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ABSTRACT

The neutron energy spectrum of the thick target Li(p,n) source is well suited to neutronics experiments on fast reactors. In such applications the angular distribution of the source energy spectrum is required in a form suitable for reactor calculations.

A method is described of using various experimental and theoretical estimates of the angular cross section of the Li(p,n) reaction to estimate the angular yield from a thick target in a form suitable for reactor calculations. The problem is discussed of determining the integration limits for angle integrated quantities appropriate to particular values of incident proton energy and target thickness.

The resulting calculational techniques have been incorporated in a computer code LIPNA written for an IBM360/65. An ancillary, interactive program, written for a NOVA computer, prepares the input data for LIPNA, obviating the need for the occasional user to remember details of the code input requirements.

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NEUTRON SPECTRA; NEUTRON BEAMS; ANGULAR DISTRIBUTION; DIFFERENTIAL CROSS SECTIONS; LITHIUM 7 TARGET; PROTON REACTIONS; NEUTRONS; BERYLLIUM 7; MEV RANGE 01-10; COMPUTER CALCULATIONS

CONTENTS

1. INTRODUCTION

The ^{7}Li(p,n)^{7}Be reaction is a relatively prolific source of neutrons when incident proton energies are less than about 3 MeV, a limit common to many existing, low energy accelerators. The neutron energy spectrum from a thick target bombarded with 3 MeV protons covers the range 0 to 1.3 MeV, which is a range of considerable interest in the neutronics of fast reactors. Hence, such a source has applications in experiments designed to investigate the neutronics of fast reactors [Rainbow & Ritchie 1975]. However, to be really useful, details of the angle-dependent neutron energy spectrum of the source must be available in a form suitable for reactor calculations.

Although it is relatively straightforward to use experimental or theoretical values of the differential cross section $\partial^2\sigma/\partial\Omega\ \partial E_p$ to evaluate the neutron yield as a function of the incident proton energy [Theobald *et al.* 1971], such information is not in a form suitable for reactor calculations, which usually require the angular distribution of neutrons emitted from a source as a function of the neutron energy. It is comparatively easy to recast the expressions required to evaluate the neutron yield so that the yield is expressed as a function of neutron energy, but actual evaluation of the yield requires interpolation of the experimental values of $\partial^2\sigma/\partial\Omega\ \partial E_p$ quoted in the literature.

This report describes a method of estimating angular yields of neutrons from thick lithium targets using various experimental and theoretical estimates of the angular cross section for the Li(p,n) reaction up to proton energies of $\sim$ 3 MeV. It indicates how the yields may be evaluated as a function of angle at a number of specified neutron energies or as a function of neutron energy at specified angles. It also shows how integration limits can be determined when either angle-integrated neutron spectra or the total yields corresponding to a given range of incident proton energies are being evaluated.

The need to recast the equation for neutron yield to allow the neutron energy, E_n, and angle of emission, θ, to be independent variables rather than θ and the proton energy, E_p, leads to some considerable simplifications. This follows largely because E_p is a single valued function of E_n and θ, whereas E_n is sometimes a double valued function of E_p and θ. Similarly, functions such as $\partial E_p/\partial E_n$ and functions relating angles in centre of mass coordinates to those in the laboratory frame (all of which are required in estimating the neutron yield) are single valued functions of E_n and θ. Since formulae for these functions, in terms of E_n or θ, are not readily available in the literature, a fairly exhaustive appendix containing them is included in this report.

A previous report, [Ritchie 1976] discusses in detail the accuracy of the cross section data used in evaluating the neutron yields and the estimated accuracy of the yields calculated by the code. That report also gives examples of angular distributions, spectra at various angles and angle-integrated spectra expected from a thick lithium source under a variety of conditions of incident proton energy and target thickness.

2. BASIC THEORY

It is easily shown [Ritchie 1976] that the number of neutrons emitted per second, due to a particular reaction, into a solid angle $d\Omega$ when a thick target is bombarded by a proton beam of i μA is

$$\frac{\partial^2 N}{\partial E_n \partial \Omega} = i\ g\ D\ \frac{\partial \sigma(E_p)}{\partial \Omega} \left(\frac{dE_p}{dx} \right)^{-1} \frac{\partial E_p}{\partial E_n} \qquad (1)$$

$$= i\ g\ F\ \frac{\partial \sigma}{\partial \Omega} \cdot \varepsilon^{-1} \frac{\partial E_p}{\partial E_n} \quad ,$$

where E_n = neutron energy,

E_p = proton energy corresponding to a neutron energy E_n emitted at an angle defined by Ω,

g = number of protons per μA,

D = atomic density of ^{7}Li,

dE_p/dx = rate of energy loss of protons in lithium,

$\partial\sigma(E_p)/\partial\Omega$ = angular cross section in the laboratory frame for the reaction,

F = factor to account for proportion of ^{7}Li in natural lithium and

ε = stopping power in units of eV cm^{-2}.

Frequently the angular cross section is quoted in the centre-of-mass frame of reference. In this case we use

$$\frac{\partial \sigma}{\partial \Omega} = \frac{\partial \sigma}{\partial \Omega'} \cdot \frac{\partial \Omega'}{\partial \Omega} \quad ,$$

where Ω' is the solid angle in the centre-of-mass frame corresponding to Ω in the laboratory frame.

The total number of neutrons emitted is a sum of the contributions from all the Li(p,n) reactions possible at the proton energy E_p. In the range 0 to 3 MeV there are two reactions to be considered: the ground state reaction with a threshold at E_p = 1.881 MeV and an excited state reaction with a threshold at E_p = 2.378 MeV.

3. CALCULATION OF NEUTRON YIELDS

3.1 Angular Neutron Yields

Information on the angular neutron yields may be presented or required in one of two ways. In the first, the quantity of interest is the angular distribution of neutrons of a particular energy. In this case, the neutron energy is specified and the yield evaluated at a number of angles in the laboratory frame of reference. This enables the various Legendre components of the source angular distribution used in reactor problems to be calculated using standard techniques. In the second case, the quantity of interest is the neutron energy spectrum as a function of angle. Here, the angle of emission is specified and the yield evaluated for a number of different neutron energies. In both cases, a choice of E_n and θ defines a unique proton energy, $E_p = E_p(E_n,\theta)$, for each of the ground and excited state reactions.

This relationship is important, since not only does it provide the basis for estimating yields for a specified E_n from data given in terms of E_p, but it also explains the shape of the angular spectra for given proton energies incident on the target. For example, the neutron energy spectrum at a given angle for each of the ground and excited state reactions is a smoothly varying function of increasing neutron energy up to some E_{nmax} where there is a sharp cut-off. This energy corresponds to the energy of a neutron emitted at the specified angle in a reaction involving a proton which has the energy of protons incident on the target. These neutrons come from reactions at the very surface of the target. The protons lose energy as they penetrate the target and, except at proton energies very close to threshold, produce neutrons of decreasing energy. It follows then, that for a particular reaction, the neutron energy spectrum at a given angle corresponding to two different incident proton energies will be exactly the same up to the cut-off corresponding to the smaller incident proton energy. There will be a similar cut-off at low energy if, for some reason, there is a minimum proton energy to be considered. This will be the case if the target is 'thin' and incident protons are capable of passing right through the target. In principle, we need to calculate the spectrum at a given angle only once for each reaction and then truncate as required at the points defined by $E_n = E_n(\theta,E_{pmax})$ and $E_n = E_n(\theta,E_{pmin})$ for particular conditions of incident proton energy and target thickness. A similar argument applies to angular distributions of neutrons of a specified neutron energy which may also exhibit sharp cut-offs for particular values of E_{pmax} and E_{pmin}. Figure 1 illustrates these points qualitatively.

Evaluation of the neutron yield at a given angle and neutron energy for a particular reaction consists simply of inserting the appropriate values of the various parameters and functions in the right hand side of equation (1). The parameters g and D, which are well defined, and the derivatives $\partial E_p/\partial E_n$ and $\partial\Omega/\partial\Omega'$, which are straightforward functions of E_n and θ, are given in Appendix A.

The adequacy of some of the functions used to describe the rate of energy loss, $\partial E_n/\partial x$, is discussed elsewhere [Ritchie 1976]. The expression given in Appendix A and used in the present calculations, results in estimates of total neutron yields consistent with those measured for the thick target Li(p,n) source [Scott 1971, Ritchie 1976]. As $\partial E_n/\partial x$ is invariably given as a function of E_p, the E_p corresponding to the required E_n and θ must be calculated (see Appendix A).

The angular cross section $\partial\sigma/\partial\Omega$ is also invariably given as a function of E_p, since it is most conveniently measured as such. In general, the angular cross section is known at a set of discrete values of E_p corresponding to the proton energies at which the cross section was measured. This means that the cross section required at E_p values different from those used in the experiments must be evaluated using some interpolation method. The interpolation methods used are discussed below.

3.2 Nuclear Data and Interpolation Methods

The accuracy and consistency of the angular cross section data for both the ground and excited state Li(p,n) reactions in the proton energy range below 3 MeV have been discussed elsewhere [Ritchie 1976]. It is sufficient for the present purpose of describing how to evaluate the centre-of-mass angular cross section at a particular value of E_p, to say that, on the basis of available nuclear data, the above energy range can be split into four intervals for the ground state reaction and one for the excited state reaction.

In the interval ($1.881 \leqslant E_p \leqslant 1.93$ MeV) the cross section in the centre-of-mass frame is assumed isotropic with the proton energy dependence given by a single Breit-Wigner formulation [Macklin & Gibbons 1958, Ritchie 1976, Stroud 1969]. When E_p lies in this energy interval, evaluation of the cross section can be carried out by straightforward evaluation of the Breit-Wigner expression.

In the energy range 1.93 to 2.977 MeV the angular cross section can be estimated using the experimental data of Bergström *et al.*[1967] which cover the range $1.928 \leqslant E_p \leqslant 2.361$ MeV, and Buccino *et al.*[1964] which cover the range $2.48 \leqslant E_p \leqslant 2.977$ MeV. When the value of $E_p = E_p(E_n,\theta)$ is such that there are at least two experimental values of E_p above it and at least two below, the values of the angular cross sections at these four E_p values and at the centre-of-mass angle defined by E_n and θ, are estimated by linear interpolation of the experimental angular data. These four values are then fitted with a third order polynomial in order to estimate the angular cross section at $E_p = E_p(E_n,\theta)$ which is the required estimate of the centre-of-mass angular cross section at the neutron energy and angle of emission specified.

When $E_p = E_p(E_n,\theta)$ lies between the lowest and second lowest of the E_p values used by Bergström *et al.* (1.928 and 2.031 MeV), the angular cross section at $E_p = 1.9$ MeV is evaluated using the Breit-Wigner expression, and the interpolation procedure carried out as before. When $E_p = E_p(E_n,\theta)$ lies between the two largest energy values used by Buccino *et al.* (2.878 and 2.977 MeV), linear interpolation is used instead of cubic interpolation to find the cross section at $E_p = E_p(E_n,\theta)$.

The angular cross section of the excited state reaction is also assumed to be described by a simple Breit-Wigner expression in the centre-of-mass frame [Buccino *et al.* 1964, Ritchie 1976]. The cross section at $E_p(E_n,\theta)$ and at the centre-of-mass angle defined by E_n and θ is obtained by simple evaluation of this expression.

At present no cross section data have been considered for proton energies greater than 2.977 MeV, and the yields for neutron energies and angles of emission, which correspond to $E_p > 2.977$ MeV, have been set to zero. Physically, this corresponds to the assumption that the incident proton energy is always less than 2.977 MeV.

3.3 Angle-integrated Yields

When the yield at a particular neutron energy has been evaluated at a number of values of $\mu = \cos\theta\,(-1 \leqslant \mu \leqslant 1)$ determination of the various angular moments of the angular distribution is comparatively simple. The only complication is the appearance of sharp cut-offs in the angular distribution at some neutron energies, depending on the minimum and maximum proton energies involved in neutron production (see 3.1 above). Because in these cases, the integration limits will not be ± 1, some scheme must be adopted to determine what the integration limits are for given E_{pmax} and E_{pmin}.

The method used to determine the integration limits is most easily discussed with reference to Figure 2 which shows qualitatively E_p as a function of E_n for typical values of $\mu = \cos\theta$. The discussion is further simplified if reference is first made to the upper diagram of Figure 2. For any given choice of E_n the quantities $\mu(E_n,E_{pmin})$ and $\mu(E_n,E_{pmax})$ are evaluated, where

$\mu = \mu(E_n, E_p)$ is the cosine of the angle of emission defined by E_n and E_p (see Appendix A). For convenience, these quantities will be called ULIM and ALIM. If ULIM is greater than + 1 for the specified E_n, then the point (E_n, E_{pmax}) lies below the figure bounded by the curves labelled $\mu = 1$ and $\mu = -1$ in Figure 2. This means physically that the minimum proton energy required to produce neutrons with the specified neutron energy, regardless of the angle of emission of these neutrons, is greater than E_{pmin}. In fact, the minimum proton energy will correspond to a neutron of the specified energy emitted in the forward direction, i.e. $\mu = 1$. The upper limit of integration is therefore, in this case, $\mu = 1$. If ULIM $\leqslant 1$, which is the case if the point $\mu(E_n, E_{pmin})$ lies within the figure bounded by the curves labelled $\mu = 1$ and $\mu = -1$, there are protons with energies less than E_{pmin} that can give rise to neutrons with the specified neutron energies, but which are emitted at angles with $\mu >$ ULIM. Since it is assumed from the target thickness and incident proton energies specified, that there are no protons with energies less than E_{pmin} available for neutron production, there will be no neutrons of the specified energy emitted into any angle greater than that defined by $\mu =$ ULIM. This is therefore the upper limit of integration.

Similar use is made of ALIM. If ALIM is less than -1 then the point (E_p, E_{pmax}) lies outside the figure bounded by the curves labelled $\mu = 1$ and $\mu = -1$ in Figure 2, and the maximum proton energy required to produce neutrons of the specified energy is less than E_{pmax}. Since the maximum proton energy for a specified E_n corresponds to neutrons emitted backwards, the lower limit of integration in this case is therefore $\mu = -1$. If ALIM is greater than -1, then protons with energies greater than E_{pmax} can give rise to neutrons with the specified energy, but these neutrons will be emitted at angles with $\mu <$ ALIM. In this case ALIM becomes the lower limit of integration.

It follows that if ULIM is $\leqslant -1$ or ALIM $\geqslant +1$, then the integral over angle vanishes. In evaluating angular moments as a function of neutron energy, it is simplest to start with the lowest neutron energy of interest. If ULIM $\leqslant -1$ or ALIM $\geqslant +1$ for this neutron energy, any integral over angle vanishes and the next higher neutron energy can be considered. For those neutron energies where neither of these conditions holds, it is computationally easier to evaluate the yield at a set of fixed μ values, then evaluate the integration limits and carry out the integration between these limits. To date, the only angular moment evaluated is the total yield into all angles for which a simple trapezoidal integration scheme proves quite adequate. As higher neutron energies are specified, the stage is reached where ALIM > +1 at a point where the gradient $\partial E_p/\partial E_n$ is positive for $\mu = 1$. It is clear from an examination of Figure 1 that ALIM will be greater than 1 for all higher neutron energies and the corresponding angular moments will be identically zero. This provides a useful check for terminating the computation for a particular set of E_{pmin} and E_{pmax}.

4. DISCUSSION

The computational schemes outlined in Section 3 above have been incorporated in an IBM360/65 computer code, LIPNA. A flow diagram for the main program of this code is shown in Figure 3 and a list of the subroutines contained in the code and the functions of these subroutines are given in Appendix B. The code has available two main options. In the first option, which is the one most likely to be used to provide source data for reactor type calculations, the yield in units of neutrons eV^{-1} μA^{-1} s^{-1} st^{-1}, is calculated as a function of angle for a set of specified neutron energies. Angle-integrated yields (neutrons eV^{-1} μA^{-1} s^{-1}) for the particular choice of incident proton energy and lithium target thickness are also calculated. If required, the integration over neutron energy can be split into intervals to give the source strengths in the energy groups required for a particular reactor type calculation.

In the second option, which is most likely to be used for, say, neutron detector calibrations or any application where the neutron spectrum is required to be known as a function of angle, the yield is calculated as a function of neutron energy for a set of specified angles. This option also proves useful in so-called 'thin target' applications where the best compromise between neutron energy spread and neutron intensity is being sought. Clearly, for a given incident proton energy, the thicker the target the greater the neutron intensity, but the larger the energy spread of neutrons from the source. For targets a few keV thick, the gradient $\partial E_n/\partial E_p$ can be used to estimate the energy spread, but in applications where a thicker target can be tolerated, a 'thick' target calculation provides a more accurate estimate of the source intensity that can be achieved for a given neutron energy spread.

Information on the angular cross section data has been incorporated in a set of subroutines in such a way that new data can easily be included. In both options, the angular cross section in the centre of mass, corresponding to the particular value of E_n and θ and hence $E_p = E_p(E_n,\theta)$, is calculated and printed. This can be compared with the experimental data as a check on the reliability of some of the numerical techniques used.

A code such as LIPNA is used infrequently but intensively to determine the source properties for specific purposes. To obviate the need for the casual user to re-acquaint himself with the input requirements of the code, an interactive code has been written for a NOVA-computer which prepares input data for LIPNA necessary for a particular job. This NOVA program directs the user to

(i) select one of the available options,
(ii) supply a table of neutron energies,
(iii) supply a table of angles, and
(iv) specify the maximum and minimum proton energies to be considered.

The data are prepared and saved in the ACL symbol table and accessed by the main code during execution as indicated in Appendix C. Appendices D, E and F are examples of the execution of the ACL-NOVA data preparation code for three of the most common situations envisaged. They correspond, respectively, to requests to obtain:

Appendix D	plots of angular distributions at specified neutron energies.
Appendix E	plots of angle-integrated energy spectra for specified incident and exit proton energies.
Appendix F	plots of energy spectra at selected angles.

The appendices D, E and F also serve to describe the input requirements and options of the code LIPNA. It is clear that modification of the code to accept more conventional input is a simple matter. It is also clear that the details of the link between the interactive code and the main code are dictated by the computer system presently in use at the AAEC Research Establishment, but nevertheless the interactive concept could be adapted to a similar computer system.

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APPENDIX A

FORMULAE AND VALUE OF CONSTANTS USED IN LIPNA

Formulae

1. $$E_p(E_n,\theta) = E_n\left\{\frac{m_y+m_n}{m_y-m_p} + 2\mu^2\frac{m_p\ m_n}{(m_y-m_p)^2}\right\} + \frac{m_z\ m_y}{(m_y+m_n)\ (m_y-m_p)}\ E_{th}$$
$$- \frac{2\mu m_n m_p}{(m_y-m_p)^2}\left\{E_n^2\left[\frac{(m_y-m_p)\ (m_y+m_n)}{m_n m_p} + \mu^2\right] + E_n E_{th}\frac{(m_y-m_p)\ (m_z m_y)}{(m_y+m_n)\ (m_n m_p)}\right\}^{\frac{1}{2}}$$

where m_p, m_n, m_y = mass of proton, neutron and ^{7}Li respectively.

$m_z = m_y+m_n-m_p$.

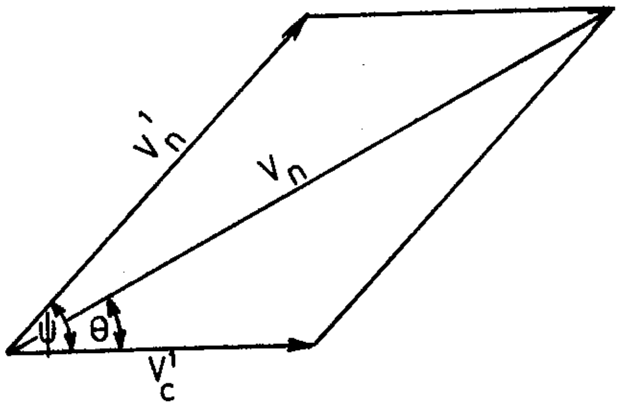
2. $$\mu(E_n,E_p) = \left[E_n - \frac{(m_y-m_p)}{(m_y+m_n)}E_p + \frac{m_y\ m_z}{(m_y+m_n)^2}\ E_{th}\right]\left\{\frac{(m_y+m_n)^2}{4m_n m_p E_n E_p}\right\}^{\frac{1}{2}}$$

3. $$\frac{\partial E_p}{\partial E_n} = \frac{(m_y+m_n) \ - \ \mu(m_n m_p)^{\frac{1}{2}}(E_p/E_n)^{\frac{1}{2}}}{(m_y-m_p) \ + \ \mu(m_n m_p)^{\frac{1}{2}}(E_n/E_p)^{\frac{1}{2}}}$$

4. $$\frac{\partial\Omega'}{\partial\Omega} = \frac{\rho^2}{[\rho^2-2\mu+1]^{\frac{1}{2}}(\rho-\mu)}$$

$$\rho^2 = \frac{(m_n+m_y)^2}{m_p m_n}\ \frac{E_n}{E_p}$$

= ratio of neutron energy in laboratory frame to energy of the neutron travelling with the centre-of-mass velocity v_c'



APPENDIX A (continued)

5. $\nu = \cos\psi$

$= $ cosine of angle of neutron emission in centre-of-mass frame

$$= \frac{\rho\mu-1}{(\rho^2-2\rho\mu+1)^{\frac{1}{2}}}$$

6. $$(d^2N)_{\text{nat. Li}} = \frac{qN_o x_7}{A_7}\,\frac{\partial\sigma_y}{\partial\Omega}\cdot\left(\frac{dE_p}{dx}\right)^{-1}\,\frac{\partial E_p}{\partial E_n}$$

$$= \frac{x_7 A_n}{A_7}\,\varepsilon^{-1}\,\frac{\partial\sigma_y}{\partial\Omega}\,\frac{\partial E_p}{\partial E_n} \text{ neutrons/proton/steradian}$$

$$= F\,\varepsilon^{-1}\,\frac{\partial\sigma}{\partial\Omega}\cdot\frac{\partial E_p}{\partial E_n}$$

where the subscripts 7 or n refer to lithium-7 and natural lithium respectively.

$\varepsilon = \frac{A_n}{qN_o}\left(\frac{dE_p}{dx}\right) = $ stopping power in eV cm^{-2}

$x_7 = $ isotopic abundance of ^{7}Li in natural lithium

$A = $ atomic weight

$q = $ density

7. $\varepsilon = 7.18 \times 10^{-16}[4.69 - \log(3/E_p)]$eV cm^{-2}. [Whaling 1958].

Values of Constants Used

$q = $ number of protons μA^{-1}

$= 10^{13}/1.602$

$F = 0.9345$

$m_p = 1.007593$ AMU

$m_n = 1.008982$ AMU

$m_y = 7.016956$ AMU

$E_{th} = 1.881$ MeV for ground state

$= 2.378$ MeV for excited state

APPENDIX B

SUBROUTINES USED IN LIPNA

AMUFN	Calculates $\mu = \mu(E_n, E_p)$ for a reaction with a given threshold energy E_{th}.
CUPOL	Performs cubic interpolation given four points.
DIFCX	Evaluates $\partial^2 N/\partial E_n \partial \Omega$ and $\partial E_p/\partial E_n$ given as input E_n, μ, E_p, $\partial\sigma/\partial\Omega'$ and E_{th}.
DSDW1N	Contains a table of the experimental results of Bergström *et al.* for $\partial\sigma/\partial\Omega'$ for the ground state reaction in the range $1.928 \leqslant E_p \leqslant 2.361$. The subroutine uses linear interpolation to find $\partial\sigma/\partial\Omega'$ at a given centre-of-mass value and at one of the experimental E_p values.
DSW2N	Contains a table of the experimental results of Buccino *et al.* for $\partial\sigma/\partial\Omega'$ for the ground state reaction in the range $2.48 \leqslant E_p \leqslant 2.977$. Performs a similar function to DSDW1N.
DSDW3	Calculates $\partial\sigma/\partial\Omega'$ as a function of E_p for the ground state reaction using a single Breit-Wigner resonance formula.
DSDW4	Calculates $\partial\sigma/\partial\Omega'$ for the excited state reaction assuming a single Breit-Wigner resonance formula.
FPCFN	Evaluates E_p for a given E_n, μ and E_{th}.
MATLIN	Performs linear interpolation over a surface which is evaluated as a doubly subscripted variable. Extrapolates to points beyond the boundaries of the surface.
TERD	Performs linear interpolation along a curve which is tabulated as a singly subscripted variable. Extrapolates to points outside the tabulated interval.
ZOID	Uses trapezoidal rule to integrate a tabulated function of one variable. Interpolates if the limits of integration are not tabulated.

APPENDIX C

OPERATION OF LIPNA USING ACL-NOVA INTERACTIVE PROGRAM

The ACL-NOVA interactive data preparation program may be called at any ACL-NOVA terminal by typing

```
#LOAD LIPNACL,MTR
```

The user may then RUN this program and prepare data for a Li(p,n) calculation. When the program has finished execution the user should save the program and symbol table under his own name by typing

```
#SAVES PROGNAME,PLS/ACCTNMBR
```

where the program name (PROGNAME) can be up to eight characters long, while the user's initials (PLS) and IBM360 account number (ACCTNMBR) are required to validate the SAVES request.

The main program for execution on the IBM360 is the member LIPNA on dataset DSN=MTR.PLIB2. The following cards should be submitted to the IBM360 for a calculation.

```
//EXEC BUFFPROG,PRG=VPLOT
//SYSUT2 DD SYSOUT=F(or C)
//EXEC FORTHG,PRG=LIPNA,DSN='MTR.PLIB2',REGION.GO=120K
//GO.ACLLIB DD DSN=AAE.ACLLIB,DISP=SHR
//GO.AEPLOT DD SYSOUT=F(or C)
//GO.SYSIN DD *
    PROGNAME,PLS
/*
```

where PROGNAME,PLS is as described above and should correspond to the user's saved ACL-NOVA program and symbol table.

For subsequent jobs the user can use his own saved ACL-NOVA program.

APPENDIX D

EXAMPLE OF DATA PREPARATION BY ACL-NOVA PROGRAM FOR CALCULATION OF ANGULAR DISTRIBUTIONS

```
    ACL-NOVA
#LOAD LIPNACL,MTR
-LIPNACL-LOADED AT 11.05 AM ON 76.237
RUN

DATA PREPARATION PROGRAM FOR LI(P,N) CALCULATIONS


    DO YOU WISH TO
(A) SELECT NEUTRON ENERGIES AND CALCULATE FOR
     A RANGE OF ANGLES, OR
(B) SELECT ANGLES AND CALCULATE FOR
     A RANGE OF NEUTRON ENERGIES? - REPLY A OR B
155 OPT←A
DO YOU WISH TO SEE THE CURRENT ENERGY TABLE
 REPLY YES OR NO
162 RPLY←YES
NEUTRON ENERGY VALUES (MEV) ARE :
5.000000E-02 1.000000E-01
DO YOU WISH TO CHANGE ANY OF THE ENERGY VALUES?
REPLY YES,NO,ALL,ADD OR ISRT (FOR INSERT)
182 RPLY←ADD
HOW MANY VALUES DO YOU WISH TO ADD?
212 ANEN←2
NOW SUPPLY THE ADDITIONAL VALUES IN MEV
215 EN(3)←0.20
215 EN( 4)←0.30
DO YOU WISH TO SEE THE CURRENT ENERGY TABLE
 REPLY YES OR NO
162 RPLY←YES
NEUTRON ENERGY VALUES (MEV) ARE :
5.000000E-02 1.000000E-01 2.000000E-01 3.000000E-01
DO YOU WISH TO CHANGE ANY OF THE ENERGY VALUES?
REPLY YES,NO,ALL,ADD OR ISRT (FOR INSERT)
182 RPLY←NO
*** NOTE *** - IF YOU HAVE CHOSEN OPTION A THE ANGLE
               COSINES ARE REQUIRED (RANGE -1 TO +1)
               IF YOU HAVE CHOSEN OPTION B THE ANGLES
               ARE REQUIRED IN DEGREES
DO YOU WISH TO SEE THE CURRENT ANGLE TABLE?
REPLY YES OR NO
241 RPLY←YES
ANGLE VALUES ARE:
-1.000000E+00 -9.000000E-01 -8.000000E-01 -7.000000E-01 -6.000000E-01
-5.000000E-01 -4.000000E-01 -2.000000E-01 -2.000000E-01 -1.000000E-01
0 1.000000E-01 2.000000E-01 3.000000E-01 4.000000E-01
5.000000E-01 6.000000E-01 7.000000E-01 8.000000E-01 9.000000E-01
1.000000E+00
DO YOU WISH TO CHANGE ANY OF THE ANGLE VALUES
REPLY YES,NO OR ALL
252 RPLY←YES
```

(continued)

APPENDIX D (continued)

```
SPECIFY THE SUBSCRIPT OF THE VALUE TO BE CHANGED
261 K←8
NOW TYPE THE VALUE
263 MU(8)←-0.30
ANY MORE TO BE CHANGED? - REPLY YES OR NO
265 PPLY←NO
DO YOU WISH TO SEE THE CURRENT ANGLE TABLE?
REPLY YES OR NO
241 RPLY←YES
ANGLE VALUES ARE:
-1.000000E+00 -9.000000E-01 -8.000000E-01 -7.000000E-01 -6.000000E-01
-5.000000E-01 -4.000000E-01 -3.000000E-01 -2.000000E-01 -1.000000E-01
0 1.000000E-01 2.000000E-01 3.000000E-01 4.000000E-01
5.000000E-01 6.000000E-01 7.000000E-01 8.000000E-01 9.000000E-01
1.000000E+00
DO YOU WISH TO CHANGE ANY OF THE ANGLE VALUES
REPLY YES,NO OR ALL
252 RPLY←NO
SPECIFY THE MINIMUM & MAXIMUM PROTON ENERGIES IN MEV
290 E1←1.80
290 E2←2.977
DO YOU WANT A PLOT OF YIELD V COSINE (NEUTRON ANGLE)
REPLY YES OR NO
293 RPLY←YES
DO YOU WANT TO SPECIFY THE PLOT SIZE & SCALE
REPLY YES OR NO
402 SCL1←YES
PLEASE SPECIFY THE PLOT SIZE IN INCHES
405 XP1←8.0
405 YP1←8.0
PLEASE SUPPLY THE LIMITS ON THE ANGLE
409 XLO1←-1.0
409 XHI1←1.0
SPECIFY THE LIMITS FOR THE GROUND STATE YIELD
411 YLO1←0.0
411 YHI1←300.0
SPECIFY THE LIMITS FOR THE EXCITED STATE YIELD
413 YLO3←0.0
413 YHI3←20.0
HOW MANY GROUPS WOULD YOU LIKE INTEGRATION OVER
THE ENERGY SPECTRUM SPLIT INTO?
298 NI←1
SPECIFY THE LIMITS IN TERMS OF NEUTRON ENERGY IN MEV
303 LM(1)←0.05
303 LM(2)←0.3
DO YOU WANT A PLOT OF ANGLE INTEGRATED YIELD VS.
NEUTRON ENERGY - REPLY YES OR NO
422 PLOP←NO
END OF RUN

NOW SAVE THIS PROGRAM AND ITS SYMBOL TABLE
(E.G. #SAVES PROGNAME,PLS/ACCTNMBR)
END ACL

SUBMIT A JOB TO THE IBM360 USING
PRG=LIPNA FROM DSN=MTR.PLIB2
ENSURE IDENTIFICATION PROGNAME,PLS
APPEARS AFTER THE GO.SYSIN  DD  * STATEMENT
330 STOP
```

APPENDIX E

EXAMPLE OF DATA PREPARATION BY ACL-NOVA PROGRAM FOR CALCULATION OF ANGLE-INTEGRATED SPECTRUM

```
    ACL-NOVA
#LOAD LIPNACL,MTR
-LIPNACL-LOADED AT 11.27 AM ON 76.237
RUN

DATA PREPARATION PROGRAM FOR LI(P,N) CALCULATIONS


    DO YOU WISH TO
(A) SELECT NEUTRON ENERGIES AND CALCULATE FOR
     A RANGE OF ANGLES, OR
(B) SELECT ANGLES AND CALCULATE FOR
     A RANGE OF NEUTRON ENERGIES? - REPLY A OR B
155 OPT←A
DO YOU WISH TO SEE THE CURRENT ENERGY TABLE
 REPLY YES OR NO
162 RPLY←YES
NEUTRON ENERGY VALUES (MEV) ARE :
2.000000E-02 3.000000E-02 4.000000E-02 5.000000E-02 7.500000E-02
1.000000E-01 1.250000E-01 1.500000E-01 1.750000E-01 3.000000E-01
3.250000E-01 3.500000E-01 3.750000E-01 4.000000E-01 4.250000E-01
4.500000E-01 4.750000E-01 5.000000E-01 5.250000E-01 5.500000E-01
5.750000E-01 6.000000E-01 6.250000E-01 6.500000E-01 6.750000E-01
7.000000E-01 7.250000E-01 7.500000E-01 7.750000E-01 8.000000E-01
8.250000E-01 8.500000E-01 8.750000E-01 9.000000E-01 9.250000E-01
9.500000E-01 9.750000E-01 1.000000E+00 1.025000E+00 1.050000E+00
1.075000E+00 1.100000E+00 1.125000E+00 1.150000E+00 1.175000E+00
1.200000E+00 1.225000E+00 1.250000E+00 1.275000E+00 1.300000E+00
DO YOU WISH TO CHANGE ANY OF THE ENERGY VALUES?
REPLY YES,NO,ALL,ADD OR ISRT (FOR INSERT)
182 RPLY←ISRT
TYPE THE SUBSCRIPT OF THE VALUE BEFORE WHICH YOU WISH
TO INSERT THE NEW VALUES, AND THE NUMBER OF NEW VALUES
222 IN←10
222 NNEW←4
NOW SUPPLY THE VALUES TO BE INSERTED IN MEV
228 EN(10)←0.20
228 EN(11)←0.2250
228 EN(12)←0.250
228 EN(13)←0.2750
DO YOU WISH TO SEE THE CURRENT ENERGY TABLE
 REPLY YES OR NO
162 RPLY←YES
```

(continued)

```
NEUTRON ENERGY VALUES (MEV) ARE :
2.000000E-02 3.000000E-02 4.000000E-02 5.000000E-02 7.500000E-02
1.000000E-01 1.250000E-01 1.500000E-01 1.750000E-01 2.000000E-01
2.250000E-01 2.500000E-01 2.750000E-01 3.000000E-01 3.250000E-01
3.500000E-01 3.750000E-01 4.000000E-01 4.250000E-01 4.500000E-01
4.750000E-01 5.000000E-01 5.250000E-01 5.500000E-01 5.750000E-01
6.000000E-01 6.250000E-01 6.500000E-01 6.750000E-01 7.000000E-01
7.250000E-01 7.500000E-01 7.750000E-01 8.000000E-01 8.250000E-01
8.500000E-01 8.750000E-01 9.000000E-01 9.250000E-01 9.500000E-01
9.750000E-01 1.000000E+00 1.025000E+00 1.050000E+00 1.075000E+00
1.100000E+00 1.125000E+00 1.150000E+00 1.175000E+00 1.200000E+00
1.225000E+00 1.250000E+00 1.275000E+00 1.300000E+00
DO YOU WISH TO CHANGE ANY OF THE ENERGY VALUES?
REPLY YES,NO,ALL,ADD OR ISRT (FOR INSERT)
182 RPLY←NO
*** NOTE *** - IF YOU HAVE CHOSEN OPTION A THE ANGLE
               COSINES ARE REQUIRED (RANGE -1 TO +1)
               IF YOU HAVE CHOSEN OPTION B THE ANGLES
               ARE REQUIRED IN DEGREES
DO YOU WISH TO SEE THE CURRENT ANGLE TABLE?
REPLY YES OR NO
241 RPLY←YES
ANGLE VALUES ARE:
-1.000000E+00 -9.000000E-01 -8.000000E-01 -7.000000E-01 -6.000000E-01
-5.000000E-01 -4.000000E-01 -3.000000E-01 -2.000000E-01 -1.000000E-01
0 1.000000E-01 2.000000E-01 3.000000E-01 4.000000E-01
5.000000E-01 6.000000E-01 7.000000E-01 8.000000E-01 9.000000E-01
1.000000E+00
DO YOU WISH TO CHANGE ANY OF THE ANGLE VALUES
REPLY YES,NO OR ALL
252 RPLY←NO
SPECIFY THE MINIMUM & MAXIMUM PROTON ENERGIES IN MEV
290 E1←1.8
290 E2←2.8
DO YOU WANT A PLOT OF YIELD V COSINE (NEUTRON ANGLE)
REPLY YES OR NO
293 RPLY←NO
HOW MANY GROUPS WOULD YOU LIKE INTEGRATION OVER
THE ENERGY SPECTRUM SPLIT INTO?
298 NI←0
DO YOU WANT A PLOT OF ANGLE INTEGRATED YIELD VS.
NEUTRON ENERGY - REPLY YES OR NO
422 PLOP←YES
DO YOU WANT TO SPECIFY THE PLOT SIZE & SCALE?
REPLY YES OR NO
426 SCL2←NO
END OF RUN

NOW SAVE THIS PROGRAM AND ITS SYMBOL TABLE
(E.G. #SAVES PROGNAME,PLS/ACCTNMBR)
END ACL

SUBMIT A JOB TO THE IBM360 USING
PRG=LIPNA FROM DSN=MTR.PLIB2
ENSURE IDENTIFICATION PROGNAME,PLS
APPEARS AFTER THE GO.SYSIN  DD  * STATEMENT
330 STOP
```

APPENDIX F

EXAMPLE OF DATA PREPARATION BY ACL-NOVA PROGRAM FOR CALCULATION OF SPECTRA AT SELECTED ANGLES

```
    ACL-NOVA
#LOAD LIPNACL,MTR
-LIPNACL-LOADED AT 11.38 AM ON 76.237
RUN

DATA PREPARATION PROGRAM FOR LI(P,N) CALCULATIONS


    DO YOU WISH TO
(A) SELECT NEUTRON ENERGIES AND CALCULATE FOR
     A RANGE OF ANGLES, OR
(B) SELECT ANGLES AND CALCULATE FOR
     A RANGE OF NEUTRON ENERGIES? - REPLY A OR B
155 OPT←B
DO YOU WISH TO SEE THE CURRENT ENERGY TABLE
 REPLY YES OR NO
162 RPLY←YES
NEUTRON ENERGY VALUES (MEV) ARE :
2.000000E-02 3.000000E-02 4.000000E-02 5.000000E-02 7.500000E-02
1.000000E-01 1.250000E-01 1.500000E-01 1.750000E-01 2.000000E-01
2.250000E-01 2.500000E-01 2.750000E-01 3.000000E-01 3.250000E-01
3.500000E-01 3.750000E-01 4.000000E-01 4.250000E-01 4.500000E-01
4.750000E-01 5.000000E-01 5.250000E-01 5.500000E-01 5.750000E-01
6.000000E-01 6.250000E-01 6.500000E-01 6.750000E-01 7.000000E-01
7.250000E-01 7.500000E-01 7.750000E-01 8.000000E-01 8.250000E-01
8.500000E-01 8.750000E-01 9.000000E-01 9.250000E-01 9.500000E-01
9.750000E-01 1.000000E+00 1.025000E+00 1.050000E+00 1.075000E+00
1.100000E+00 1.125000E+00 1.150000E+00 1.175000E+00 1.200000E+00
1.225000E+00 1.250000E+00 1.275000E+00 1.300000E+00
DO YOU WISH TO CHANGE ANY OF THE ENERGY VALUES?
REPLY YES,NO,ALL,ADD OR ISRT (FOR INSERT)
182 RPLY←NO
*** NOTE *** - IF YOU HAVE CHOSEN OPTION A THE ANGLE
               COSINES ARE REQUIRED (RANGE -1 TO +1)
               IF YOU HAVE CHOSEN OPTION B THE ANGLES
               ARE REQUIRED IN DEGREES
DO YOU WISH TO SEE THE CURRENT ANGLE TABLE?
REPLY YES OR NO
241 RPLY←NO
DO YOU WISH TO CHANGE ANY OF THE ANGLE VALUES
REPLY YES,NO OR ALL
252 RPLY←ALL
SPECIFY THE NEW NUMBER OF ANGLE VALUES
271 NMU←6
```

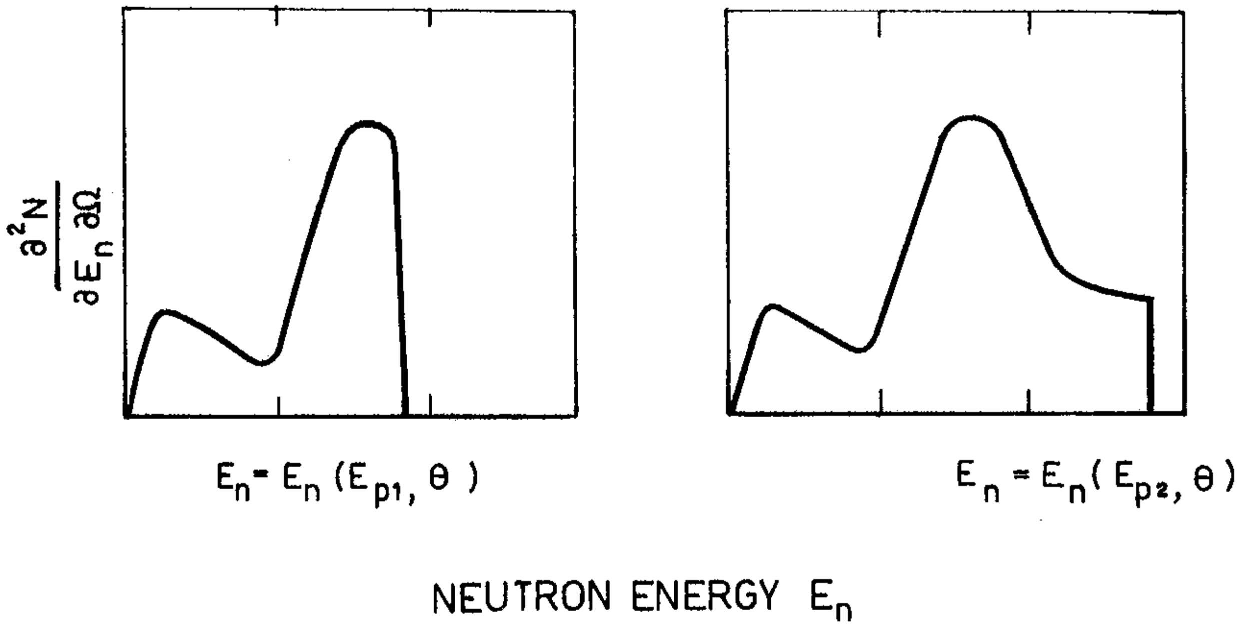
(continued)

APPENDIX F (continued)

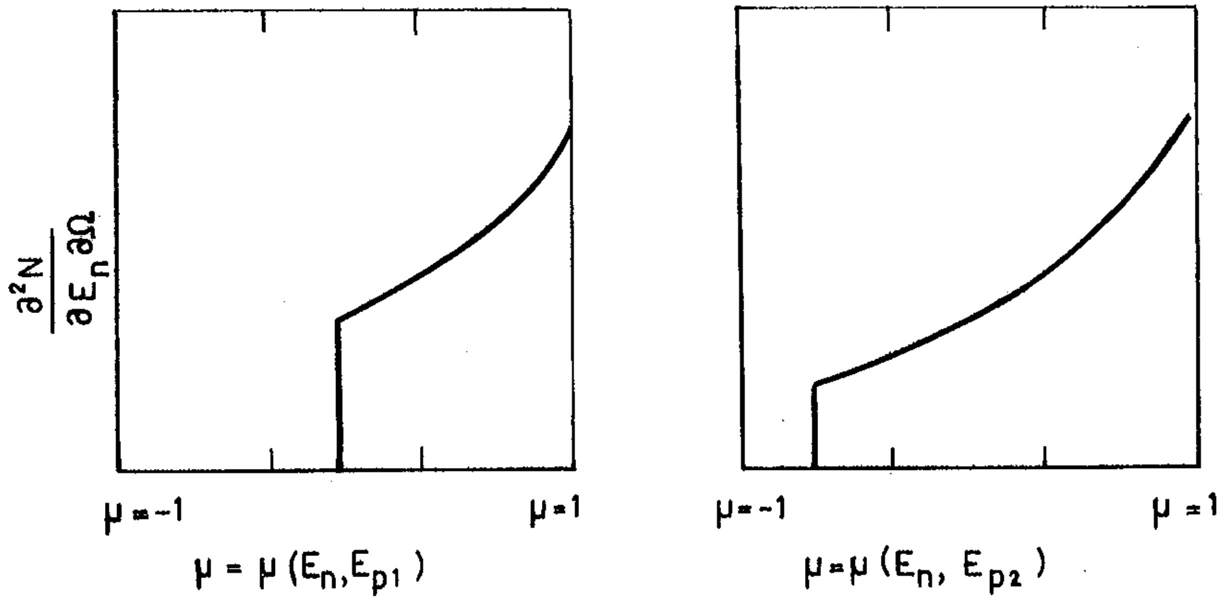
```
NOW SUPPLY THE NEW ANGLES
274 MU(1)←0.0
274 MU(2)←30.0
274 MU(3)←60.0
274 MU(4)←90.0
274 MU(5)←120.0
274 MU(6)←150.0
DO YOU WISH TO SEE THE CURRENT ANGLE TABLE?
REPLY YES OR NO
241 RPLY←NO
DO YOU WISH TO CHANGE ANY OF THE ANGLE VALUES
REPLY YES,NO OR ALL
252 RPLY←NO
    DO YOU WANT TO PLOT (AT CONSTANT ANGLE)
(A) YIELD VS NEUTRON ENERGY,(B) YIELD VERSUS PROTON ENERGY
OR (C) NO PLOT - REPLY A,B OR C
313 RPLY←B
DO YOU WANT TO SPECIFY THE PLOT SIZE & SCALE?
REPLY YES OR NO
442 SCL1←YES
PLEASE SPECIFY THE PLOT SIZE IN INCHES
445 XP1←8.0
445 YP1←8.0
SPECIFY THE LIMITS ON THE NEUTRON/PROTON ENERGY IN MEV
449 XLO1←1.80
449 XHI1←3.0
SPECIFY THE LIMITS FOR THE GROUND STATE YIELD
451 YLO1←0.0
451 YHI1←700.0
SPECIFY THE LIMITS FOR THE EXCITED STATE YIELD
453 YLO2←0.0
453 YHI2←30.0
END OF RUN

NOW SAVE THIS PROGRAM AND ITS SYMBOL TABLE
(E.G. #SAVES PROGNAME,PLS/ACCTNMBR)
END ACL

SUBMIT A JOB TO THE IBM360 USING
PRG=LIPNA FROM DSN=MTR.PLIB2
ENSURE IDENTIFICATION PROGNAME,PLS
APPEARS AFTER THE GO.SYSIN  DD  * STATEMENT
330 STOP
```



(a) $\partial^2 N/\partial E_n \partial \Omega$ AS A FUNCTION OF NEUTRON ENERGY E_n FOR A GIVEN ANGLE OF EMISSION θ



(b) $\partial^2 N/\partial E_n \partial \Omega$ AS A FUNCTION OF $\mu = \cos\theta$ FOR A GIVEN NEUTRON ENERGY

FIGURE 1. THE SHAPE OF $\partial^2 N/\partial E_n \partial \Omega$ FOR A PARTICULAR REACTION AND FOR TWO INCIDENT PROTON ENERGIES E_{p1} AND E_{p2}, $E_{p2} > E_{p1}$

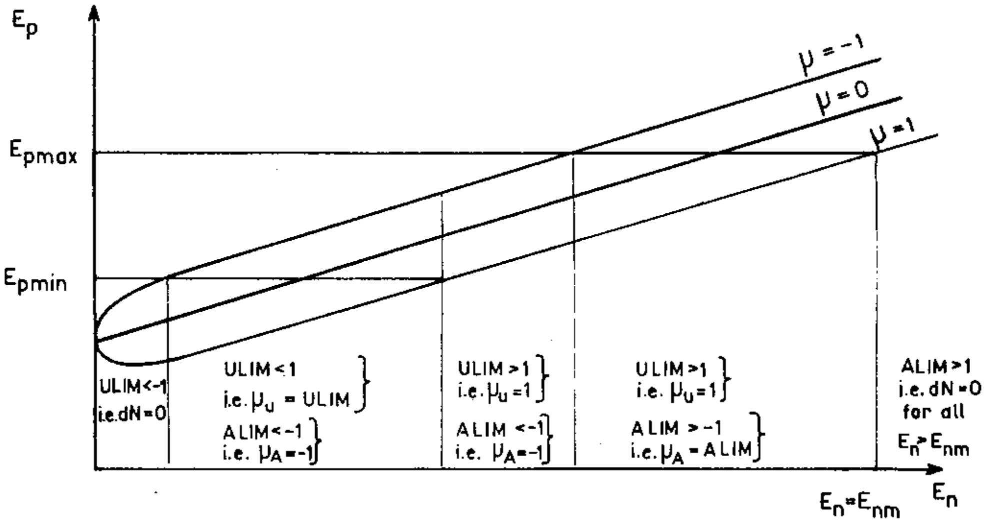
$$dN = 2\pi \int_{\mu_A}^{\mu_u} (\partial^2 N/\partial E_n \, \partial\Omega)\, d\mu$$

$$\mu_u = ULIM = \mu(E_n, E_{pmin}), \quad \mu_A = ALIM = \mu(E_n, E_{pmax}), \quad E_{nm} = E_n\,(E_{pmax}, \mu = 1)$$

$\quad$ or $\qquad\qquad\qquad\qquad$ or

$\quad = 1 \qquad\qquad\qquad\qquad = -1$

Case $\frac{\partial E_p}{\partial E_n}(E_{nm}, \mu = 1) > 0$



Case $\frac{DE_p}{OE_n}(E_{nm}, \mu = 1) < 0$ or > 0

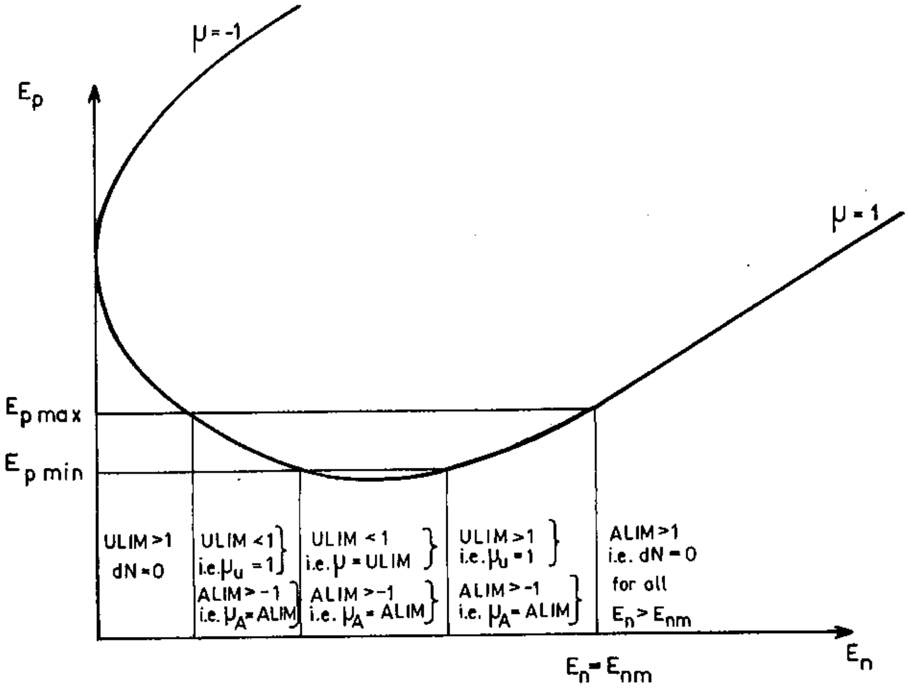
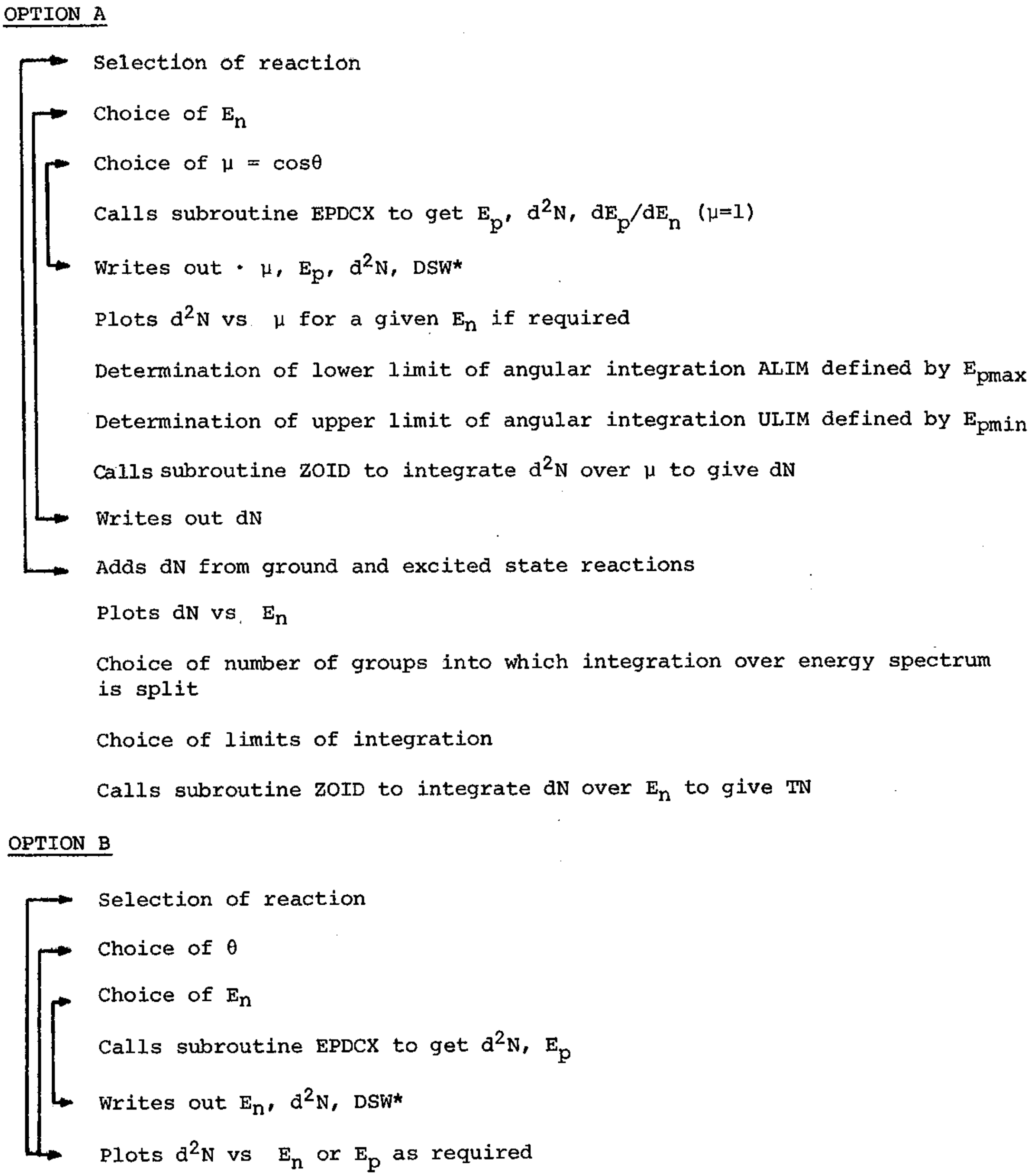


FIGURE 2. DETERMINATION OF INTEGRATION LIMITS FOR VARIOUS VALUES OF E_n



OPTION A

Selection of reaction

Choice of E_n

Choice of $\mu = \cos\theta$

Calls subroutine EPDCX to get E_p, d^2N, dE_p/dE_n (μ=1)

Writes out · μ, E_p, d^2N, DSW*

Plots d^2N vs μ for a given E_n if required

Determination of lower limit of angular integration ALIM defined by E_{pmax}

Determination of upper limit of angular integration ULIM defined by E_{pmin}

Calls subroutine ZOID to integrate d^2N over μ to give dN

Writes out dN

Adds dN from ground and excited state reactions

Plots dN vs E_n

Choice of number of groups into which integration over energy spectrum is split

Choice of limits of integration

Calls subroutine ZOID to integrate dN over E_n to give TN

OPTION B

Selection of reaction

Choice of θ

Choice of E_n

Calls subroutine EPDCX to get d^2N, E_p

Writes out E_n, d^2N, DSW*

Plots d^2N vs E_n or E_p as required

* Note: DSW = 4π x estimate of the angular cross section in the centre-of-mass frame.

FIGURE 3. FLOW OF MAIN PROGRAM OF LIPNA