



**AUSTRALIAN ATOMIC ENERGY COMMISSION  
RESEARCH ESTABLISHMENT  
LUCAS HEIGHTS**

**ANALOGUE STUDIES OF POWER TRANSIENTS PRODUCED BY REACTIVITY  
CHANGES IN THE MATERIALS TESTING REACTOR HIFAR  
LOADED WITH MK III 150 GRAM FUEL ELEMENTS**

by

**D.J. WILSON**

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ABSTRACT

The HIFAR reactor loaded with a core of 150 g Mark III fuel elements was simulated using the PACE analogue computer to assess expected power transients following positive reactivity additions.

If five coarse control arms operate, and the operational power is 85% of the trip level, the power transient from an initial 15 MW will not exceed 30 MW if the rate of addition of reactivity does not exceed 0.535%  $\delta k/k$  per second, or a step addition does not exceed 0.342  $\delta k/k$ .

The peak power attained is influenced by the delay in releasing, and by the initial setting of, the coarse control arms.

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**FUEL ELEMENTS; HIFAR REACTOR; POWER; REACTIVITY; REACTIVITY WORTHS;  
REACTOR FUELLING; REACTOR KINETICS; REACTOR SIMULATORS; SHIM PODS**

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## 1. INTRODUCTION

Changes in rig load may necessitate the uprating of the materials testing reactor HIFAR to 15 MW. Maximum allowable step and ramp reactivity changes must be defined to ensure that the reactor does not suffer a power transient sufficiently large to damage the fuel when at this power. These maximum changes may then be used in the design of rigs, or in calculating the rate of removal of control absorber mechanisms etc. and in formulating operational standing orders and administrative controls.

The present study was made using the P.A.C.E. analogue computer to simulate HIFAR when loaded with 150 g Mk III fuel elements to obtain the power response to a range of step and ramp additions of reactivity.

## 2. KINETIC EQUATIONS

These may be written as:

$$\frac{dP}{dt} = \left( \frac{\rho - \beta}{\ell^*} \right) P + \sum_i \lambda_i C_i \quad \dots(a)$$

$$\frac{dC_i}{dt} = \frac{\beta_i P}{\ell^*} - \lambda_i C_i \quad \dots(b)$$

$$\rho = \rho_a f(t) + \rho_{cca} g(t) \quad \dots(c)$$

These equations are derived in Appendix I.

## 3. DATA

### 3.1 The Power

This was assumed to be 15 MW at the start of each transient, but as heating effects were ignored, all changes in power can in fact be considered as ratios, i.e., a peak power of 20 MW following a transient from an initial steady power of 15 MW can be considered as a change of 1.333 and may be applied to any initial power. This is a very conservative assumption, since the reactor temperature coefficient of reactivity has been measured to be  $-0.034 \pm 0.003\% \delta k/k$  per degree C (Connolly et al. 1968), which would provide a significant negative feedback mechanism to reduce the peak power in positive transients. In the model, a step or ramp reactivity change (the possible result of an accident) is added, causing the power to rise. At a power determined by the trip setting on the shut-down amplifiers, a negative reactivity  $\rho_{caa}$  is introduced, simulating the dropping of coarse control arms which eventually reduces the power to a low value. The shut-down amplifier trip setting in the model can be varied.

### 3.2 Reactor Kinetic Parameters

Previous assessments of the effectiveness of these constants (Corran and McCulloch 1968) based on the data of Keepin (1962) and condensed using the method of Skinner and Cohen (1959) led to the use of the following data:

- (a) Mean prompt neutron lifetime

$$l^* = 4 \times 10^{-4} \text{ sec}$$

- (b) Delayed neutron group data

<u>Group i</u>	<u><math>\lambda_i</math> (sec<sup>-1</sup>)</u>	<u><math>\beta_i</math></u>
1	0.01708	$1.461 \times 10^{-3}$
2	0.41863	$5.371 \times 10^{-3}$
3	3.8723	$0.169 \times 10^{-3}$
		<u><math>\beta = 7.001 \times 10^{-3}</math></u>

### 3.3 Reactivity Worth of the Coarse Control Arms

The reactivity worth of the coarse control arm (C.C.A.) bank is measured whenever there is a major change to the HIFAR core (e.g., refuelling with a new type of fuel element or fuel elements of higher fuel content), or if routine differential measurements suggest that the present value is in error following the replacement of a C.C.A. blade. To measure this worth, the C.C.A.'s are raised a small angle  $\Delta\theta$  from the critical angle  $\theta_c$  ( $\theta_c$  is that C.C.A. angle necessary for constant reactor power). The resulting rate of change of neutron flux is used to calculate the change in reactivity  $\Delta\rho$  corresponding to  $\Delta\theta$ .  $\theta_c$  can be changed by adding or removing poisons from the core, enabling a series of values for  $\Delta\rho/\Delta\theta$  to be obtained. Figure 1 is the result of integrating a graph of  $\Delta\rho/\Delta\theta$  versus  $\theta_c$  (Bolton and Parsons, 1966 unpublished data).

The worth of the C.C.A. bank ( $\rho_{cca}$ ) is at present 17%  $\delta k/k$  with a core containing 3.25 kg of <sup>235</sup>U. Increasing HIFAR power from 10 MW to 15 MW will increase the rate of depletion of <sup>235</sup>U if the rig load does not change. Consequently the number of fuel elements replaced each reactor shutdown will increase, and the average fuel content of the core will rise to about 3.4 kg. The maximum possible fuel content, corresponding to twenty-five 150 gram elements, is 3.75 kg and to avoid future changes in the allowable core inventory, this figure was used in the calculations.

Since the reactivity worth of the C.C.A.'s is approximately inversely proportional to the fuel content of the core, the worth of the C.C.A. bank with a core of 3.75 kg <sup>235</sup>U is about 14.8%  $\delta k/k$ .

The reactivity invested in the C.C.A. bank is divided in the ratios 26(C.C.A's 2, 5) : 17(C.C.A's 3, 6) : 7(C.C.A's 1, 4), (Parsons, 1965, 1966, unpublished data). If it is assumed that one arm may fail to drop, the reactivity insertion to be relied upon is 11%, that is the minimum worth of five of the six C.C.A's. This is the value used in the calculations.

Each time a coarse control arm or its mechanism is changed, the drop time from various angles is measured to ensure that it lies between the acceptable laid-down limits. During the normal operation of the reactor, routine tests are made which include the measurement of the C.C.A. drop time from the current C.C.A. angle. The rate of injection of C.C.A. reactivity into the system used in these calculations has been derived from a statistical analysis of these drop times and the measured shape and worth of the C.C.A. bank. This rate of injection is shown in Figure 2, together with the analogue simulation (see 4.2.1).

It has been shown (Crick et al. 1968) that C.C.A. drop times are considerably reduced in a core comprising Mark 4 fuel elements and consequently the above arguments are extremely conservative if applied to such a core.

#### 3.4 Positive Addition of Reactivity

The maximum increase in reactivity used in the calculations is 1.5% since this is currently the limit imposed on any moveable item within the HIFAR reactor system. Reactivity was added in the form of a ramp with 1.5% as a maximum, though in practice this value was never reached before the trips occurred. The ramp rate was varied from 0.1 to 1.5% per sec., but a special case of 0.0337% per second was examined as this represents the fastest ramp possible when moving out a C.C.A. bank of total worth 17%.

The reactor sealed building circular crane has a maximum vertical speed of 1.6 inches per second. At this speed, an infinitely small rig worth 1.5%  $\delta k/k$  would have a maximum ramp rate of about 0.16%  $\delta k/k$  per second occurring as it passes through the core boundary, and is therefore within the limits examined with the model.

Various step additions of reactivity were also examined to deduce maximum allowable changes.

The method of production of the ramps and steps is given in Section 4.

4. THE ANALOGUE MODEL

To solve the equations of Section 2, the parameters are scaled in such a way that the maximum variation of each parameter covers the maximum or near maximum range of the analogue unit in use.

The analogue circuit used is shown in Figure 3 together with the functions handled by each unit. Where units which generate special functions were introduced, these are described below.

Potentiometer settings for static tests and for each measurement are given in Appendices II and III.

4.1 The Scaled Equations

The method of scaling is given in Appendix IV.

The scaled equations to be solved are :

$$\begin{aligned} d \left( \frac{P}{2500} \right) &= (0.75) 10 \left[ \frac{\rho P}{7.5} \right] - (0.35) \left[ \frac{P}{50} \right] + (0.07306) \left[ \frac{X_1}{50} \right] + \\ &+ (0.2686) \left[ \frac{X_2}{50} \right] + (0.00845) \left[ \frac{X_3}{50} \right] \quad \dots(1) \end{aligned}$$

$$\frac{d \left( \frac{X_1}{50} \right)}{dt} = \frac{1}{10} \left\{ (0.1708) \left[ \frac{P}{50} \right] - (0.1708) \left[ \frac{X_1}{50} \right] \right\} \quad \dots(2)$$

$$\frac{d \left( \frac{X_2}{50} \right)}{dt} = 1 \left\{ (0.4186) \left[ \frac{P}{50} \right] - (0.4186) \left[ \frac{X_2}{50} \right] \right\} \quad \dots(3)$$

$$\frac{d \left( \frac{X_3}{50} \right)}{dt} = 10 \left\{ (0.3872) \left[ \frac{P}{50} \right] - (0.3872) \left[ \frac{X_3}{50} \right] \right\} \quad \dots(4)$$

$$\left[ \frac{P_{total}}{0.15} \right] = \left[ \frac{P_{cca}}{0.15} \right] + (0.1) \left[ \frac{P_a}{0.015} \right] \quad \dots(5)$$

## 4.2 Special Functions

### 4.2.1 Coarse Control Arm Reactivity Input

To allow simulation of the change of reactivity when inserting the coarse control arms, the C.C.A. reactivity v. time curve was divided into a series of straight lines (Figure 2). The slope changes at points called break points which are tabulated as the time in seconds following the initiation of the C.C.A. drop and the scaled reactivity and scaled time in the tables of Appendix II.  $\rho_{cca} g(t)$  of Eqn. 3 is simulated by unit P62, Figure 3.

Following the instant at which the HIFAR reactor power reaches the trip level, there is a delay of approximately 0.24 seconds due to the finite time required to operate relays and for the magnetic flux to decay sufficiently to release the C.C.A. clutches. The first break point was made 0.48 seconds on the analogue computer, and the effect of varying this release delay between 0 and 0.48 seconds was investigated by operating potentiometer P10 (Figure 3). A setting of 0.1200 on P10 will set the normal delay of 0.24 seconds (see Table X).

### 4.2.2 Ramp or Step Reactivity Input

The ramp is effected by integrator 11 which produces  $\rho_a f(t)$  of Eqn. 3. Altering the input Potentiometer Q09 alters the slope. Potentiometer P11 sets the starting reactivity of the ramp.

Similarly a step is simulated by having a ramp slope of zero; the worth of the step is produced by the setting of P11.

With the normal two  $D_2O$  pumps operating, the onset of flow instability would occur at a power of 30.7 MW (Crick et al. 1968). Provided this flow instability power is not exceeded in a transient, the fuel elements will be undamaged. Should this power be exceeded, the damage occurring will depend upon the energy released after a power of 30.7 MW has been exceeded.

The present study was carried out to determine allowable ramp rates of addition of reactivity or step additions of reactivity at different settings of the shut-down amplifiers in order that a peak power of 30.7 MW should not be exceeded when the transient is terminated by 5 only of the C.C.A.'s dropping into the core; the remaining C.C.A. is assumed to have jammed when called upon to scram.

The recorded outputs of the analogue computer are in the form shown in Figure 4 and these results are transformed into the graphs of Figures 5-14.

## 5. RESULTS

### 5.1 Ramp Addition of Reactivity

A typical setting for the shut-down amplifier is 0.85 which means that the reactor is operating at a power of 0.85 of the shut-down power.

With this setting the effect of a ramp input is shown in Figure 5 and it is clear that the peak power attained is very strongly influenced by the initial position of the C.C.A.'s. During normal 15 MW operation the C.C.A.'s are likely to be between about  $12^\circ$  at the beginning and  $30^\circ$  at the end of an operational programme (typically 23 days). Within these limits the variation in the peak power attained is quite small. It is when operation occurs outside these limits (e.g., after a mid-cycle shut-down it may be necessary to raise the C.C.A.'s further to overcome Xe poisoning) that the peak power becomes very high since at high C.C.A. angles the reactivity worth of the C.C.A.'s varies only very slowly with C.C.A. angle. Note that the effect of initial C.C.A. position practically disappears for slow ramps as in the case of C.C.A. movement under operator control.

Figure 6 shows the peak power reached with a ramp rate of 0.5% per second as a function of the trip amplifier setting. Over the normal working range of the C.C.A.'s the peak power reached at this ramp rate is not greatly affected by the initial C.C.A. position. The peak would be considerably higher if the C.C.A.'s were at their maximum angle of  $56^\circ$  because from this position the time taken to start effective negative reactivity insertion is greater. The C.C.A.'s are never taken this far out in practice.

Figure 7 was derived using the expression:

$$\left(\frac{\partial R}{\partial S}\right)_P \left(\frac{\partial P}{\partial R}\right)_S \left(\frac{\partial S}{\partial P}\right)_R = -1$$

This shows quite clearly the limitations on shut-down amplifier setting necessary to prevent a peak power of 30 MW being exceeded due to a ramp addition of reactivity.

Because of local conditions within the reactor, for example the inconvenience of having to reset the position of the ion chambers each time the flux is perturbed by a change in rig load, it is unlikely that a shut-down amplifier setting higher than 0.9 would be acceptable.

With the shut-down amplifier set at 0.85 the maximum safe ramp rate is 0.535%  $\delta k/k$  per second with no restrictions on the C.C.A. angle. If, however, the maximum C.C.A. angle were administratively limited to  $40^\circ$ , a ramp rate of 0.77%  $\delta k/k$  per second could be tolerated. Such a restriction would create minimal operational inconvenience since for C.C.A. angles greater than about  $45^\circ$  a trip would in any case cause the reactor to poison out because the rate of increase of negative reactivity due to the accumulation of xenon and samarium is more rapid than the positive increase which can be obtained from raising the C.C.A.'s from this angle.

## 5.2 The Step Addition of Reactivity

Figure 8 shows the effect of a step addition of reactivity with a typical shut-down amplifier setting of 0.85. Note that the most effective initial C.C.A. position is no longer  $10^\circ$  but somewhere around  $30^\circ$  (see also Figures 11 and 12).

The effect of the initial C.C.A. position is less marked than in the case of the ramp addition. This is because the reactivity step is over and a stable doubling time essentially established before the C.C.A.'s start to drop. Once the drop is started, the excess reactivity of the core is progressively reduced until it becomes negative.

In the case of the ramp addition, however, the continued increase of the ramp can compensate or even over-compensate the initial slow negative injection of reactivity by the falling C.C.A.'s. The higher the initial C.C.A. angle the more pronounced this becomes. Figure 2 shows that the early injection of negative reactivity when the C.C.A.'s drop from  $56^\circ$  is about 0.5%  $\delta k/k$  per second so that if the C.C.A. drop has been initiated by a positive ramp of 0.5%  $\delta k/k$  per second, the reactivity of the system will remain substantially constant at the positive reactivity attained by the ramp at the time the C.C.A.'s start to move. This situation persists until the rate of injection of negative reactivity by the C.C.A.'s increases; this is a few tenths of a second, which doubles or trebles the effective C.C.A. delay.

Figure 9 shows the peak power attained following a 0.4% step change in reactivity as a function of the shut-down amplifier setting and the initial C.C.A. position. Also included are the curves for steps of 0.2% and 0.3% with initial C.C.A. positions of  $10^\circ$  and  $20^\circ$  which are typical of normal operations. At these smaller steps the effect of initial position becomes less important and because of this they are similar to ramp additions.

Figure 10 is derived in the same way as Figure 7 and shows that the step addition must be limited to 0.342% to limit the peak power to 30 MW with a shut-down amplifier setting of 0.85. If the maximum C.C.A. angle is restricted to  $40^\circ$  as suggested in considering the ramp case, then the limiting condition for the step reactivity addition occurs at a C.C.A. angle of  $10^\circ$  and allows a step of 0.363% for 30 MW peak power.

### 5.3 The Effect of the Initial C.C.A. Position

Figures 11 and 12 show the effect of initial C.C.A. position in terms of the ramp or step reactivity required to reach a specified peak power with the shut-down amplifiers at 0.85.

It is seen that for a ramp input the effectiveness varies monotonically with C.C.A. angle and is greatest at  $10^\circ$  (the lowest angle tested) whilst for a step addition the most effective initial C.C.A. position is about  $35^\circ$ .

### 5.4 The Effect of Delaying the Release of the C.C.A's

This delay is the time which elapses from the instant the power reaches the shut-down power, until the time the C.C.A. starts to fall. The delay is due to the time required for relays to move and the magnetic clutch to de-energise. The current time for this in HIFAR varies with C.C.A. angle presumably because the further out the C.C.A's are, the more the turning moment and the more quickly the magnetic clutch breaks. The measured values are as follows:

<u>C.C.A. position</u>	<u>Delay time (seconds)</u>
$4^\circ$	$0.134 \pm 0.036$
$12^\circ$	$0.110 \pm 0.036$
$20^\circ$	$0.122 \pm 0.035$
$28^\circ$	$0.088 \pm 0.032$
$36^\circ$	$0.065 \pm 0.022$

A histogram showing the frequency distribution of all the measured C.C.A. delays is shown in Figure 15.

Times have not been measured from other angles. For the sake of simplicity a conservative value of maximum mean value ( $0.134$ ) + 3 standard deviations was taken (= 0.24 seconds) and used in each case so that the peak power reached in Figures 13 and 14 is over-estimated by about 1.5 MW.

It is apparent from Figures 13 and 14 that if the delay in the release of the C.C.A's could be reduced from 0.1 to 0.05 the peak power reached would be reduced by about 3 MW.

6. CONCLUSION

The calculations presented are based on a very conservative model of the HIFAR reactor which takes no account of negative reactivity feedback mechanisms. The results show that the acceptable peak power of 30 MW will not be exceeded at any position of the C.C.A's provided that the shut-down amplifiers are set at 0.85 or above, and the ramp addition of reactivity does not exceed 0.535%  $\delta k/k$  per second or the step change does not exceed 0.342%  $\delta k/k$ .

If the upper position of the C.C.A's is limited to  $40^\circ$  then the maximum acceptable ramp rate becomes 0.77%  $\delta k/k$  per second and the maximum step change becomes 0.363%  $\delta k/k$ .

Should it become necessary to reduce the setting of the shut-down amplifiers for any reason, then a re-assessment of the safe addition rates of reactivity must be made using Figures 7-10.

These results provide a basis for the safety assessment of reactivity changes in HIFAR whether induced accidentally or by the planned removal of rigs or targets during reactor operation at power. The relation between acceptable ramp rates of reactivity addition and C.C.A. angle is such that a substantial increase in permissible ramp rate would be available if the maximum C.C.A. angle was restricted to about  $40^\circ$ . Such a restriction would inconvenience reactor operation very little.

7. ACKNOWLEDGEMENTS

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## APPENDIX I

### Derivation of the Equations

Using simple one-group theory, the general diffusion equation is

$$D\nabla^2\phi - \Sigma_a\phi + S = \frac{dn}{dt} \quad \dots(1)$$

where the source term  $S = k_{\text{eff}}\Sigma_a\phi$ .

Because of the time variation effects of the neutron emissions it is necessary to divide the emitted neutrons into the prompt neutron fraction  $(1-\beta)$ , and the delayed fraction  $\beta$ .

The rate of production of delayed neutrons is  $k_{\infty}\Sigma_a(1-\beta)$  and is thus the prompt contribution to the source term.

If there are  $i$  groups of delayed neutrons, the rate of formation of a group is equal to the rate of decay of its precursor, i.e. to  $\lambda_i C_i$  neutrons  $\text{cm}^{-2} \text{sec}^{-1}$  where  $C_i$  nuclei  $\text{cm}^{-3}$  is the concentration of these precursors at any time and  $\lambda_i$  is the appropriate decay constant. Then the total rate of formation of neutrons =  $\Sigma_i \lambda_i C_i$  neutrons  $\text{cm}^{-2} \text{sec}^{-1}$ . Inserting prompt and delayed neutron groups into the diffusion equation gives

$$D\nabla^2\phi - \Sigma_a\phi + k_{\infty}\Sigma_a(1-\beta) + \Sigma_i \lambda_i C_i = \frac{dn}{dt} \quad \dots(2)$$

The neutron flux  $\phi$ , neutron density  $n$ , and the precursor concentration  $C_i$  are all functions of space and time in a finite reactor. However, in general the variables are separable and thus  $\nabla^2\phi$  can be replaced by  $-B^2\phi$  provided the system is close to critical.

$$\text{Since } \phi = nv \text{ and } \frac{D}{\Sigma_a} = L^2,$$

equation (2) can be written:

$$k_{\infty}v\Sigma_a \left[ (1-\beta) - \frac{1+L^2 B^2}{k_{\infty}} \right] n + \Sigma_i \lambda_i C_i = \frac{dn}{dt} \quad \dots(3)$$

and for the one-group model  $k_{\text{eff}} = \frac{k_{\infty}}{1+L^2 B^2}$  so that equation (3) may be written:

(cont....)

Appendix I (cont.)

$$k_{\infty} v \Sigma_a \left[ \left( 1 - \frac{1}{k_{\text{eff}}} \right) - \beta \right] n + \Sigma_i \lambda_i C_i = \frac{dn}{dt} \quad \dots(4)$$

Now  $l = \frac{\lambda_a}{v} = \frac{1}{v \Sigma_a}$  ; \dots(5)

hence  $\frac{1}{k_{\infty} v \Sigma_a} = \frac{l}{k_{\infty}} = l^*$

where  $l^*$  is the prompt neutron generation time.

Reactivity is defined as  $\rho = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}} = 1 - \frac{1}{k_{\text{eff}}}$  \dots(6)

and substituting in equation (4)

$$\frac{dn}{dt} = \left( \frac{\rho - \beta}{l^*} \right) n + \Sigma_i \lambda_i C_i \quad \dots(7)$$

To solve this equation it is necessary to have an expression for the variation of the concentration  $C_i$  of the delayed neutron precursors with time. The rate of formation of the  $i$  th group is equal to  $\beta_i k_{\infty} \Sigma_a \phi$  where  $k_{\infty} \Sigma_a \phi$  is the total rate of production of fission neutrons and  $\beta_i$  the fraction of neutrons in the  $i$  th delayed group. Radioactive decay occurs simultaneously with the production and at a rate  $\lambda_i C_i$  and the net rate of formation of the  $i$  th group precursor is given by

$$\frac{dC_i}{dt} = \beta_i k_{\infty} \Sigma_a \phi - \lambda_i C_i$$

Since  $\phi = nv$

and  $k_{\infty} v \Sigma_a = \frac{1}{l^*}$

then  $\frac{dC_i}{dt} = \frac{\beta_i}{l^*} n - \lambda_i C_i$  \dots(8)

In terms of reactor power, the equations to be solved are then

$$\frac{dP}{dt} = \left( \frac{\rho - \beta}{l^*} \right) P + \Sigma_i \lambda_i C_i \quad \dots(9)$$

and  $\frac{dC_i}{dt} = \frac{\beta_i}{l^*} P - \lambda_i C_i$  \dots(10)

(cont....)

Appendix 1 (cont.)

where

$$\begin{aligned}\rho &= \text{total reactivity} \\ &= \rho_{\text{injected}} + \rho_{\text{coarse control arms}} \\ &= \rho_a + \rho_{cca}\end{aligned}$$

Note that the source term contains a spontaneous fission source but this has been neglected since it represents only about  $10^{-6}$  of the normal operating power.



APPENDIX II

Analogue Circuit Parameters

1. BREAK POINTS FOR THE CCA REACTIVITY INSERTION

TABLE I

CCA's Falling from 56°

t (sec) No delay	t <sub>D</sub> (sec) Max delay	t <sub>D</sub> Scaled = t <sub>D</sub> /2 = x*	ρ <sub>cca</sub> %	ρ <sub>cca</sub> Scaled = y*
0.0	0.0	0.0	0.0	0.0
0.0	0.48	0.24	0.0	0.0
0.585	1.065	0.5325	-0.3	-0.0200
0.805	1.285	0.6425	-0.85	-0.0566
0.880	1.360	0.68	-1.65	-0.11
0.930	1.410	0.705	-2.95	-0.1966
0.980	1.430	0.715	-4.05	-0.27
0.950	1.460	0.730	-7.25	-0.483
1.044	1.524	0.762	-11.00	-0.7333

\* x and y refer to controls of the function generator F62.

TABLE II

CCA's Falling from 40°

t (sec) No delay	t <sub>D</sub> (sec) Max delay	t <sub>D</sub> Scaled = t <sub>D</sub> /2 = x	ρ <sub>cca</sub> %	ρ <sub>cca</sub> Scaled = y
0.0	0.0	0.0	0.0	0.0
0.0	0.48	0.24	0.0	0.0
0.310	0.79	0.395	-0.28	-0.0186
0.465	0.945	0.4725	-0.93	-0.062
0.610	1.090	0.545	-2.23	-0.1466
0.735	1.215	0.6075	-3.90	-0.26
0.810	1.29	0.645	-5.60	-0.3733
0.875	1.355	0.6775	-8.00	-0.533
0.945	1.425	0.7125	-10.46	-0.6973

TABLE III

CCA's Falling from 30°

t (sec) No delay	t <sub>D</sub> (sec) Max delay	t <sub>D</sub> Scaled = t <sub>D</sub> /2 = x	ρ <sub>cca</sub> %	ρ <sub>cca</sub> Scaled = y
0.0	0.0	0.0	0.0	0.0
0.0	0.48	0.24	0.0	0.0
0.135	0.615	0.3075	-0.3	-0.02
0.245	0.725	0.3625	-0.9	-0.06
0.36	0.84	0.42	-2.05	-0.1366
0.47	0.95	0.475	-3.6	-0.24
0.62	1.10	0.55	-6.3	-0.42
0.715	1.195	0.5975	-8.4	-0.56
0.825	1.305	0.6525	-10.38	-0.692

TABLE IV

CCA's Falling from 20°

t (sec) No delay	t <sub>D</sub> (sec) Max delay	t <sub>D</sub> Scaled = t <sub>D</sub> /2 = x	ρ <sub>cca</sub> %	ρ <sub>cca</sub> Scaled = y
0.0	0.0	0.0	0.0	0.0
0.0	0.48	0.24	0.0	0.0
0.085	0.565	0.2825	-0.57	-0.38
0.017	0.65	0.325	-1.49	-0.993
0.265	0.745	0.3725	-2.95	-0.1966
0.4	0.88	0.44	-5.7	-0.38
0.46	0.94	0.47	-6.95	-0.4633
0.52	1.00	0.50	-8.0	-0.5333
0.588	1.068	0.534	-8.76	-0.5840

TABLE V

CCA's Falling from  $10^0$ 

t (sec) No delay	$t_D$ (sec) Max delay	$t_D$ Scaled = $t_D/2$ = x	$\rho_{cca}$ %	$\rho_{cca}$ Scaled = y
0.0	0.0	0.0	0.0	0.0
0.0	0.48	0.24	0.0	0.0
0.085	0.565	0.2825	-1.175	-0.0653
0.17	0.65	0.325	-2.82	-0.1566
0.238	0.718	0.359	-4.536	-0.252
0.255	0.735	0.3675	-5.04	-0.28
0.275	0.755	0.3775	-5.375	-0.2986
0.295	0.775	0.3875	-5.58	-0.31
0.32	0.8	0.4	-5.675	-0.3153

TABLE VI

Potentiometer Settings

Number	Value	Remarks
P00	0.2703	P00 x P08 = 0.07306
P01	0.5	
P02	0.1708	10 $\lambda_1$
P03	0.75	
P08	0.2703	
P04	0.3	
P05	0.2686	
P06	0.0919	P06 x P07 = 0.00845
P07	0.0919	
P10	0.12	This is the normal value. Altered to change CCA delay (Table X).
P11	Variable	0.0000 during ramp, varied for step (Table VIII).
Q00	0.1708	10 $\lambda_1$
Q01	0.4186	$\lambda_2$
Q02	0.35	
Q03	0.3872	$\lambda_3/10$
Q04	Variable	Shut down amplifier setting (Table IX).
Q05	0.4186	$\lambda_2$
Q06	0.3872	$\lambda_3/10$
Q09	Variable	0.0000 for step. Varied for ramp rate (Table VII).
Q10	0.5000	
Q11	0.1000	
Q15	Variable	Used to prevent overload of integrator 15
Q24	Variable	Keeps CCA value constant when fully in

2. VARIABLE POTENTIOMETER / AMPLIFIER SETTINGS

TABLE VII

Production of Ramp Input of Reactivity

Ramp Required % $\delta k/k$ per sec.	Potentiometer Q09 Setting	Amplifier 11 Time Constant
0.0337	0.2247	10.0
0.1	0.6667	10.0
0.5	0.3333	1.0
0.6	0.4000	1.0
0.675	0.4500	1.0
0.713	0.4750	1.0
0.75	0.5000	1.0
0.9	0.6000	1.0
1.0	0.6667	1.0
1.05	0.7000	1.0
1.2	0.8000	1.0
1.35	0.9000	1.0
1.5	1.0000	1.0
2.0	0.1333	0.1
5.0	0.3333	0.1

TABLE VIII

Production of Step Input of Reactivity  
(Integrator 11 time constant = 1.0 second)

Step Required % $\delta k/k$	Potentiometer P11 Setting
0.1	0.0667
0.2	0.1333
0.3	0.2000
0.4	0.2667
0.5	0.3333
0.6	0.4000

TABLE IX

Production of the Shut Down Amplifier Setting

Trip Setting Required	Power at this trip (Mw)	Potentiometer Q04 Setting
0.95	15.79	0.3157
0.85	17.65	0.3529
0.80	18.75	0.3750
0.75	20.00	0.4000
0.65	23.08	0.4615
0.60	25.0	0.5000
0.55	27.27	0.5454
0.50	30.0	0.6000
0.45	33.33	0.6667
0.428	35.0	0.7000
0.375	40.0	0.8000
0.3333	45.0	0.9000
0.30	50.0	1.0000

TABLE X

Production of the CCA Delay Time

Delay Required (Seconds)	Potentiometer P10 Setting
0.0	0.24
0.12	0.18
0.24	0.12
0.30	0.09
0.36	0.06

TABLE XI

Amplifier List

Unit No.	Mode	Time-Const. (Sec.)	Function
00	Integrator	10	to produce $X_1/50$ delayed neutron cct.
01	Integrator	0.1	to produce $P/50$
02	Amp.	-	inverter producing - $P/50$
03	Amp.	-	inverter producing - $\frac{d}{dt} \left( \frac{P}{2500} \right)$
04	Amp.	-	to produce the 15 MW starting power
05	Integrator	1.0	to produce $X_2/50$ delayed neutron cct.
06	Integrator	0.1	to produce $X_3/50$ delayed neutron cct.
10	Integrator	10	to produce the time scale
11	Integrator	1	to produce the ramp or step reactivity input
12	Amp.	-	to sum the reactivity inputs
13	Amp.	-	connected by SJ's to 03 to provide an extra $X_1$ input
15	Integrator	10	to produce $\int P dt$
Mo	Comparator		to compare $P$ with $P_{trip}$ and to actuate the dropping of the CCA's
M1	Comparator		to limit the ramp input maximum to 1.5% $\delta k/k$
M2	Comparator		to limit the worth of the CCA's
F62	Function Generator		to simulate the insertion of the CCA's



APPENDIX III

Static Test

$$\frac{d}{dt} \left( \frac{P}{2500} \right) = 0.75 \times 10 \times \left[ \frac{\rho_{\text{total}}}{0.15} \right] \left[ \frac{P}{50} \right] - 0.35 \left[ \frac{P}{50} \right] + \\ + 0.07306 \left[ \frac{X_1}{50} \right] + 0.2686 \left[ \frac{X_2}{50} \right] + 0.00845 \left[ \frac{X_3}{50} \right] .$$

With the function generator set up for the CCA's to drop from  $56^\circ$ , setting potentiometer P10 to 0.715 produces a value for  $\frac{\rho_{\text{cca}}}{0.15}$  of -0.27.

Setting P11 to 0.5 produces a value of  $\frac{\rho_a}{0.015}$  of 0.5.

$$\text{When } \rho_{\text{total}} = -0.27 + 0.1 \times 0.5 \\ = -0.22$$

$$\frac{d}{dt} \left( \frac{P}{2500} \right) = 7.5 (-0.22) \left( \frac{15}{50} \right) - 0.35 \left( \frac{15}{50} \right) \\ + 0.07306 \left( \frac{15}{50} \right) + 0.2686 \left( \frac{15}{50} \right) + 0.00845 \left( \frac{15}{50} \right) \\ = -0.49586$$

Thus with the settings of the potentiometer above, the digital volt meter connected to the output of amplifier O3 should read 0.4959.



APPENDIX IV

Derivation of the Analogue Scaling

The equations to be solved are:-

$$\frac{dP}{dt} = \left( \frac{\rho - \beta}{\ell^*} \right) P + \sum_i \lambda_i C_i \quad , \quad \dots(1)$$

$$\frac{dC_i}{dt} = \frac{\beta_i P}{\ell^*} - \lambda_i C_i \quad , \quad \dots(2)$$

$$\rho = \rho_a f(t) + \rho_{cca} g(t) \quad . \quad \dots(3)$$

Defining a new variable  $X_i = \frac{\lambda_i \ell^*}{\beta_i} C_i \quad \dots(4)$

when  $\frac{dX_i}{dt} = 0, X_i = P$

and substituting for  $C_i$  in equations (1) and (2)

$$\frac{dP}{dt} = \left( \frac{\rho - \beta}{\ell^*} \right) P + \sum_i \frac{\beta_i X_i}{\ell^*} \quad . \quad \dots(5)$$

The equations may now be scaled making the assumptions that the maximum power attained is 50 MW, the maximum reactivity held in the C.C.A. bank is 15 percent (allowing for a little flexibility if required later) and that the maximum injected reactivity is 1.5 percent.

Expanding equation (5),

$$\frac{dP}{dt} = \frac{\rho P}{\ell^*} - \frac{\beta P}{\ell^*} + \frac{\beta_1 X_1}{\ell^*} + \frac{\beta_2 X_2}{\ell^*} + \frac{\beta_3 X_3}{\ell^*}$$

Scaling equation (5) using the assumptions made

$$\begin{aligned} \frac{d(P/2500)}{dt} = \frac{1}{50} \left\{ \frac{1}{4 \times 10^{-4}} \left[ \frac{\rho}{0.15} \right] 0.15 \left[ \frac{P}{50} \right] 50 - \right. \\ - \frac{7.001 \times 10^{-3}}{4 \times 10^{-4}} \left[ \frac{P}{50} \right] 50 + \frac{1.461 \times 10^{-3}}{4 \times 10^{-4}} \left[ \frac{X_1}{50} \right] 50 + \\ \left. + \frac{5.3711 \times 10^{-3}}{4 \times 10^{-4}} \left[ \frac{X_2}{50} \right] 50 + \frac{0.169 \times 10^{-3}}{4 \times 10^{-4}} \left[ \frac{X_3}{50} \right] 50 \right\} \end{aligned}$$

(cont...)

Appendix IV (cont.)

$$\begin{aligned}
 &= 7.5 \left[ \frac{\rho P}{7.5} \right] - 0.3500 \left[ \frac{P}{50} \right] + \\
 &\quad + 0.07306 \left[ \frac{X_1}{50} \right] + 0.26856 \left[ \frac{X_2}{50} \right] + 0.00845 \left[ \frac{X_3}{50} \right] \quad \dots(6)
 \end{aligned}$$

Using the inputs and potentiometers available in the PACE units the equation may be re-written as:

$$\begin{aligned}
 \frac{d \left( \frac{P}{2500} \right)}{dt} &= (0.75) \cdot 10 \left[ \frac{\rho P}{7.5} \right] - (0.3500) \left[ \frac{P}{50} \right] + \\
 &\quad + (0.07306) \left[ \frac{X_1}{50} \right] + (0.2686) \left[ \frac{X_2}{50} \right] + \\
 &\quad + (0.00845) \left[ \frac{X_3}{50} \right] \quad \dots(7)
 \end{aligned}$$

from equations (2) and (4)

$$\frac{dC_i}{dt} = \frac{\beta_i}{\ell^*} P - \lambda_i C_i \quad \dots(8)$$

$$\frac{dX_i}{dt} = \frac{dX_i}{dC_i} \cdot \frac{dC_i}{dt} \quad \dots(9)$$

$$\begin{aligned}
 &= \frac{\lambda_i \ell^*}{\beta_i} \left[ \frac{\beta_i}{\ell^*} P - \lambda_i C_i \right] \\
 &= \lambda_i P - \lambda_i X_i \quad \dots(10)
 \end{aligned}$$

Scaling as described above

$$\frac{d \left( \frac{X_1}{50} \right)}{dt} = \lambda_i \left[ \frac{P}{50} \right] - \lambda_i \left[ \frac{X_1}{50} \right] \quad \dots(11)$$

(cont...)

## Appendix IV (cont.)

Inserting the data

$$\begin{aligned}\frac{d\left(\frac{X_1}{50}\right)}{dt} &= (0.01708) \left[\frac{P}{50}\right] - (0.01708) \left[\frac{X_1}{50}\right] \\ &= \frac{1}{10} \left\{ (0.1708) \left[\frac{P}{50}\right] - (0.1708) \left[\frac{X_1}{50}\right] \right\} \quad \dots(12)\end{aligned}$$

The factor  $\frac{1}{10}$  is obtained by making the time constant of the integrator equal to 10 seconds.

Similarly

$$\frac{d\left(\frac{X_2}{50}\right)}{dt} = (0.4186) \left[\frac{P}{50}\right] - (0.4186) \left[\frac{X_2}{50}\right] \quad \dots(13)$$

$$\text{and } \frac{d\left(\frac{X_3}{50}\right)}{dt} = 10 \left\{ (0.3872) \left[\frac{P}{50}\right] - (0.3872) \left[\frac{X_3}{50}\right] \right\} \quad \dots(14)$$

The factor 10 is obtained by an amplifier time constant of 0.1 seconds.

From equation (3)

$$\rho = \rho_{\text{total}} = \rho_{\text{cca}} + \rho_a$$

where  $\rho_a$  is the ramp or step injected reactivity

$$\begin{aligned}\left[\frac{\rho}{0.15}\right] &= \left[\frac{\rho_{\text{cca}}}{0.15}\right] + \left(\frac{0.015}{0.15}\right) \left[\frac{\rho_a}{0.015}\right] \\ &= \left[\frac{\rho_{\text{cca}}}{0.15}\right] + (0.1) \left[\frac{\rho_a}{0.015}\right] \quad \dots(15)\end{aligned}$$



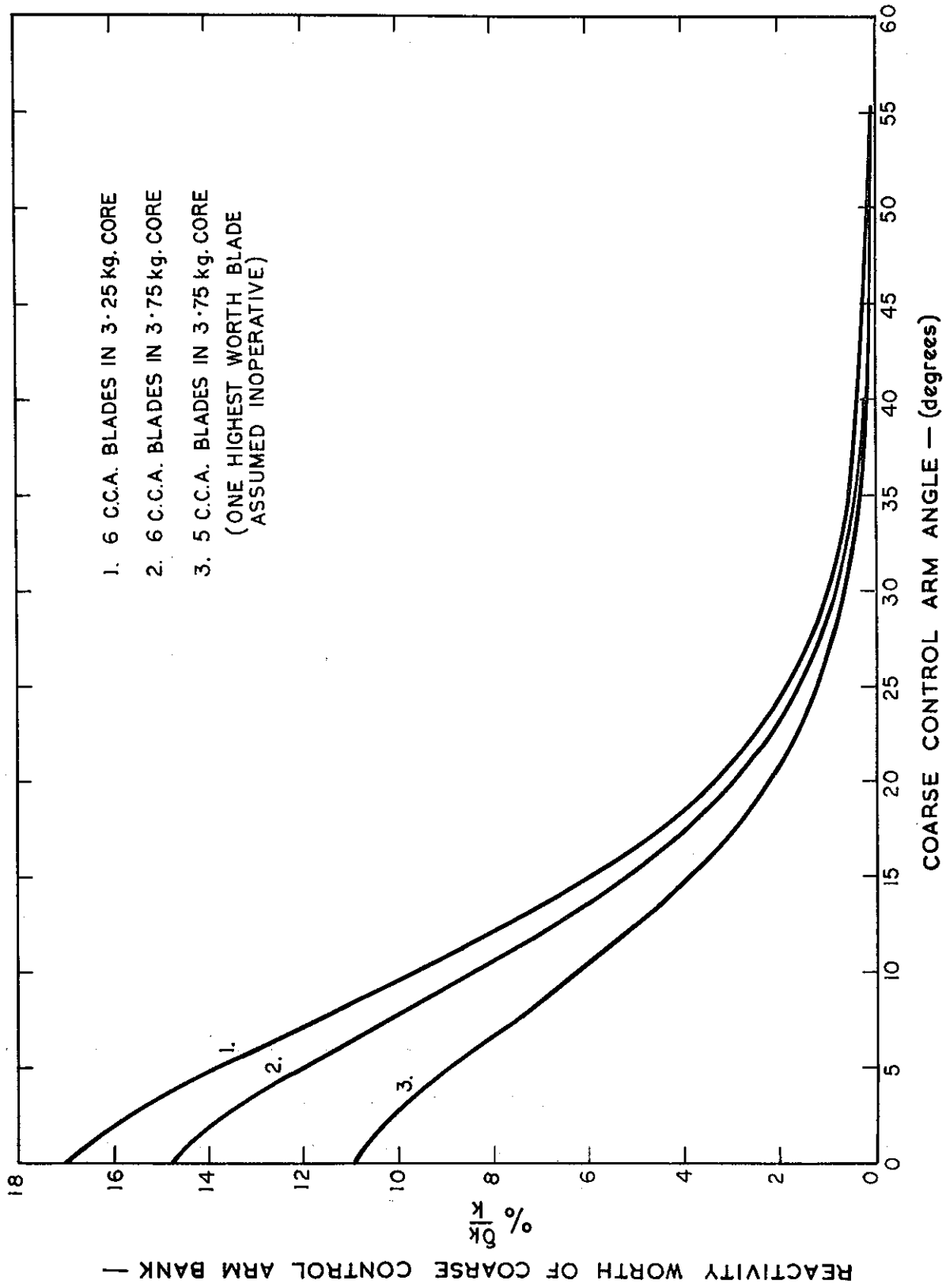
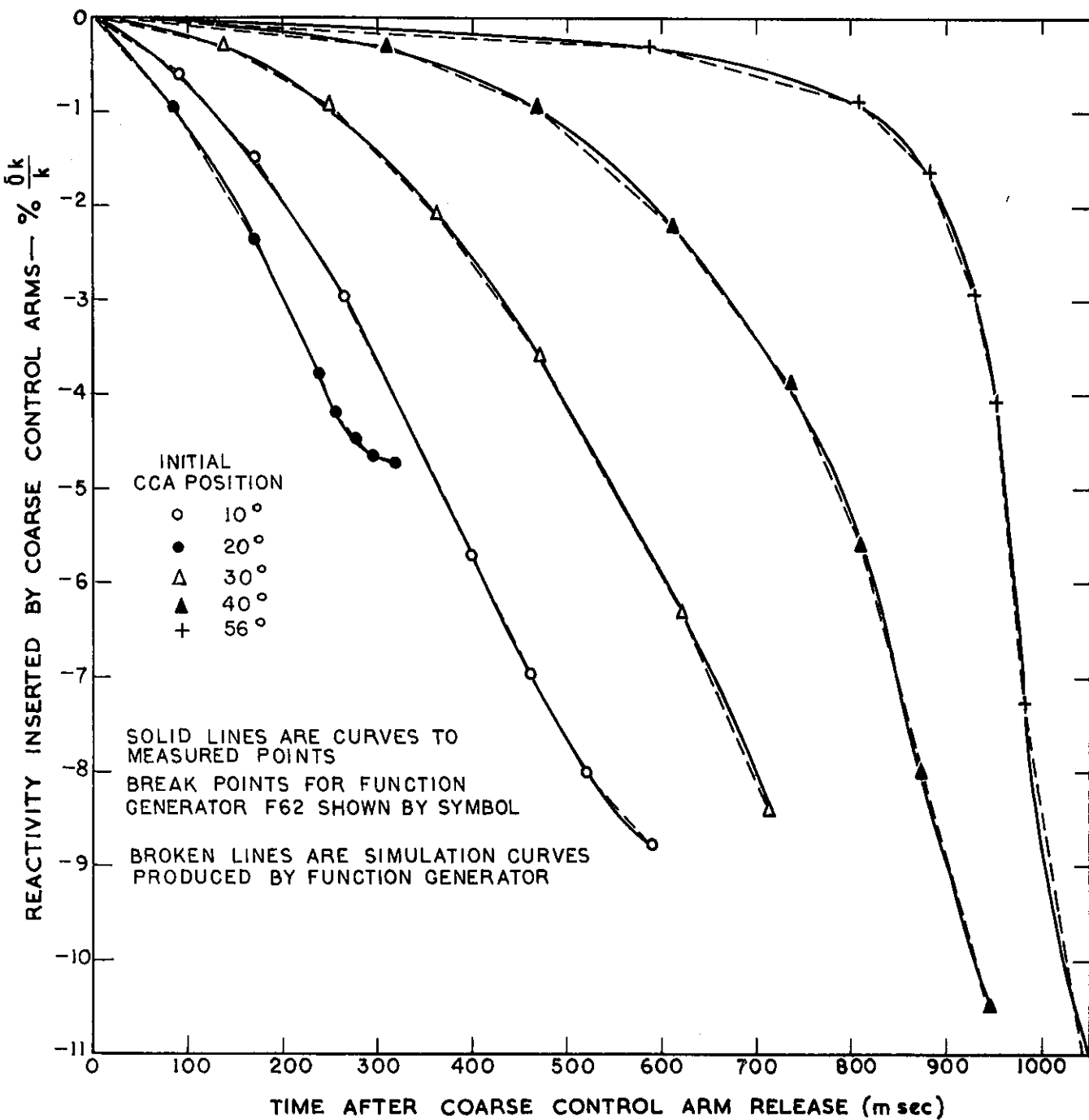


FIGURE 1. C.C.A. WORTH AT VARIOUS CORE LOADINGS



**FIGURE 2. INSERTION OF REACTIVITY WHEN COARSE CONTROL ARMS ARE DROPPED FROM VARIOUS INITIAL POSITIONS**

HIFAR ANALOG SIMULATION FOR  
POWER TRANSIENT STUDIES

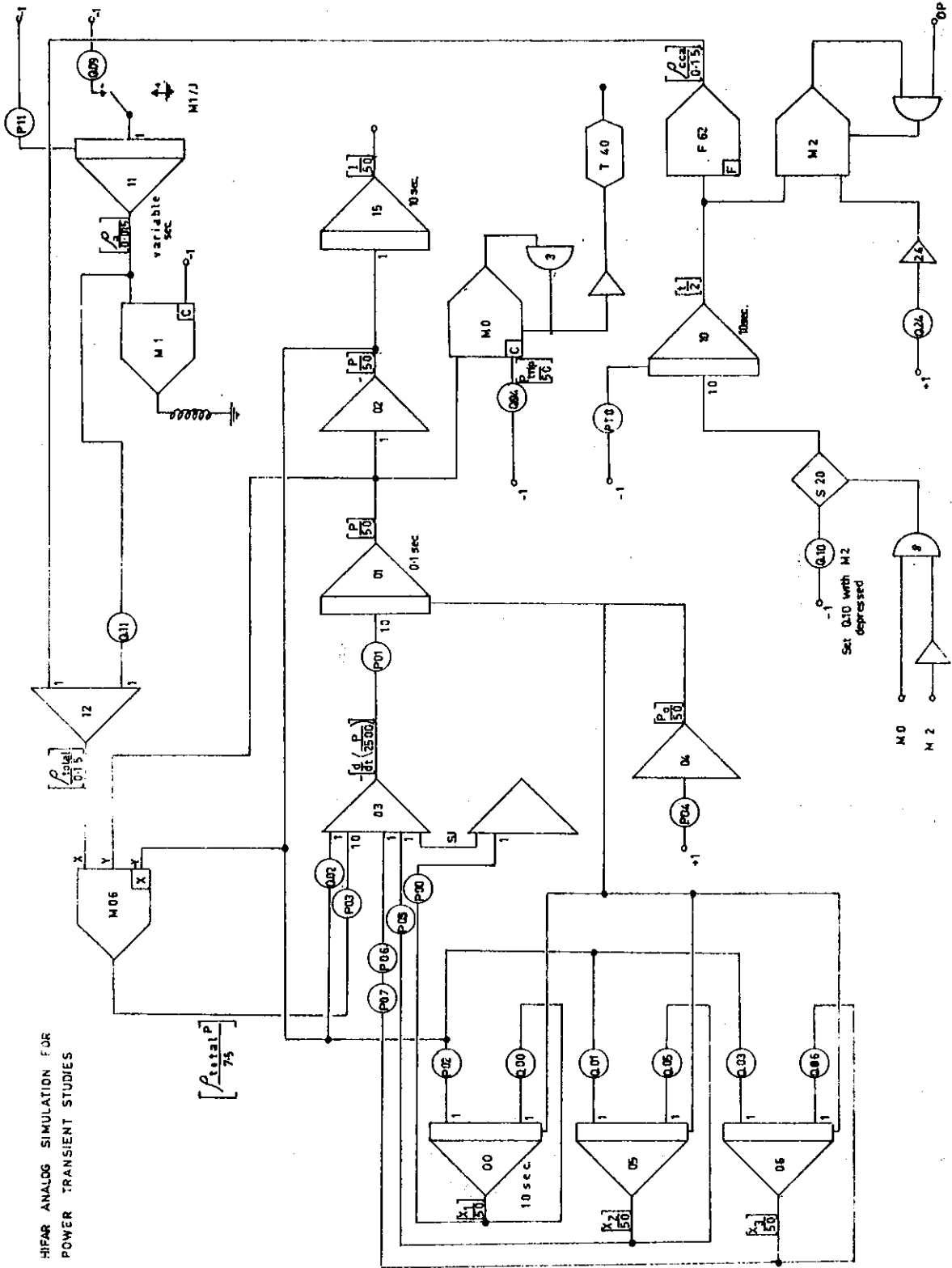


FIGURE 3. THE ANALOGUE CIRCUIT

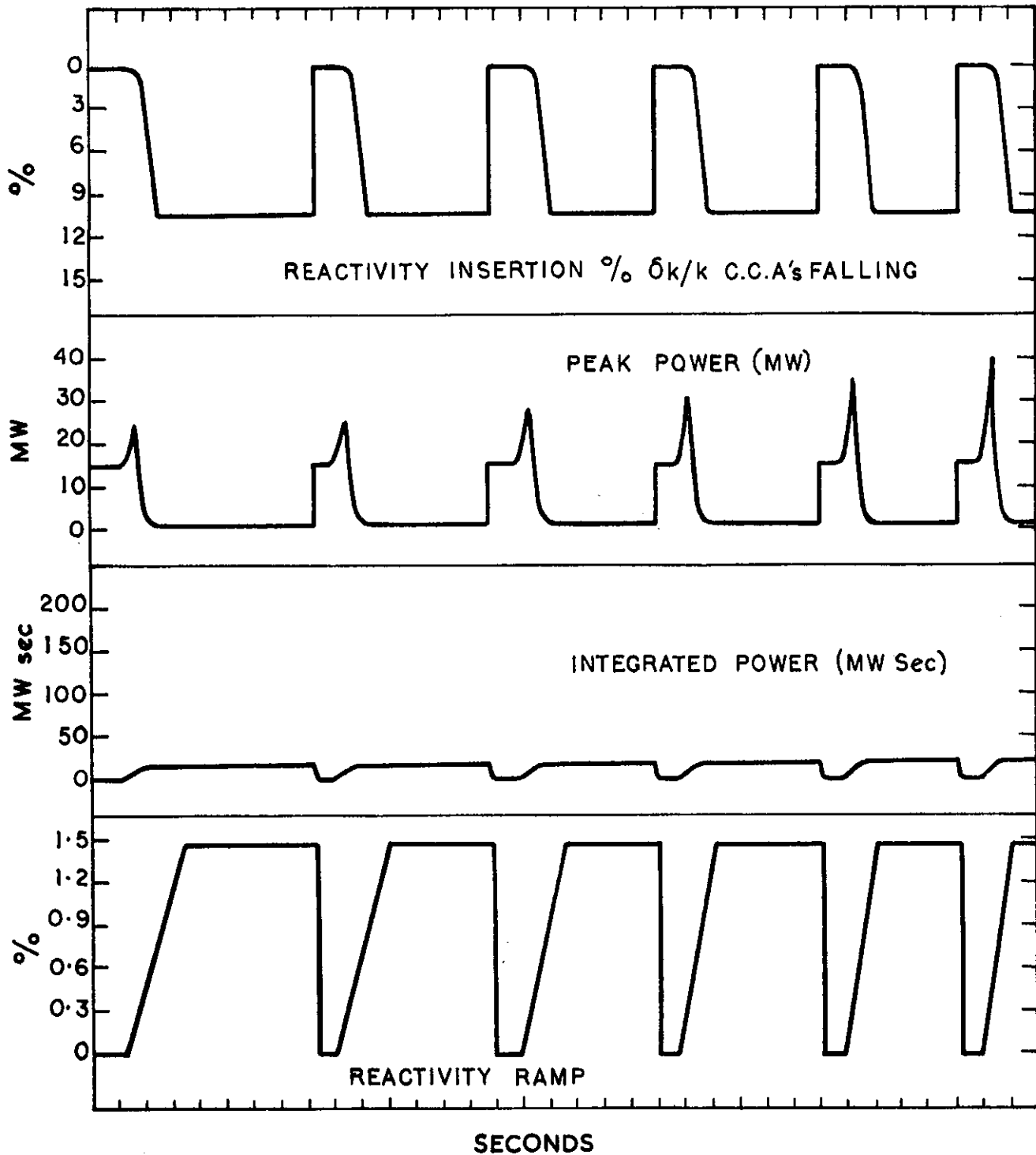
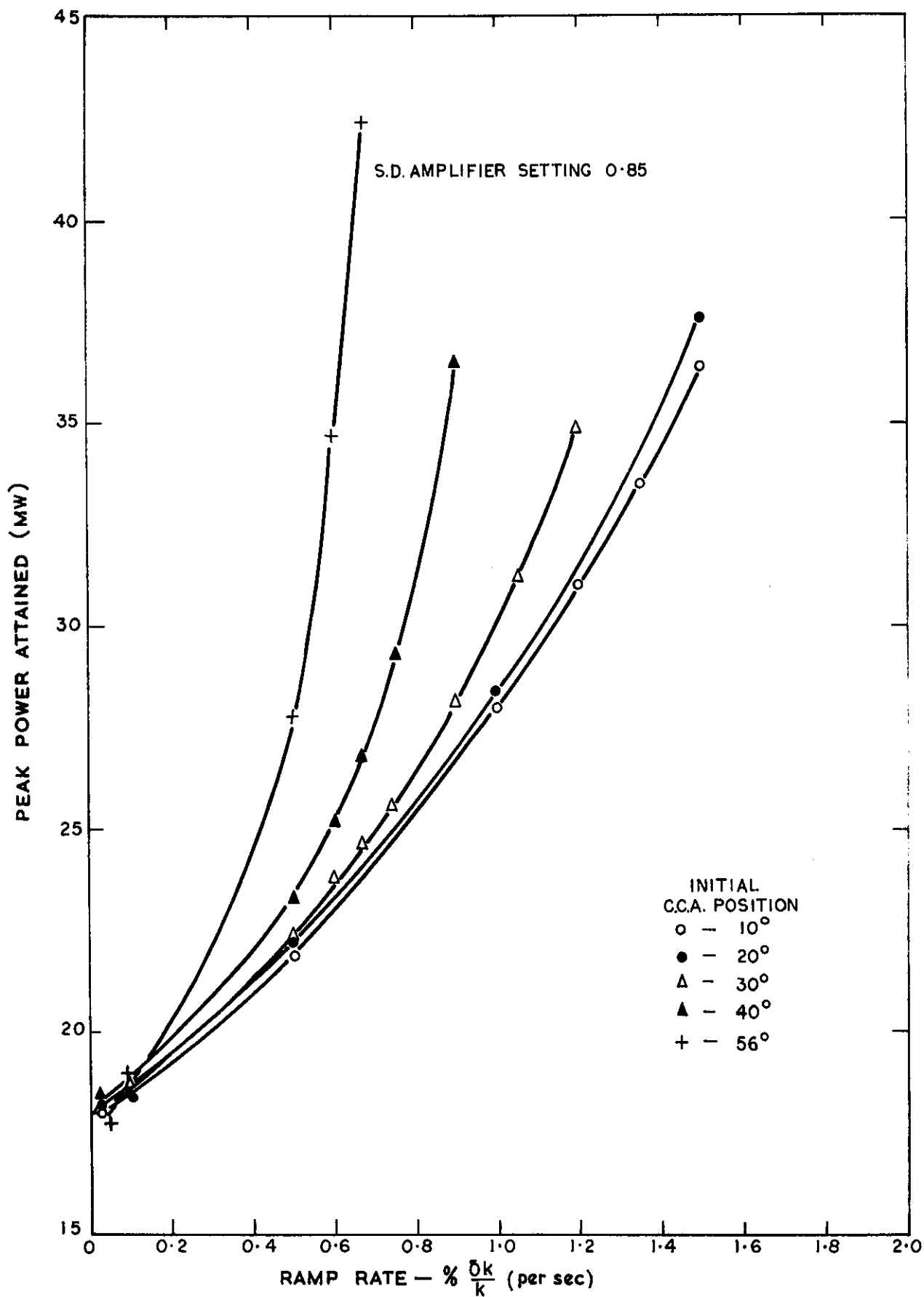


FIGURE 4. TYPICAL ANALOGUE OUTPUT



**FIGURE 5. PEAK POWER ATTAINED ON INCREASING REACTIVITY AT A STEADY RATE. EFFECTS OF RATE OF REACTIVITY INCREASE AND INITIAL C.C.A. POSITION**

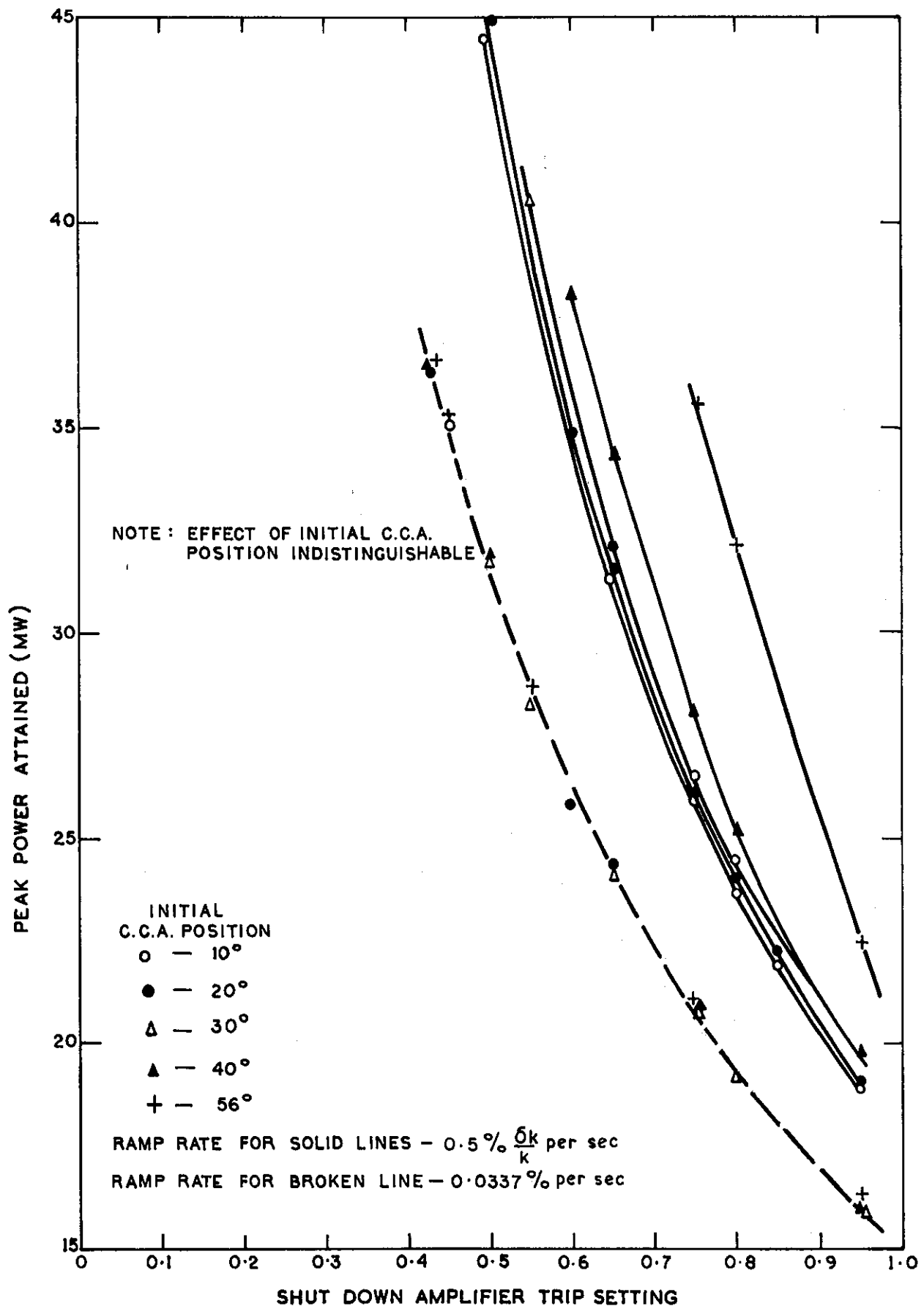
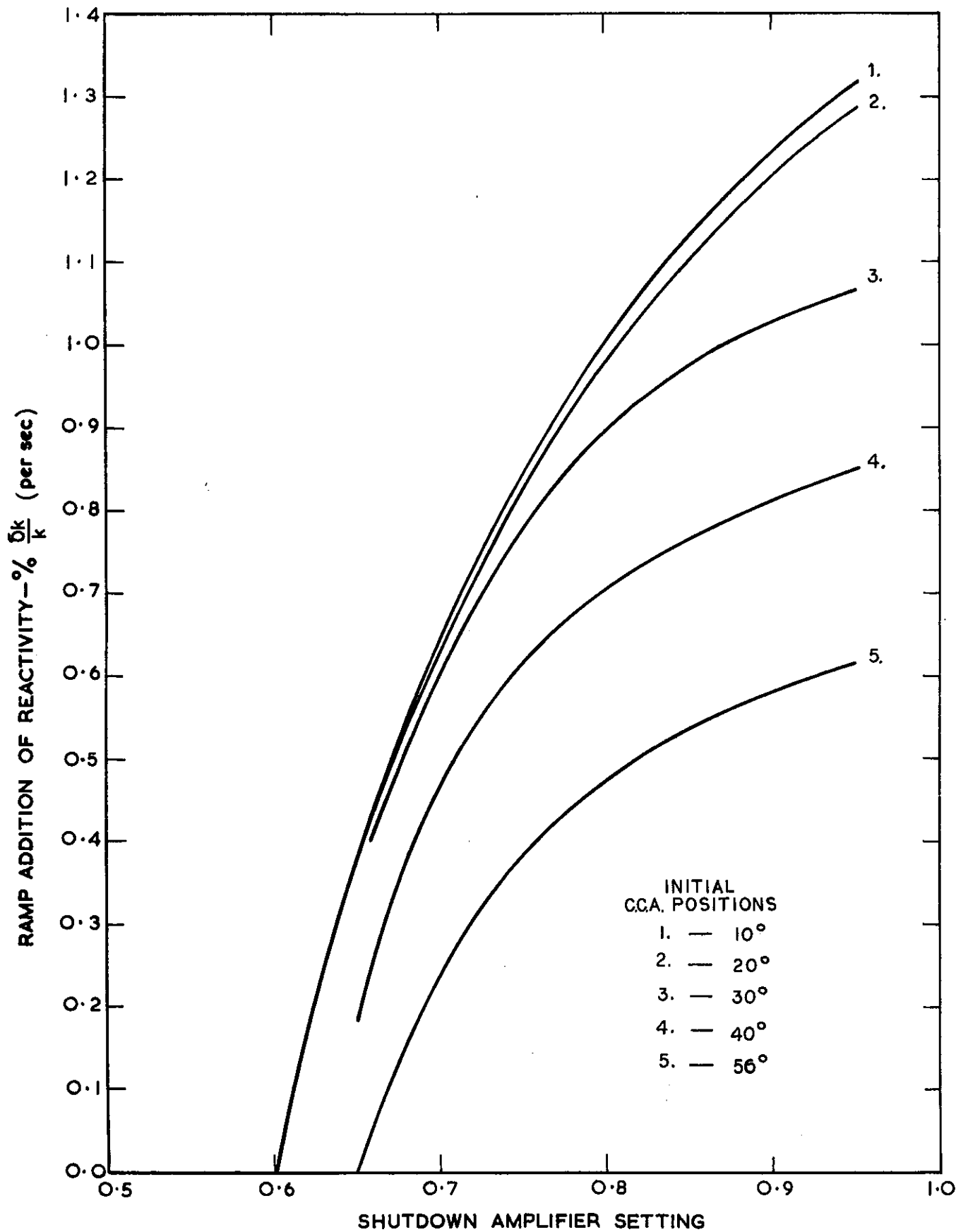
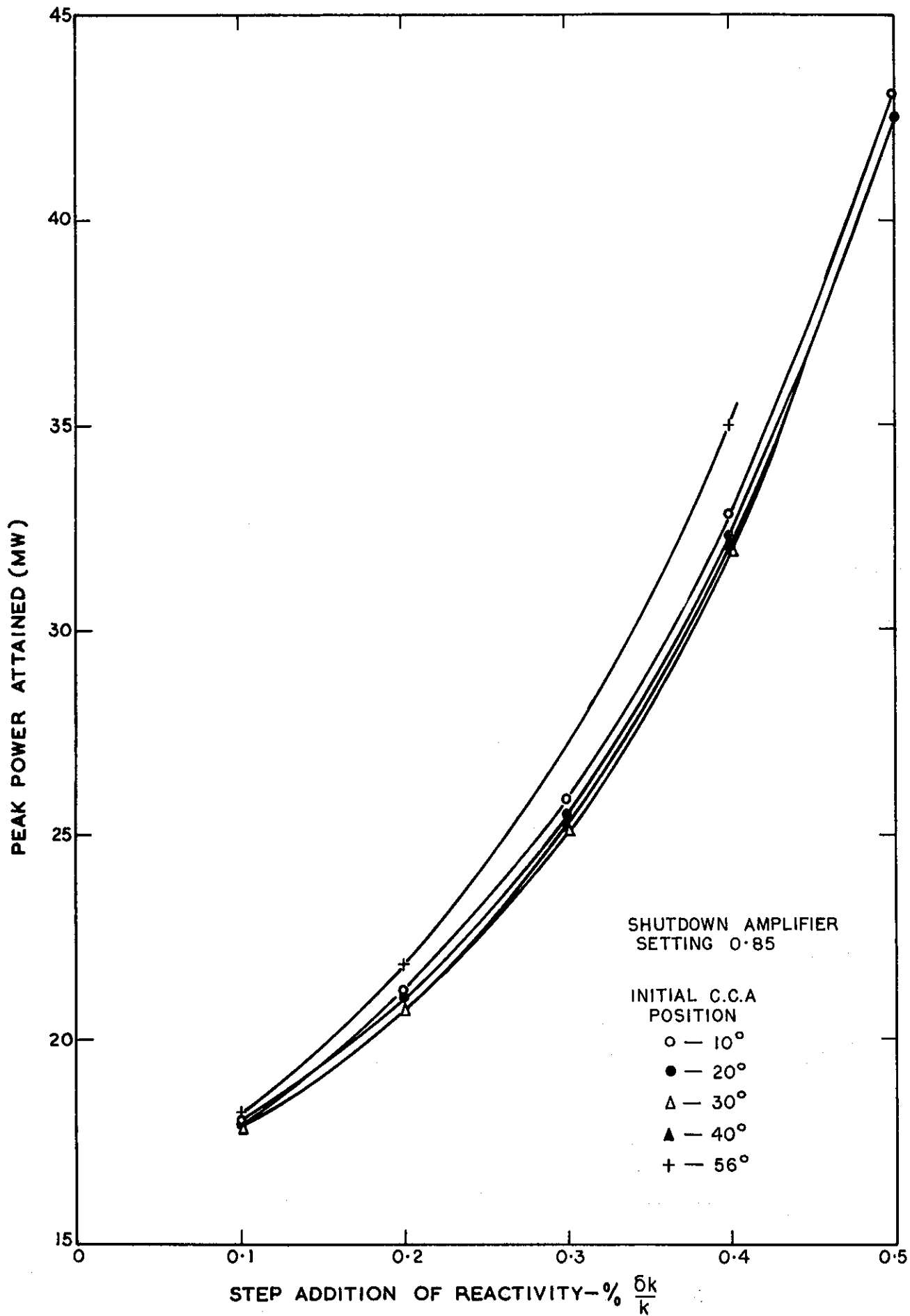


FIGURE 6. PEAK POWER ATTAINED ON INCREASING REACTIVITY AT A STEADY RATE EFFECTS OF SHUTDOWN AMPLIFIER TRIP SETTING AND INITIAL C.C.A. POSITION



**FIGURE 7. RAMP ADDITION OF REACTIVITY REQUIRED TO PRODUCE A PEAK POWER OF 30 MW AT VARIOUS SHUTDOWN AMPLIFIER SETTINGS**



**FIGURE 8. PEAK POWER ATTAINED FOLLOWING A STEP ADDITION OF REACTIVITY. EFFECT OF STEP SIZE AND INITIAL C.C.A. POSITION**

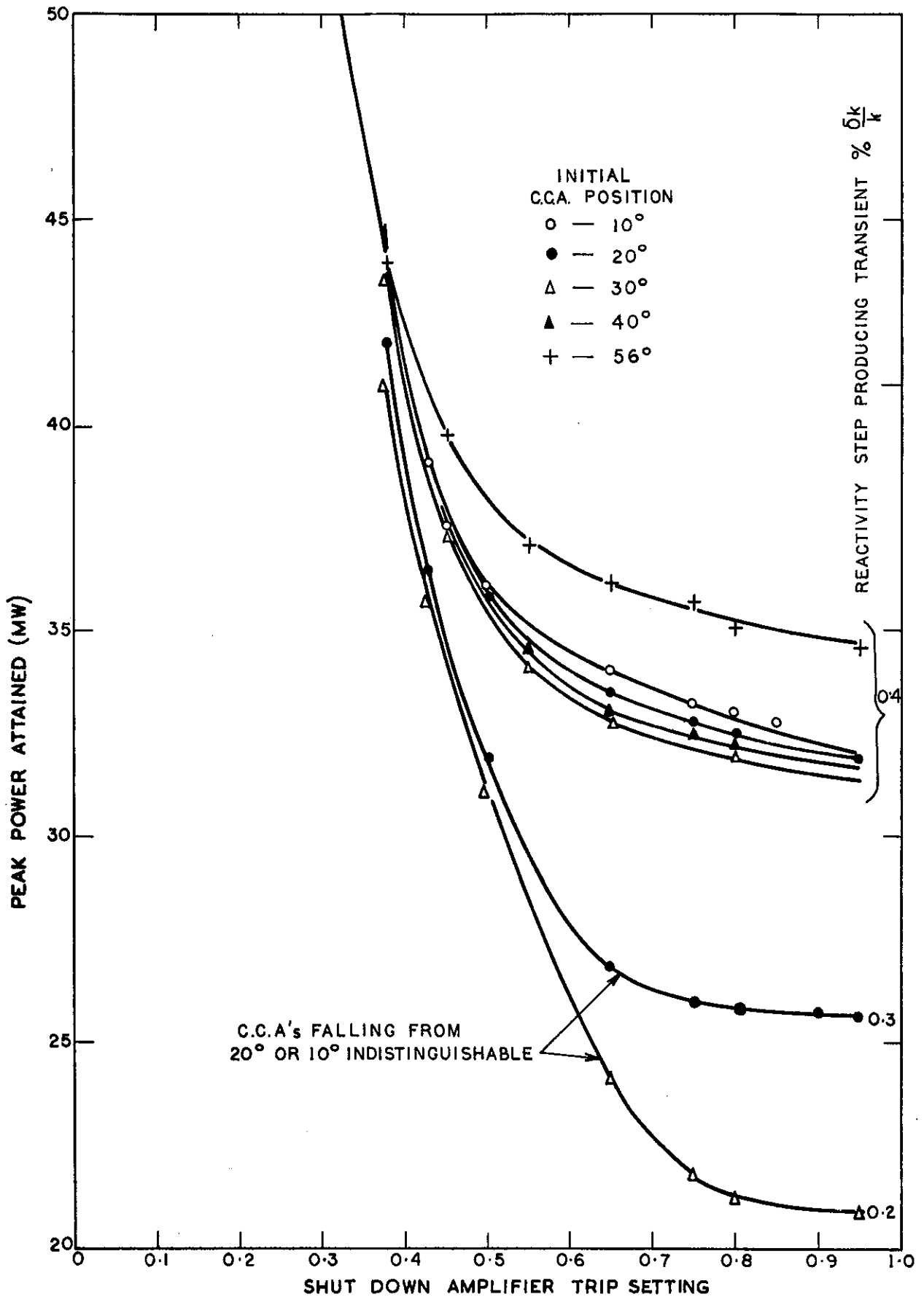
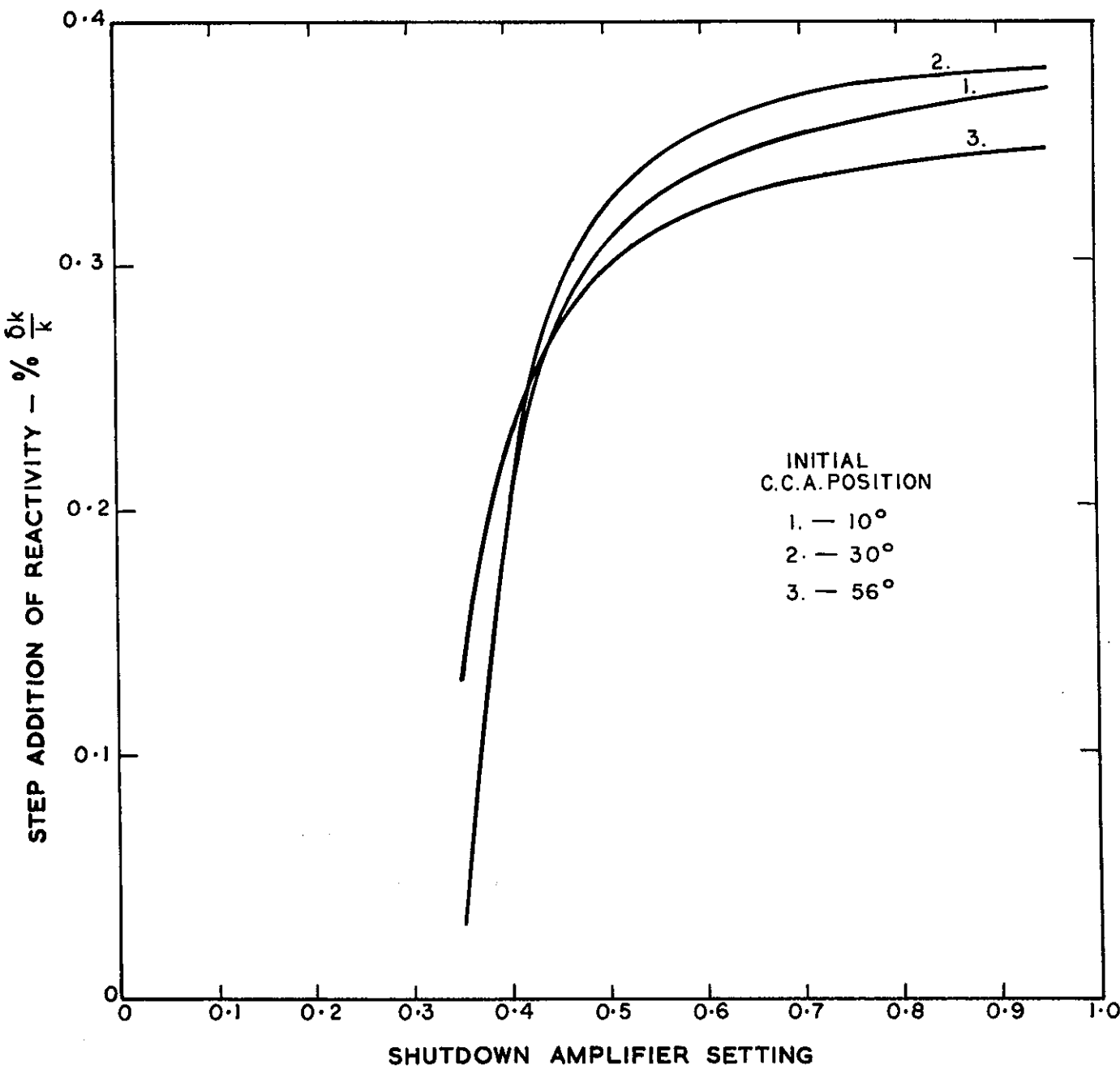


FIGURE 9. PEAK POWER ATTAINED FOLLOWING A STEP ADDITION OF REACTIVITY. EFFECT OF SHUTDOWN AMPLIFIER SETTINGS, INITIAL C.C.A. POSITION AND SIZE OF STEP



**FIGURE 10. STEP ADDITION OF REACTIVITY REQUIRED TO PRODUCE A PEAK POWER OF 30 MW AT VARIOUS SHUTDOWN AMPLIFIER SETTINGS**

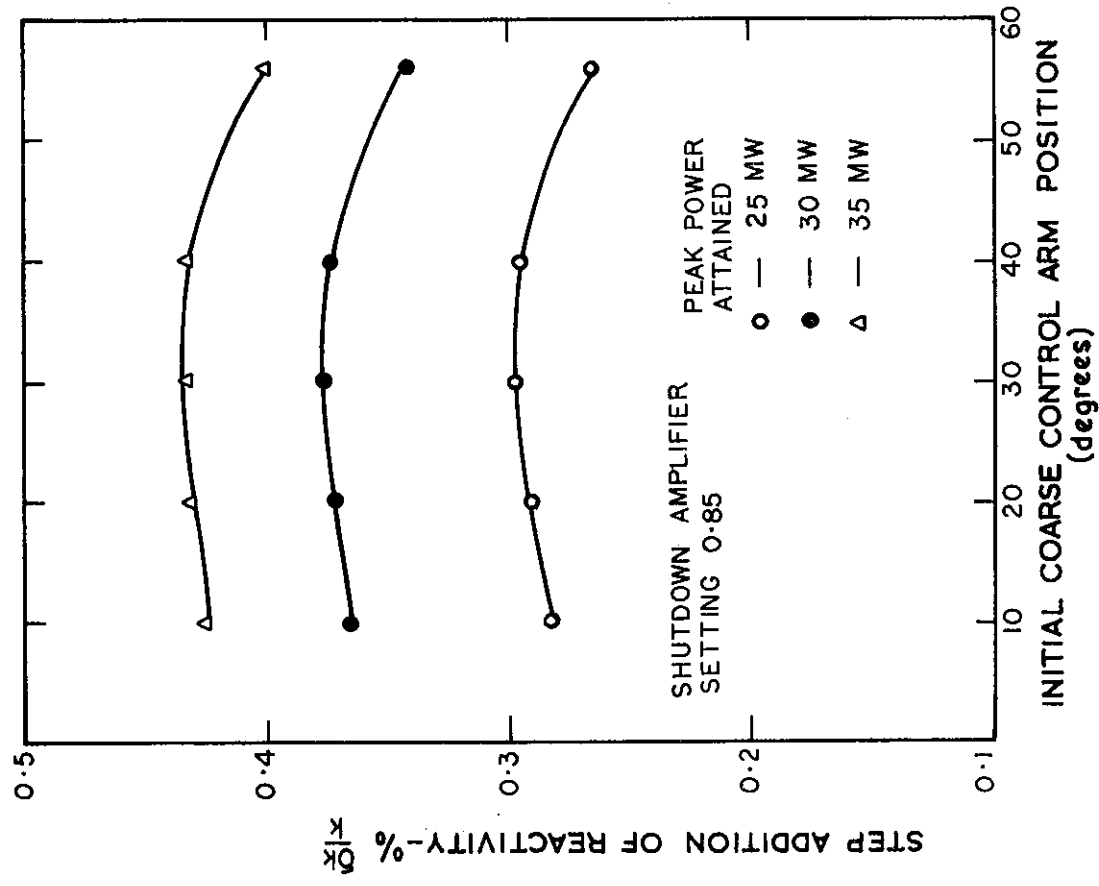


FIGURE 12. STEP ADDITION OF REACTIVITY v. INITIAL C.C.A. POSITION TO PRODUCE A PEAK POWER OF 25, 30 OR 35 MW

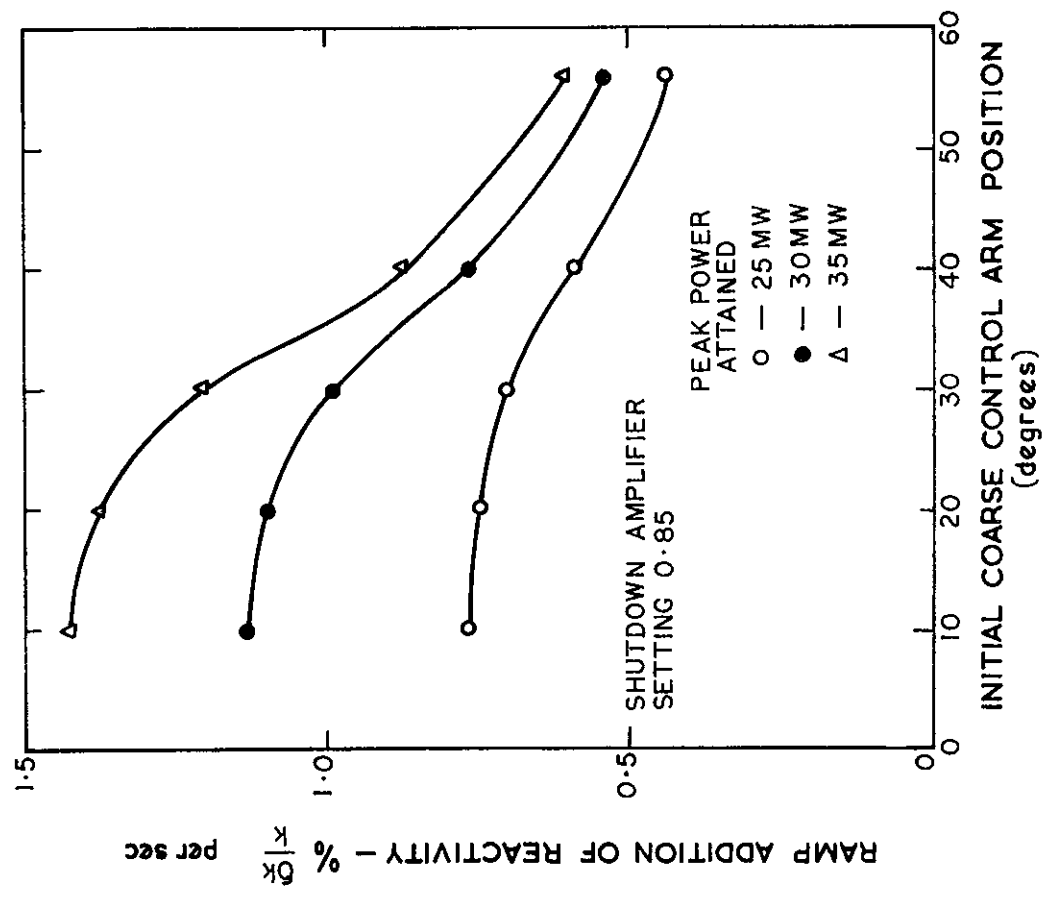


FIGURE 11. RAMP ADDITION OF REACTIVITY v. INITIAL C.C.A. POSITION TO PRODUCE A PEAK POWER OF 25, 30 OR 35 MW

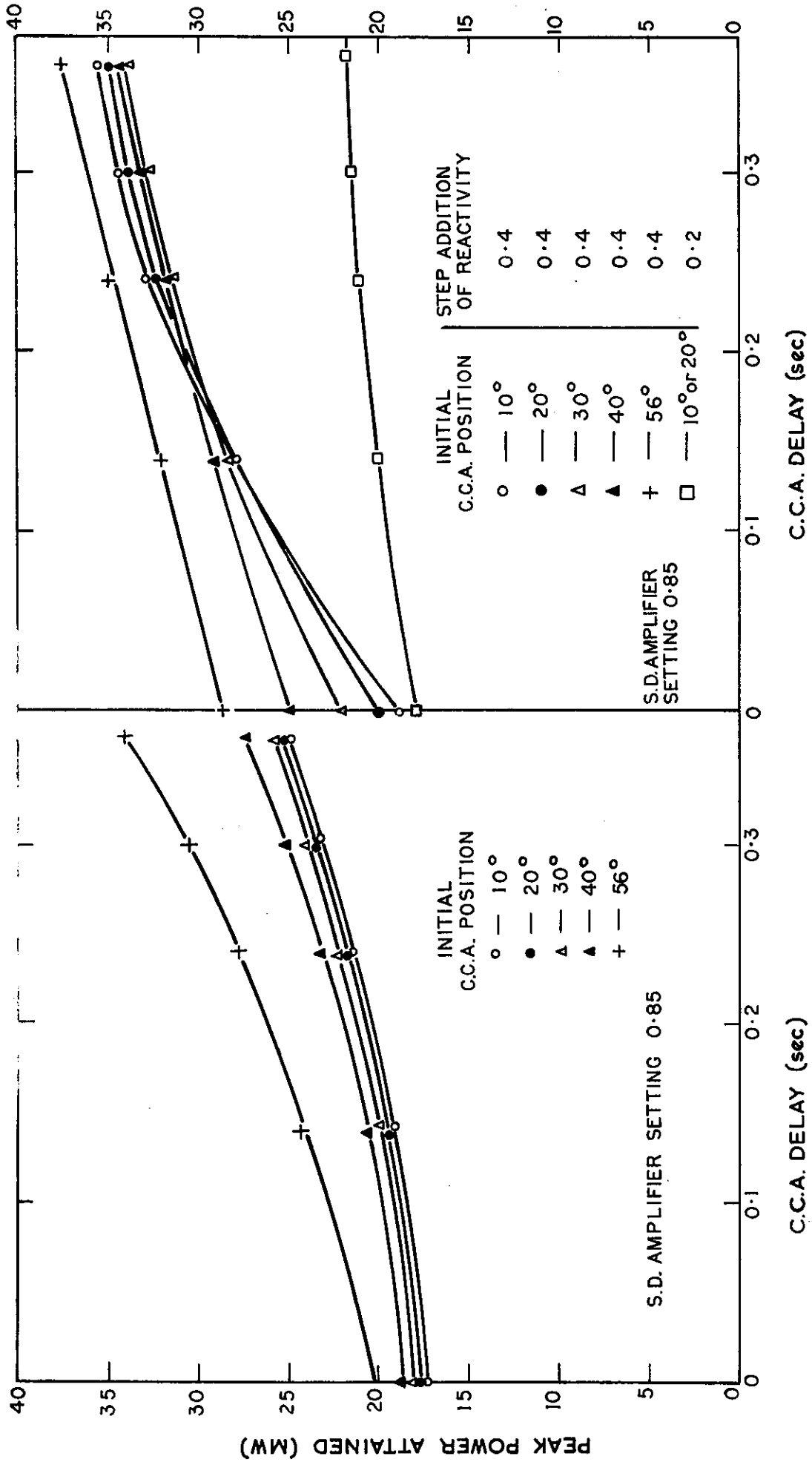


FIGURE 13. PEAK POWER ATTAINED FOLLOWING A RAMP ADDITION OF REACTIVITY OF 0.5%  $\delta k/k$  PER SECOND. EFFECT OF C.C.A. DELAY AND C.C.A. INITIAL POSITION

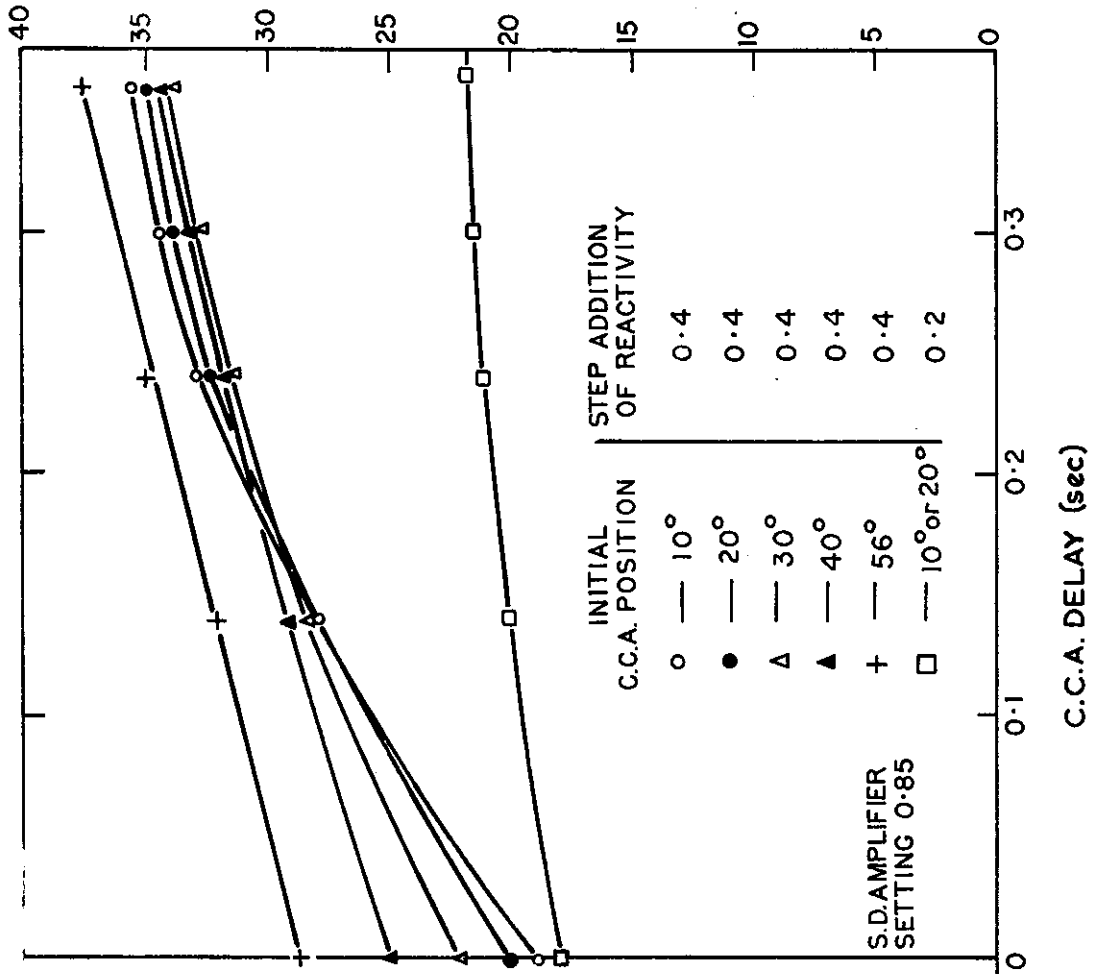
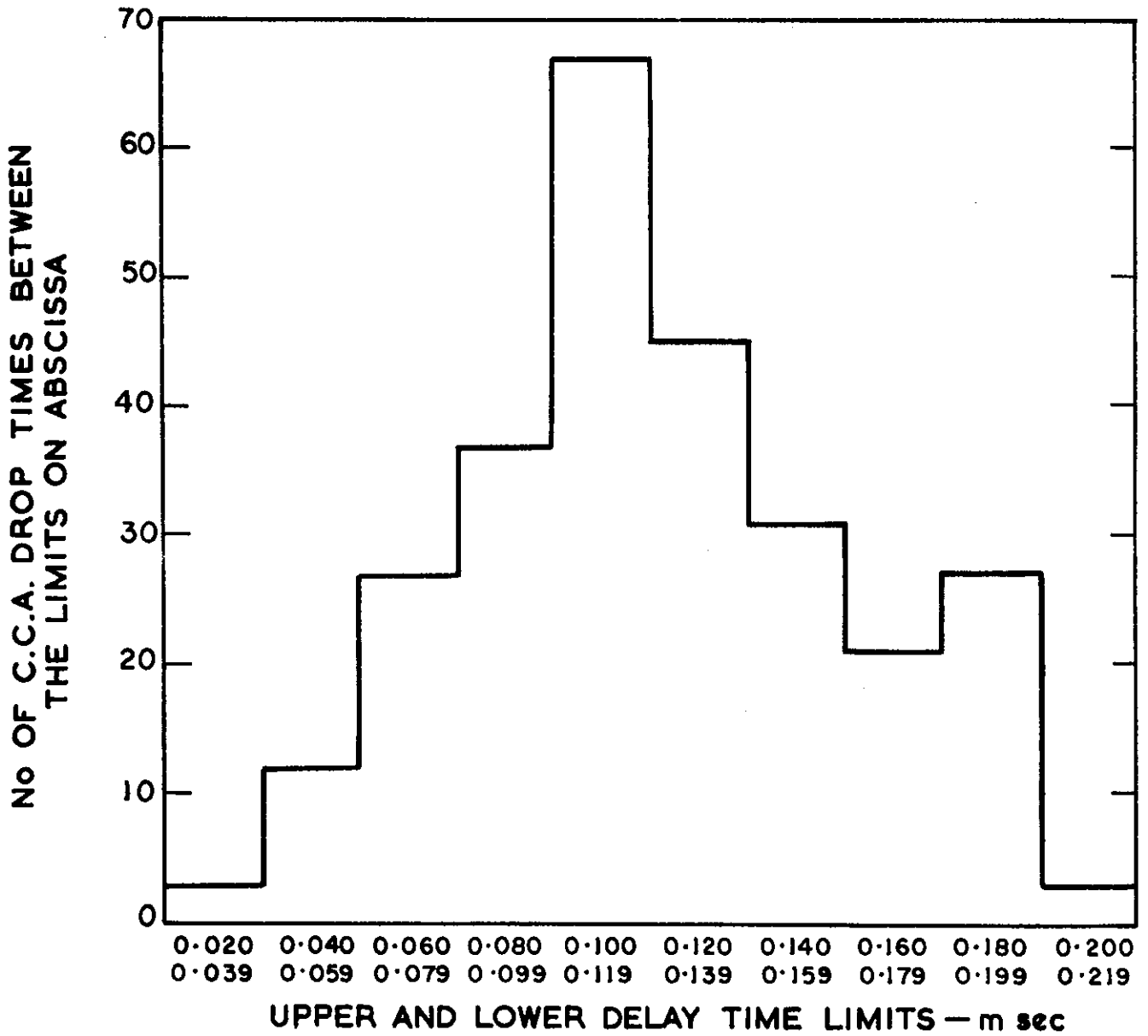


FIGURE 14. PEAK POWER ATTAINED FOLLOWING A STEP ADDITION OF REACTIVITY OF 0.4%  $\delta k/k$  AND 0.2%  $\delta k/k$ . EFFECT OF C.C.A. DELAY AND C.C.A. INITIAL POSITION



**FIGURE 15. FREQUENCY DISTRIBUTION OF ALL MEASURED DELAYS IN DROPPING C.C.A.'S**

