AAEC/TM580

AUSTRALIAN ATOMIC ENERGY COMMISSION
RESEARCH ESTABLISHMENT
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March 1971

ISBN 0 642 99411 0

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ABSTRACT

Using a cyclic temperature technique, average heat transfer coefficients were determined for gas flow through packed beds. Materials of widely differing thermal diffusivities were used, comprising, in turn, spheres of zircon sand, glass, steel and aluminium.

The results agreed with data in the literature obtained by using a conventional heated sphere method.

Pressure losses associated with flow of air through the beds were also measured, and the corresponding friction factors were evaluated for a considerable range of Reynolds numbers (2,500-35,000).

National Library of Australia card number and ISBN 0 642 99411 0

CONTENTS

1. INTRODUCTION

In the measurement of average heat transfer coefficients for such complex heat transfer surfaces as packed beds, the use of the conventional method of measuring steady state surface and fluid temperatures calls for the use of complicated electrically heated replicas. However, if the cyclic temperature method is used, the average heat transfer performance of complete beds can be measured using unheated models which are naturally much simpler and cheaper to manufacture.

The cyclic temperature method depends on the fact that variation of the fluid inlet temperature sinusoidally with time allows the average heat transfer coefficients of complex test assemblies, such as packed beds, to be determined from a knowledge of various physical parameters of the system and the resultant temperature attenuation in the fluid.

As large temperature variations are not necessary, the method has the great advantage that it can be conducted under nearly isothermal conditions. Another important advantage is that there is no need to measure the temperature difference between the heat transfer surface and the fluid. For complex surfaces this measurement can present practical difficulties.

In the work reported here the cyclic temperature technique was used to measure average heat transfer coefficients for a series of packed beds using, in turn, spheres of zircon sand, glass, steel and aluminium.

2. THEORY

The cyclic heat transfer method has been developed by Bell and Katz (1949), Dayton et al (1952), Meek (1962), Hart and Szomanski (1968) and Manins and Szomanski (1968) for idealised heat transfer models making assumptions concerning thermal conductivity and other physical parameters of the test assemblies.

These theoretical investigations were based on equations governing heat balance in a differential volume of the solid, heat balance in a differential volume of the fluid, and heat conduction in the solid for a sinusoidally varying temperature at the fluid inlet to the test section.

The heat transfer coefficient is defined as the ratio between the time rate of heat transfer per unit area of surface and the difference between the surface temperature and the average gas temperature over a cross section normal to the direction of gas flow. It is assumed that the surface heat transfer coefficient for packed beds, as measured by the cyclic temperature technique, is equivalent to the coefficient as determined in a steady state heat flow measurement and is constant through the bed.

According to Meek (1962) the following equation relating the average heat transfer coefficient to the temperature attenuation factor and other measurable properties of the system is relevant to packed beds of spheres for finite Biot numbers:

$$r_s = \exp\left\{ -p \left[(C_- - XS_+ + \tfrac{1}{2}X^2C_+) + Bi(\tfrac{1}{2}XS_+ - C_-) \right] \left[(C_- - XS_+ + \tfrac{1}{2}X^2C_+) + 2Bi(\tfrac{1}{2}XS_+ - C_-) + Bi^2C_- \right]^{-1} \right\} , \quad \ldots(1)$$

where

$$X = (\tfrac{1}{2}d^2 \rho_f C_s \omega K^{-1})^{\frac{1}{2}}$$

$$p = h S (A_f \rho_f C_p u)^{-1}$$

$$C_+ = \cosh X + \cos X$$

$$C_- = \cosh X - \cos X$$

and

$$S_+ = \sinh X + \sin X .$$

Although the following assumptions were made in deriving this equation, the solution is apparently closely approximated in actual pebble beds, for finite Biot numbers:

(1) the thermal conductivity of the solid and fluid are assumed to be zero in the longitudinal direction, and

(2) the relevant value of the thermal conductivity of the solid is accepted in the transverse direction.

For Biot numbers $\leqslant 0.1$, a good approximation to Equation 1 is given by the following equation (Hart and Szomanski 1968) :

$$r_s = \exp\left(\frac{-p}{1 + m^2 p^2}\right) , \quad \ldots(2)$$

where

$$p = \frac{h S}{A_f \rho_f C_p u}$$

and

$$m = \frac{A_f \rho_f C_p u}{A_s \rho_s C_s L\omega} .$$

Equation 2 is in fact the exact solution applicable to packed beds assuming that the conductivities of the solid and fluid are zero in the longitudinal direction, while the conductivity of the solid in the transverse direction is infinite.

Infinite conductivity, however, can be considered approximated in an experimental sense if $\frac{K}{\omega \rho_s C_s L^2}$ is sufficiently large, where L is the length of the specimen.

By solving for p in Equation 2, we obtain the physically realistic solution:

$$p = \frac{1 + \sqrt{1 - 4 m^2 \left(\ell n \frac{1}{r_s}\right)^2}}{2 m^2 \left(\ell n \frac{1}{r_s}\right)} . \qquad \ldots(3)$$

As

$$\frac{h}{\rho_f C_p V} = St ,$$

we have

$$St = \frac{A_f}{S} \frac{\left(1 + \sqrt{1 - 4 m^2 \left(\ell n \frac{1}{r_s}\right)^2}\right)}{2 m^2 \left(\ell n \frac{1}{r_s}\right)} . \qquad \ldots(4)$$

Conditions for the beds of aluminium and steel spheres corresponded to this case.

For large values of Biot numbers, X assumes large values; also, $C_+ = C_- = S_+$ and the first order terms in Equation 1 can be neglected; thus

$$Bi \rightarrow \infty \quad r_s = \exp\left\{ \frac{-p \left(\frac{1}{2}X^2 + \frac{1}{2}X\ Bi\right)}{\frac{1}{2}X^2 + X\cdot Bi + Bi^2} \right\} . \qquad \ldots(5)$$

Substituting

$$q = \frac{Bi}{X} ,$$

Equation 5 can be written in the form

$$r_s = \exp\left[\frac{-p\ (1 + q)}{1 + 2q + 2q^2}\right]. \qquad \ldots(6)$$

For Biot numbers in excess of 10, Equation 1 can be approximated reasonably well by Equation 6.

Equations 1 and 2 are compared in Figure 1 for Bi = 0.1, 1 and 5 by plotting $\frac{(\ell n \frac{1}{r_s})}{p}$ against q. The curves demonstrate clearly that Equation 2 closely approximates Equation 1 for Bi $\leqslant$ 0.1 but large divergences can occur for larger Biot numbers. The difference between Equations 1 and 2 depends not only upon the Biot number but also the value of q (that is, X).

Equation 6, which corresponds to Bi $\rightarrow \infty$, is also plotted; it can be seen that the larger the value of the Biot number the more closely does it approximate Equation 1.

It can also be seen from Figure 1 that as $q \rightarrow o$, Equations 1, 2 and 6 tend to the same limit, namely $r_s = \exp(-p)$.

Conditions for the beds of glass and zircon sand spheres were found to correspond to intermediate values of Biot number (see Table 1). A practical solution was obtained by extrapolating $\ell n\ (\frac{1}{r_s})$ as a function of frequency to obtain the value corresponding to infinite frequency which would then be equal to p (see Figure 2). In these experiments a suitable range of cyclic frequency values was used ($\frac{1}{4}$ to 1 cycle per second) corresponding to small values of q (<0.1).

3. EXPERIMENTAL WORK

3.1 Description of the Apparatus

The schematic layout of the cyclic heat transfer apparatus is shown in Figure 3. It can be described under four categories; flow equipment, cyclic heat generation, instrumentation and test sections.

3.1.1 Flow equipment

The atmospheric air rig used in these experiments is supplied with air from a 60HP centrifugal blower at a pressure slightly above atmospheric pressure, and a temperature of about 50°C, via an orifice plate and 90° bend. The latter is fitted with turning vanes to aid in developing a uniform velocity profile in the duct leading to the test section. The air then passes through a cyclic heater into the test section from which it is exhausted to the atmosphere.

Air velocity measurements taken in the approach ducting to the test section with a micro-pitot tube and tilting manometer showed that the air velocity had an approximately uniform profile over the section.

The total air flow rate was obtained from measurements at the orifice plate to British Standard BS:1042 (1943). By measuring the temperature and pressure of the air in the test section, and applying the law of continuity to the flow through the air rig, the average velocity of the air flow at the test section was computed (assuming steady state conditions applied).

During the initial stages of the experiment a 0.150 inch pitot tube and micromanometer were used to measure the flow at the test bed and the results were compared with the corresponding values of air flow rate determined by the orifice plate method. Good agreement was obtained and the orifice plate method was adopted to monitor flow for the various sets of experiments undertaken with the packed beds of spheres.

3.1.2 Cyclic heat generation

The heater consists of fine stainless steel screens (250 mesh) mounted in the 6 in. square cross section. The eight meshes of the heater are connected in a series-parallel arrangement shown in Figure 4.

Such a heater has the following advantages:

(1) small heat capacity,

(2) the time constant of the heater is small (of the order of 100-200 milliseconds),

(3) the heater design allows for a uniform current density, and therefore uniform power generation across the screens,

(4) the screens impose an effectively uniform velocity profile over the fluid stream, and

(5) the temperature profile across the air stream is smoothed out.

The uniformity of the temperature profile was shown to be satisfactory by making measurements with a traversing chromel-alumel thermocouple inserted at various places downstream of the heater.

The method of supplying cyclic power to the heaters is shown in Figure 4. The wiper arm of a variable autotransformer is oscillated in simple harmonic motion by a scotch yoke mechanism.

A variable speed motor driving the yoke enables the frequency to be altered while the variable power amplitude is obtained by regulating the voltage output from the auto transformer. The limit to the frequency with which the wiper arm can be safely oscillated is about one cycle per second.

The cyclic output of the auto transformer is applied to the primary of a 20:1 stepdown transformer rated at 3 kVA. The low resistance of the mesh heaters allows currents up to 225A at 2.7 kVA.

3.1.3 Instrumentation

The layout of the equipment used for measuring the upstream and downstream temperature is shown in Figure 4. Each temperature pick-up serves as one leg of a Wheatstone Bridge circuit operating near balance point.

An important advantage of the method of cyclic temperature variations is that it is not necessary to know the absolute values of the temperature amplitudes. It is necessary, however, to measure the ratio of sensitivities of the thermometer amplifiers so that a direct comparison can be made of the trace amplitudes in order to obtain the value of the attenuation factor r.

The diameter of the platinum wire used in both upstream and downstream resistance thermometers is 5/1000 in. Since the resistance wires have the same temperature coefficient of resistance and since both bridges are driven by the same stabilised power supply, the ratio of the temperature amplitudes may be taken as the ratio of the voltage amplitudes across the two bridges. The bridge signal in each case is fed into an ultra-violet lamp recording galvanometer.

The resistance thermometers were calibrated and found to have equal sensitivities such that a change of 1 degC corresponded to a deflection of approximately 1 inch on the recording paper.

The following measurements were also made:

(1) Pressure drop across the specimen, using a water-filled manometer.

(2) Pressure drop across the orifice plate, pressure upstream of the orifice plate, and the temperature of the air at the orifice plate, to determine the mass flow of air through the test bed.

(3) Temperature and static pressure of air in the test section.

(4) Atmospheric humidity and pressure.

3.1.4 Test sections

Specimens of randomly packed balls of zircon sand, glass, steel or aluminium, in stainless steel sample holders, were used. The zircon sand bed was obtained from a 30 inch diameter and 15 inch high bed of randomly packed

[illegible]

spheres frozen in wax (Manins and Szomanski 1968). For the other types of balls, the samples were prepared by simply pouring the balls into the container and fastening the end screens to keep the balls in place in the sample holder. The internal dimensions of the sample holders were 6 in. x 6 in. x 12 in. The average voidage of each bed was determined from weight measurements of the actual packed test bed in question, the sample holder and end screens, and the density of the sphere material.

The total pressure drop across the randomly packed pebble beds was measured during the experimental study of the heat transfer characteristics of the beds.

These particular experiments in no way constituted a systematic study of the fluid resistance properties of the randomly packed beds as the fundamental interest in the cyclic temperature heat transfer technique dictated the choice of the type of test section used.

4. EXPERIMENTAL PROCEDURE

With the flow controls adjusted at a predetermined flow rate and the temperature bridges balanced at this flow, power was supplied to the heaters at a given frequency. About 20 heating cycles were allowed for the cyclic heat transfer conditions in the packed beds to reach a quasi-steady state. By this is meant that the cyclic temperature variations repeat themselves reproducibly. The next 10 to 20 cycles were recorded using the ultra-violet lamp recorder for determining the temperature attenuation factor.

To obviate the effect of bridge sensitivity and amplifier characteristics, this procedure was repeated with the temperature connections to the equipment reversed. The true amplitude ratio was accepted as the mean between the measured values with the temperature indicators in the normal and reversed positions.

The fluid temperature attenuation r_E due to the sample holder and associated end screens was determined from measurements on the empty test section. The fluid temperature attenuation for the specimen was then determined from the relationship:

$$r_s = r_T / r_E \quad .$$

The pressure drops across the test sections investigated were measured by using a manometer connected to appropriate pressure taps provided. There are 4 pressure tappings at each axial position at distances of 1.25 inches upstream and downstream from the test section assembly.

5. ANALYSIS OF THE EXPERIMENTAL RESULTS

The test beds were not geometrically identical as they were randomly packed and had somewhat different voidages. Hence the heat transfer characteristics would be expected to vary.

Each bed was tested over a range of flow rates and cyclic heating frequencies. The attenuation ratios r_s obtained were tabulated against the corresponding frequencies for each run and it was found convenient to plot experimental values of $\ln\left(\frac{1}{r_s}\right)$ against the corresponding values of $\omega^{-1/2}$. A typical curve is shown in Figure 2.

By extrapolating to $\omega^{-1/2} = 0$, the corresponding value of $\ln\left(\frac{1}{r_s}\right)$, designated p_o, gives an estimate of the correct value of p for each run.

Since

$$p_o = \frac{hS}{A_f \rho_f C_p u}$$

and

$$St = \frac{h}{\rho_f C_p V} ,$$

Stanton number (for various Reynolds numbers) were then found from the relation

$$St = \frac{p_o A}{S} ,$$

and were then plotted against the corresponding values of the Reynolds number, Re.

Computer programmes were used to determine suitable relationships to fit the data by the least squares method in a manner similar to that described by Manins and Szomanski (1968).

The pressure drop through the beds of randomly packed spheres was correlated, firstly by plotting a friction factor f versus a Reynolds number Re, where

$$f = \frac{\Delta P \ \rho_f \ d}{L \ 2G^2}$$

and

$$Re = \frac{d\ G}{\mu} ,$$

and secondly, by plotting a modified friction factor f' versus a modified

Reynolds number Re', where

$$f' = \frac{\Delta P}{L} \cdot \frac{\rho_f \; d \; \epsilon^3}{2G^2 \; (1 - \epsilon)}$$

and

$$Re' = \frac{d \; G}{\mu \; (1 - \epsilon)} .$$

6. DISCUSSION OF RESULTS

6.1 Heat Transfer

The results of heat transfer measurements on the packed bed assemblies are plotted in Figure 5. These results show that the heat transfer characteristics, for the packed beds investigated, can be expressed by the following relationships:

	Material	Diameter	Voidage	St - Re Relationship	r.m.s. Error
(1)	glass spheres	17 mm	0.39	$St = 0.841 \; Re^{-0.331}$	0.9%
(2)	glass spheres	1 inch	0.39	$St = 0.73 \; Re^{-0.295}$	3.0%
(3)	aluminium spheres	1 inch	0.40	$St = 0.749 \; Re^{-0.299}$	4.4%
(4)	steel spheres	1 inch	0.42	$St = 1.05 \; Re^{-0.341}$	3.1%
(5)	zircon sand spheres	1 inch	0.38	$St = 0.57 \; Re^{-0.247}$	1.2%
	(The general range of Reynolds numbers 3500-25000).				

It can also be seen from Figure 5 that the results obtained are in reasonable agreement with the heat transfer data obtained by Denton et al (1963) using a conventional heated sphere method.

The somewhat higher values of h for the test section filled with zircon sand balls are in all probability due to the fact that the section was cut from a large bed of randomly packed 1 in. zircon sand spheres (which had previously been frozen in wax for the purpose), and did not suffer from the 'wall effect' which was present in the other beds investigated.

6.2 Pressure Drop

The variation of friction factor with Reynolds number for the packed beds of spheres (Figure 6) shows reasonable agreement with the results of Denton et al (1963). In these curves the pressure losses generally flatten out over much of the range of Reynolds numbers investigated.

Further pressure drop measurements were made with 12 mm glass spheres and the results are included in Figure 6.

In Figure 7, modified friction factor is plotted against modified Reynolds number because the results are generally more suitably interpreted by considering the fluid flow in packed beds as a modification of the case of flow over an isolated sphere using a suitable value for the fluid velocity.

7. CONCLUSION

Measurements of average heat transfer coefficient of packed bed assemblies using the cyclic temperature technique agree reasonably well with those obtained by conventional heating method. This demonstrates that the technique, as used in this work, is a convenient method for measuring the heat transfer characteristics of such assemblies for a wide range of materials.

Determinations were also made of the friction factor f and modified friction factor f' for the packed beds over a range of Reynolds numbers similar to that investigated for the heat transfer characteristics of the beds. Over much of the range, f reaches an approximately constant value in each case.

8. ACKNOWLEDGEMENTS

The author acknowledges the assistance given throughout the project by Mr. C. Evans in the manufacture and repair of equipment, the experimentation and initial data analysis and by Mr. R. Titler, vacation student (University of Hobart), who carried out some of the data analysis. The author is also grateful for helpful discussions with Dr. K. Lawther and Professor J. J. Thompson.

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APPENDIX 1

NOTATION

A	total area of cross section normal to flow
A_f	area of cross section open to flow
A_s	area of cross section occupied by solid
Bi	Biot number = $Hd/2K$
C_p	specific heat of fluid at constant pressure
C_s	specific heat of solid
C_+	$\cosh \overline{X} + \cos \overline{X}$
C_-	$\cosh \overline{X} - \cos \overline{X}$
d	sphere diameter
ϵ	specimen voidage fraction
f	friction factor = $(\Delta P \rho_f d)(L\, 2G^2)^{-1}$
f'	modified friction factor $(\Delta P \rho_f d \epsilon^3)\left[L\, 2G^2 (1 - \epsilon)\right]^{-1}$
G	superficial mass velocity of fluid $\rho_f V$
h	average heat transfer coefficient
K	thermal conductivity of solid
L	length of specimen
m	$A_f \rho_f C_p u (A_s \rho_s C_s \omega L)^{-1}$
p	$hS (A_f \rho_f C_p u)^{-1}$
p_o	extrapolated value of p at $\omega \to \infty$
q	$h (2K \rho_s C_s \omega)^{-1/2}$
Re	Reynolds number = dG/μ
Re'	modified Reynolds number = $dG \left[\mu(1 - \epsilon)\right]^{-1}$
r	fluid temperature attenuation factor
r_E	fluid temperature attenuation factor due to sample holder losses
r_s	fluid temperature attenuation factor due to specimen
r_T	total fluid temperature attenuation factor
S	total heat transfer area in the specimen
S_+	$\sinh X + \sin X$

St	Stanton number $h/\rho_f C_p V$
u	fluid velocity through the specimen in axial direction
v	superficial fluid velocity
X	$(1/2\ d^2\ \rho_s\ C_s\ \omega K^{-1})^{1/2}$
μ	fluid viscosity
ρ_f	fluid density
ρ_s	solid density
ΔP	pressure drop across bed
ω	angular frequency

TABLE 1

RELEVANT EXPERIMENTAL DATA FOR PACKED BEDS

	TYPE AND SIZE OF BALL				
	17mm Glass	1 in. Glass	1 in. Aluminium	1 in. Steel	1 in. Zirconite Sand
Biot Number $hd/2K$	2.17 - 4.598	3.28 - 10.35	0.0124 - 0.0322	0.056 - 0.139	6.7 - 12.15
h(cal $sec^{-1} cm^{-2}$ $°C^{-1}$)	0.0046 - 0.0097	0.0046 - 0.0148	0.00147 - 0.0049	0.0047 - 0.0117	0.0054 - 0.0095
$q = \frac{h}{\sqrt{2K\, C_s\, \rho_s\, \omega}}$	0.0468 - 0.174	0.047 - 0.265	0.0027 - 0.014	0.0049 - 0.021	0.067 - 0.214
density ρ_s g cm^{-3}	2.53	2.53	2.71	7.7	2.9
Specific heat C_s (Cal g^{-1} $°C^{-1}$)	0.199	0.199	0.23	0.107	0.20
Thermal conductivity K (cal cm^{-1} sec^{-1} $°C^{-1}$)	0.0018	0.0018	0.500	0.107	0.001
Thermal diffusivity $K/\rho_s\, C_s$	0.00358	0.00358	0.802	0.1299	0.0017
Angular cyclic frequency ω (rad sec^{-1})	1.72 - 5.3	1.72 - 5.3	1.72 - 5.3	1.72 - 5.3	1.72 - 5.3

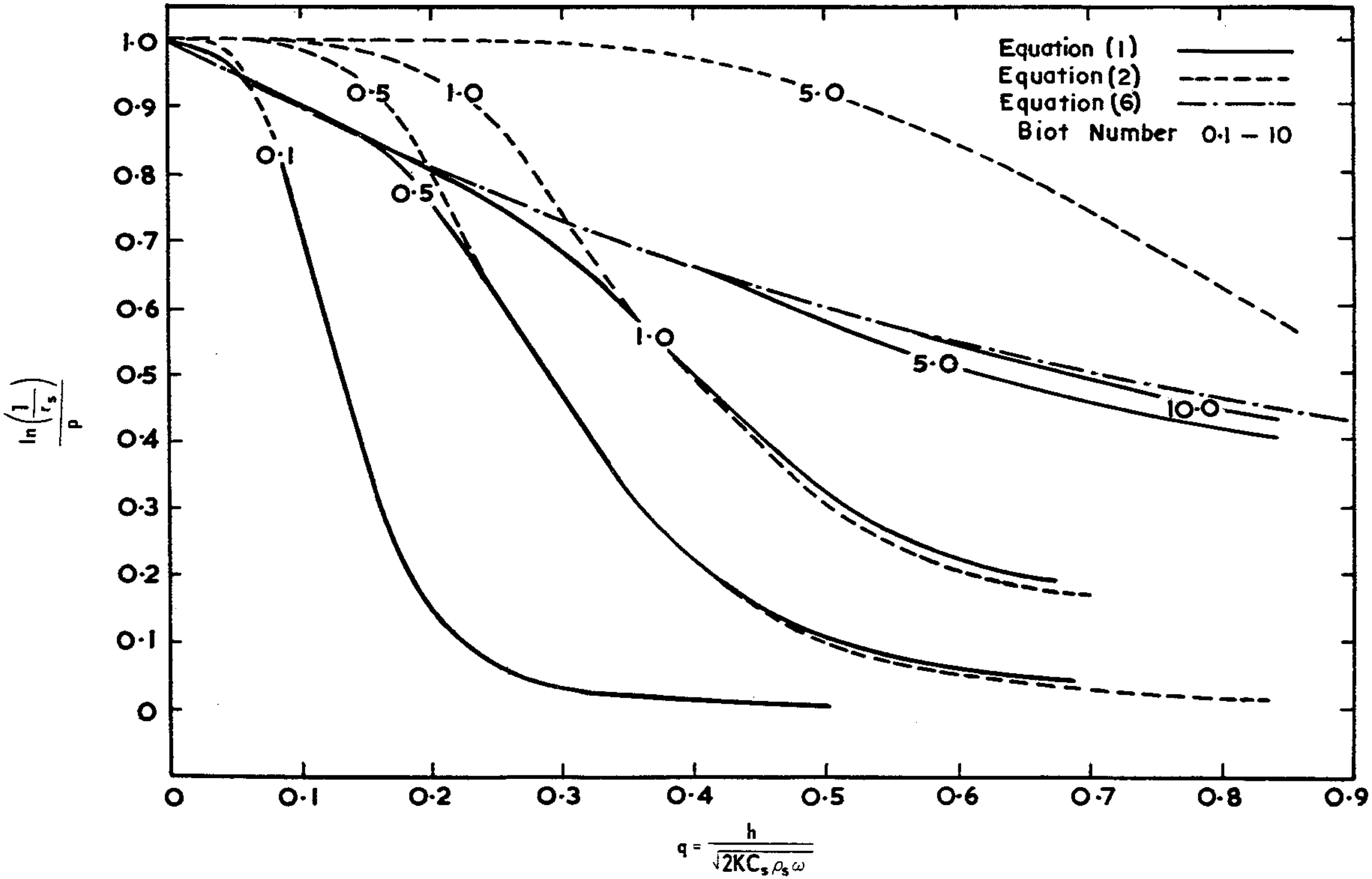

FIGURE 1. COMPARISON OF SOLUTIONS FOR SINUSOIDAL FLUID TEMPERATURE SIGNAL

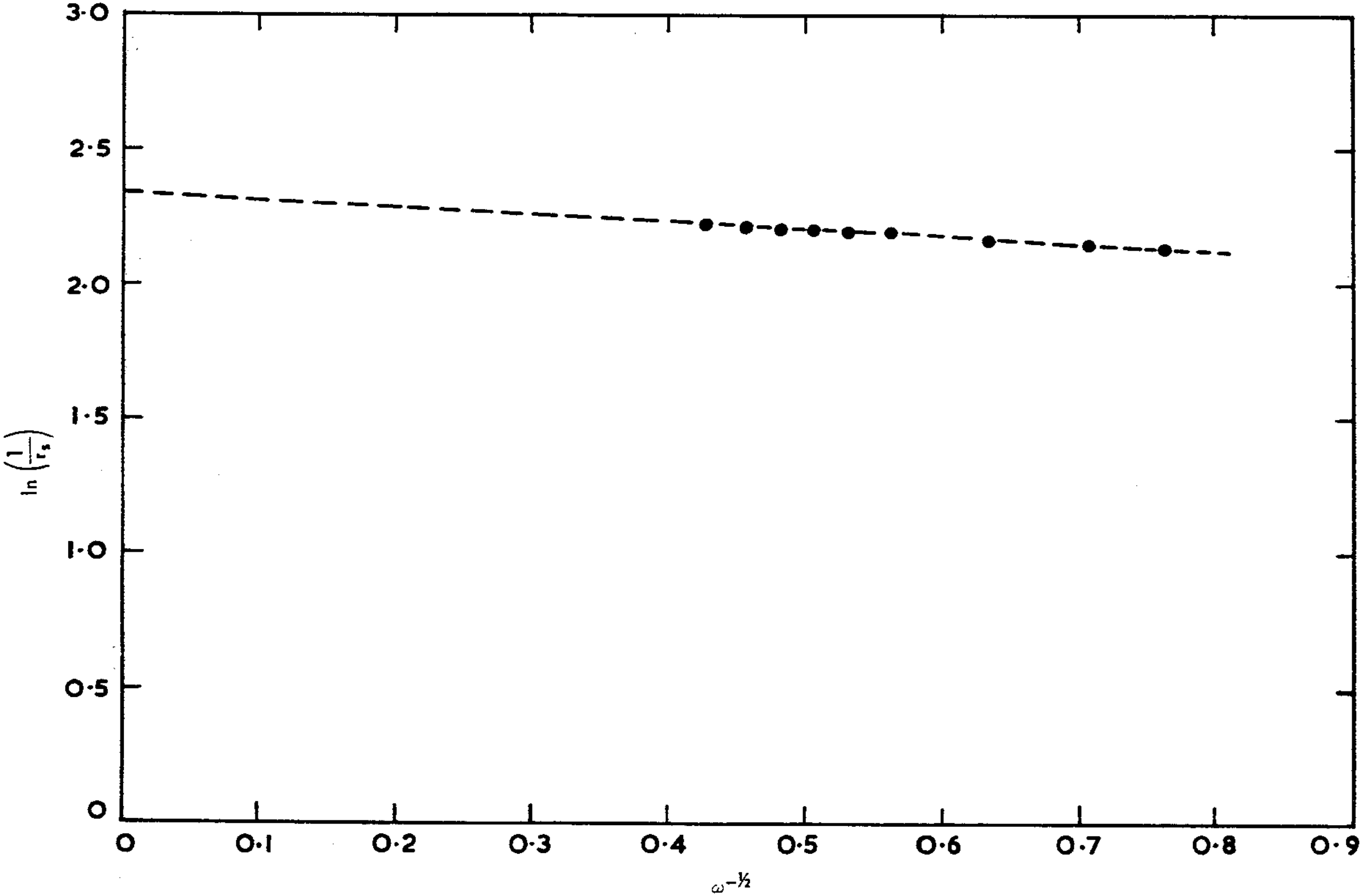

FIGURE 2. EXTRAPOLATION GRAPH OF FREQUENCY RESPONSE CURVE

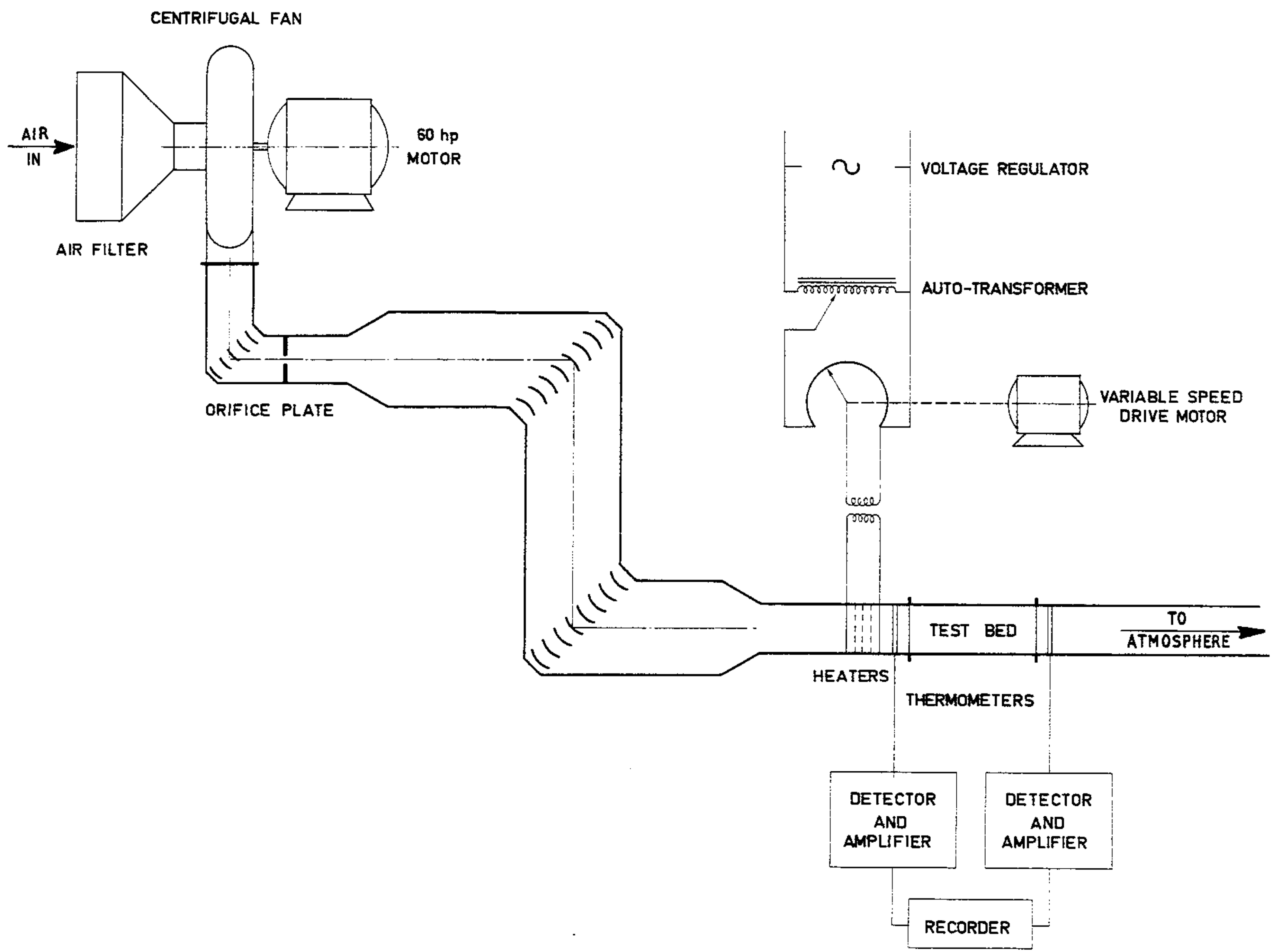

FIGURE 3. A SCHEMATIC LAYOUT OF THE CYCLIC TEMPERATURE APPARATUS

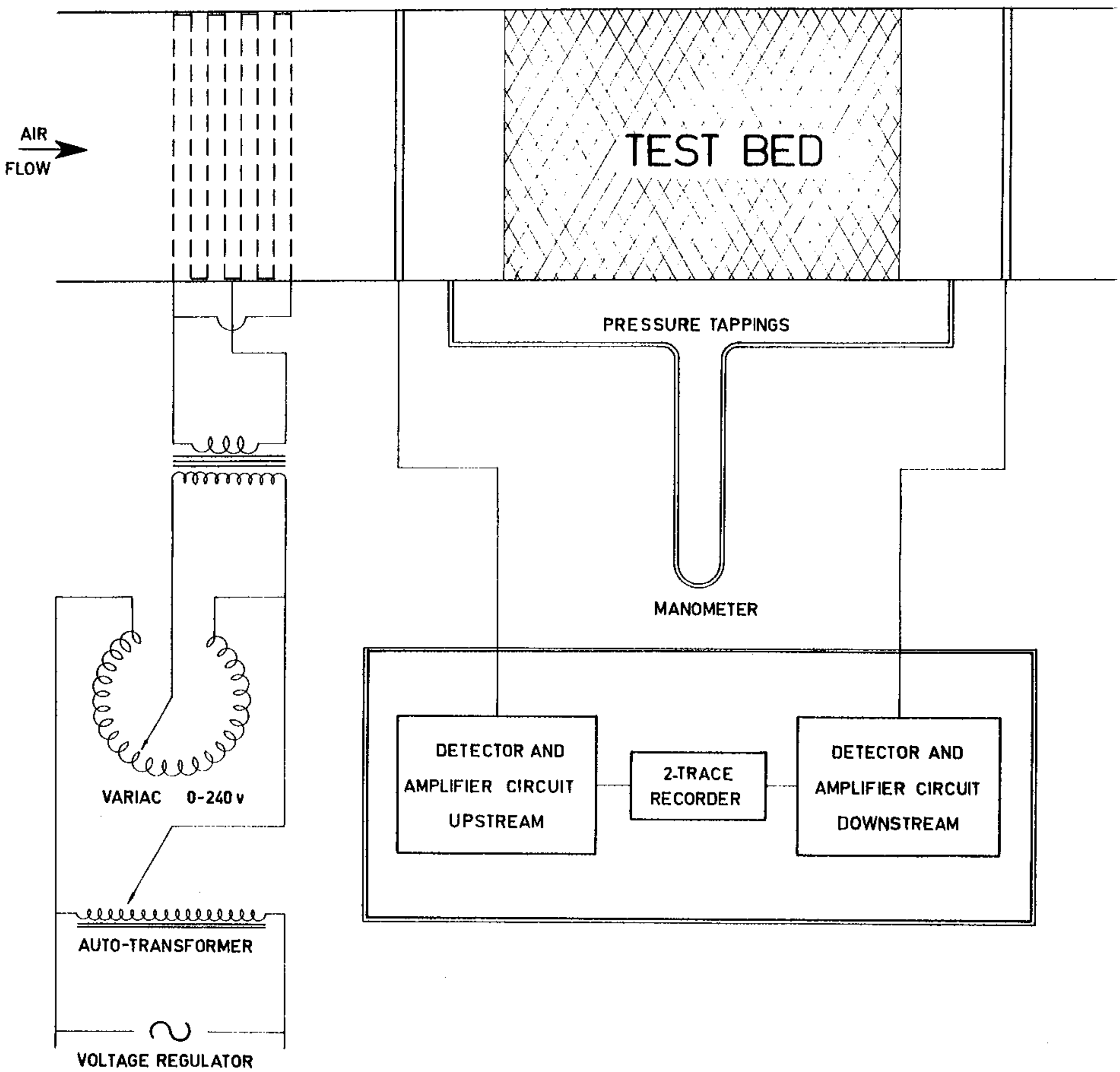

FIGURE 4. CYCLIC HEATER GRID ARRANGEMENT

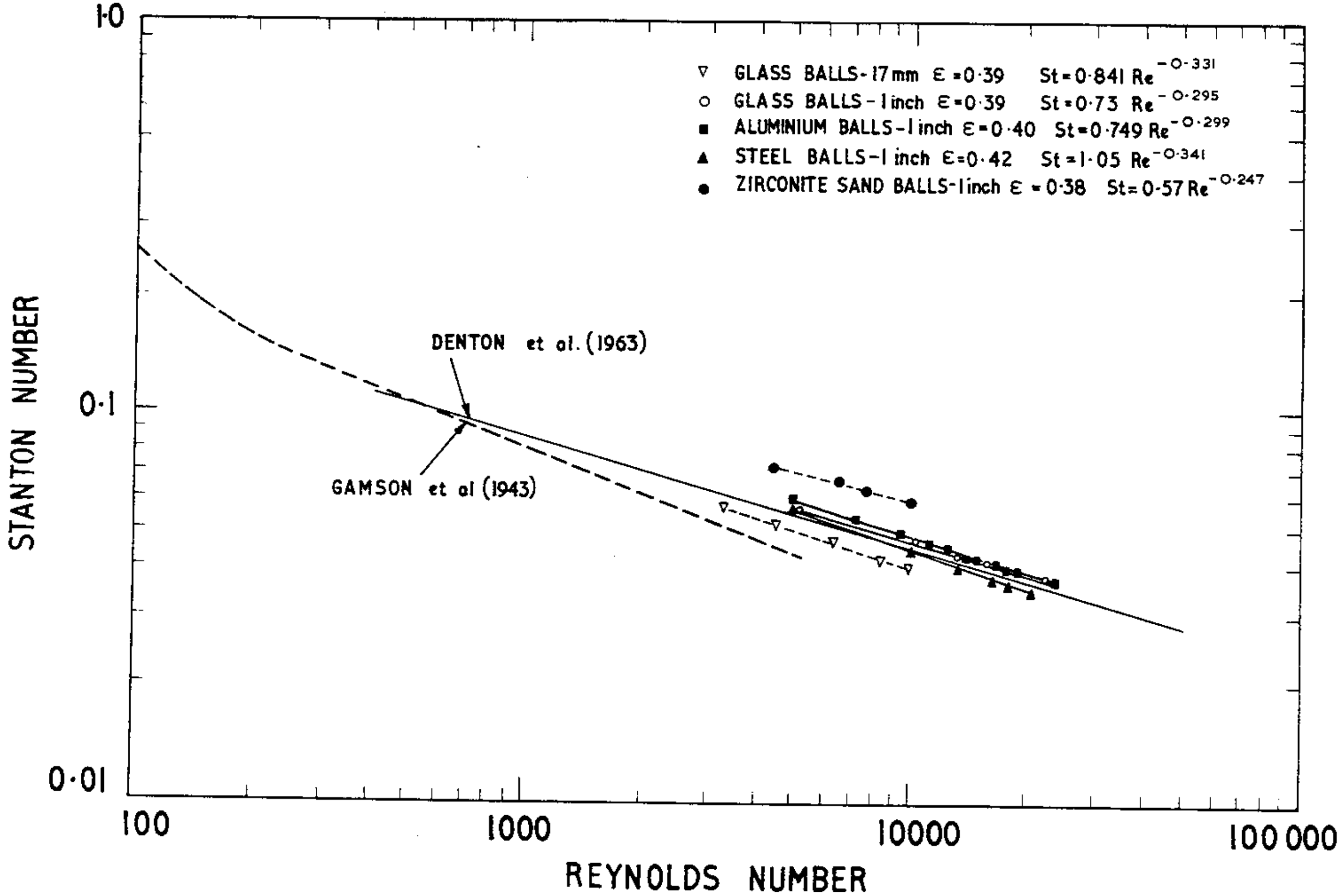

FIGURE 5. STANTON NUMBER VERSUS REYNOLDS NUMBER
VARIOUS PACKED BEDS

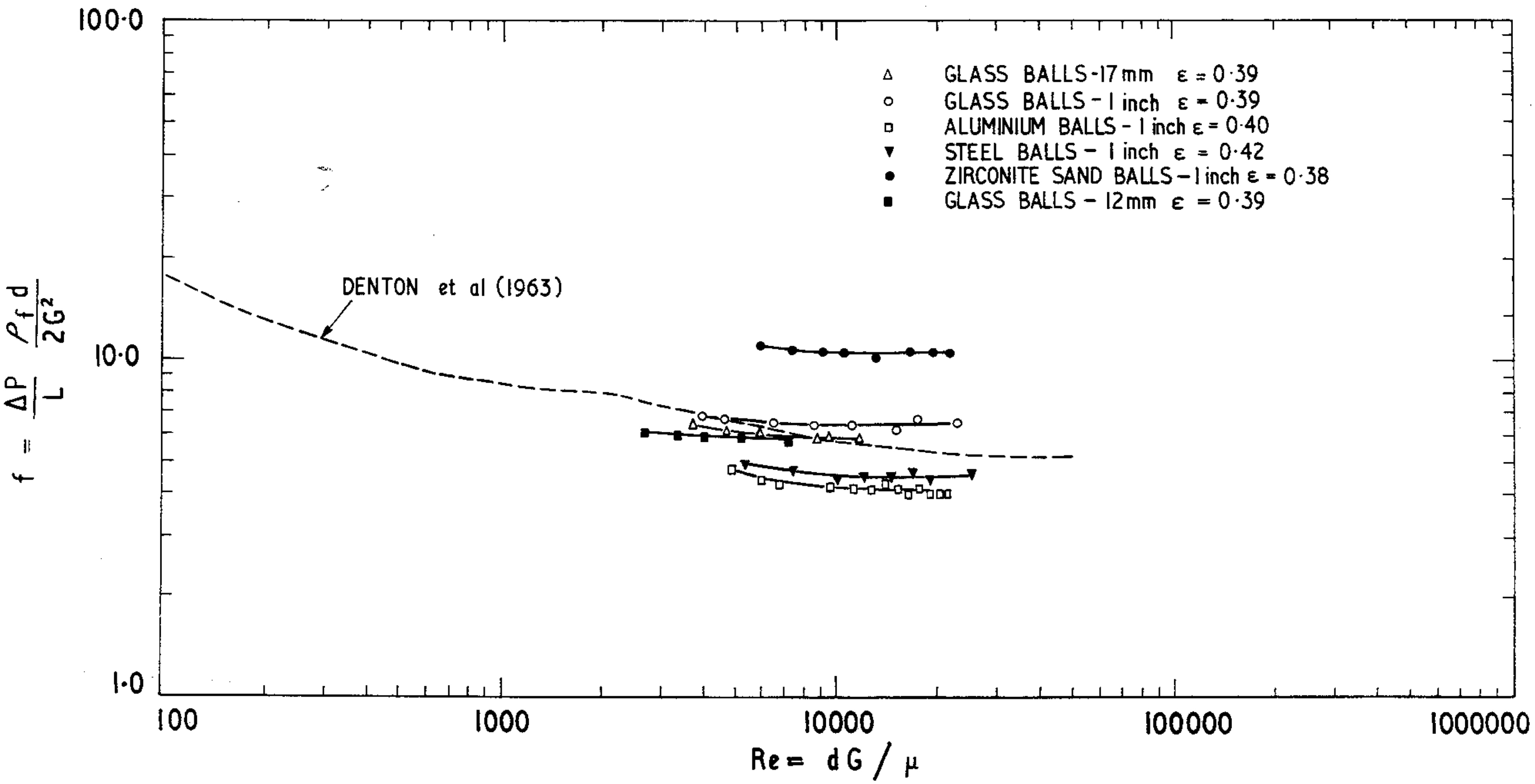

FIGURE 6. FRICTION FACTOR VERSUS REYNOLDS NUMBER FOR VARIOUS PACKED BEDS

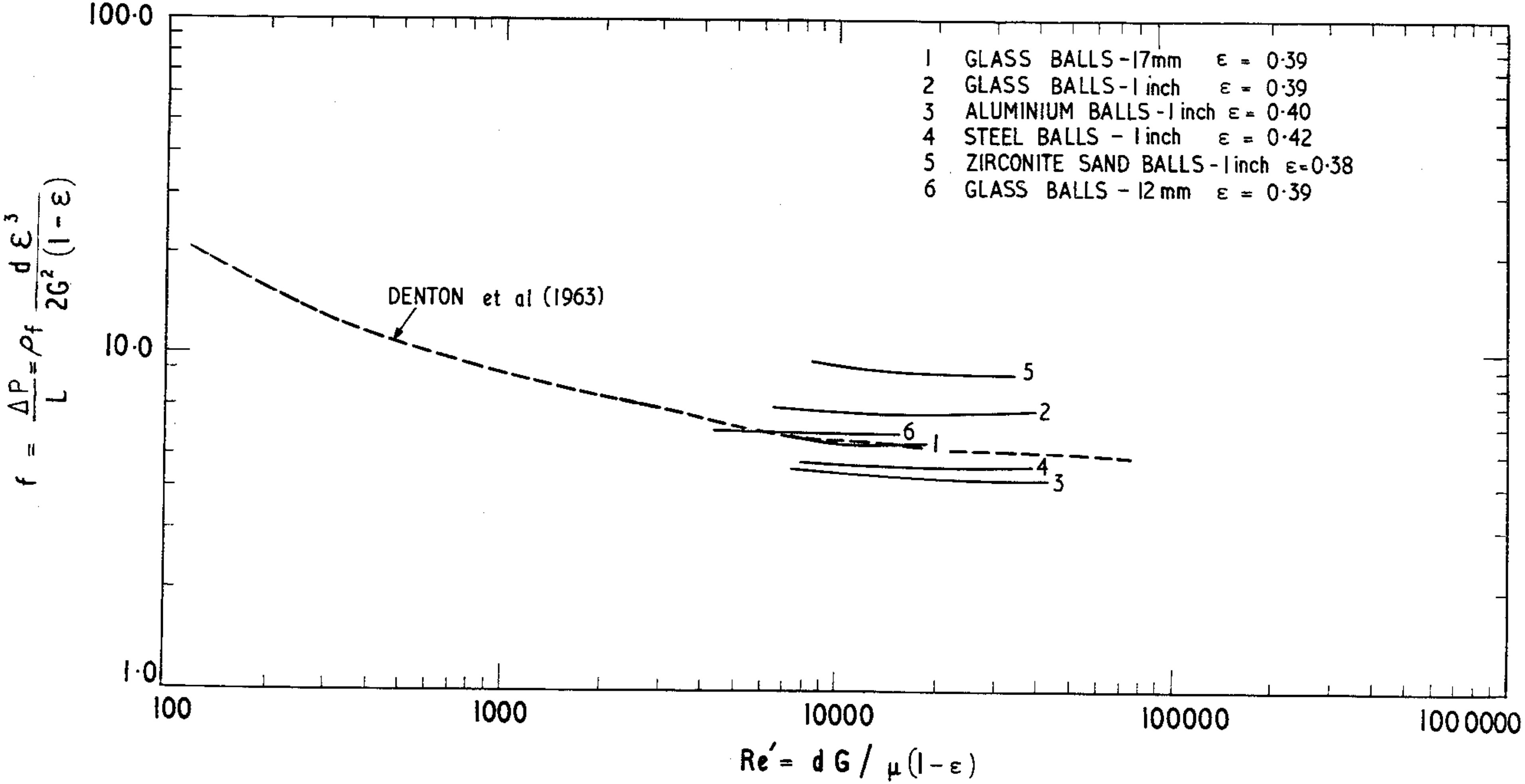

FIGURE 7. MODIFIED FRICTION FACTOR VERSUS MODIFIED REYNOLDS NUMBER FOR VARIOUS PACKED BEDS