



**AUSTRALIAN ATOMIC ENERGY COMMISSION
RESEARCH ESTABLISHMENT
LUCAS HEIGHTS**

**TRAJECTORY CALCULATIONS FOR LIGHT-PARTICLES EMITTED
IN SPONTANEOUS TERNARY FISSION**

by

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ABSTRACT

Trajectory calculations have been carried out for ^{252}Cf fission accompanied by ^1H , ^2H , ^3H , ^4He and ^3He particles. Initial dynamic variables of the fissioning system were found which gave satisfactory fits to the observed energy spectra. It was found that the heavy fragment separation at scission increases as the mass of the third particle decreases.

The energy cost in releasing each of the particles was calculated and three factors influencing the fractional yields were identified.

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1. INTRODUCTION

In recent years several investigators have discovered evidence for the occurrence of spontaneous ternary fission modes of ^{252}Cf in which light charged particles with $Z \leq 6$ are produced. (Whetstone and Thomas 1967, Cosper, Cerni and Gatti 1967, and Raisbeck and Thomas 1968.) The angular distributions for these particles, where measured, show a characteristic peaking at right angles to the fission axis. It follows that the particles are emitted from the region between the heavy fragments within a short time interval ($\sim 10^{-21}$ sec) of the moment of scission.

The localisation in both time and place of the light particle emission makes these particles an important source of knowledge about the nuclear configuration and dynamics at the moment of scission. It is the aim of trajectory calculations to obtain an initial set of distributed dynamic variables which reproduces the experimental energy and angular distributions of the emitted particle.

The α -particle accompanied mode has been the most frequently studied and trajectory computations have been carried out by Boneh, Fraenkel and Nebenzahl (1967), Raisbeck and Thomas (1968) and Musgrove (1971). Raisbeck and Thomas found that the initial conditions which gave agreement with the final alpha-particle energy spectrum, did not in general reproduce the measured energy spectra for other particles emitted in ternary fission of ^{252}Cf . Blocki and Krogulski (1968) arrived at a similar conclusion in respect of the ternary fission of ^{236}U . Nardi, Boneh and Fraenkel (1969) fitted the observed energy spectra of ^1H , ^2H , ^3H , ^4He , ^6He and ^8He from ^{252}Cf fission and obtained different initial conditions for each mode.

The calculations reported here were performed before the author was aware of the paper by Nardi et al. (1969) and since our results are in substantial agreement we give independent confirmation of the earlier calculations.

Calculations have also been carried out for the average energy required to emit each of the light particles previously mentioned, in ternary fission. If the various particles are released by a common mechanism, the observed fractional yield for each particle could be expected to be anti-correlated with this energy.

2. THE MODEL

A classical three point charge model of the fissioning system is employed in the trajectory calculations and the equations of motion are integrated numerically. The final quantities obtained are the energies of the three particles and the angle between the ejected particle direction and the light fragment direction. In the initial configuration the three charges are assumed to be collinear and subsequent motion therefore occurs in a plane.

The dynamic variables used to fit the final particle energy spectra in these

calculations are \bar{E}_0 , the initial average energy of particle, V_H , the initial velocity of the heavy fragment and D , the initial separation of the fission fragments.

The initial kinetic energy of the particle was assumed to have a Maxwellian distribution and the point of materialisation of the particle was taken to be normally distributed about the position of minimum potential energy. The standard deviation in initial position was linked with the standard deviation in initial momentum through the uncertainty principle:

$$\sigma_x \sigma_p = n\hbar \quad \dots(1)$$

The value of n assumed for the present calculation was 1.5 which had been found to give satisfactory agreement for the α -particle energy distribution (Musgrove 1971).

In all calculations the distributions of V_H and D were assumed to be δ functions. Values of V_H and D were found which reproduce the average final kinetic energy of the heavy fragments. This quantity has been measured by Nardi, Gazit and Katcoff (1969) for proton, triton and α -particle accompanied fission. For deuteron accompanied fission we use the value measured for the triton accompanied mode while for the ^6He and ^8He modes a value of ~ 170 MeV has been assumed for the final heavy fragment energy.

Trajectory calculations were performed for three mass ratios $R = 1.2, 1.4$ and 1.6 and these were added in the ratio 1:2:2 in the final calculated energy and angular distributions.

To calculate the energies required to emit the various particles at the moment of scission the model described by Feather (1969) was used. We examined only the two modes in which the particle separates pre-formed from one or other of the heavy fragments. According to Feather (1969) the energy required for an exoergic separation is the sum of the conventional binding energy of the particle to the heavy nucleus in its ground state and a coulomb potential energy term. In addition, there is the initial kinetic energy of the third particle which is found from the trajectory calculations.

The particle binding energies were calculated using the mass formula of Myers and Swiatecki (1966, 1968) and were averaged over the mass distribution for the light and heavy fragments respectively. The coulomb energy term was calculated for three point charges, the heavy fragments having the separation D found from the trajectory calculations with the third particle at the position of minimum potential energy. This calculation provides no more than a crude estimate for the coulomb term which ought to be calculated for the deformed charge distributions before and after separation of the third particle.

3. RESULTS

In Table 1, values for the initial dynamic variables of the system which give satisfactory agreement with the measured particle kinetic energy spectra are given. The numbers given for the α -particle accompanied mode come from Musgrove (1971). The standard deviation in the initial position of the particle is also given along with the most probable final angle with respect to the light fragment direction and the FWHM of the angular distribution.

Figures 1-5 compare the calculated and experimental energy spectra. The experimental data are those of Cosper et al. (1967) with additional measurements by Raisbeck and Thomas (1968) and Nardi et al. (1969). The proton spectrum given by Raisbeck and Thomas (1968) was gathered at 90° to the fission axis and we have compared a calculated energy spectrum for protons having final angles in the range $85^\circ - 95^\circ$ to the fission axis.

Except for the calculated spectrum for ^8He , reasonable agreement was obtained with experiment. For ^8He the calculated spectrum is considerably broader than experiment, a difficulty also encountered by Nardi et al. (1969). The ^6He spectrum also appears to be too broad on the low energy side of the peak. It was found impossible with our model to decrease the width of these distributions appreciably without at the same time obtaining extremely narrow angular distributions. However Raisbeck and Thomas (1968) found that the widths of the angular distributions for ^4He and ^6He were approximately equal.

The initial parameters obtained here are in good qualitative agreement with those found by Nardi et al. (1969) although individually our values throughout are slightly lower than theirs. The main point of difference occurs for triton emission for which Nardi et al. (1969) found an initial separation of the heavy fragments equal to that found for alpha particles. We require a somewhat smaller heavy fragment separation for triton emission relative to that for α -particles. In all other respects however, we confirm the trend found by Nardi et al. (1969) that the value of D increases as the mass of the particle decreases. It is, of course, impossible to say how much this conclusion depends on the classical model which has been assumed, remembering especially that the model appears to break down for the heavier mass particles ^6He and ^8He .

The calculated average energy required to emit each of the particles considered from the light and heavy fragments in turn is given in Table 2. In addition we have calculated the energy required to emit ternary neutrons. The angular distribution of prompt neutrons emitted in fission cannot be fully explained by assuming that all neutrons are evaporated from the fully accelerated fission fragments. A residue, upon detailed analysis, appears to have been emitted isotropically in the laboratory system with somewhat greater energy, on average,

than the neutrons evaporated later from the moving fragments. Bowman et al. (1962) found that their measured angular distribution could be fitted by assuming a scission neutron component of at least 10 per cent of the total. The fractional yield of neutron accompanied ternary fissions would accordingly be about 0.4 of all fissions in ^{252}Cf . The fractional yields for the other particles were obtained by averaging the measurements of Whetstone and Thomas (1967), Cospér et al. (1967) and Raisbeck and Thomas (1968) after normalising to an α -particle yield of 3.27×10^{-3} per fission.

In all cases less energy is expended, on average, when the third particle is emitted entirely from the nascent heavy fragment than when it comes out of the light fragment. The minimum energy release is plotted in Figure 6 against the logarithm of the fractional yield and a strong measure of anti-correlation between these two quantities can be observed. The observed fractional yields will also depend on a factor expressing the relative probability for reaching the configurations appropriate to the emission of each light particle. Thus, while α -particles are emitted 60 times more frequently in ternary fission than protons, our calculated energies favour proton emission by 0.6 MeV on average. Competition with binary fission and ternary fission in other channels would make relatively unlikely the configuration in which the heavy fragments separate sufficiently to be able to emit a proton.

We have yet to deal with the main conclusion which emerged from the trajectory calculations, namely that the heavy fragment separation increases as the mass of the emitted particle decreases. Naively interpreted it would appear that the increased stretching of the neck progressively breaks down the structure of the nascent third particle until finally at large separations single protons can be emitted. The calculations of Feather (1971) who found that alpha-particle release appears to take place near a critical heavy fragment separation about which only a small variation is allowed, lend some weight to this interpretation. Evidently the probability for pre-formation of an alpha-particle remains significant over only a small range of heavy fragment separations.

We have therefore identified three factors which are expected to influence the observed fractional yields; the energy required to emit the particle, the intrinsic pre-formation probability of the particle in the neck of the fissioning system and the probability that a configuration favourable to particle release be reached in competition with binary fission. Trajectory calculations indicate that the second factor peaks at different heavy fragment separations for different particles while the calculations of Feather (1971) show that for alpha particles, at least, its width may be narrow.

Because of the large yield of the neutron-accompanied ternary mode, it appears unlikely that neutron emission can occur solely when the heavy fragment separation is large. Neutron emission may therefore be 'anomalous' in terms of the trend

uncovered here for the other particles of an increasingly stretched configuration as the particle mass decreases.

4. REFERENCES

- Blocki, J. and Krogulski, T. (1968). - Nucl. Phys. A122:417.
- Boneh, Y., Fraenkel, Z. and Nebenzahl, I. (1967). - Phys. Rev. 156:1305.
- Bowman, H.R., Thompson, S.G., Milton, J.C.D. and Swiatecki, W.J. (1962). - Phys. Rev. 126:2120.
- Cosper, S.W., Cerny, J. and Gatti, R.G. (1967). - Phys. Rev. 154:1193.
- Feather, N. (1969). - Proc. Roy. Soc. (Edinburgh) 68A:229.
- Feather, N. (1971). - Correlations in alpha-particle-accompanied fission. - Preprint.
- Musgrove, A.R. de L. (1971). - Aust. J. Phys. 24:129.
- Myers, W.D. and Swiatecki, W.J. (1966). - Nucl. Phys. 81:1.
- Myers, W.D. and Swiatecki, W.J. (1968). - UCRL 17070.
- Nardi, E., Boneh, Y. and Fraenkel, Z. (1969). - Physics and Chemistry of Fission (IAEA, Vienna) p.143.
- Nardi, E., Gazit, Y. and Katcoff, S. (1969). - Phys. Rev. 182:1244.
- Raisbeck, G.M. and Thomas, T.D. (1968). - Phys. Rev. 172:1272.
- Whetstone, S.L. and Thomas, T.D. (1967). - Phys. Rev. 154:1174.

TABLE 1

THE INITIAL CONDITIONS GIVING AGREEMENT WITH
EXPERIMENTAL ENERGY SPECTRA

Nuclide	D fermis	V_H cm/sec	\bar{E}_0 MeV	σ_x fermis	$\bar{\theta}_f$ degrees	FWHM degrees
^1H	29.0	5.50×10^8	1.0	7.6	81	31
^2H	25.0	4.50×10^8	2.0	3.8	82	30
^3H	22.5	3.25×10^8	1.75	3.3	81	28
^4He	23.7	3.75×10^8	2.75	2.3	83	23
^6He	22.2	2.40×10^8	1.25	2.8	82.5	20
^8He	21.0	1.10×10^8	0.75	3.1	79	20

TABLE 2

CALCULATED ENERGY REQUIRED TO EMIT THE PARTICLE
FROM THE LIGHT OR HEAVY MASS FRAGMENT

Nuclide	\bar{E}_0 MeV	Light Fragment			Heavy Fragment			Average Yield
		Binding MeV	Coulomb MeV	Total MeV	Binding MeV	Coulomb MeV	Total MeV	
^1H	1.0	12.5	4.4	17.9	10.2	5.1	16.3	5.3×10^{-5}
^2H	2.0	15.3	5.1	22.4	12.6	5.9	20.5	2.1×10^{-5}
^3H	1.75	14.3	5.7	21.8	11.3	6.5	19.6	2.3×10^{-4}
^4He	2.75	7.2	10.6	20.6	1.95	12.2	16.9	3.27×10^{-3}
^6He	1.25	16.5	11.3	29.1	10.9	13.1	25.3	8.4×10^{-5}
^8He	0.75	26.5	11.9	39.2	20.8	13.8	35.4	3.3×10^{-6}
n	2.6*	5.4	-	8.0	5.0	-	7.6	~ 0.4

* This value assumed by Bowman et al. (1962)

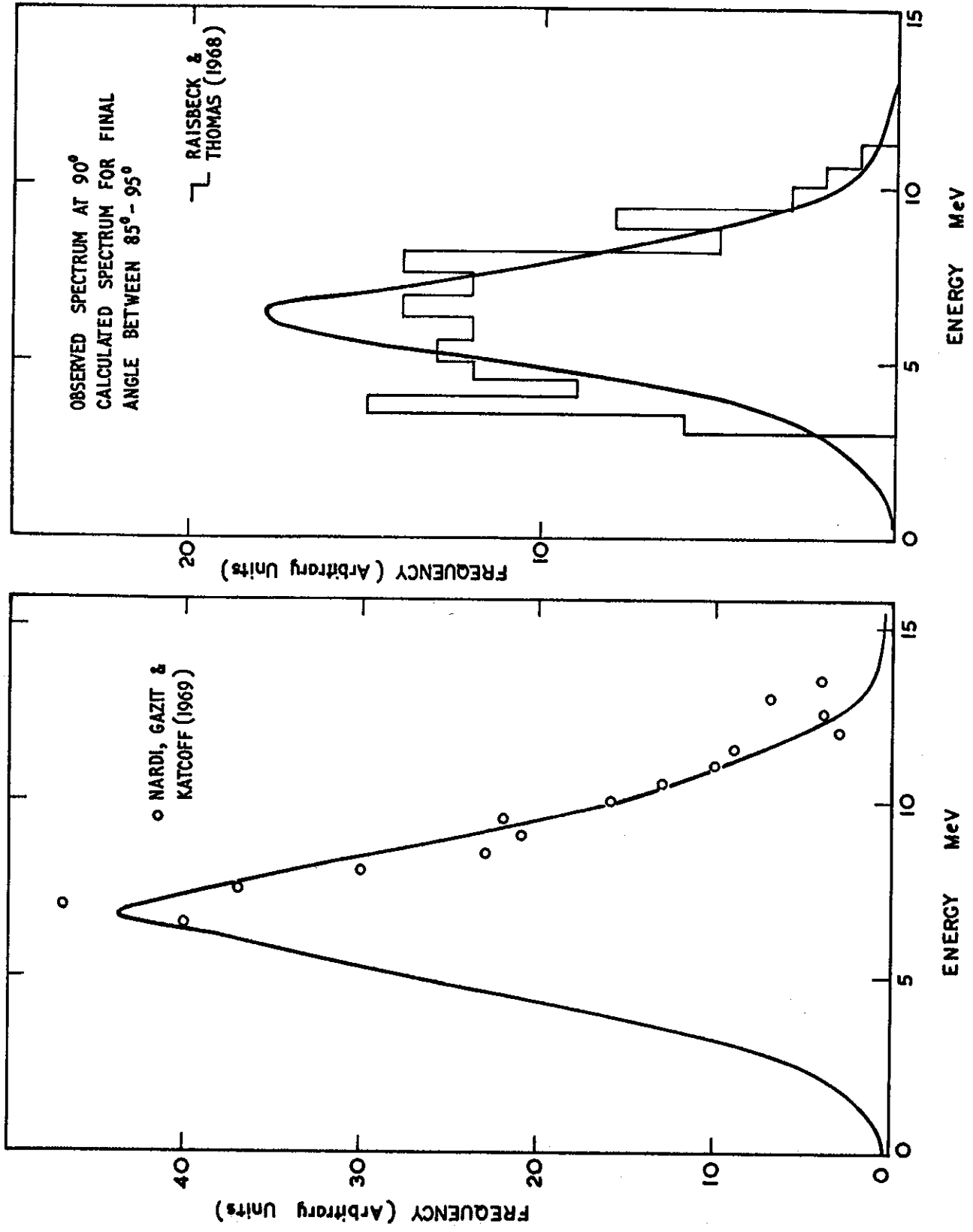


FIGURE 1. CALCULATED AND OBSERVED PROTON ENERGY SPECTRA

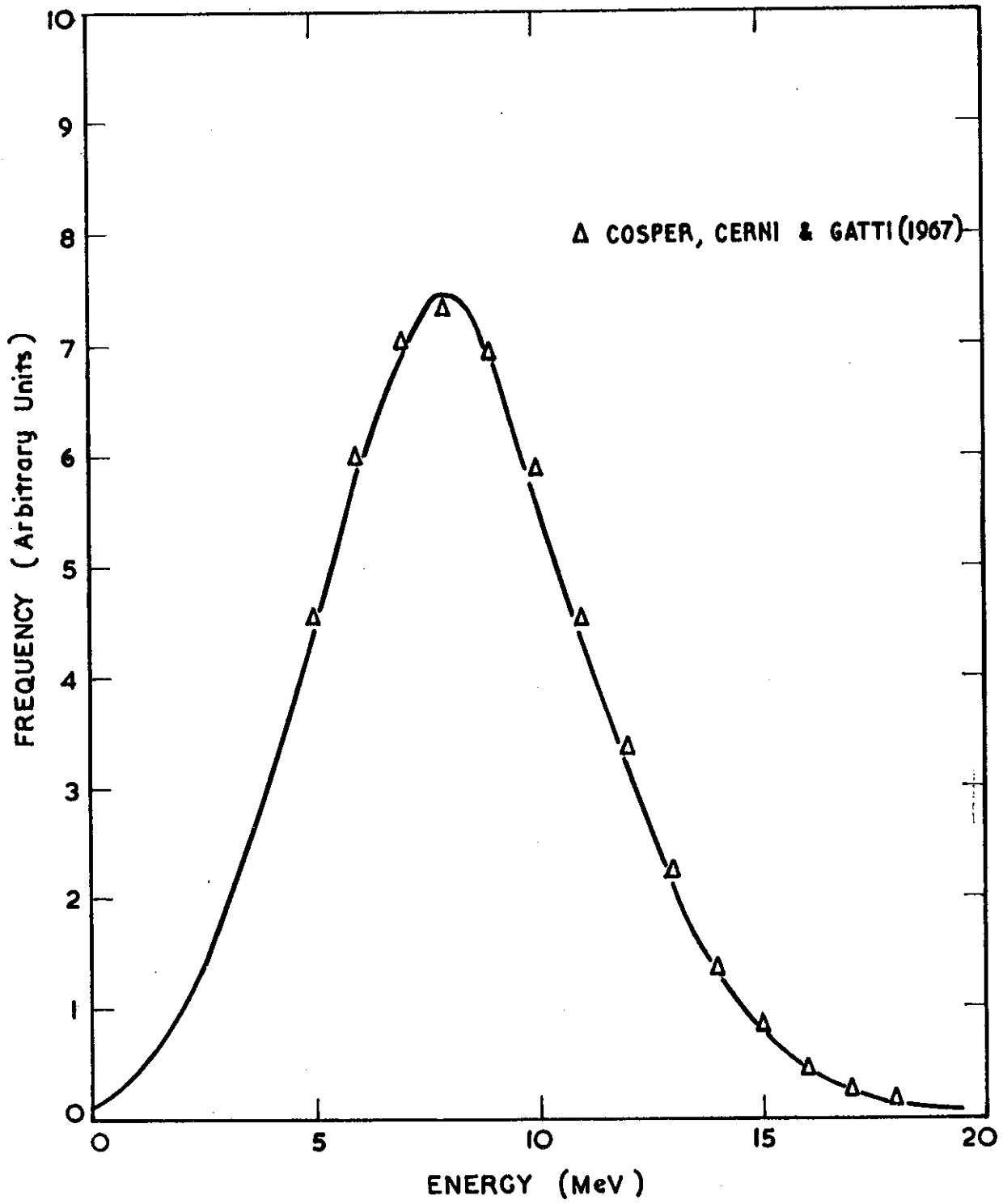


FIGURE 2. CALCULATED AND OBSERVED DEUTERON ENERGY SPECTRUM

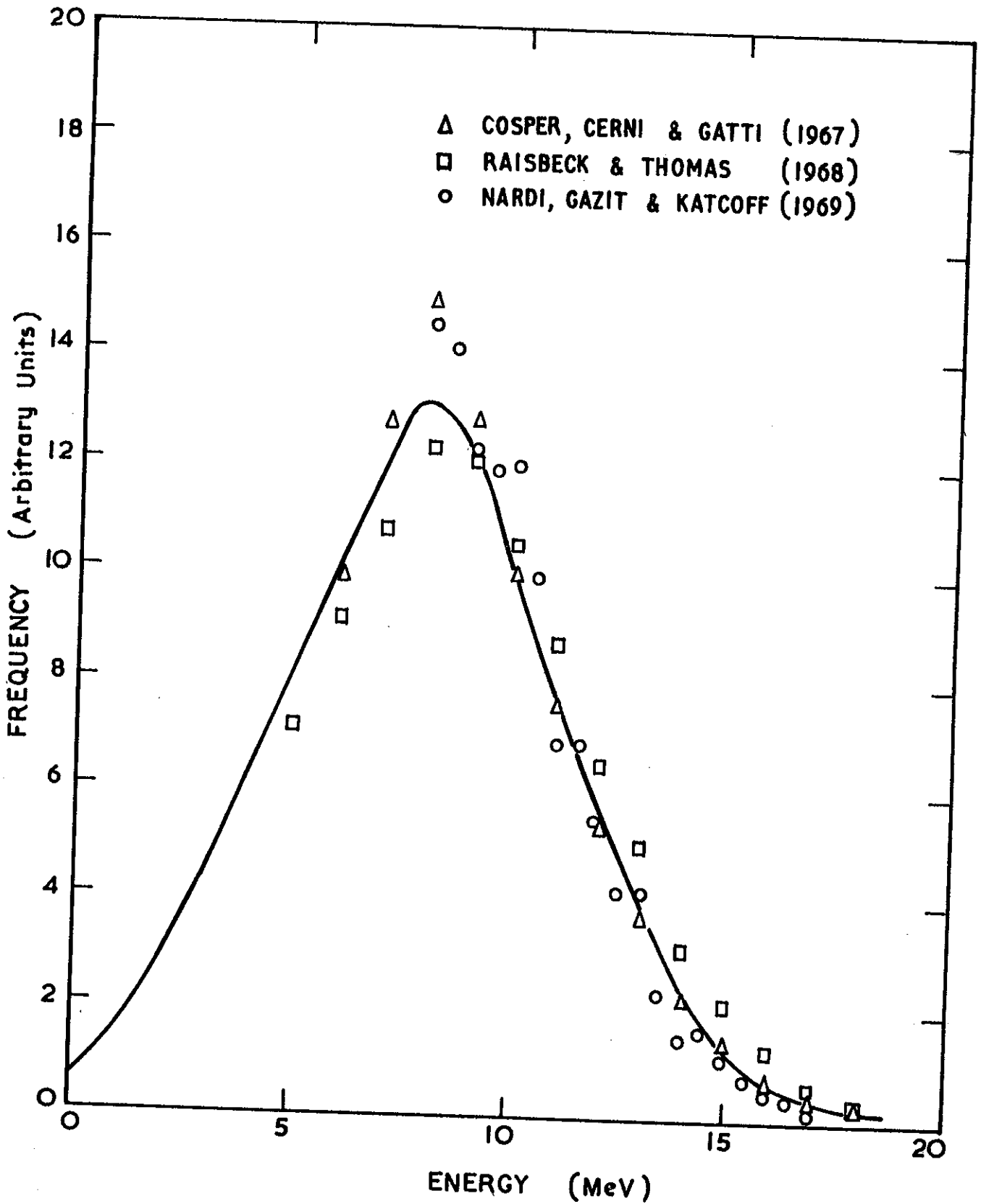


FIGURE 3. CALCULATED AND OBSERVED TRITON ENERGY SPECTRUM

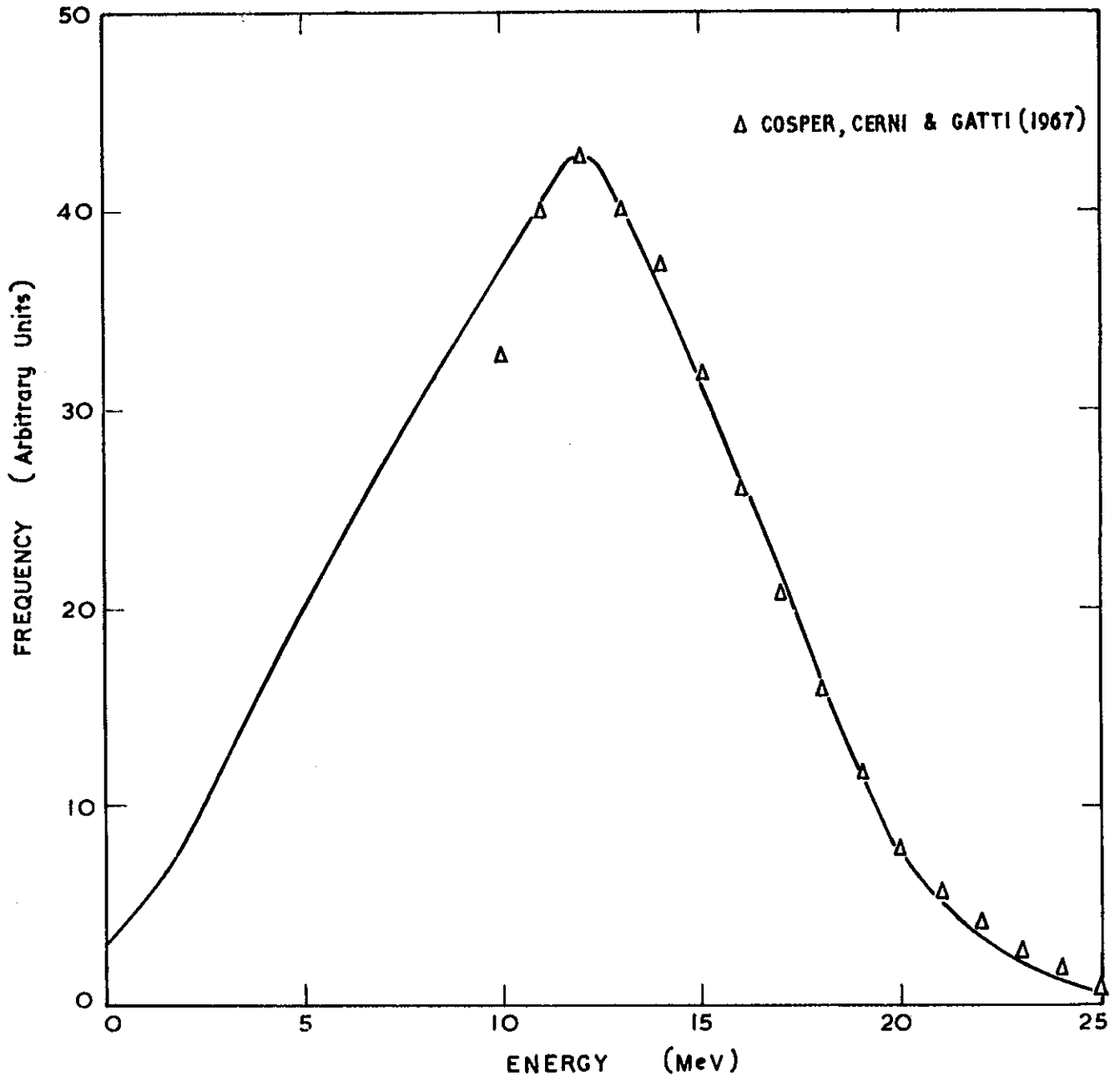


FIGURE 4. CALCULATED AND OBSERVED ${}^6\text{He}$ ENERGY SPECTRUM

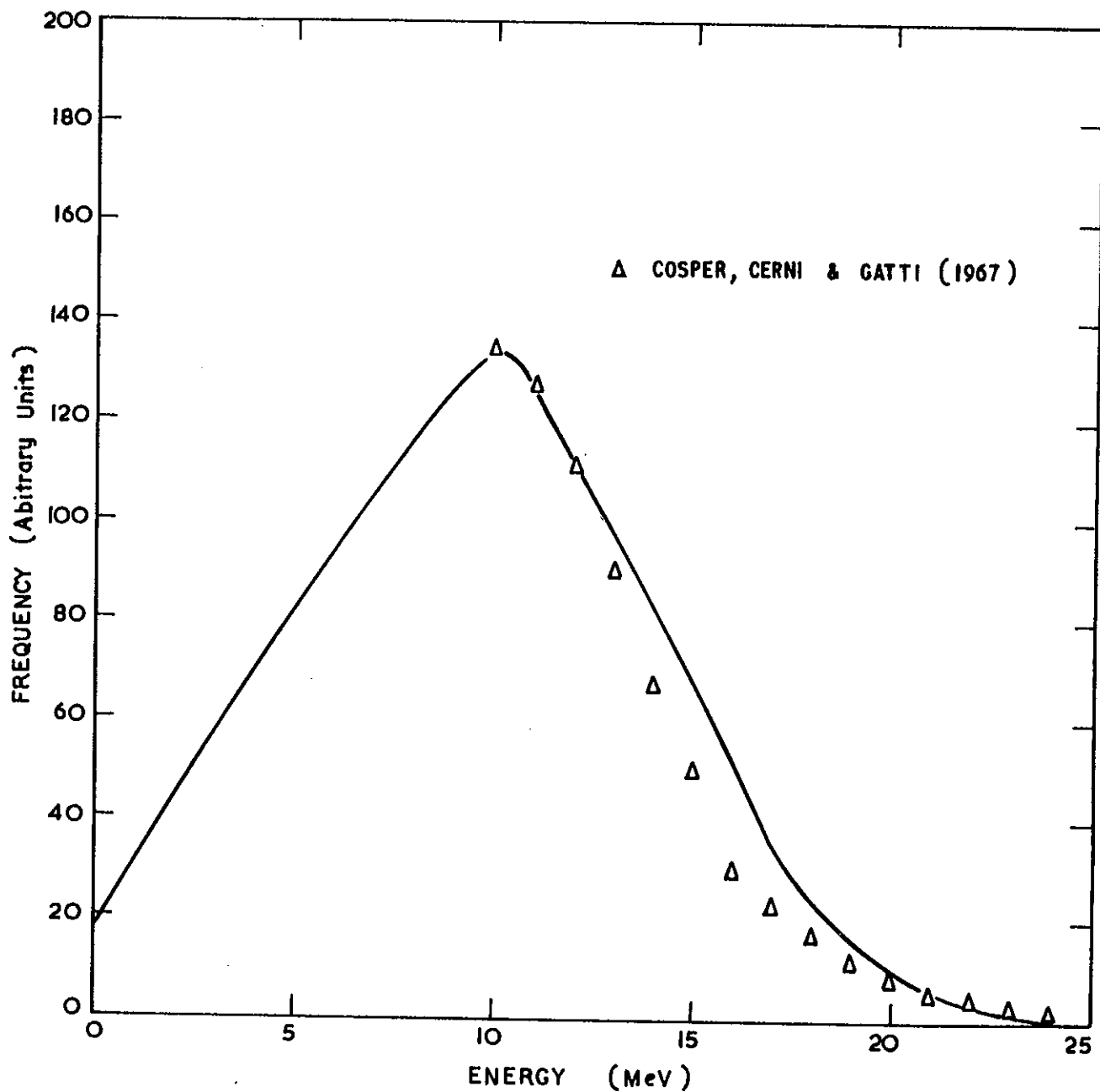


FIGURE 5. CALCULATED AND OBSERVED ^8He ENERGY SPECTRUM

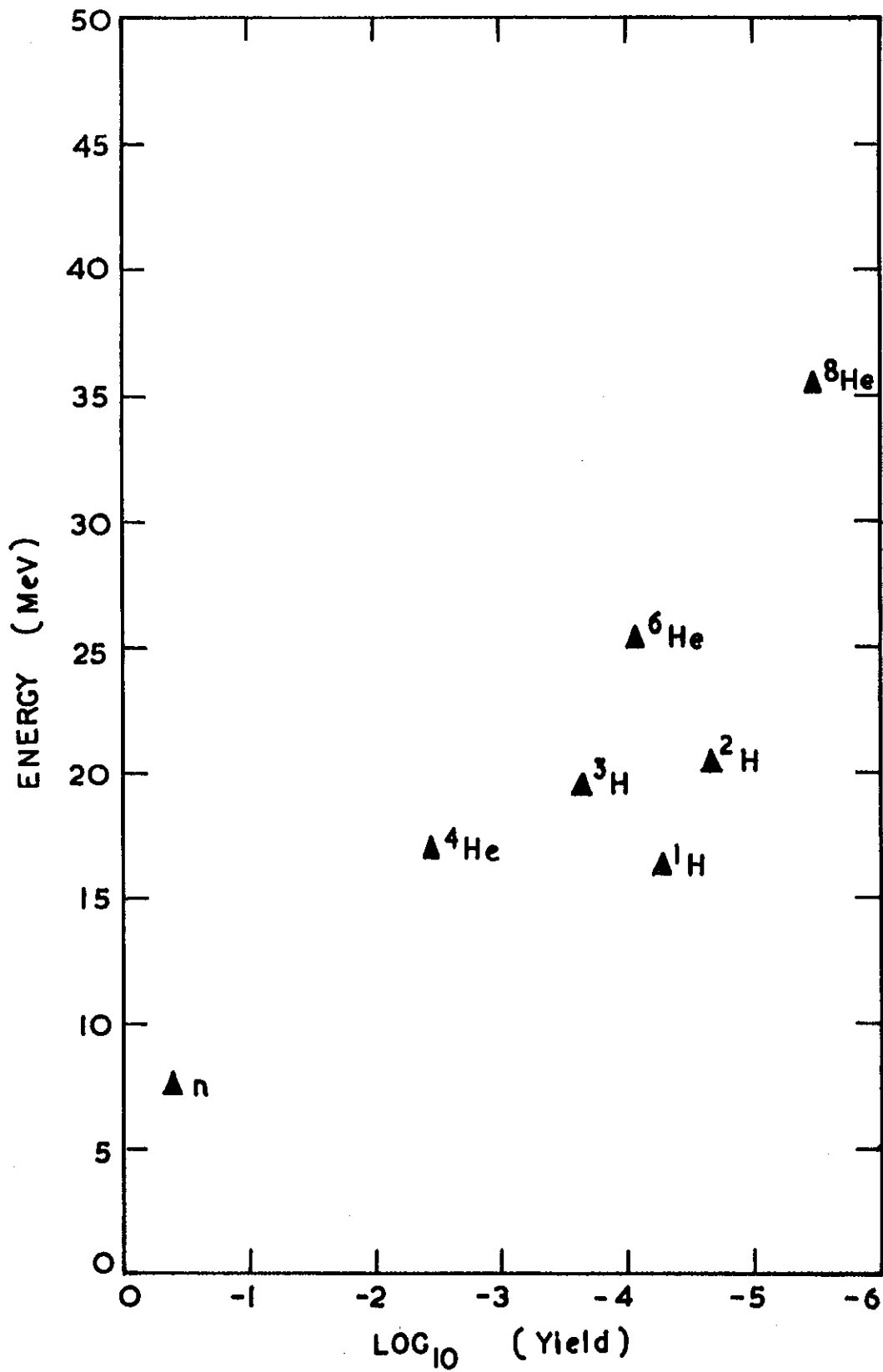


FIGURE 6. ENERGY REQUIRED FOR PARTICLE RELEASE v.
LOGARITHM OF OBSERVED YIELD