Digital Converter.
β DISC	β discriminator.
DDDG	Detected-Disintegration Distribution Graph (i. e., MCA Graph).
G&DG	Gate-and-Delay Generator.
I	Integral. Value is given on MCA screen.
i	MCA Channel Number (between 1 and 160).
Itet1	Value of the Integral when an MCA Graph is acquired for the 1st of a tetrad of disintegrations. The other 6 Integral Symbols are similarly defined.
MCA	Multi-Channel Analyser.
Par. Unit	Paralysis Unit
q	Only the Multi-Channel Analyser of TAC-1 is in use.
SCA	Single-Channel Analyser.
TAC	Time-to-Analogue Converter.
Valid Conv.	Valid Conversion.

2. INTRODUCTION

Systems for counting radioactive sources experience an intrinsic Dead-Time immediately after a nuclear event is detected. During this period a subsequent event cannot be recorded.

The pulse emitted from the detector requires a finite time to be processed by the associated electronics.

To facilitate the making of an accurate correction for events lost during the Dead-Time, a paralysis unit is added to the system, with its Dead-Time set higher than the intrinsic Dead-Time of the preceding components. The paralysis unit imposes, for correction calculations, a set Dead-Time which is exactly the same for each event detected. This set Dead-Time can be measured with considerable accuracy.

3. OUTLINE OF THE REPORT

The first part of this report deals with the determination of the percentage of second and third detected-disintegrations which occurred during many β Dead-Times. The β Dead-Time was set at 105 μs. The Co 60 β count-rate was approximately 2800 counts per second.

The system used (see Figure 1) consists of two Time-to-Amplitude Converters operating in tandem (abbreviated to "Tandem TAC").

The disintegrations were detected by a 4πβ proportional-counter. Its output pulses went to the β-discriminator input. The β Dead-Time was set in the Paralysis Unit. The system recorded (in the channels of the two multi-channel analysers) up to 3 disintegrations detected during the 105 μs β Dead-Time.

The system was later extended, as shown in Figure 2, and used to determine the percentage of second, third and fourth detected-disintegrations.

Using the computer program TetD4.BAS, D4(i), denoting the total number of detected-disintegrations in channel i, was computed for each channel (from i = 1 to i = 160), and tabulated in tables 6, 7, 8 and 9.

It can be seem that D4(i) was virtually constant from i = 4 to i = 141. Based on this assumption, the final part of this report gives a derivation of the Dead-Time Correction Equation for a single-detector counting system.

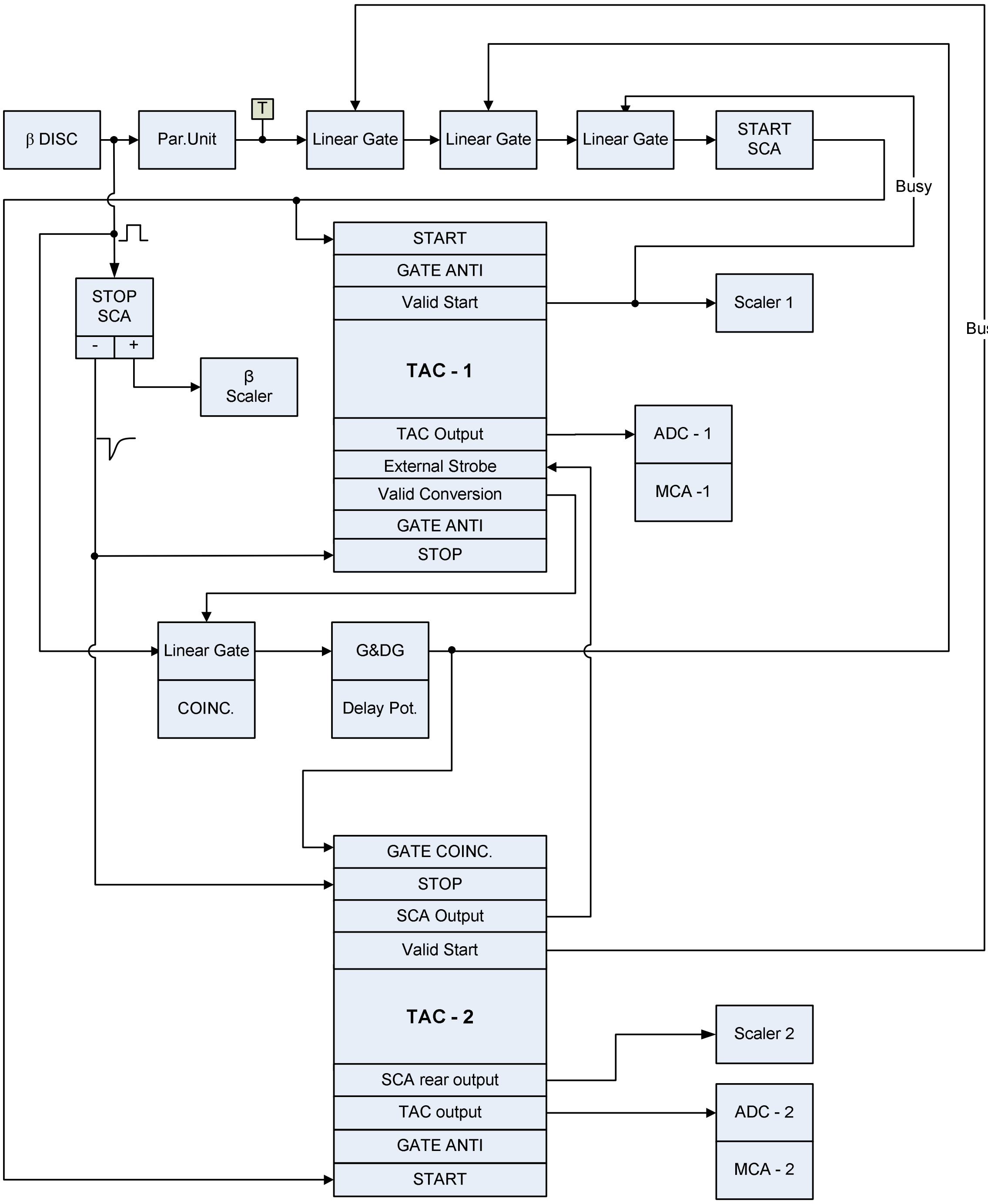


Figure 1. System for recording 2nd and 3rd STOP-pulses during the β Dead-Time.

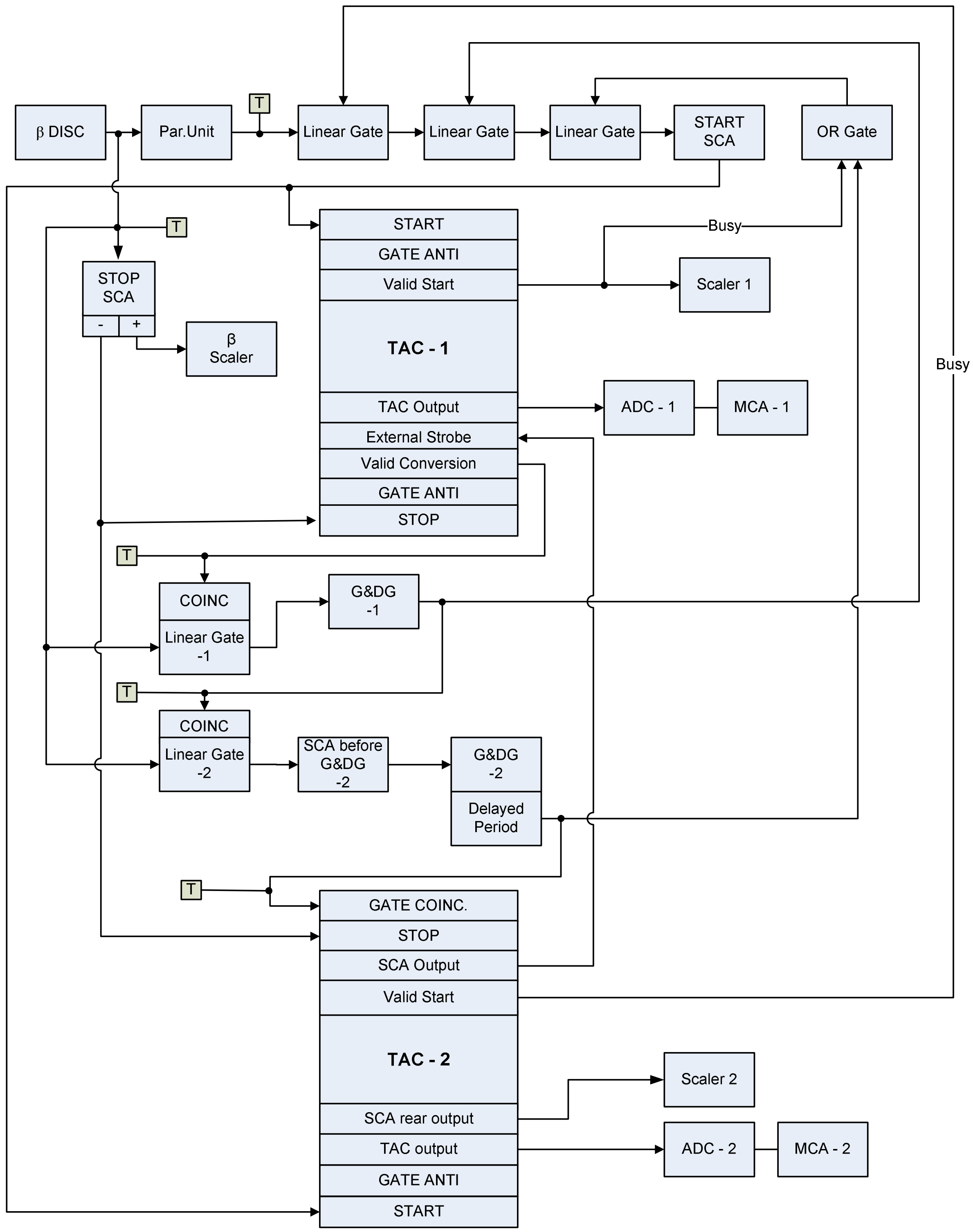


Figure 2. System for recording 2nd, 3rd and 4th STOP-pulses during the β Dead-Time.

4. DESCRIPTION OF THE ELECTRONIC MODULES

a) The Linear Gates permit the passage of positive pulses. The three anti-coincidence linear-gates at the top of the two Tandem-TAC diagrams (Figure 1 and Figure 2) ensure that when part of a Tandem-TAC System is busy, there cannot be any output from its START-SCA.

b) The Single-Channel Analysers in the two Tandem-TACs are Ortec Models 551. The fast, narrow, negative, output-pulses from the START and STOP-SCAs become input-pulses to TAC-1 and TAC-2.

c) In the two block diagrams (Figures 1 and 2) the symbol [T] represents a terminator.

d) "G&DG" denotes an Ortec Gate and Delay Generator. G&DG-1 has been internally adjusted to generate an output-pulse 105 μs wide. The output from G&DG-2 is a Delayed-Period pulse 105 μs wide. The module labelled "SCA before G&DG-2" delays the G&DG-2 output.

e) An Ortec model 567 Time-to-Amplitude Converter (TAC-1 & TAC-2 in Figure 1 and Figure 2) measures the interval between two SCA negative output-pulses which are accepted, respectively, at the TAC START and STOP. Acceptance occurs

   i. when the TAC Gate has been switched to ANTI and there is no pulse entering the Gate input, or

   ii. when the TAC Gate has been switched to COINC, and a positive pulse is simultaneously entering the Gate input.

The amplitude of the "TAC Output" pulse (of TAC-1 and TAC-2) is proportional to the interval being measured, and it is represented on the Multi-Channel-Analyser Graph by a channel number. The number of counts which accumulate in a channel is the number of measured intervals whose magnitudes are within the interval-range represented by that particular channel.

An Ortec TAC is "busy" during the generation of the Valid-START output-pulse, i.e., from the instant of an accepted START to the end of Reset.

The duration of the Valid-Conversion output-pulse extends from the end of the internal delay after STOP to the end of Reset.

In the pulse diagrams (Figures 3, 4, 5 & 6) "TAC Windows (T)" is equal to 105 μs**.** On the MCA Graphs (Figures 7 to 21) 105 μs was represented by Channel 141.

This arrangement of MCA channel-number versus measured interval was obtained by selecting on TAC-1 and TAC-2 the appropriate "Range", "SCA Lower Level" and "SCA Window".

When TAC-1 and TAC-2 are operating in tandem, the "TAC-Output" of TAC-1 is externally strobed by an "SCA-Output" pulse from TAC-2.

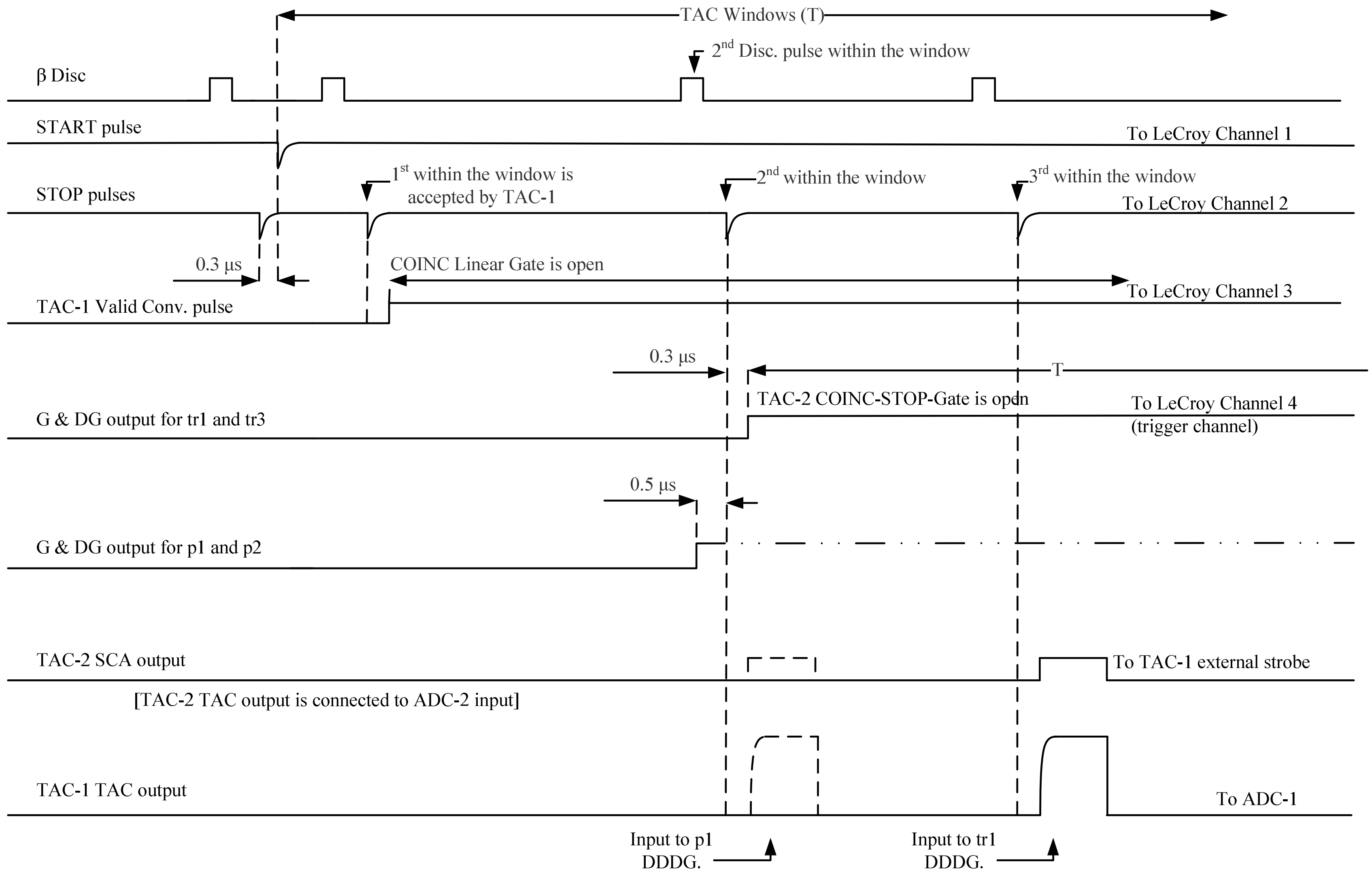


Figure 3. Pulse diagram obtained by the Figure 1 System.

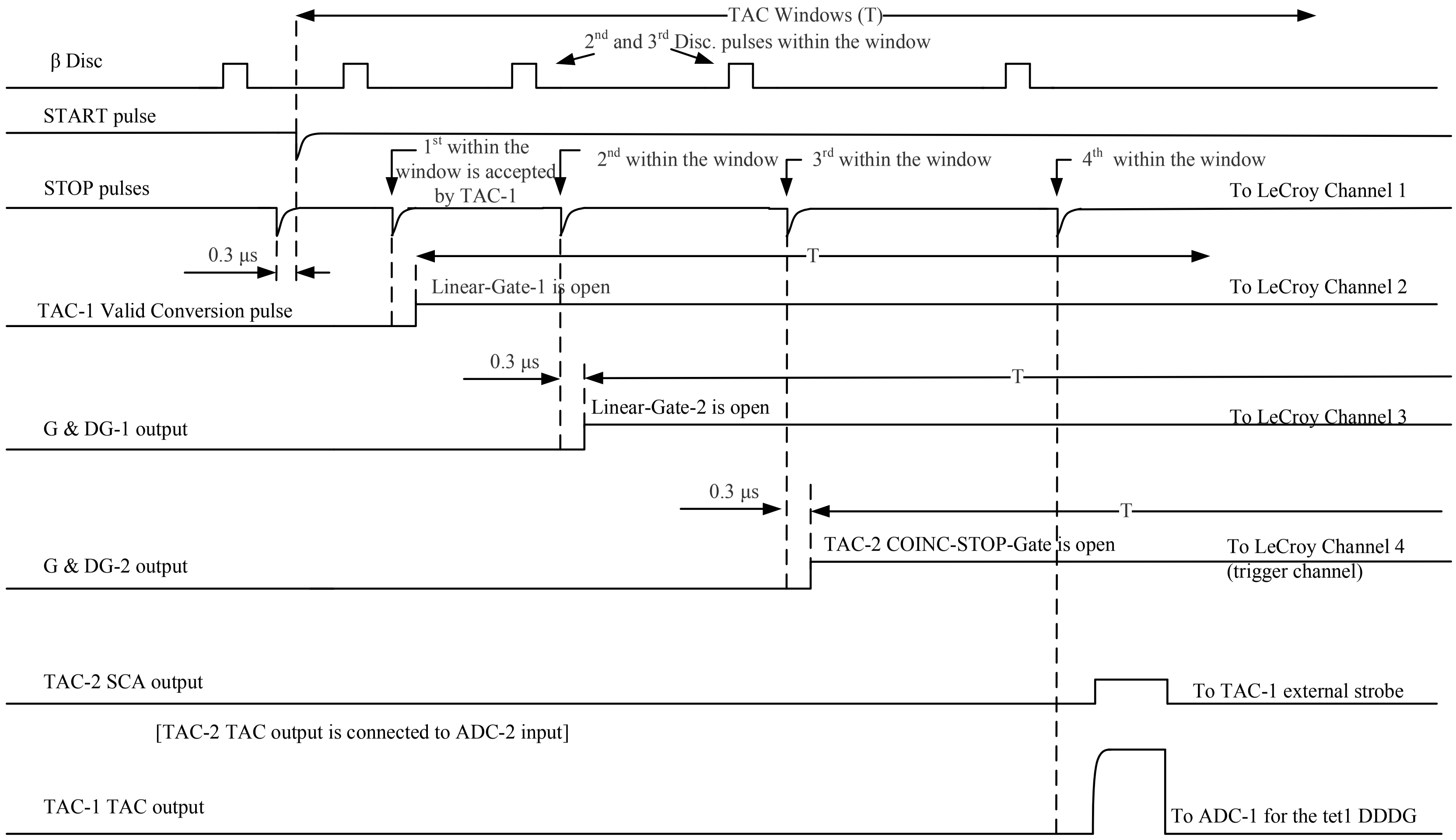


Figure 4. Arrangement for recording tet1 and tet4 counts during the β Dead-Time.

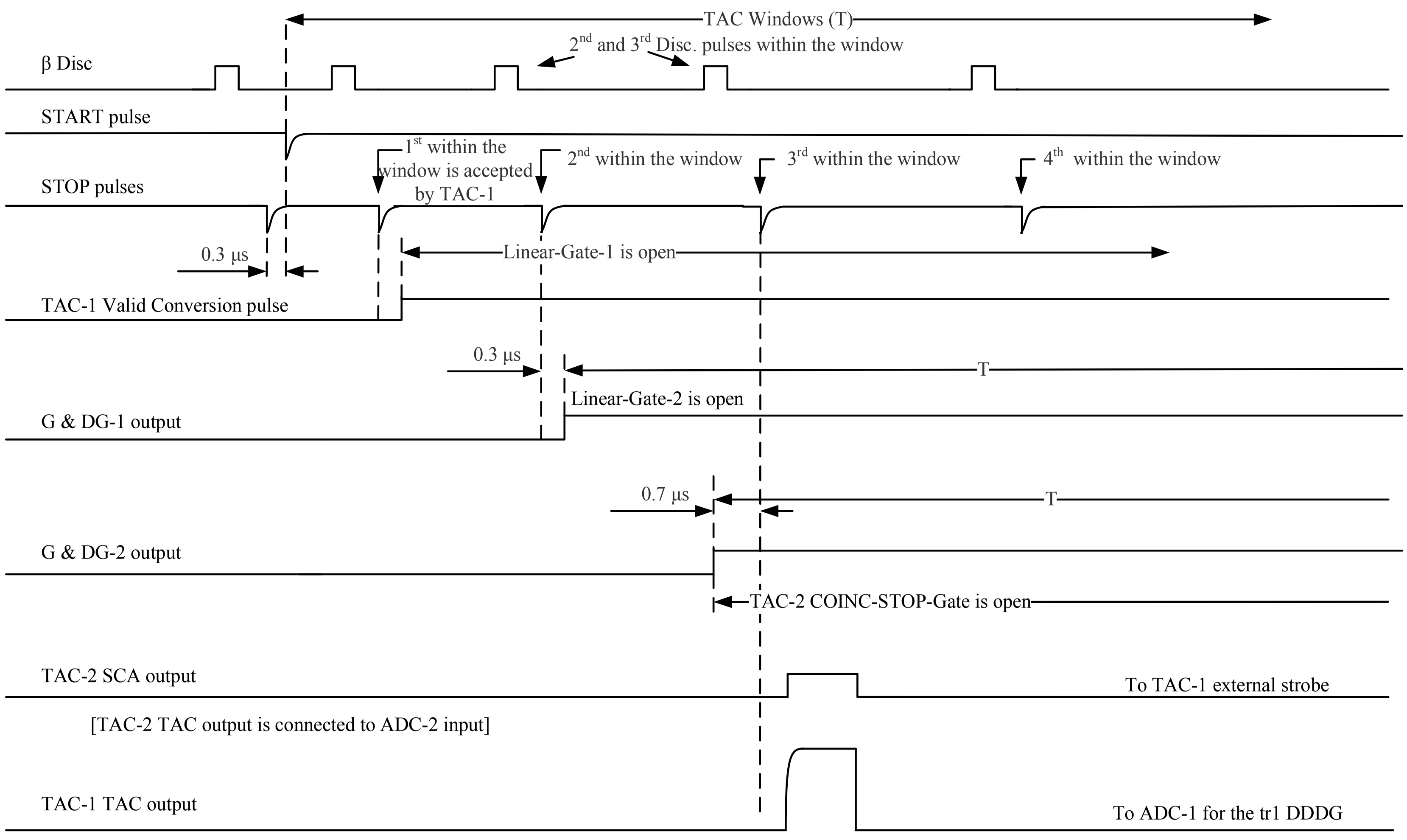


Figure 5. Arrangement for recording tr1 and tr3 counts during the β Dead-Time.

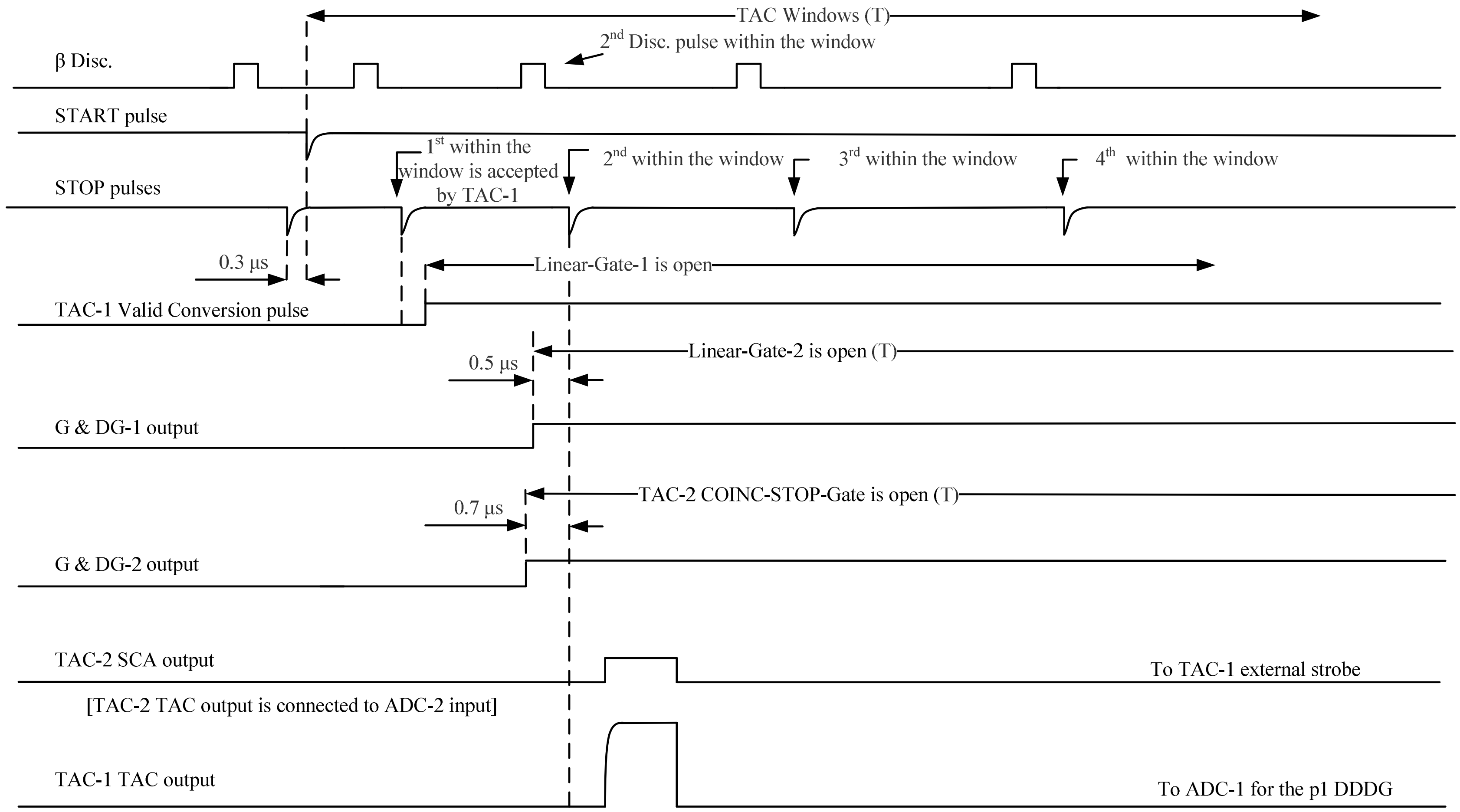


Figure 6. Arrangement for recording p1 and p2 counts during the β Dead-Time.

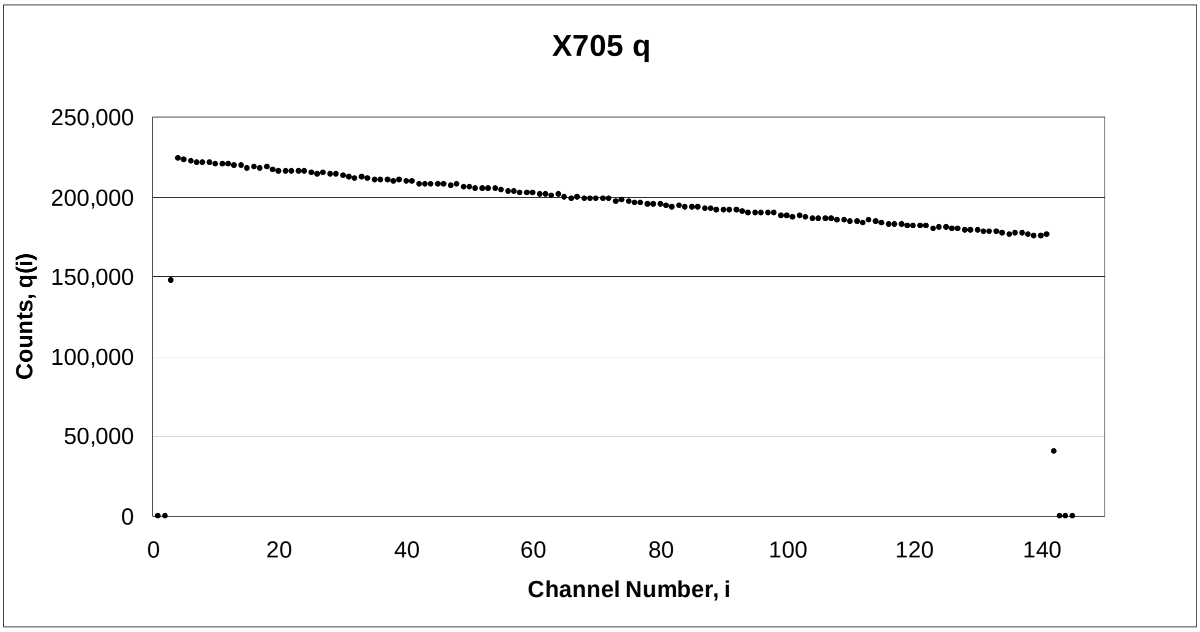
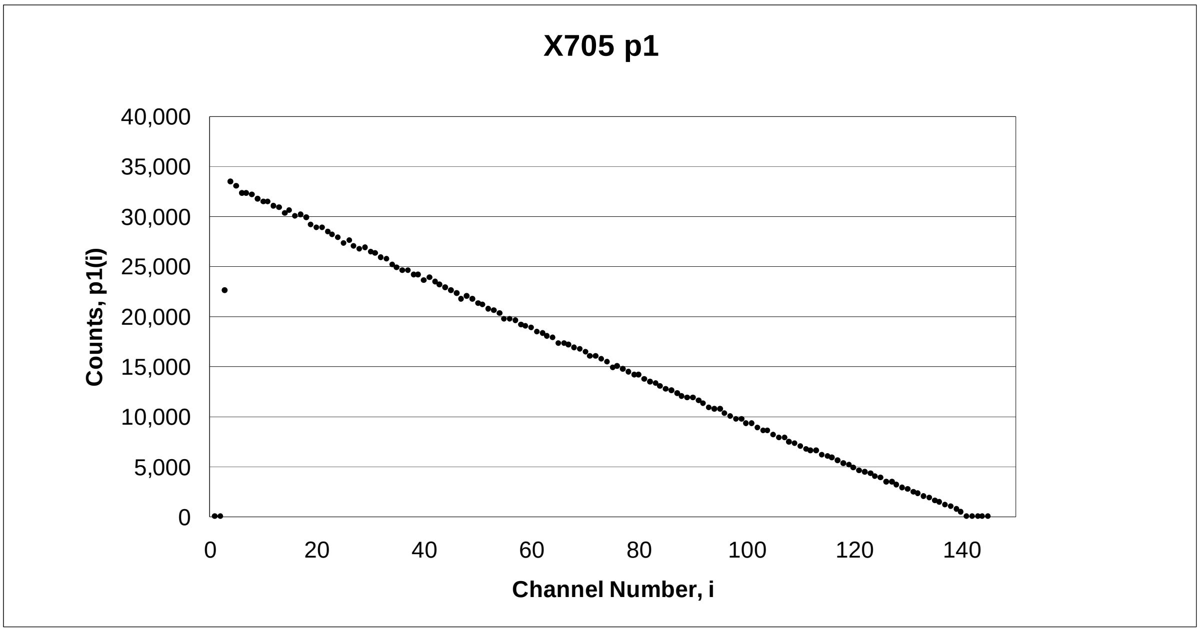


Figure 7. Experiment X705:- MCA Graph of q(i) versus i.



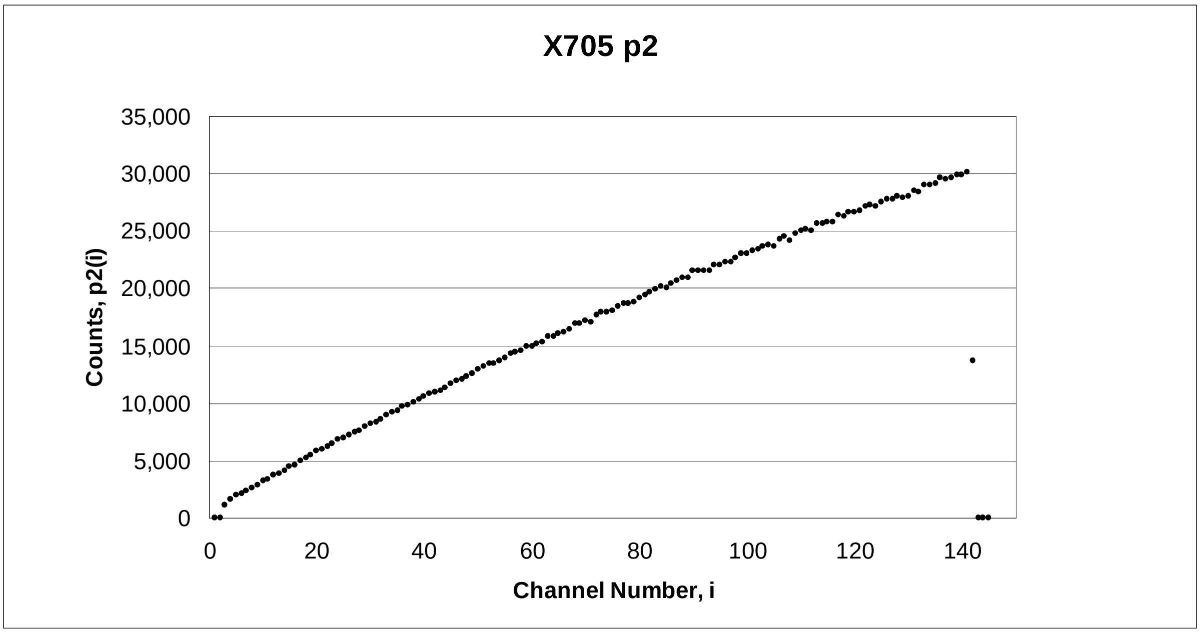
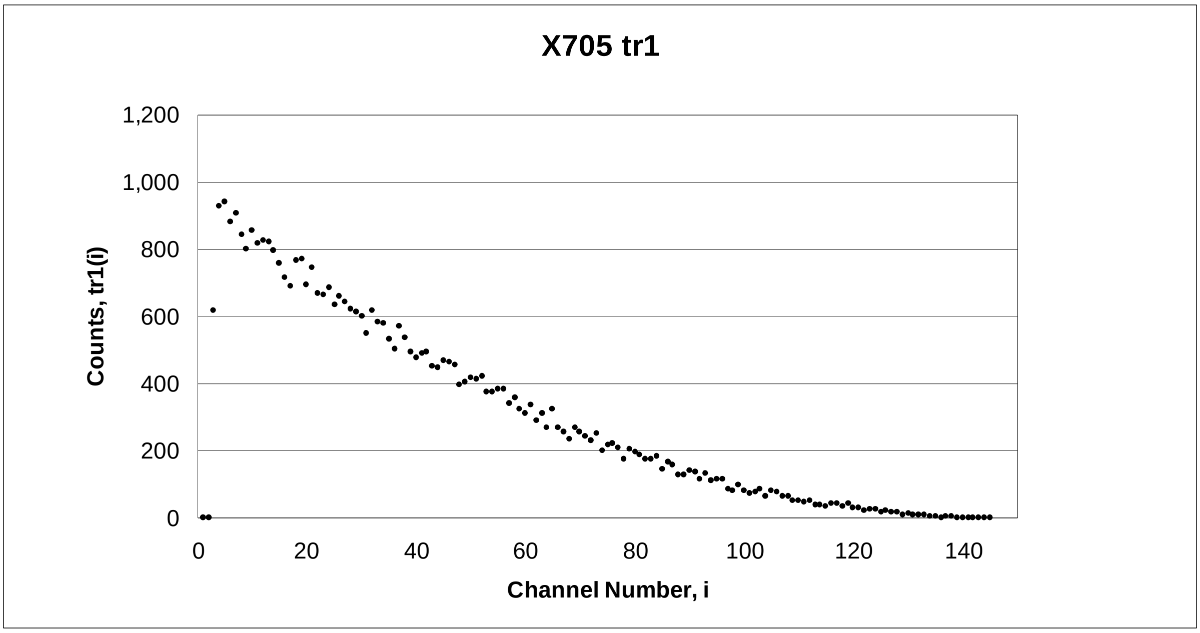


Figure 8. Experiment X705:- MCA Graphs of $p1(i)$ and $p2(i)$ versus i.



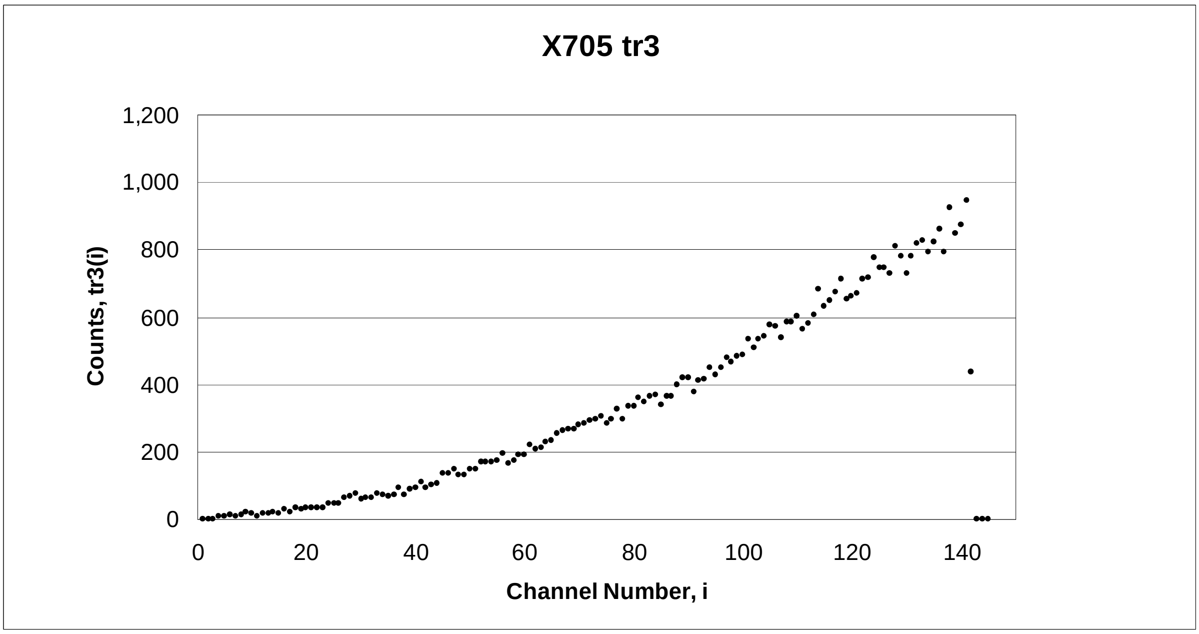


Figure 9. Experiment X705:- MCA Graphs of tr1(i) and tr3(i) versus i.

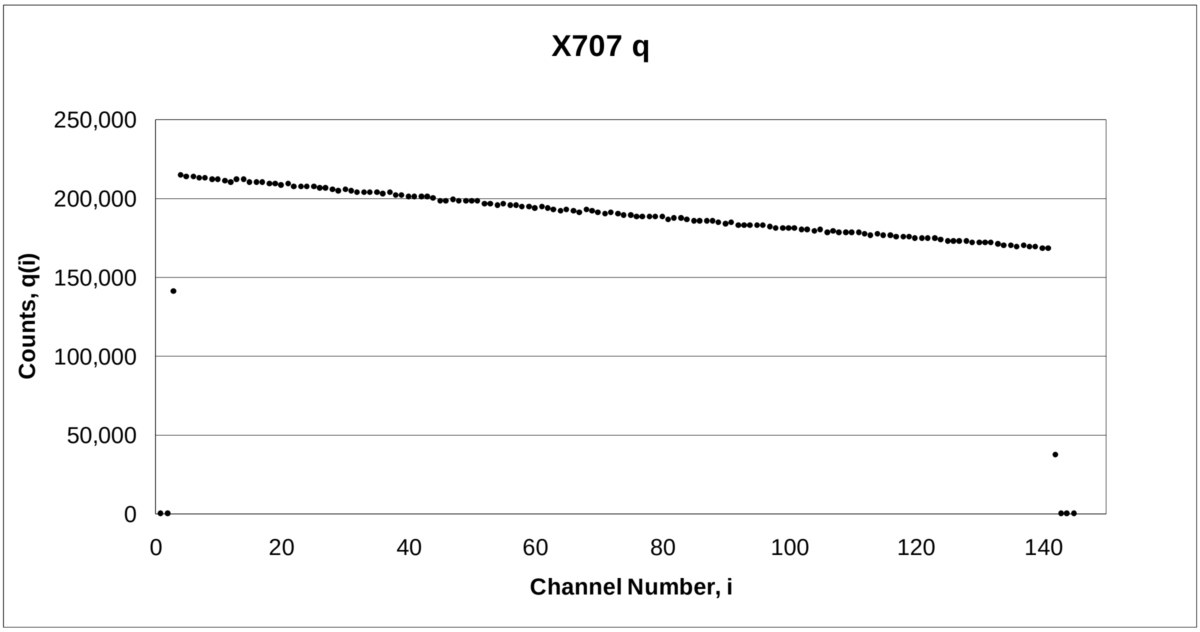
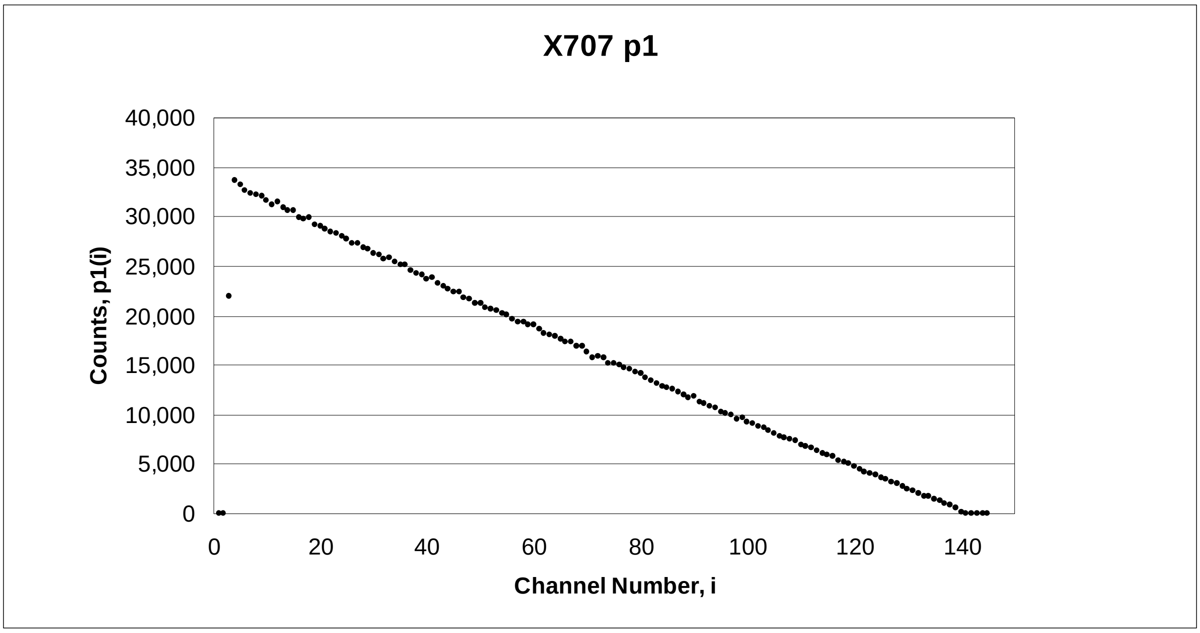


Figure 10. Experiment X707:- MCA Graph of q(i) versus i.



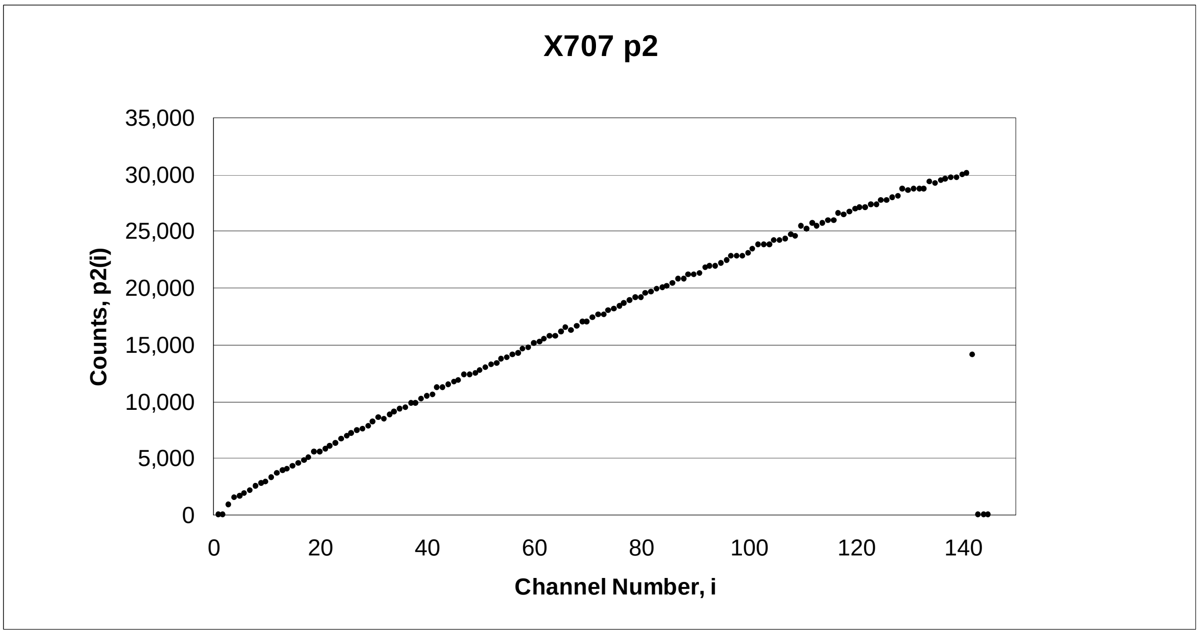
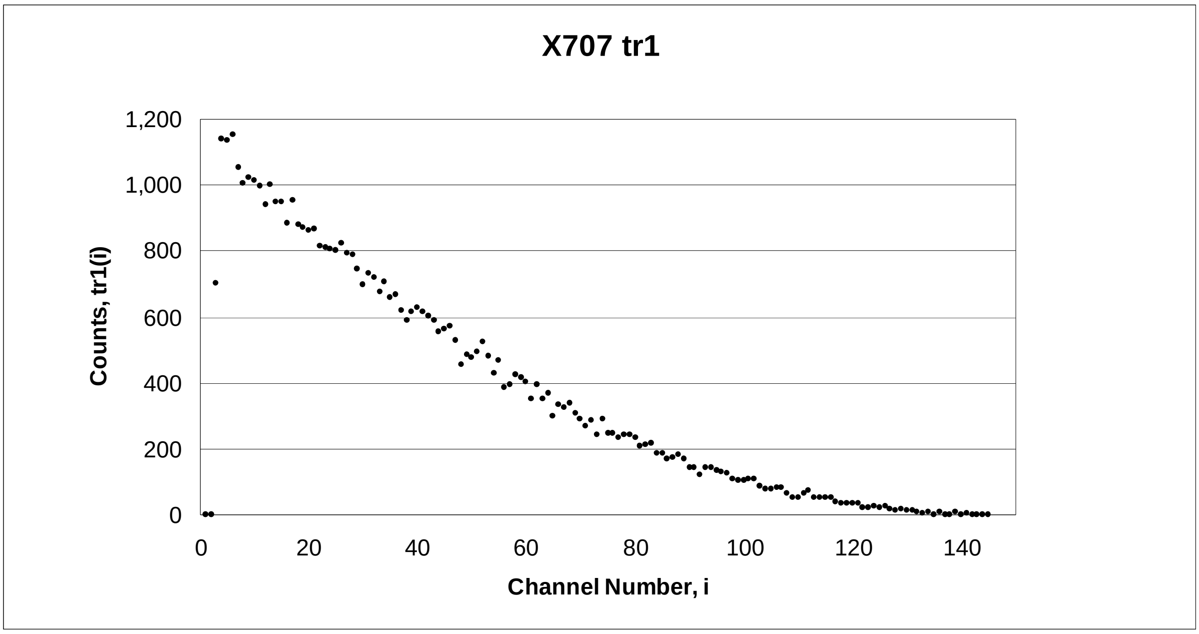


Figure 11. Experiment X707:- MCA Graphs of p1(i) and p2(i) versus i.



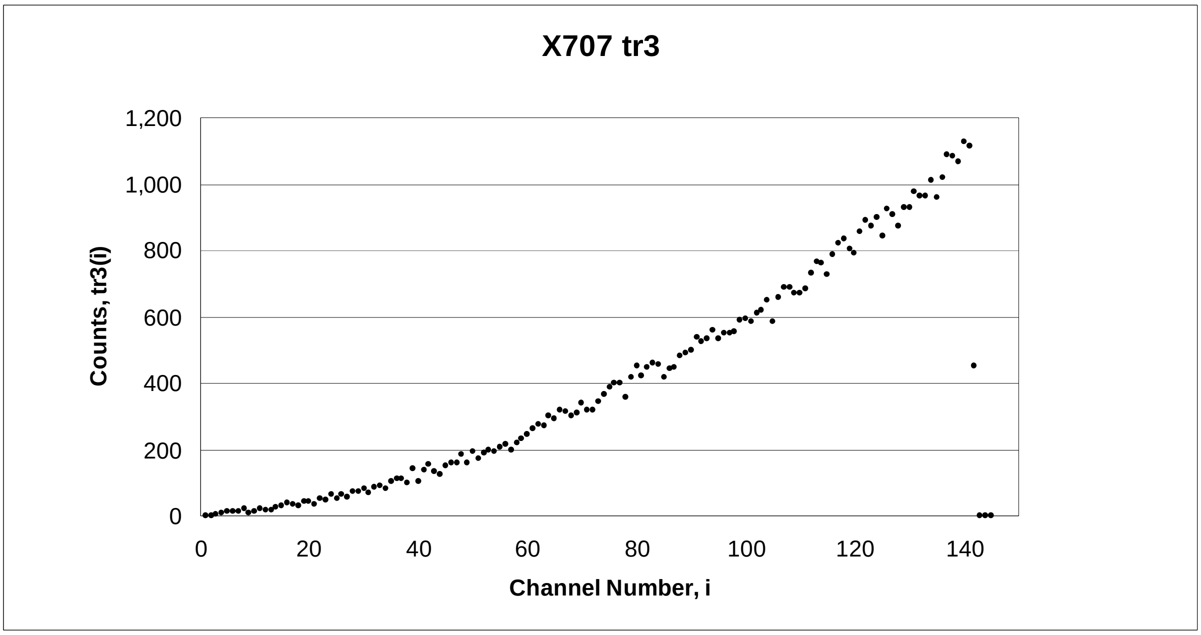
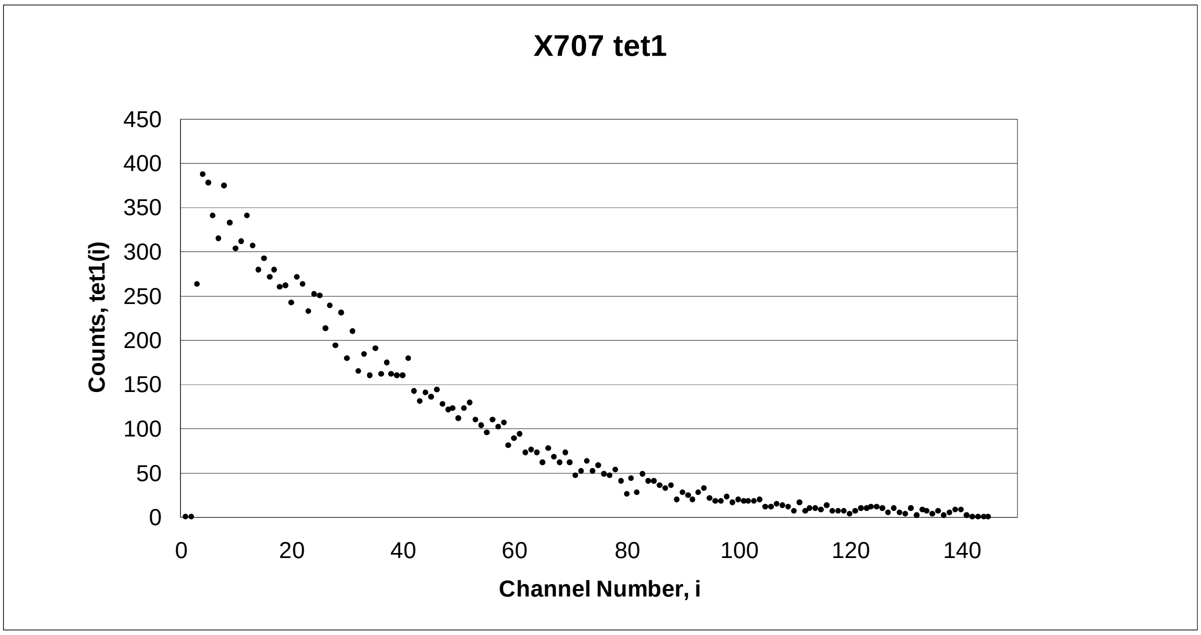


Figure 12. Experiment X707:- MCA Graphs of tr1(i) and tr3(i) versus i.



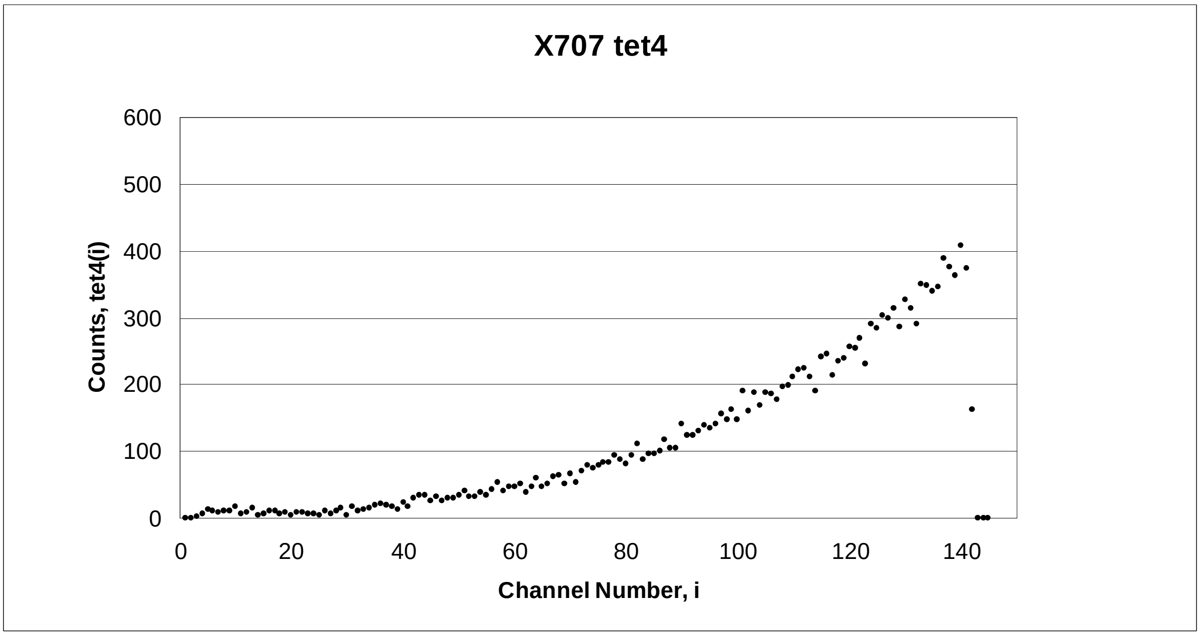


Figure 13. Experiment X707:- MCA Graphs of tet1(i) and tet4(i) versus i.

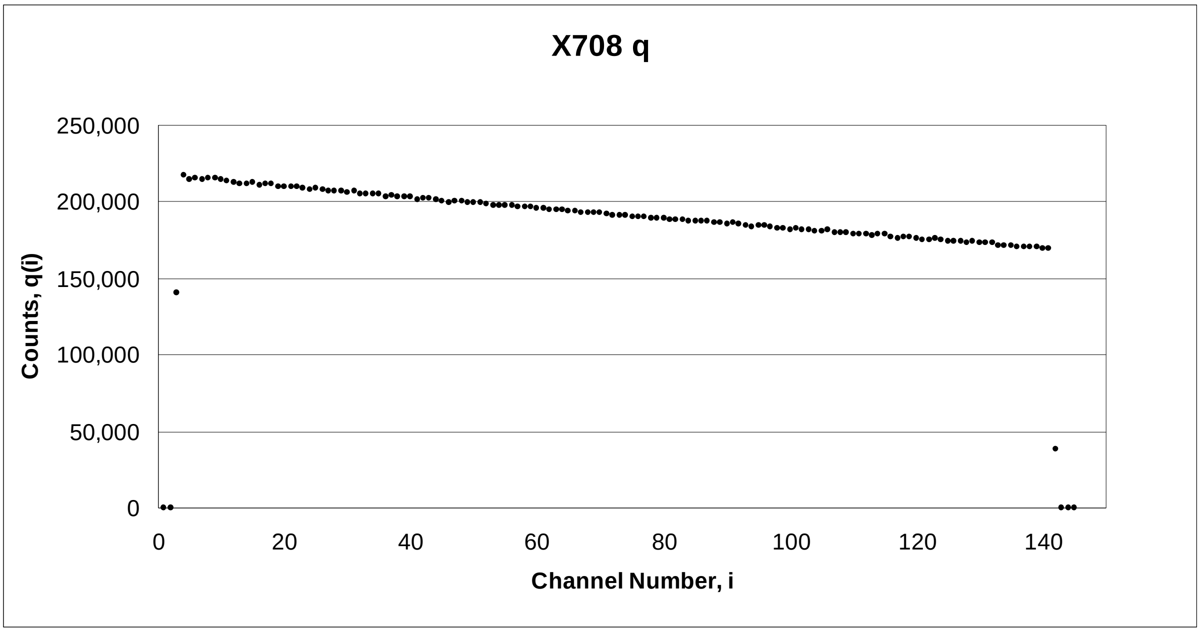
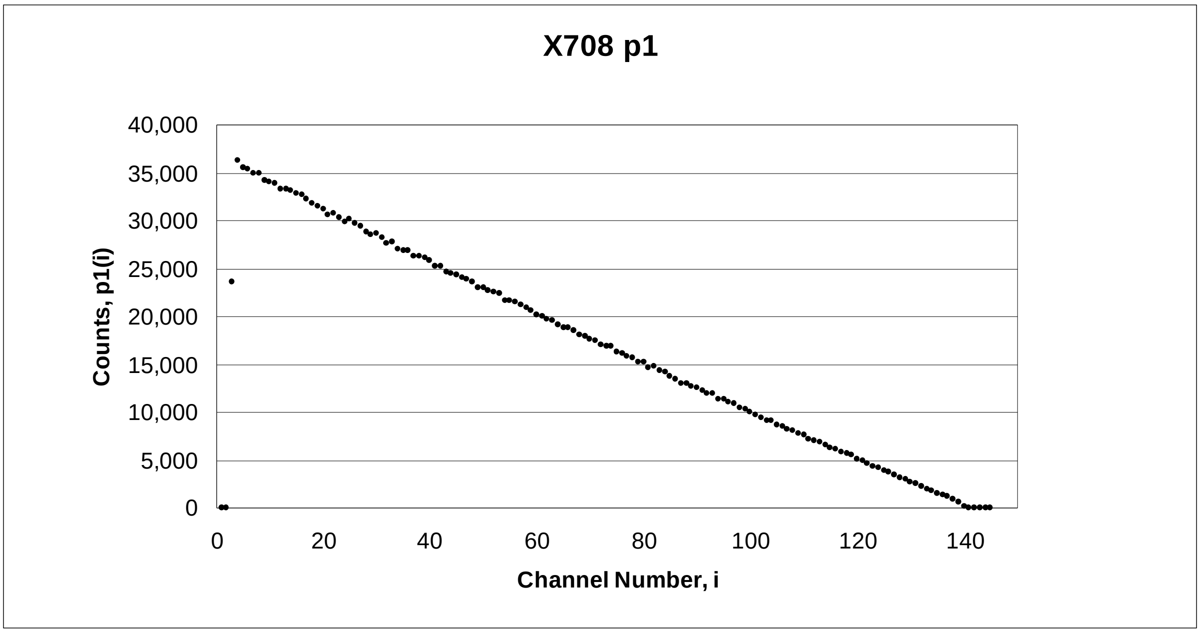
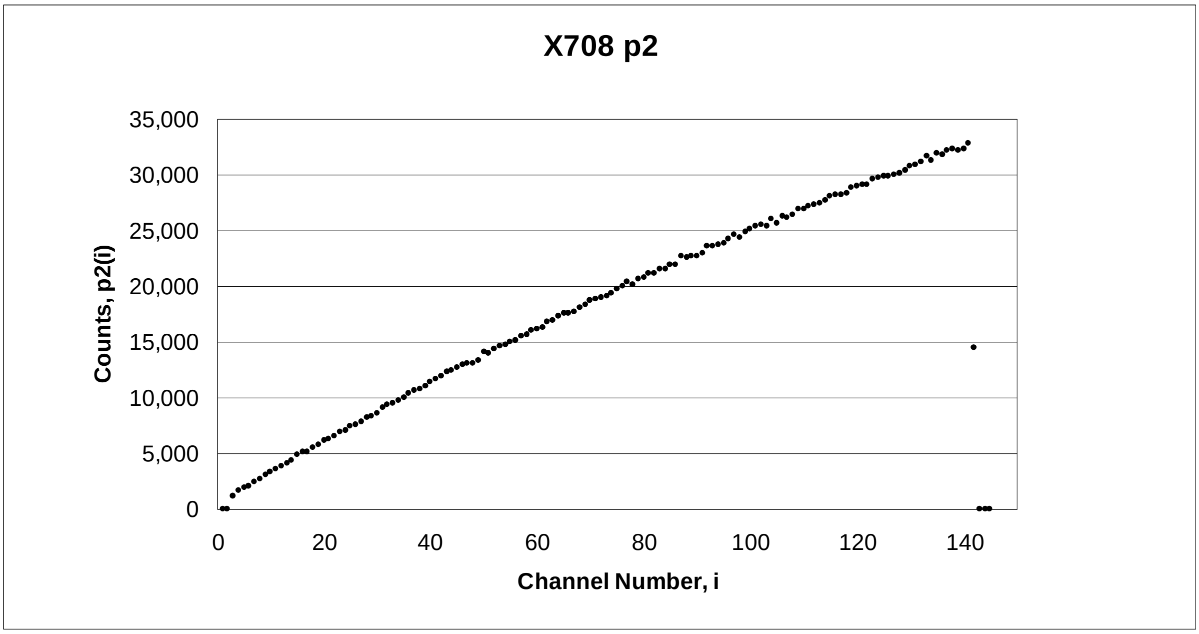


Figure 14. Experiment X708:- MCA Graph of q(i) versus i.





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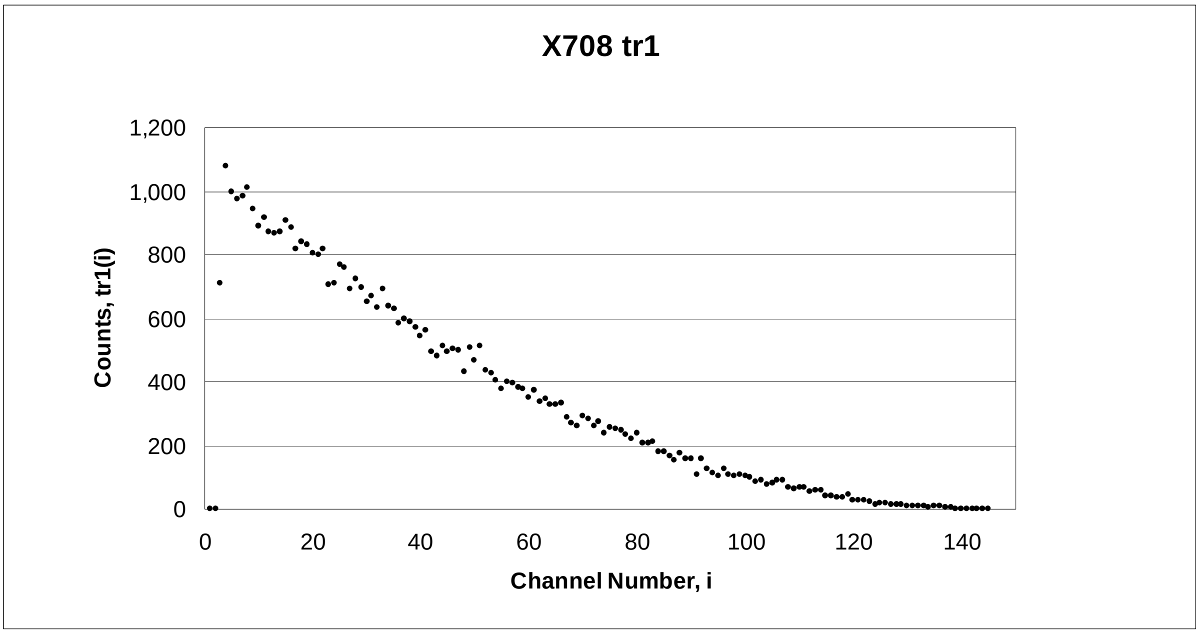


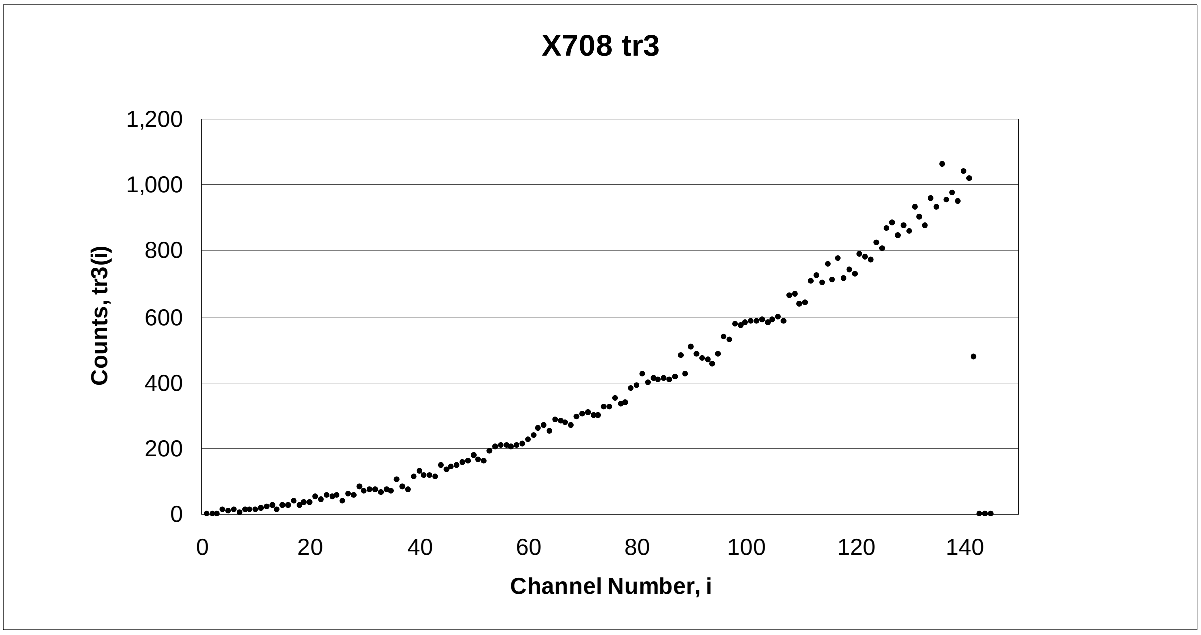
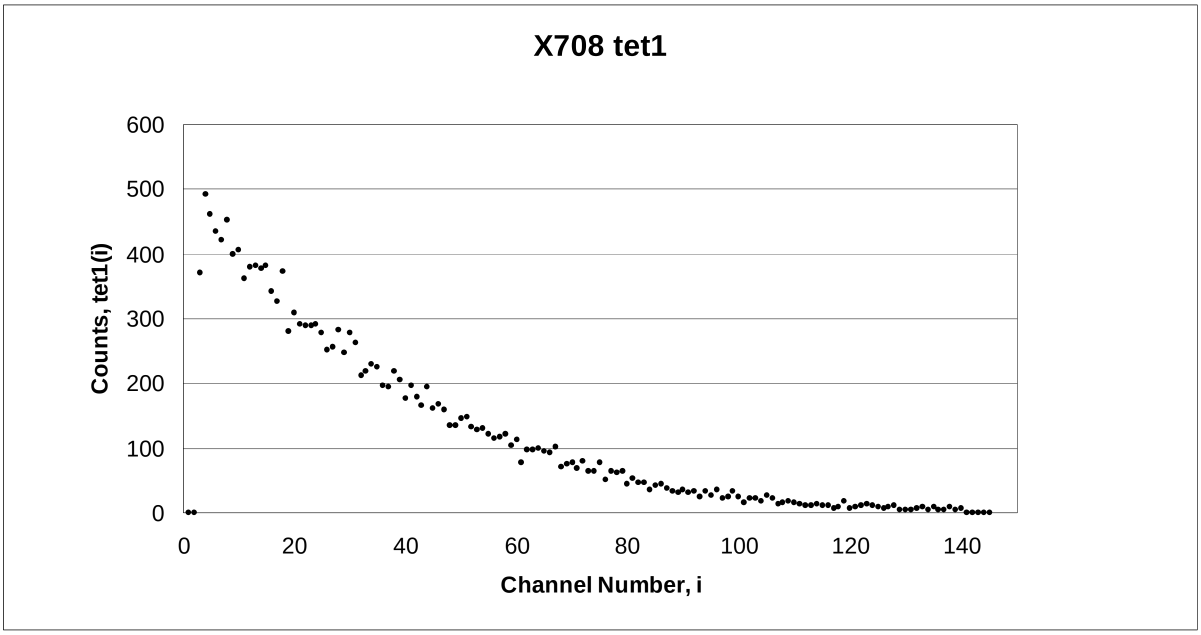


Figure 16. Experiment X708:- MCA Graphs of tr1(i) and tr3(i) versus i.



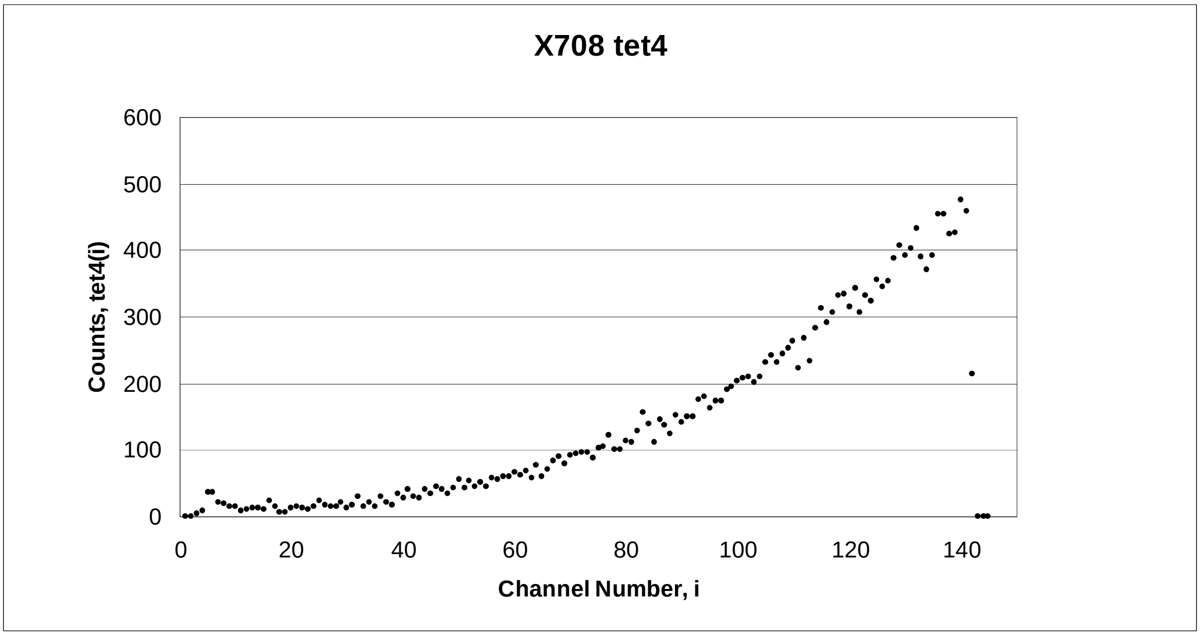


Figure 17. Experiment X708:- MCA Graphs of tet1(i) and tet4(i) versus i.

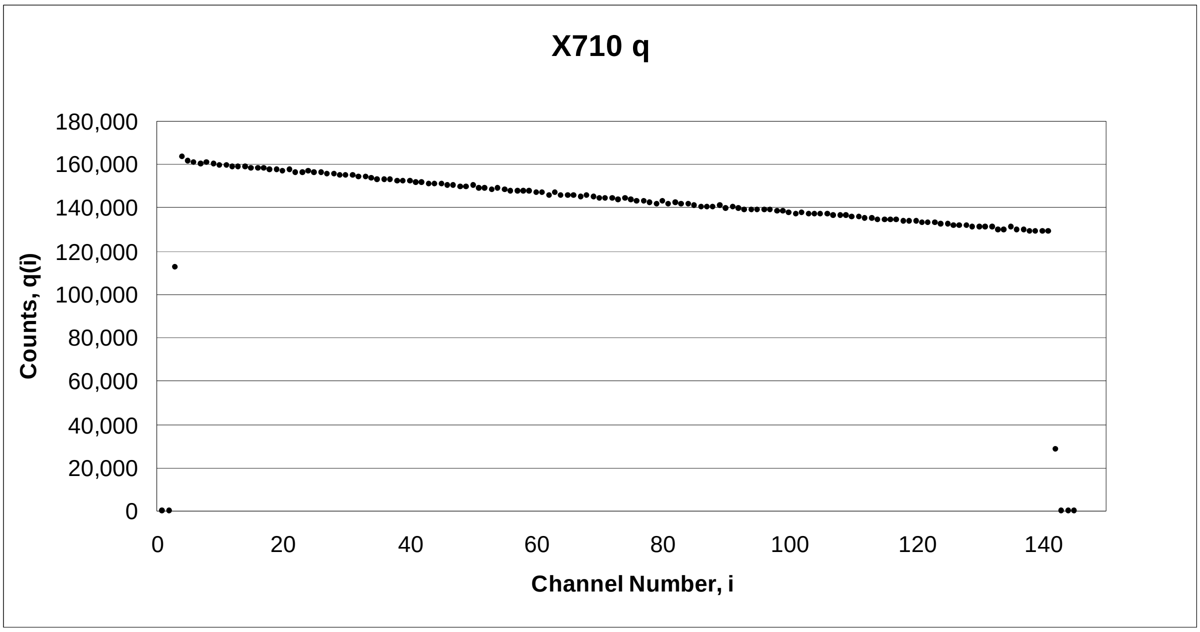
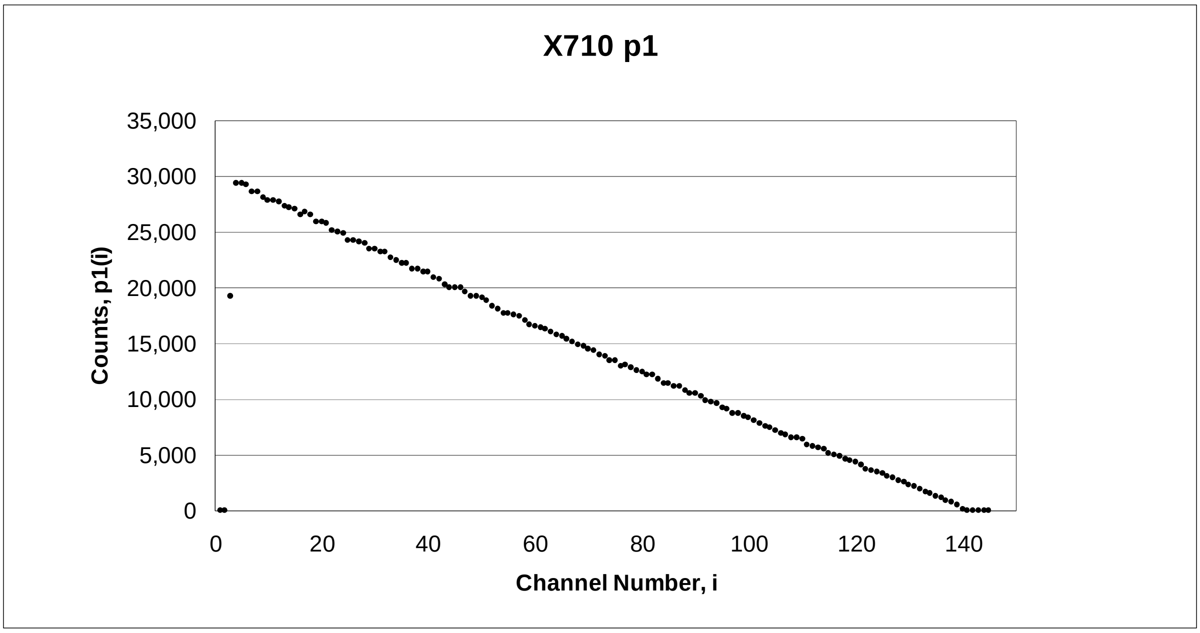


Figure 18. Experiment X710:- MCA Graph of q(i) versus i.



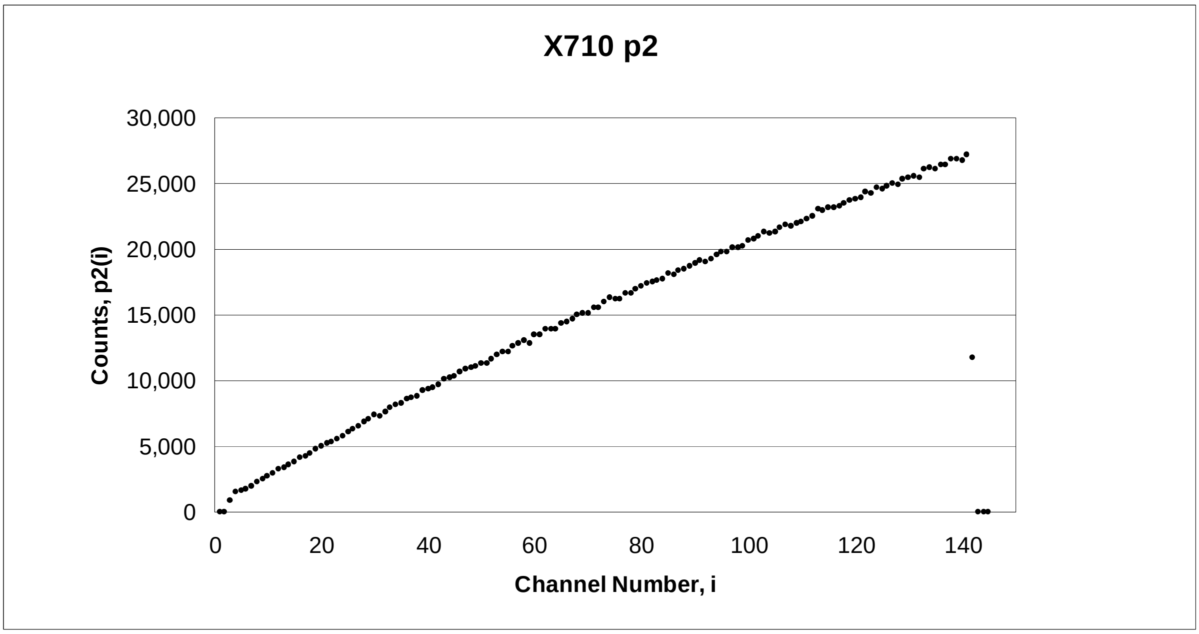
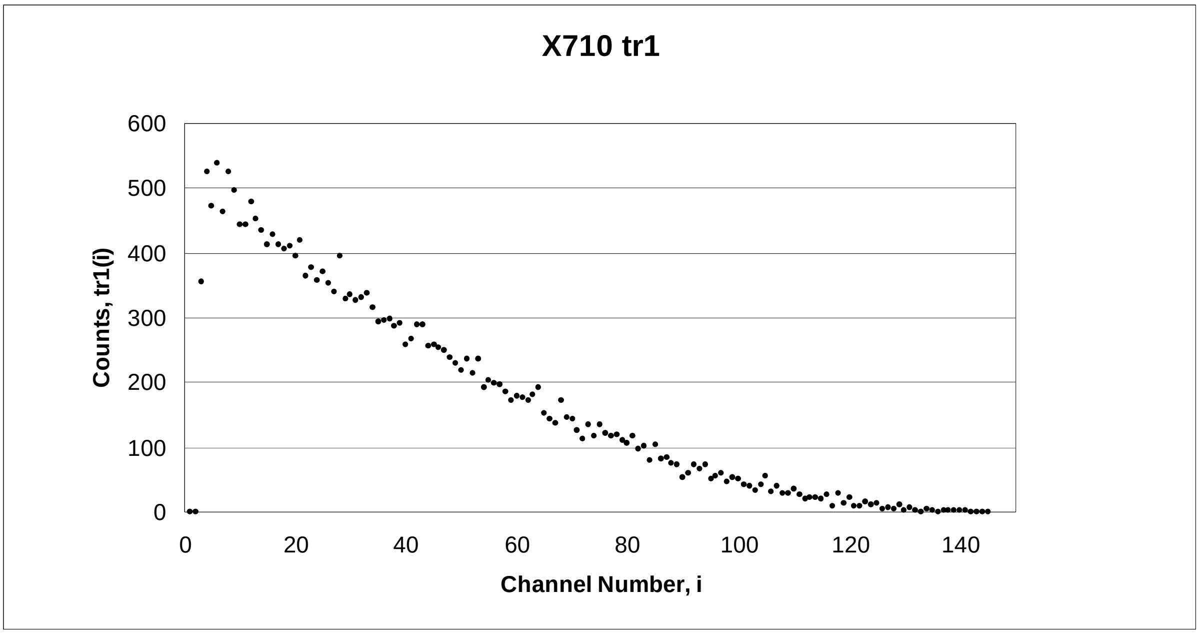


Figure 19. Experiment X710:- MCA Graphs of p1(i) and p2(i) versus i.



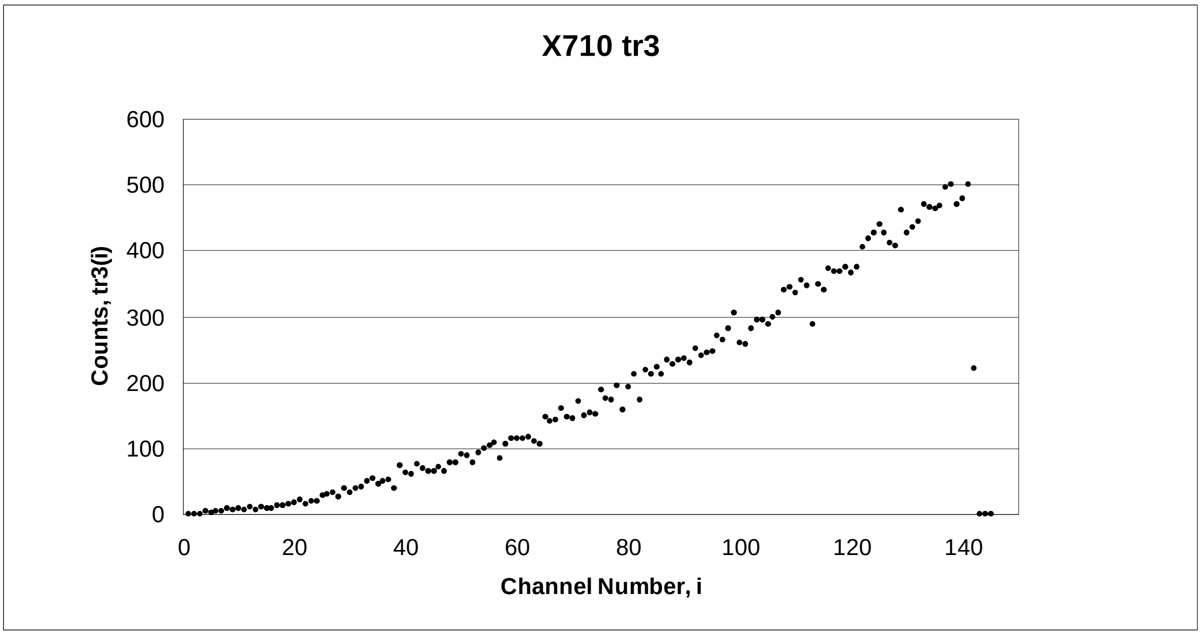
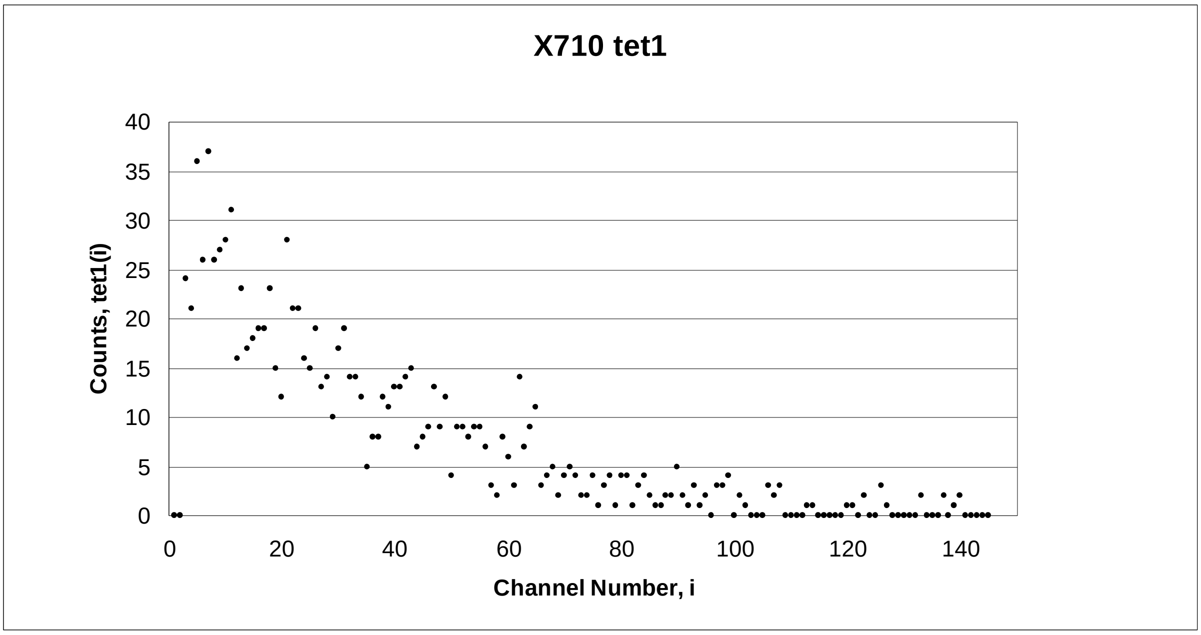


Figure 20. Experiment X710:- MCA Graphs of tr1(i) and tr3(i) versus i.



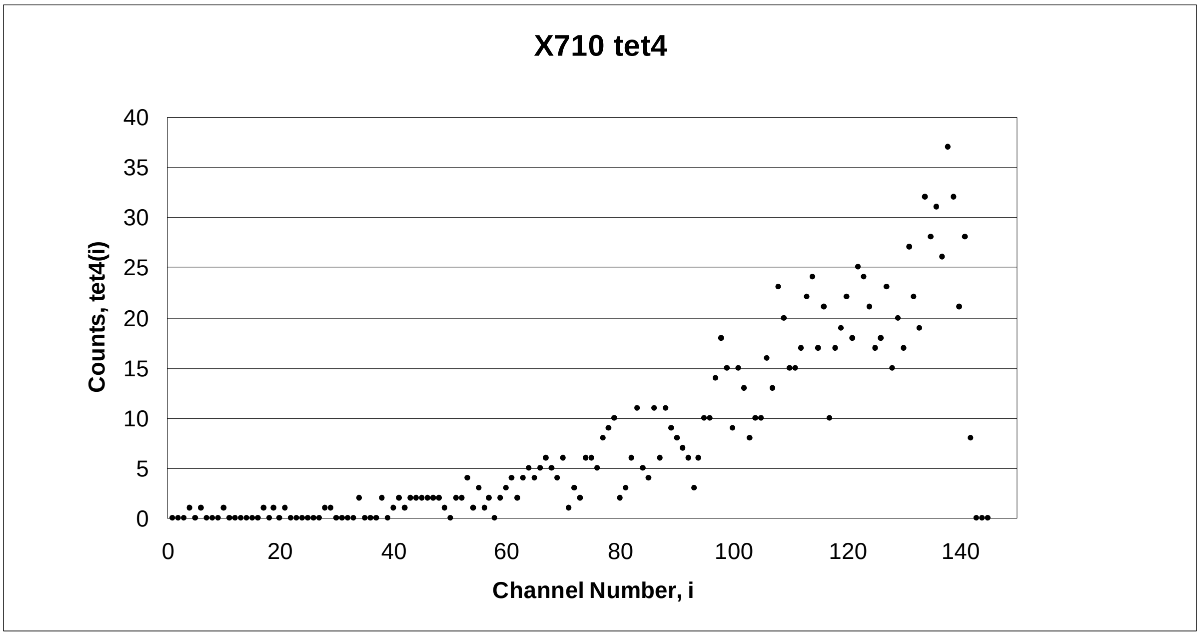


Figure 21. Experiment X710:- MCA Graphs of tet1(i) and tet4(i) versus i.

5. MULTI-CHANNEL-ANALYSER GRAPHS

Using TAC-1 (with internal strobing) a single pulse, recorded on MCA-1 during a 105 μs β Dead-Time period, may have been generated by the 1st of several disintegrations which were detected during that period, or by a single (sole) detected-disintegration. The number of such pulses, recorded as counts in MCA-1's channel-number i, is designated $q(i)$. A graph of $q(i)$ versus i is shown in Figure 7. It was plotted from a table of q(i) versus i. The table was saved as A:\DATA\X705q.txt on a floppy disc (for portability). The tables from which the graphs in Figures 8 to 21 were plotted, were similarly saved on A discs. The data saved on the A discs were used in the computer program TetD4.BAS.

Using TAC-1 (with external strobing) and TAC-2, the 1st and 2nd of a pair of detected-disintegrations were recorded on MCA-1 and MCA-2, respectively, during a 105 μs period. The number of 1st pulses recorded in MCA-1's channel-number i is designated $p1(i)$, and the number of second pulses recorded in MCA-2's channel-number i is designated $p2(i)$. A graph of $p1(i)$ versus i, and a graph of $p2(i)$ versus i are shown in Figure 8.

Less frequently, a trio of disintegrations is detected during a 105 μs period, giving rise to a trio of pulses. The 1st and 3rd pulses of the trio are recorded on MCA-1 and MCA-2, respectively. The number of 1st pulses recorded in MCA-1's channel-number i is designated $tr1(i)$, and the number of 3rd pulses recorded in MCA-2's channel-number i is designated $tr3(i)$. A graph of $tr1(i)$ versus i and a graph of $tr3(i)$ versus i are shown in Figure 9.

Still less frequently, a tetrad of disintegrations is detected during a 105 μs period, giving rise to a tetrad of pulses. This occurred in experiment X707. Referring to Figure 13, the number of 1st tetrad pulses which are recorded in channel-number i of MCA-1 is denoted by $tet1(i)$. The number of 4th tetrad pulses which are recorded in channel-number i of MCA-2 is denoted by $tet4(i)$. Referring to Figure 10, the $q(i)$ pulses recorded in channel-number i of

MCA-1 consist of the “sole” pulses recorded in channel-number i, plus p1(i), plus tr1(i), plus tet1(i).

6. SYSTEM FOR RECORDING 2ND AND 3RD STOP-PULSES IN A β SET-DEAD-TIME

See Block diagram (Figure 1) and Pulse diagram (Figure 3).

The pulse diagram shows pulses produced by 4 successive, detected disintegrations. Each β-discriminator positive pulse is followed by a negative STOP-pulse (from the STOP SCA). The START-SCA’s negative START-pulse is generated by the 1st detected disintegration. Delays are arranged so that the STOP-pulse generated by the 1st detected disintegration occurs 0.3 μs before the START-pulse (generated by the 1st disintegration) and is therefore not accepted.

Following the START-pulse, the 1st STOP pulse within the window is accepted at the TAC-1 STOP, giving rise to a Valid-Conv. pulse which opens the COINC. Linear-gate. The 2nd DISC.-pulse within the window goes through the COINC. Linear-gate to the G&DG. Two alternative G&DG Outputs are used:-

G&DG 0utput for tr1 and tr3. The G&DG output-pulse commences 0.3 μs after the 2nd STOP-pulse within the window, and opens the TAC-2 COINC.- STOP-Gate. The 3rd STOP-pulse within the window is accepted at the TAC-2 STOP; TAC-2 is internally strobed and TAC-1 is externally strobed. As a result, the tr1 and tr3 DDDGs are each augmented by one count.

G&DG Output for p1 and p2. The G&DG output-pulse commences 0.5 μs before the 2nd STOP-pulse (within the window) arrives at the TAC-2 STOP. (This happens as a result of a prior delay-adjustment.) Therefore, the TAC-2 STOP COINC.-Gate opens 0.5 μs before the arrival of the 2nd STOP-pulse (within the window) at TAC-2 STOP. As a result, the 2nd STOP-pulse (within the window) is accepted; TAC-2 is internally strobed, TAC-1 is externally strobed, and the p1 and p2 DDDGs are each augmented by one count.

7. SYSTEM FOR RECORDING 2ND, 3RD AND 4TH STOP-PULSES IN A β SET-DEAD-TIME.

See Block diagram (Figure 2) and Pulse diagram for the recording of tet1 and tet4 counts (Figure 4).

After the START pulse, the 1st STOP-pulse within the window is accepted at TAC-1 STOP, giving rise to a Valid-Conv. pulse which opens COINC. Linear-Gate-1. This is followed by Sequence (a), (b) or (c):-

Sequence (a). The 2nd DISC-pulse, (within the window) goes through COINC. Linear-Gate-1, giving rise to a G&DG-1 pulse which opens COINC. Linear-Gate-2. The 3rd DISC.-pulse (within the window) goes through COINC. Linear-Gate-2, giving rise to a G&DG-2 pulse (of 105 μs width) which opens the TAC-2 COINC. STOP-Gate. The 4th STOP-pulse (within the window) is accepted at TAC-2 STOP, and TAC-1 is externally strobed. As a result, the tet1 and tet4 DDDGs are each augmented by one count.

Sequence (b). See pulse diagram for the recording of tr1 and tr3 counts (Figure 5). The 2nd DISC.-pulse (within the window) goes through COINC. Linear-Gate-1, giving rise to a G&DG-1 pulse which opens COINC. Linear-Gate-2. The 3rd DISC.-pulse (within the window) goes through COINC. Linear-Gate-2. It then by-passes the "SCA before G&DG-2", giving rise to a G&DG-2 output-pulse which, commencing 1.0 μs earlier than in Sequence (a), opens the TAC-2 COINC. STOP-Gate 0.7 μs before the 3rd STOP-pulse (within the window) arrives at TAC-2 STOP. This 3rd STOP-pulse (within the window) is therefore accepted at TAC-2 STOP, and TAC-1 is externally strobed. As a result, the tr1 and tr3 DDDGs are each augmented by one count.

Sequence (c). See Pulse diagram for the recording of p1 and p2 counts (Figure 6). The 2nd DISC.-pulse (within the window) goes through Linear-Gate-1 to the G&DG-1 input. The

G&DG-1 output commences 0.5 μs <u>before</u>* the 2nd STOP-pulse (within the window). Linear-Gate-2 opens; the 2nd DISC.-pulse (within the window) goes through it, and the G&DG-2 output commences, causing the TAC-2 COINC STOP-Gate to commence opening 0.7 μs before the arrival of the 2nd STOP-pulse; the latter is therefore accepted at TAC-2 STOP.

8. COMMENTS ON THE RECORDING PROCEDURES

The accuracy of the Detected-Disintegration Distribution-Graphs is limited by the following:

a) The minimum SCA-Lower-Level of the TAC-2 is 0.05 V.
b) In both ADCs, the settings of the Lower-Level-Discriminator permit:
    i. no counts to be recorded in channels 1 and 2;
    ii. a reduced number of counts to be recorded in channel 3.
c) The Tandem-TAC Intrinsic Dead-Time is not corrected for. It is equal to the AAEC Type 494 β-amplifier output-pulse width, which is approximately 0.6 μs.

9. THE PERCENTAGE OF 3RD AND 4TH DETECTED-DISINTEGRATIONS IN THE 105 μs β DEAD-TIMES

The Tandem-TAC data observed during Experiments X707, X708, X709 and X710 are presented in Tables 1, 2, 3 and 4, respectively.

* Increasing the STOP-SCA delay allows the 2nd STOP-pulse output to be delayed until after the commencement of the output-pulses from the G&DG-1 and the G&DG-2.

Table 1. Experiment Number X707.

Integral		TAC-2 SCA Output	TAC-1 Valid Starts		β Scaler	γ Scaler	Real Time
Symbol	**Value**		**Symbol**	**Scaler**			**Seconds**
Itet1	14,117	---	**---**	---	---	---	---
Itet4	15,952	15,977	**Vtet4**	97,985,311	158,250,764	9,782,600	67,000
Itr1	52,489	---	**----**	---	---	---	----
Itr3	57,356	57,364	**Vtr3**	24,869,695	40,145,986	2,497,200	17,000
Ip1	2,240,625	---	**---**	---	---	---	---
Ip2	2,370,949	2,372,500	**V2**	89,521,865	143,896,469	8,989,100	61,000
Iq	26,459,885	---	**V3**	123,301,195	195,814,328	12,353,300	83,000

Table 2. Experiment Number X708.

Integral		TAC-2 SCA Output	TAC-1 Valid Starts		β Scaler	γ Scaler	Real Time
Symbol	**Value**		**Symbol**	**Scaler**			**Seconds**
Itet1	17,351	---	**---**	---	---	---	---
Itet4	20,088	20,108	**Vtet4**	116,631,568	188,221,696	11,540,400	80,000
Itr1	48,841	---	**----**	---	---	---	----
Itr3	53,356	53,361	**Vtr3**	23,358,392	37,648,057	2,332,800	16,000
Ip1	2,416,247	---	**---**	---	---	---	---
Ip2	2,560,865	2,562,600	**V2**	96,736,753	155,380,925	9,668,000	66,000
Iq	26,666,011	---	**V3**	124,557,786	197,563,621	12,381,100	84,000

Table 3. Experiment Number X709.

Integral		TAC-2 SCA Output	TAC-1 Valid Starts		β Scaler	γ Scaler	Real Time
Symbol	Value		Symbol	Scaler			Seconds
Itet1	850	---	**---**	---	---	---	---
Itet4	939	940	**Vtet4**	9,476,127	14,561,419	895,634	7,000
Itr1	19,654	---	**----**	---	---	---	----
Itr3	21,343	21,342	**Vtr3**	13,541,260	20,809,120	1,281,900	10,000
Ip1	1,757,465	---	**---**	---	---	---	---
Ip2	1,865,438	1,866,900	**V2**	88,213,757	135,251,178	8,257,500	65,000
Iq	16,846.685	---	**V3**	87,660,569	133,172,674	8,217,600	64,000

Table 4. Experiment Number X710.

Integral		TAC-2 SCA Output	TAC-1 Valid Starts		β Scaler	γ Scaler	Real Time
Symbol	Value		Symbol	Scaler			Seconds
Itet1	1,046	---	**---**	---	---	---	---
Itet4	1,154	1,155	**Vtet4**	9,814,043	15,409,222	930,249	7,000
Itr1	24,279	---	**----**	---	---	---	----
Itr3	26,629	26,636	**Vtr3**	14,024,659	22,021,795	1,330,800	10,000
Ip1	1,989,181	---	**---**	---	---	---	---
Ip2	2,110,334	2,111,900	**V2**	90,034,644	140,938,020	8,572,100	64,000
Iq	20,074,683	---	**V3**	99,529,670	154,208,395	9,240,400	70,000

Consider (in Figure 10) the X707 MCA graph of $q(i)$ and the X707 results in Table 1. The total number of detected-disintegrations recorded on the MCA graph of $q(i)$ versus i equals $\sum_i q(i)$, = Iq, the Integral value shown on the MCA screen.

Similarly, consider in turn the MCA graph of $p2(i)$ in Figure 11, $tr3(i)$ in Figure 12, and $tet4(i)$ in Figure 13, together with the X707 results in Table 1. We then obtain:

$$\sum_i p2(i) = Ip2\,;\ \sum_i tr3(i) = Itr3\,;\ \sum_i tet4(i) = Itet4\,.$$

Taking into account that the number of Valid Starts varies from run to run in an experiment, it can be seen that:-

The Percentage of 3rd Recorded-Disintegrations equals

$$Itr3 \div \left[Itr3 + \left(Ip2 \times \frac{Vtr3}{V2} \right) + \left(Iq \times \frac{Vtr3}{V3} \right) \right] \times 100\,.$$

The Percentage of 4th Recorded-Disintegrations equals

$$Itet4 \div \left[Itet4 + \left(Itr3 \times \frac{Vtet4}{Vtr3} \right) + \left(Ip2 \times \frac{Vtet4}{V2} \right) + \left(Iq \times \frac{Vtet4}{V3} \right) \right] \times 100$$

The percentages of the 3rd and 4th detected-disintegrations are listed in Table 5.

Table 5. The percentage of 3rd and 4th detected-disintegrations in the 105 μs β Dead-Times.

Expt. No.	**Detected-Disintegrations which could be recorded**				**Percentage of 3rd Detected-Disintegrations**	**Percentage of 4th Detected–Disintegrations**
X705	1st	2nd	3rd		0.945	---
X706	1st	2nd	3rd		0.764	---
X707	1st	2nd	3rd	4th	0.948	0.067
X708	1st	2nd	3rd	4th	0.941	0.071
X709	1st	2nd	3rd	4th	0.733	0.046
X710	1st	2nd	3rd	4th	0.836	0.052

10. TABULATION OF D4(i) VERSUS THE MCA CHANNEL-NUMBER, i

The tabulation is carried out by means of Computer Program TetD4.BAS (see Appendix A). The MCA channel-numbers in use extend from 1 to 160. During an experiment (see, e.g., Table 1) the number of TAC-1 Valid Starts (symbolised by Vtet4, Vtr3, V2 and V3) varies between the 4 runs. We therefore now calculate from tet4(i), tr3(i), p2(i) and q(i), the counts (denoted by $\text{tet4d(i)}, \text{tr3d(i)}$, p2d(i) and qd(i), respectively) which would have accumulated in Channel i during a period of V0 Valid Starts (V0 is usually the lowest of the TAC-1 Valid Starts).

Thus :-

$$\text{tet4d(i)} = \text{tet4(i)} \times \frac{\text{V0}}{\text{Vtet4}}$$

$$\text{tr3d(i)} = \text{tr3(i)} \times \frac{\text{V0}}{\text{Vtr3}}$$

$$\text{p2d(i)} = \text{p2(i)} \times \frac{\text{V0}}{\text{V2}}$$

$$\text{qd(i)} = \text{q(i)} \times \frac{\text{V0}}{\text{V3}}$$

For each of the 160 values of i, let $D4(i)$ denote the total number of counts accumulated in Channel i during a period of V0 Valid Starts. Thus, for each of the 160 values of i,

$$D4(i) = tet4d(i) + tr3d(i) + p2d(i) + qd(i) \, .$$

$D4(i)$ versus i (listed in tables 6, 7, 8 and 9) shows the distribution, among the 160 channels, of the detected-disintegrations (neglecting the occurrence of the 5^{th} and later detected-disintegrations during a 105 μs period). It can be seen that $D4(i)$ is almost constant with respect to i, from i = 4 to i = 141.

It may here be noted that in the computer program, there is a replacement of the symbol

$tet4(i)$ by $tet4dd(i)$,

$tr3(i)$ by $tr3dd(i)$,

$p2(i)$ by $p2dd(i)$, and

$q(i)$ by $qdd(i)$.

Table 6. Values of D4(i) for Experiment Number X707.

0	0	28746	43729	43570	43669	43592	43568
43457	43635	43551	43432	43736	43850	43635	43673
43732	43694	43760	43601	43743	43557	43644	43675
43782	43719	43720	43588	43507	43757	43657	43573
43526	43652	43743	43586	43854	43469	43639	43595
43556	43737	43738	43630	43366	43420	43767	43541
43569	43680	43704	43490	43518	43398	43645	43537
43590	43466	43542	43499	43699	43727	43562	43467
43607	43640	43416	43779	43669	43627	43515	43688
43626	43590	43546	43505	43458	43600	43780	43743
43477	43620	43691	43583	43464	43503	43696	43720
43588	43530	43617	43543	43452	43554	43618	43604
43656	43424	43494	43492	43537	43579	43576	43448
43606	43441	43595	43472	43415	43734	43689	43626
43496	43742	43543	43576	43642	43541	43706	43592
43690	43611	43658	43583	43481	43557	43618	43561
43621	43617	43708	43575	43491	43606	43398	43454
43687	43575	43455	43446	43469	11997	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

m = -.6830775055450445

b = 43645.54168111065

The x coordinate of the centroid = 72.5

The y coordinate of the centroid = 43596.01856195863

The ordinate of channel 141 = 43549.2277528288

V2 = 89521865

V3 = 123301195

Vtet4 = 97985311

Vtr3 = 24869695

V0 = 24869695

Table 7. Values of D4(i) for Experiment Number X708.

0	0	26536	41187	40775	40845	40754	41019
41065	41004	40916	40778	40750	40814	41019	40804
40943	40976	40788	40852	40909	40880	40904	40811
40959	40910	40805	40886	40997	40778	40999	40843
40890	40842	40935	40792	40983	40807	40850	40958
40698	40858	40966	40898	40757	40678	40839	40875
40772	40999	40875	40924	40850	40818	40947	40852
40896	40809	40985	40765	40822	40894	40819	40915
40863	40818	40751	40794	40943	40947	40834	40738
40770	40758	40826	40861	40879	40629	40918	40943
40902	40740	41007	40843	40933	40804	41051	40885
40833	40820	40911	40999	40803	40640	40804	40965
40954	40810	40872	40768	40931	40950	40778	40790
40807	40965	40689	40775	40928	40822	40821	40911
40804	40921	41148	40791	40737	40844	40894	40803
40774	40749	40904	40968	40725	40773	40872	40708
40911	40809	40961	40927	40808	40710	40804	40867
40851	40853	40817	40793	40872	11170	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

m = -.4449982467277501

b = 40892.55479849014

The x coordinate of the centroid = 72.5

The y coordinate of the centroid = 40860.29242560238

The ordinate of channel 141 = 40829.81004570153

V2 = 96736753

V3 = 124557786

Vtet4 = 116631568

Vtr3 = 23358392

V0 = 23358392

Table 8. Values of D4(i) for Experiment Number X709.

0	0	9382	14816	14736	14699	14694	14713
14697	14686	14696	14698	14640	14719	14668	14642
14659	14749	14683	14762	14646	14711	14683	14671
14780	14693	14704	14701	14661	14684	14666	14635
14679	14674	14695	14716	14597	14686	14713	14723
14667	14700	14674	14633	14681	14664	14697	14680
14668	14743	14639	14644	14670	14639	14665	14659
14739	14722	14606	14664	14655	14643	14697	14614
14708	14713	14667	14725	14633	14625	14684	14725
14660	14524	14657	14624	14662	14613	14670	14630
14660	14641	14622	14675	14650	14638	14650	14680
14675	14670	14638	14636	14670	14643	14617	14712
14634	14632	14654	14724	14591	14644	14633	14657
14595	14677	14627	14627	14634	14684	14647	14702
14704	14663	14600	14661	14654	14632	14677	14680
14631	14665	14606	14623	14670	14718	14652	14697
14699	14642	14617	14711	14649	14682	14624	14610
14542	14625	14642	14716	14660	3962	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

m = -.4249733303666094

b = 14697.84006347628

The x coordinate of the centroid = 72.5

The y coordinate of the centroid = 14667.0294970247

The ordinate of channel 141 = 14637.91882389459

V2 = 88213757

V3 = 87660569

Vtet4 = 9476127

Vtr3 = 13541260

V0 = 9476127

Table 9. Values of D4(i) for Experiment Number X710.

0	0	11197	16280	16070	16050	16045	16140
16075	16016	16055	16046	16065	16048	16028	16074
16083	16020	16070	16027	16094	15991	16020	16084
16092	16093	16038	16103	16054	16090	16061	16052
16071	16049	16031	16037	16088	16000	16051	16098
16017	16070	16015	16015	16047	16062	16051	16004
16042	16081	15952	15990	16006	16089	16046	16018
15989	16013	16005	16047	16050	15973	16054	15970
16028	16028	16024	16092	16036	16008	16067	16051
16019	16091	16055	15995	16013	16012	15960	16076
16035	16045	16051	16039	16032	15965	16022	16011
16083	16003	16078	16022	15989	16007	16021	16032
16064	16054	16098	16030	15992	16059	16040	16034
16063	16070	16035	16087	16079	16009	16046	16036
16032	16017	16045	16063	16048	16025	16035	16046
15987	16062	16079	16082	16066	16034	16013	15978
16029	16019	16050	16003	15967	16029	16093	16059
16050	16060	16024	15978	16042	4275	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

m = -.239743735577295

b = 16059.38490809608

The x coordinate of the centroid = 72.5

The y coordinate of the centroid = 16042.00348726673

The ordinate of channel 141 = 16025.58104137968

V2 = 90034644

V3 = 99529670

Vtet4 = 9814043

Vtr3 = 14024659

V0 = 9814043

11. DESCRIPTION OF THE COMPUTER PROGRAM TETD4.BAS

Part 1. The table of D4(i) versus i.

As noted in Section 5, the A disc contains files of the type :- A:\DATA\X705q.txt.

The symbol expt$ represents the number of an experiment, e.g., 710 (omitting the experiment-book prefix "X"). An experiment's file-name in the floppy disc is denoted by mcaFileName$. The program proper starts with :-

mcaFileName$ = "A:\DATA\X" + expt$ + "tet4.TXT"

This file is opened and sent to the sub-routine named mcaDATA, in which, using a FOR-NEXT loop, the MCA Report is converted into the Q-Basic-array $\mathrm{cts(i)}$, in which i has all the integer values from 1 to 160. Using a FOR-NEXT loop, the name of the array $\mathrm{cts(i)}$ is changed to $\mathrm{tet4dd(i)}$.

Following the same procedure, we obtain the arrays $\mathrm{tr3dd(i)}$, $\mathrm{p2dd(i)}$ and $\mathrm{qdd(i)}$. Each of the latter 4 arrays results from a different number of Valid Starts.

An expanded version of the seven-line FOR-NEXT loop used in the calculation of $\mathrm{D4(i)}$ is given below:-

FOR i = 1 to 160

$$\mathrm{D4(i)} = \left[\mathrm{qdd(i)} \times \frac{\mathrm{V0}}{\mathrm{V3}}\right] + \left[\mathrm{p2dd(i)} \times \frac{\mathrm{V0}}{\mathrm{V2}}\right] + \left[\mathrm{tr3dd(i)} \times \frac{\mathrm{V0}}{\mathrm{Vtr3}}\right] + \left[\mathrm{tet4dd(i)} \times \frac{\mathrm{V0}}{\mathrm{Vtet4}}\right]$$

NEXT i

Part 2. The Least-Squares-Line Equation: y = mx + b

The coordinates of a set of points, for which a least squares line is required, are usually denoted by (x_i, y_i).

Let the number of points in the set be denoted by n. Let $\sum_{i=1}^{n}$ be represented by Σ.

Let

$$\bar{x} = \frac{\sum x_i}{n} \text{ and } \bar{y} = \frac{\sum y_i}{n}.$$

Then,

$$m = \frac{\sum y_i x_i - \frac{\sum x_i \sum y_i}{n}}{\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}},$$

$$b = \bar{y} - m\bar{x}.$$

In Part 2, we replace the above coordinate x_i by i (the MCA channel-number) which has all the integer values from 4 to141. Thus n = 138. We replace the above coordinate y_i by the Array-element D4(i), which is as described at the end of Part 1, but with i restricted to all the integer values from 4 to 141.

A graph of $y = mx + b$ for Experiment X710 is shown in Figure 22. The line passes through three points:-

a) The intercept point, (0, b),
b) The centroid for the 138 points whose abscissae are the channel numbers from 4 to141,
c) The point (x_i, y_i), where x_i = 141.

The graph is produced by entering the coordinates of these three points into the Excel “Chart Wizard”.

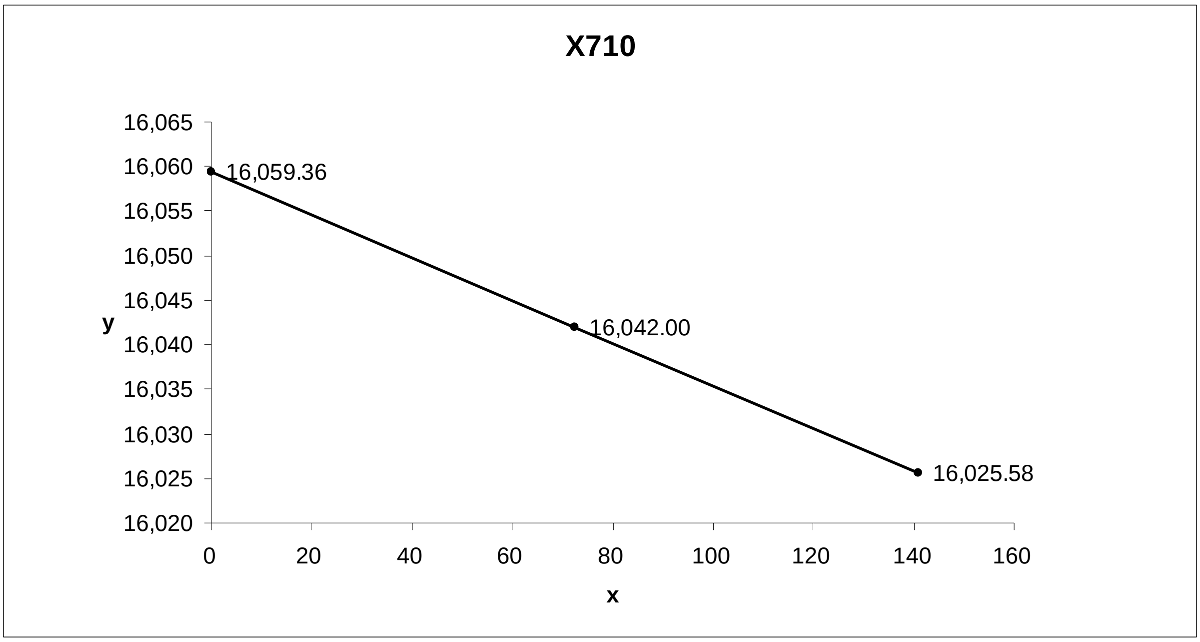


Figure 22 Graph of $y = mx + b$ for Experiment X710

12. THE VERTICAL COMPONENT OF THE SLOPE OF FOUR LEAST - SQUARES LINES

Tables (6, 7, 8 and 9) of $D4(i)$ versus i appear in Section 10. Referring to the graph in Figure 23, which represents (not to scale) the Least-Squares Lines for Experiments X707, X708, X709 and X710, the Vertical Component of the Least-Squares-Line Slope is equal to the fraction $\frac{b - Y}{b}$, where Y is the ordinate for channel 141. Table 10 gives the value of $\frac{b - Y}{b}$ for these four experiments.

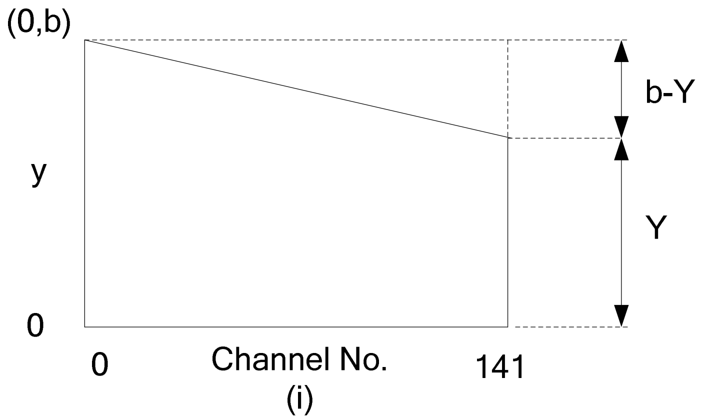


Figure 23. This graph represents the Least-Squares Lines for Experiments 707, 708, 709 and 710.

Table 10. Experiment Number and Vertical Component of the Least-Squares-Line Slope

Experiment No.	$\frac{b-Y}{b}$
X707	0. 00221
X708	0. 00153
X709	0. 00408
X710	0. 00210

13. THE DEAD-TIME CORRECTION EQUATION FOR ONE CHANNEL

In this section a new derivation is given of the non-extending-Dead-Time correction equation for a single counting channel:

$$NE_A = \frac{A'}{1 - A'T}\text{, where} \quad (1)$$

N = the disintegration rate of the source being measured;

E_A = the Efficiency of Channel A

= the Probability that Channel A, when live, will record a disintegration;

A' = the source count-rate, with the background rate not included; and

T = the non-extending Dead-Time imposed on the channel by the paralysis unit.

Equation (1), with background included, becomes

$$NE_A + A_b = \frac{A''}{1 - A''T}, \text{ where} \tag{2}$$

A_b = the background count-rate; and

A'' = the count-rate of the source plus background.

Equation (2) can be "simply" derived [NCRP 1985: 62] by assuming that the lost counts per second are equal to the product of the total Dead-Time per second, $A''T$, and ($NE_A + A_b$). Thus,

$NE_A + A_b = A'' + A''T(NE_A + A_b)$, which, when rearranged, gives equation (2).

Equation (2), neglecting background, can be derived more elaborately: e.g., Parzen [1962].

The new derivation of equation (1) considers the <u>states</u> of a single-channel counting system. The system can be in one of the two states shown in Figure 24. In state L the channel is Live, and in state D it is Dead for time T. The probabilities of a disintegration occurring in states L and D are denoted by P_L and P_D, respectively.

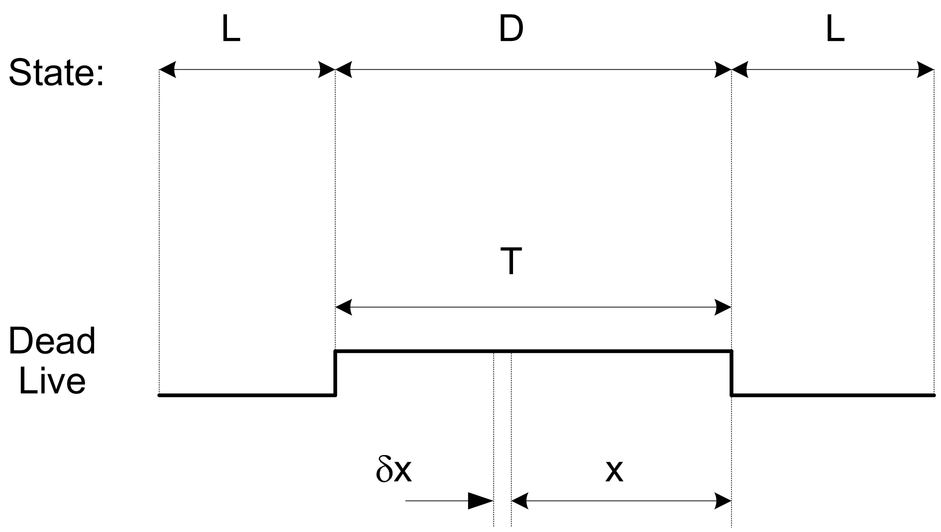


Figure 24. The two States of the System.

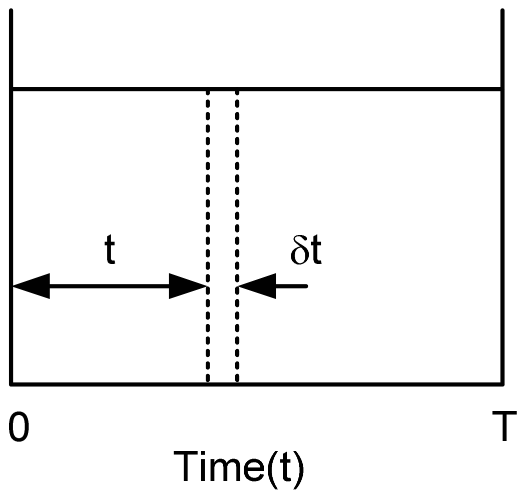


Figure 25. Distribution Graph during the Dead-Time Intervals.

Figure 25 shows the distribution graph of the disintegrations that occur during the Dead-Time intervals. Each Dead-Time interval starts at the instant of a disintegration which is detected in state L. We can assume from Table 10 (in Section 12) that the distribution graph of the disintegrations is virtually horizontal. Thus, the constant ordinate is equal to $\frac{P_D}{T}$; the Dead-Time starts and finishes at 0 and T, respectively; and δt denotes an increment of Time. The probability of a disintegration occurring during δt is $\delta t \times \frac{P_D}{T}$.

Adapting a procedure used by J. Bryant [1963], we now consider a pair of consecutive disintegrations. Suppose that a counting system is in state r immediately before the first disintegration, and in state s immediately before the second disintegration. The probability of this occurrence is denoted by (r.s). In a single-channel system, there are four (r.s) probabilities, as shown in Table 11. P_D is equal to the sum of the two (r.s) probabilities in the bottom row of the table:

$$P_D = (D.L) + (D.D). \tag{3}$$

P_D is also equal to the sum of the two (r.s) probabilities in the right-hand column of the table:

$$P_D = (L.D) + (D.D). \tag{4}$$

Equation (4) is discussed in Appendix B. Equating the right-hand sides of equations (3) and (4),

$$(D.L) = (L.D). \tag{5}$$

Table 11. The four (r.s) probabilities of a single-channel system.

Second state / First state	L	D
L	(L.L)	(L.D)
D	(D.L)	(D.D)

In Figure 24, x is the interval between the instant of the first of a pair of consecutive disintegrations and the end of the Dead-Time. The probability that no disintegration occurs during the interval x is e^{-Nx}. The probability that the first of the pair occurs between x and $x+\delta x$, and that the second disintegration does not occur during the Dead-Time is $P_D \frac{\delta x}{T}\left(e^{-Nx}\right)$.Thus,

$$\left(D.L\right)=\int_0^T P_D \frac{1}{T} e^{-Nx} dx = P_D \frac{1}{T}\left[-\frac{1}{N}e^{-Nx}\right]_0^T = P_D \frac{1}{NT}\left(1-e^{-NT}\right).$$

The probability, (L.D), that the first disintegration occurs during state L, that it is detected, and that at least one disintegration occurs during the Dead-Time (T) is $P_L E_A\left(1-e^{-NT}\right)$.

Substituting the above formulae for (D.L) and (L.D) into equation (5), $P_D \frac{1}{NT} = P_L E_A$. Simultaneously, $P_L + P_D = 1$. Combining these two simultaneous equations, $P_D = P_L E_A NT = 1 - P_L$. Dividing by P_L and rearranging,

$$NE_A T = \frac{1}{P_L} - 1. \tag{6}$$

$NP_L E_A$ is the number of disintegrations per second which (a) occur when the channel is live, (b) are detected, and recorded by the scaler; i.e., $NP_L E_A = A'$ (Count-rate when background is negligible).

Thus,

$$P_L = \frac{A'}{NE_A} \tag{7}$$

In equation (6) P_L is replaced by the right-hand side of equation (7):

$$NE_A T = \frac{NE_A}{A'} - 1 .$$

Multiplying both sides by $-A'$,

$$-NE_A TA' = -NE_A + A' .$$

Rearranging twice,

$$NE_A (1 - TA') = A' ,$$

$$NE_A = \frac{A'}{1 - A'T} . \tag{1}$$

14. REFERENCES

Bryant, J. [1963] - "Coincidence counting corrections for dead-time loss and accidental coincidences".

Int. J. Appl. Radiat. Isot., **14**: 143.

NCRP [1985] – "A Handbook of Radioactivity Measurements Procedures, NCRP Report No. 58". 2nd Edition. National Council on Radiation Protection and Measurements, Bethesda, Maryland.

Parzen, Emanuel [1962] – "Stochastic Processes".

Holden - Day, Inc., San Francisco. Page 181.

APPENDIX A – THE PROGRAM TETD4.BAS

```
DECLARE SUB mcaData ()

CLS

REM      The file name of this program is C:\BASIC\TetD4.BAS

REM   This program is an amalgamation of Tetrad.BAS and Ba88Pts.BAS.
REM   It compiles a table of the MCA-channel counts, D4(i), versus the
REM   MCA-channel number, i. The Least-Squares-Line constants, m and b,
REM   are then derived for the set of points whose x and y coordinates
REM   are i and D4(i).

DEFDBL A-Z

READ chMax       'from the DATA statement.
REM      chMax denotes the maximum channel-number used in the calculations.
REM      See REM in the sub-routine.

DIM tet4dd(chMax)   AS DOUBLE
DIM tr3dd(chMax)   AS DOUBLE
DIM p2dd(chMax)   AS DOUBLE
DIM qdd(chMax)    AS DOUBLE
DIM D4(chMax)      AS DOUBLE
DIM cts(chMax)    AS DOUBLE

INPUT "Type the number of the experiment :  "; expt$
INPUT "Type V2"; V2
INPUT "Type V3"; V3
INPUT "Type Vtet4"; Vtet4
INPUT "Type Vtr3"; Vtr3
INPUT "Type V0"; V0

LPRINT
LPRINT
LPRINT
LPRINT "      The file name of this program is C:\BASIC\TetD4.BAS"
LPRINT
LPRINT "   This program is an amalgamation of Tetrad.BAS and Ba88Pts.BAS. "
LPRINT "   It compiles a table of the MCA-channel counts, D4(i), versus the "
LPRINT "   MCA-channel number, i. The Least-Squares-Line constants, m and b, "
LPRINT "   are then derived for the set of points whose x and y coordinates "
LPRINT "   are i and D4(i). "
LPRINT

mcaFileName$ = "A:\DATA\X" + expt$ + "tet4.TXT"
OPEN mcaFileName$ FOR INPUT AS #1 LEN = 360 '= 180 integers multiplied by 2 bytes
```

```
REM                                                  (see the last REM in the sub-routine).
mcaData
CLOSE #1
REM                                     We now have the array cts().

FOR i = 1 TO chMax
   tet4dd(i) = cts(i)
NEXT i

mcaFileName$ = "A:\DATA\X" + expt$ + "tr3.TXT"
OPEN mcaFileName$ FOR INPUT AS #1 LEN = 360
mcaData
CLOSE #1

FOR i = 1 TO chMax
   tr3dd(i) = cts(i)
NEXT i

mcaFileName$ = "A:\DATA\X" + expt$ + "p2.TXT"
OPEN mcaFileName$ FOR INPUT AS #1 LEN = 360
mcaData
CLOSE #1

FOR i = 1 TO chMax
   p2dd(i) = cts(i)
NEXT i

mcaFileName$ = "A:\DATA\X" + expt$ + "q.TXT"
OPEN mcaFileName$ FOR INPUT AS #1 LEN = 360
mcaData
CLOSE #1

FOR i = 1 TO chMax
   qdd(i) = cts(i)
NEXT i

FOR i = 1 TO chMax
   tet4d = tet4dd(i) * V0 / Vtet4
   tr3d = tr3dd(i) * V0 / Vtr3
   p2d = p2dd(i) * V0 / V2
    qd = qdd(i) * V0 / V3
   D4(i) = qd + p2d + tr3d + tet4d
NEXT i

LPRINT
LPRINT "            Number of experiment :   "; expt$
LPRINT
LPRINT
```

```
LPRINT "         The values of D4(I) are :   "
LPRINT
    i = 1
DO
    FOR add = 0 TO 7
      LPRINT USING "######## "; D4(i + add);
    NEXT add
    i = i + 8
    LPRINT " "
LOOP UNTIL i = chMax + 1

DATA 160

REM  This next part was written by H. A. Wyllie during Autumn, 2006.
REM  The equation for the Least-Squares Line is :-  y = m x + b.
REM  The data in this program are values of y and x. The values of
REM  m and b are calculated when the program is run.
LPRINT : LPRINT
LPRINT

REM  The number of points on the graph = n
       n = 138

DIM Y(170)   AS DOUBLE
DIM X(170)   AS DOUBLE
DIM YX(170)   AS DOUBLE
DIM XSqd(170)   AS DOUBLE

REM       SigmaY = Y(1) + Y(2) + ... + Y(n)
REM       SigmaX = X(1) + X(2) + ... + X(n)
REM        YX(i) = Y(i) * X(i)
REM      SigmaYX = YX(1) + YX(2) + ... + YX(n)
REM      XSQD(i) = The square of X(i)
REM    SigmaXSqd = XSQD(1) + XSQD(2) + ... + XSQD(n)

FOR i = 1 TO 141
   Y(i) = D4(i)
NEXT i

FOR i = 1 TO 141
   X(i) = i
NEXT i

 SigmaY = 0
 FOR i = 4 TO 141
    SigmaY = SigmaY + Y(i)
 NEXT i
```

```
SigmaX = 0
FOR i = 4 TO 141
   SigmaX = SigmaX + X(i)
NEXT i

FOR i = 4 TO 141
   YX(i) = Y(i) * X(i)
NEXT i

SigmaYX = 0
FOR i = 4 TO 141
   SigmaYX = SigmaYX + YX(i)
NEXT i

FOR i = 4 TO 141
   XSqd(i) = X(i) ^ 2
NEXT i

SigmaXSqd = 0
FOR i = 4 TO 141
   SigmaXSqd = SigmaXSqd + XSqd(i)
NEXT i

m = (SigmaYX - ((SigmaX * SigmaY) / n)) / (SigmaXSqd - ((SigmaX ^ 2) / n))

b = (SigmaY / n) - ((m * SigmaX) / n)

LPRINT "                         m =  "; m
LPRINT
LPRINT "                         b =  "; b
LPRINT
LPRINT "            The x coordinate of the centroid =  "; SigmaX / n
LPRINT
LPRINT "            The y coordinate of the centroid =  "; SigmaY / n
LPRINT
LPRINT "            The ordinate of channel 141 =  "; (m * 141) + b
LPRINT
LPRINT "                        V2 =  "; V2
LPRINT
LPRINT "                        V3 =  "; V3
LPRINT
LPRINT "                     Vtet4 =  "; Vtet4
LPRINT
LPRINT "                      Vtr3 =  "; Vtr3
LPRINT
LPRINT "                        V0 =  "; V0

LPRINT CHR$(12)
```

```
END

SUB mcaData
SHARED cts() AS DOUBLE, i

REM  A Genie-2000 mca-report is converted into a Q-Basic array. There are no
REM  counts in the mca-channels whose channel-numbers exceed 160. Hence, the
REM  last member of the array is cts(160), i.e., cts(chMax). The array is
REM  compiled from the first 20 rows of the report; each row contains the
REM  counts recorded in 8 consecutive mca-channels.

  i = 1

   FOR rowNum = 1 TO 20
      INPUT #1, chNum, cts(i), cts(i + 1), cts(i + 2), cts(i + 3), cts(i + 4), cts(i + 5), cts(i + 6),
cts(i + 7)
      i = i + 8
   NEXT rowNum

REM   Including chNum, there are 9 integers in each row;  9 * 20 = 180 integers.
END SUB
```

APPENDIX B – THE EQUATION: (L.D) + (D.D) = P_D

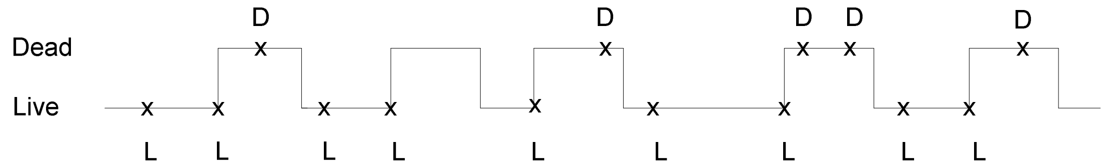


Figure 26.

In Figure 26, x signifies the instant of a disintegration, L indicates that the disintegration occurs when the system is Live, and D indicates that the disintegration occurs when the system is Dead. During the period represented in Figure 26, there are 14 disintegrations.

P_L = the fraction of the disintegrations which occur when the system is Live = 9/14.

P_D = the fraction of the disintegrations which occur when the system is Dead = 5/14.

Let (L.D) denote the fraction of the following type of disintegration:-

a) It occurs when the system is Dead;
b) It is the 2nd disintegration of a consecutive pair, the 1st disintegration of which occurs when the system is Live.

Thus, $(L.D) = \frac{4}{14}$.

Let (D.D) denote the fraction of the following type of disintegration:-

a) It occurs when the system is Dead;
b) It is the 2nd disintegration of a consecutive pair, the 1st disintegration of which occurs when the system is Dead.

Thus, $(D.D) = \frac{1}{14}$.

Therefore $(L.D) + (D.D) = \frac{5}{14} = P_D$.