AAEC/E387

AAEC/E387

# AUSTRALIAN ATOMIC ENERGY COMMISSION

## RESEARCH ESTABLISHMENT

## LUCAS HEIGHTS

# AUS DIFFUSION MODULE POW CHECKOUT – 1– AND 2– DIMENSIONAL KINETICS CALCULATIONS

by

J. P. POLLARD

January 1977

ISBN 0 642 99733 0

AUSTRALIAN ATOMIC ENERGY COMMISSION
RESEARCH ESTABLISHMENT
LUCAS HEIGHTS

# AUS DIFFUSION MODULE POW CHECKOUT - 1- AND 2-DIMENSIONAL KINETICS CALCULATIONS

by

J.P. POLLARD

## ABSTRACT

POW is the diffusion module 'workhorse' of the AUS reactor neutronics modular code system; its steady state calculations have been checked out against other diffusion codes (particularly CRAM and GOG). Checkout of kinetic aspects, however, is difficult as kinetic codes are not freely available. In this report POW has been checked against three benchmark calculations as well as a calculation on the 100 kW Argonaut reactor Moata.

National Library of Australia card number and ISBN 0 642 99733 0

The following descriptors have been selected from the INIS Thesaurus to describe the subject content of this report for information retrieval purposes. For further details please refer to IAEA-INIS-12 (INIS: Manual for Indexing) and IAEA-INIS-13(INIS: Thesaurus) published in Vienna by the International Atomic Energy Agency.

A CODES; COMPUTER CALCULATIONS; MULTIGROUP THEORY; NEUTRON DIFFUSION EQUATION; NEUTRON FLUX; ONE-DIMENSIONAL CALCULATIONS; P CODES; REACTOR KINETICS; STEADY-STATE CONDITIONS; TWO-DIMENSIONAL CALCULATIONS

# CONTENTS



(Continued)

## CONTENTS (Continued)

## 1. INTRODUCTION

For several years the AUS reactor neutronics modular code system [Robinson 1975] has been used on the AAEC's IBM360/50 computer at the Research Establishment, Lucas Heights. Presently, some of the peripheral modules are being written; however, the main 'workhorse' modules are available and have been tested. Even so, extensive crucial testing of the scheme is yet to be undertaken, particularly of the system as a whole.

This report describes the checkout of the diffusion module 'workhorse' POW [Pollard 1974] for multidimensional kinetics calculations. Extensive earlier checkout of POW has been carried out for steady state comparisons [J. Pollard, unpublished data], particularly against the codes CRAM [Hassitt 1962] and GOG [Hopkins & Oakes 1968]. However, checkout of kinetic aspects is difficult as kinetic codes are not freely available. A further difficulty is encountered in trying to repeat published multidimensional kinetic calculations since the reported calculations frequently do not list all of the required data. The main reason for the oversights is probably that the authors present comparisons of different methods and are not particularly concerned with presenting 'benchmark' calculations. Another difficulty is that sometimes authors are using undefined, although apparently universal, notation. As an example, even with commonly used notation, the 2-group fast removal cross section is sometimes the downscattering cross section, sometimes the absorption cross section and sometimes the sum of the two (as it is with POW).

The kinetic checkout adopted here uses three well-documented benchmark type calculations and reports a calculation on the AAEC research reactor Moata. The Moata calculation is well-documented but, on account of the low computing power at the time of writing, extensive verification of dependence of the results on choice of groups, mesh spacing, time step *etc.* is not pursued. Even so, the results would be of use to others pursuing 'benchmark' type kinetic calculations.

## 2. THE MULTIGROUP NEUTRON DIFFUSION EQUATION

To present some of the quantities required by POW, the reader needs to be acquainted with the types of equations that POW solves. The time, t, dependent equations to be solved at a general point $\underline{r}$ of the reactor under study [Stacey 1969; Pollard 1975] are as follows:

(i) The neutron flux diffusion equation which gives the connection: the rate of decrease of neutron flux (velocity times neutron density) for energy group g, $\phi_g(\underline{r},t)$ (the sought after quantity) = that due to leakage and removal *via* absorption reactions and out scattering reactions, less production from in scattering reactions and prompt fission from all groups, less production from decay of precursors. Mathematically these are

$$-\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\underline{r},t) = \{-\nabla.D_g(\underline{r},t)\nabla+\sigma_{remg}(\underline{r},t)\}\phi_g(\underline{r},t) -$$

$$-\sum_{g'}[\sigma_{sg'\to g}(\underline{r},t)+\chi_{Pg}(1-\beta)\frac{\nu}{k}\sigma_{fg'}(\underline{r},t)]\phi_{g'}(\underline{r},t) -$$

$$-\sum_{d}\chi_{dg}\lambda_d C_d(\underline{r},t) \quad , \quad g = 1,2,\ldots,G \; . \qquad (1)$$

(ii) The precursor concentration equation, which gives the relation: the rate of increase of precursor concentration for the delayed group d, $C_d(\underline{r},t)$ (an intermediate quantity only) = that due to production from fission less decay of the precursor. Mathematically, this is

$$\frac{\partial}{\partial t}C_d(\underline{r},t) = \beta_d \sum_{g'}\frac{\nu}{k}\sigma_{fg'}(\underline{r},t)\phi_{g'}(\underline{r},t) - \lambda_d C_d(\underline{r},t),$$

$$d = 1,2,\ldots, \text{ usually } 6 \quad , \qquad (2)$$

where the following are supplied data:

$\chi_{Pg}$ is the prompt fission spectrum (normalised to unit group sum);

$\chi_{dg}$ is the delayed fission spectrum (normalised to unit sum) for delayed group d with decay constant $\lambda_d$ and which yields fraction $\beta_d$ of total fission emissions, of which $\beta(=\Sigma_d\beta_d)$ are delayed ($\beta$ is usually about 0.7 per cent of total fissions);

$\sigma_g(\underline{r},t)$ denotes various macroscopic cross sections (spatially constant for one material and possibly subject to step or ramp time variation);

$\sigma_{sg'\to g}$ = scattering matrix from group g' to g (with g' to g term set to zero);

$\sigma_{remg}$ = removal
$= \sigma_{ag} + \sum_{g'} \sigma_{sg\to g'}$ (absorption + outscatters);
$\nu\sigma_{fg}$ = fission emission;

$D_g(\underline{r},t)$ denotes diffusion coefficient which is usually given as $D_g(\underline{r},t) = 1/\{3\sigma_{trg}(\underline{r},t)\}$, $\sigma_{trg}$ being the transport cross section;

$v_g$ denotes group g average velocity; and

k is the effective steady state multiplication eigenvalue supplied from a preceding calculation (equation (5b)), and sums are taken over all groups (1,2,...,G for energy groups and 1,2,..., usually 6 for delayed groups).

Solution of the equations is subject to the following boundary conditions:

(iii) the outer boundary condition

$$D_g(\underline{r},t)\,\frac{\partial}{\partial n}\phi_g(\underline{r},t) + \frac{\phi_g}{3c}(\underline{r},t) = 0 \text{ for each group g,} \qquad (3)$$

where n = outward normal,

c = transport (mean free path) extrapolation distance,

$$= \begin{cases} 0.71 \text{ for an actual outer boundary of the reactor,} \\ \infty \text{ for a symmetric boundary (with POW, input of c = 0 sets this condition).} \end{cases}$$

(iv) The internal boundary conditions for boundaries separating different materials are all assumed to lie parallel to axes, say L|R:

(a) continuity of flux

$$\phi_g(\underline{r},t)\big|_L - \phi_g(\underline{r},t)\big|_R = 0 \text{ for each group g ;} \qquad (4a)$$

(b) continuity of current

$$n.\{D_g(\underline{r},t)\nabla\phi_g(\underline{r},t)\big|_L - D_g(\underline{r},t)\nabla\phi_g(\underline{r},t)\big|_R\} = 0$$

for each group g. (4b)

(v) In addition the initial conditions are

$$\frac{\partial}{\partial t}\phi_g(\underline{r},t) = 0\ ,\ \frac{\partial}{\partial t}C_d(\underline{r},t) = 0\ ,\ t \leqslant 0 \text{ for all groups} \qquad (5a)$$

which give the steady state equation (dropping time dependence)

$$\{-\nabla.D_g(\underline{r})\nabla+\sigma_{remg}(\underline{r})\}\phi_g(\underline{r}) - \sum_{g'}[\sigma_{sg'\to g}(\underline{r}) + \chi_g \frac{\nu}{k}\sigma_{fg'}(\underline{r})]\phi_{g'}(\underline{r}) = 0 \quad , \tag{5b}$$

where $\chi_g = \chi_{Pg}(1-\beta) + \sum_d \chi_{dg}\beta_d$ is the equilibrium fission spectrum, k is the effective steady state multiplication (eigenvalue), $\phi_g(\underline{r})$ is the steady state (eigenfunction) flux and $\phi_g(\underline{r},0) = \phi_g(\underline{r})$, except for an arbitrary normalising factor.

## 3. POW CALCULATION PROCEDURE IN BRIEF

POW uses a finite difference approximation for the spatial derivative involved in equation (1) and requires calculation of group fluxes at the intersection of chosen grid lines i = 1,2,...,I and j = 1,2,...,J. Temporal integration from time $t_{p-1}$ to $t_p$, a step of $\delta t_p$, is carried out using the following assumptions:

(i) for the temporal variation of the cross sections a constant value is taken

$$\sigma_g(\underline{r},t) = \int_{t_{p-1}}^{t_p} \sigma_g(\underline{r},t')dt'/\delta t_p \quad , \tag{6}$$

(ii) and for the temporal variation of the flux, a linear form is assumed

$$\phi_g(\underline{r},t) = \{(t_p-t)\phi_g(\underline{r},t_{p-1}) + (t-t_{p-1})\phi_g(\underline{r},t_p)\}/\delta t_p \quad . \tag{7}$$

When we do this, the precursor concentration equation (2) may be (time) integrated analytically and the result substituted into the diffusion equation (1).

Leaving aside essentially all the details [see Stacey 1969; Pollard 1973, 1974] we arrive at the matrix equation

$$M\phi(p) = -M'\phi(p-1) + LC(p-1) \quad , \tag{8}$$

where M and M' are matrices of order IJG, L is a matrix of order IJ times the number of delayed groups (usually 6), and $\phi(p-1)$ and $C(p-1)$ are flux and precursor concentration vectors. The precursor concentration is updated using the solution of equation (8) as

$$C(p) = HC(p-1) + F\phi(p) \ , \qquad (9)$$

where H and F are matrices containing precursor decay and fission emission elements respectively.

TABLE 1

POW FLOW PROCEDURE FOR SOLUTION OF THE DIFFUSION EQUATION

| | |
|---|---|
| DO 1 p=0,1,...,P | temporal loop<br>no iteration required,<br>calculate $t_p$, cross sections and<br>coefficient matrix |
| IF(p=0) CONTINUE | steady state trial solution<br>generate for bare homogeneous system |
| IF(p=0) DO 5 $\ell_o$=1,2,...,* | outer (fission) loop<br>calculate total fission source from<br>last flux $\phi$ and last eigenvalue k and<br>combine with earlier sources<br>(Chebyshev extrapolation) |
| IF(p≠0) $\ell_o$=1 | right hand calculation<br>calculate right hand of equation (8) |
| $g_l$ = 1<br>DO 7 $\ell_u$ = 1,2,...,*<br>DO 6 g = $g_l$,$g_l$+1,...,G | upscatter (and fission p > 0) loop<br>update groups using Gauss-Seidel<br>method |
| IF(g=$g_u$, 1st upscatter group) | group rebalance<br>rebalance groups on a whole plane<br>basis when 1st upscatter group (or<br>1st group p > 0) encountered |
| | region rebalance<br>rebalance regions of the whole plane<br>for a group |
| DO 4 $\ell_I$ = 1,2,...,* | inner plane solution for a group |
| DO 4 j = 1,2,...,J | line solution, SLOR<br>solve directly for a line (i=1,2...,I)<br>as matrix is tridiagonal (i and j<br>could be swapped here) |
| 4 CONTINUE | combine lines using SLOR method |
| 6 CONTINUE | |
| 7 $g_l$ = $g_u$ | |
| 5 CONTINUE | |
| IF(p=0) CONTINUE | precursor concentration initial condition<br>calculate precursor concentration |
| IF(p≠0) CONTINUE | precursor concentration update<br>update precursor concentration following<br>equation (9) |
| 1 CONTINUE<br>STOP | output results (disk, printer, plotter,<br>etc.) |

** these loops terminate on convergence tests*

The solution procedure adopted by POW employs successive line over-relaxation (SLOR) for the inner plane solution for each group and relies on a region and group rebalance technique [Pollard 1975] to hasten convergence for the whole plane (Table 1).

## 4. 1D BENCHMARK-SPATIALLY ASYMMETRIC NEGATIVE RAMP OF 1/10 \$ $s^{-1}$

The need for well documented reactor calculations able to serve as 'benchmark' standards for the development and testing of codes has been recognised for many years. Mostly, the calculations are taken to higher accuracy than normally required for everyday work to show up possible small discrepancies between the different codes. The Argonne Code Center has produced two books on benchmark problems [Argonne National Laboratory 1968, 1972] and from the latest book we draw our first example (also reported by Fuller 1972). Results are compared from the four codes:

RAUMZEIT [Adams & Stacey 1967] - best benchmark solutions;

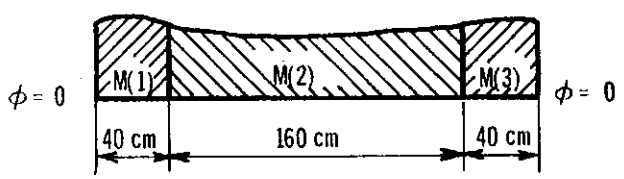
WIGLE [Cadwell, Henry & Vigilotti 1964];

QX1 [Meneley, Ott & Wiener 1971]; and

POW [Pollard 1974].

### 4.1 Problem Description

Following the data of W.M. Stacey [Argonne National Laboratory 1972] a very loosely coupled 1D slab reactor is subject to a negative reactivity ramp in one part only of an otherwise symmetric 3-region system.




M(3) = material 3; M(2) = material 2; M(1) = M(3) initially.

### 4.2 Initial Data Required

#### 4.2.1 Mesh spacing

A mesh spacing of $\delta x = 2$ cm is used throughout the system. This results in 121 mesh points for POW (an edge of mesh code). The boundary condition $\phi = 0$ is taken to be essentially at the reactor extremes with a transport extrapolation distance of $c = 10^{-6}$ rather than the usual value of $c = 0.71$.

4.2.2 Initial nuclear data

Two (energy) group data are used as indicated (according to POW requirements).

| | M(1) | M(2) | M(3) | |
|---|---|---|---|---|
| $\sigma_{tr}$ $cm^{-1}$, g=1 | 0.222222 | 0.333333 | same | D=1.5 1 for M(1), M(2) |
| =2 | 0.666667 | 0.666667 | as | 0.5 0.5 for M(1), M(2) |
| $\sigma_{rem}$ $cm^{-1}$, g=1 | 2.6-2 | 2.-2 | M(1) | rem = absorption |
| =2 | 1.8-1 | 8.-2 | ↓ | plus outscatters |
| $\nu\sigma_f$ $cm^{-1}$, g=1 | 1.-2 | 5.-3 | | 2.6-2 = 2.6 x $10^{-2}$ |
| =2 | 2.-1 | 9.9-2 | | |
| $\sigma_{sg'\to g}$ $cm^{-1}$, g'=1 | 0,1.5-2 | 0,1.-2 | | downscatters from |
| =2 | 0,0 | 0,0 | | group 1=1.5-2 for M(1) |
| $\chi_p$, g=1 | 1 | same as | | |
| =2 | 0 | M(1) | | |
| v cm $s^{-1}$, g=1 | 1.+7 | ↓ | | |
| =2 | 3.+5 | | | |

4.2.3 Delayed group data

Six delayed group data are used:

| Family d | Effective Delay Fraction ($\beta_d$) | Decay Constant ($\lambda_d$ $s^{-1}$) |
|---|---|---|
| 1 | 2.50-4 | 1.24-2 |
| 2 | 1.64-3 | 3.05-2 |
| 3 | 1.47-3 | 1.11-1 |
| 4 | 2.96-3 | 3.01-1 |
| 5 | 8.60-4 | 1.14 |
| 6 | 3.20-4 | 3.01 |
| | $\beta$=7.5-3 | |

The delayed group fission spectra are taken to be the same as the prompt spectrum, $\chi_p$.

4.3 Steady State Calculation, $t \leq 0$

The steady state calculation, solution of equation (5b), is compared for different codes following the Argonne National Laboratory

[1972] proposals. In the comparison, power, p(t), is taken to be

$$p(t) = f \sum_g \int_{reactor} \nu\sigma_{fg}(\underline{r},t)\phi_g(\underline{r},t)d\underline{r} \quad , \tag{10}$$

where f is an arbitrary normalising factor, and group g variation of $\nu_g$ (the number of neutrons released per fission) is ignored. (This power expression is apparently that adopted by the benchmark proposers.) From this calculation we have the following results:

| | RAUMZEIT | WIGLE | QX1 | POW |
|---|---|---|---|---|
| multiplication, k | 0.90156 | 0.90156 | 0.90156 | 0.90156 |
| fractional power, M(1) | 0.2790 | 0.2790 | 0.2790 | 0.2790 |
| M(2) | 0.4421 | 0.4421 | 0.4421 | 0.4421 |
| M(3) | 0.2790 | 0.2790 | 0.2790 | 0.2790 |

Starting from a symmetric cosine distribution trial flux, POW took 31 outer iterations to drop the error indicator down to $10^{-5}$. (The error indicator used in POW is

$$S_{err}^{(n)} = \max_{i,j} S_{ij}^{(n)}/S_{ij}^{(n-1)} - \min_{i,j} S_{ij}^{(n)}/S_{ij}^{(n-1)} \quad , \tag{11}$$

where $S_{ij}^{(n)}(t) = \sum_g \int_{i,j} \nu\sigma_{fg}(\underline{r},t)\phi_g(\underline{r},t)d\underline{r}$ , (12)

the integration being taken over a box about the point i,j and which has edges midway between mesh lines.) The number of outer iterations is not surprising for the very loosely coupled core considering the effective dominance ratio (ratio of successive k-eigenvalues corresponding to symmetric eigenfunctions) is 0.937. (Starting with an asymmetric trial flux, the convergence is far worse, since the dominance ratio d = 0.9984.)

4.3.1 Steady state normalisation, t=0

POW has several different normalisation procedures by which it can generate

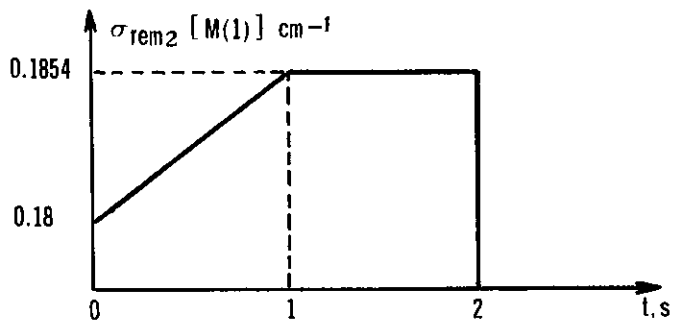
$$\phi_g(\underline{r},0) = a\phi_g(\underline{r}) \quad , \tag{13}$$

where a is a constant. The normalisation adopted in the present calculation is

$$a \sum_g \int_{reactor} \phi_g(\underline{r})d\underline{r} = \int_{reactor} d\underline{r} \quad . \tag{14}$$

4.4 Time Dependent Data

The only quantity varied with time is the group 2 removal cross section (equivalent to the absorption cross section) for M(1). The variation is a ramp of 3 per cent as illustrated.




The asymptotic reactivity corresponding to the full ramp

$$\rho = \frac{\delta k}{(k+\delta k)\beta} \$ \qquad (15)$$

is -0.094\$ which is obtained from a second steady state calculation with $\sigma_{rem2}$[M(1)] = 0.1854.

4.4.1 Temporal step length, $\delta t$

The temporal step length, $\delta t$, adopted is:

| | RAUMZEIT | WIGLE | QX1 | POW |
|---|---|---|---|---|
| $\delta t$,s | $10^{-3}$ | $10^{-3}$ | ave. $5 \times 10^{-2}$ | $10^{-2}$ |

POW convergence for the kinetics part corresponds to the condition:

$S^{(n)}_{err}(t) \leqslant 10^{-4}$ (see equation (11)).

4.5 Kinetics Calculation, $t \geqslant 0$

Results of the calculation are plotted in Figure 1 (thermal flux) and summarised in Tables 2 and 3.

4.6 Observation on Results

From the results, it is clear that POW is able to calculate the 1D benchmark job to acceptable accuracy (normally a factor of 10 lower than the accuracy implied by the tables).

5. 2D TWIGL TEST-POSITIVE RAMP OF $2\frac{1}{2}$\$ s$^{-1}$

Documentation of 2D kinetics calculations in the literature unfortunately tends to be incomplete. Sometimes vital basic data are not given (or worse, wrongly given) and, at other times, the calculated quantities

## TABLE 2

## TOTAL POWER

| Time, seconds | RAUMZEIT | WIGLE | QX1 | POW | |
|---|---|---|---|---|---|
| 0. | 1.0000 | 1.0000 | 1.0000 | 1.0000 | ←normalised here |
| 0.1 | 0.9299 | 0.9298 | 0.9298 | 0.9298 | |
| 0.2 | 0.8733 | 0.8732 | 0.8733 | 0.8732 | |
| 0.5 | 0.7597 | 0.7596 | 0.7597 | 0.7596 | |
| 1.0 | 0.6588 | 0.6588 | 0.6588 | 0.6587 | |
| 1.5 | 0.6432 | 0.6432 | 0.6433 | 0.6432 | |
| 2.0 | 0.6307 | 0.6306 | 0.6307 | 0.6306 | |

## TABLE 3

## RELATIVE REGIONAL POWER

| | Time, seconds | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0. | 0.1 | 0.2 | 0.5 | 1.0 | 1.5 | 2.0 | |
| Region 1 | | | | | | | | |
| RAUMZEIT | 1.0000 | 0.8621 | 0.7521 | 0.5337 | 0.3453 | 0.3236 | 0.3066 | |
| WIGLE | 1.0000 | 0.8621 | 0.7520 | 0.5336 | 0.3452 | 0.3235 | 0.3066 | Note the tilt in power |
| QX1 | 1.0000 | 0.8621 | 0.7521 | 0.5336 | 0.3452 | 0.3235 | 0.3066 | |
| POW | 1.0000 | 0.8622 | 0.7522 | 0.5337 | 0.3453 | 0.3236 | 0.3066 | |
| Region 2 | | | | | | | | |
| RAUMZEIT | 1.0000 | 0.9340 | 0.8805 | 0.7724 | 0.6753 | 0.6588 | 0.6455 | |
| WIGLE | 1.0000 | 0.9339 | 0.8804 | 0.7724 | 0.6753 | 0.6587 | 0.6455 | |
| QX1 | 1.0000 | 0.9340 | 0.8805 | 0.7724 | 0.6753 | 0.6588 | 0.6455 | |
| POW | 1.0000 | 0.9339 | 0.8805 | 0.7723 | 0.6752 | 0.6588 | 0.6455 | |
| Region 3 | | | | | | | | |
| RAUMZEIT | 1.0000 | 0.9910 | 0.9831 | 0.9655 | 0.9463 | 0.9382 | 0.9312 | |
| WIGLE | 1.0000 | 0.9910 | 0.9830 | 0.9655 | 0.9462 | 0.9381 | 0.9311 | |
| QX1 | 1.0000 | 0.9910 | 0.9831 | 0.9656 | 0.9463 | 0.9383 | 0.9312 | |
| POW | 1.0000 | 0.9908 | 0.9829 | 0.9653 | 0.9460 | 0.9380 | 0.9310 | |

↑
normalised here

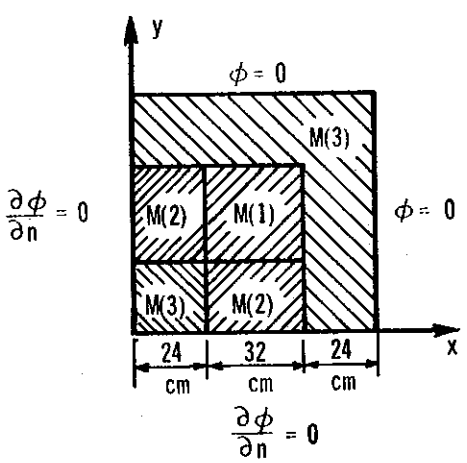
being compared are imprecise. Test cases are offered in ENEA/EURATOM [1972], but the present author has had little success with them. There is thus a real need for 2D benchmark kinetics problems to be defined.

Some problems originally set up to test the 2D, 2-group TWIGL code [Yasinsky, Natelson & Hageman 1968] and reported by Hageman & Yasinsky [1969] and Wight, Hansen & Ferguson [1971] are discussed in Sections 5.1 and 5.2. Results are compared from the five codes:

TWIGL [Yasinsky, *et al.* 1968] - best comparison solutions;
LUMAC [McCormick 1969];
MITKIN [Reed & Hansen 1970];
SADI [Wight, *et al.* 1971]; and
POW .

5.1 Problem Description

A 2D symmetric square section reactor is subject to a positive reactivity ramp in part of the system. The quarter reactor layout is as follows:




M(3) = blanket material; M(2) = core material;
M(1) = driver material
= M(2) initially.

5.2 Initial Data Required

5.2.1 Mesh spacing

A mesh spacing of $\delta x = \delta y = 8$ cm is taken throughout the system. The result is an 11 x 11 spatial mesh arrangement for POW. The boundary condition $\phi = 0$ is taken close to the actual boundary, using a transport extrapolation distance of $c = 10^{-3}$.

5.2.2 Initial nuclear data

POW 2-group data for the problem are given below:

| | M(1) | M(2) | M(3) | | |
|---|---|---|---|---|---|
| $\sigma_{tr}$ $cm^{-1}$, g=1 | 0.238095 | same | 0.256410 | D=1.4 | 1.3 for M(1),M(3) |
| =2 | 0.833333 | as | 0.666667 | 0.4 | 0.5 for M(1),M(3) |
| $\sigma_{rem}$ $cm^{-1}$, g=1 | 2.-2 | M(1) | 1.8-2 | | |
| =2 | 1.5-1 | ↓ | 5.-2 | | |
| $\nu\sigma_f$ $cm^{-1}$, g=1 | 7.-3 | | 3.-3 | | |
| =2 | 2.-1 | | 6.-2 | | |
| $\sigma_{sg'\to g}$ $cm^{-1}$, g'=1 | 0,1.-2 | | 0,1.-2 | | |
| =2 | 0,0 | | 0,0 | | |
| $\chi_p$, g=1 | 1 | | same | | |
| =2 | 0 | | as | | |
| v cm $s^{-1}$, g=1 | 1.+7 | | M(1) | | |
| =2 | 2.+5 | | ↓ | | |

5.2.3 Delayed group data

A one delayed group data set is used:

$$\beta = 7.5\text{-}3,\ \lambda = 8.\text{-}2\ s^{-1}\ .$$

The delayed group fission spectrum is the same as the prompt spectrum, $\chi_p$.

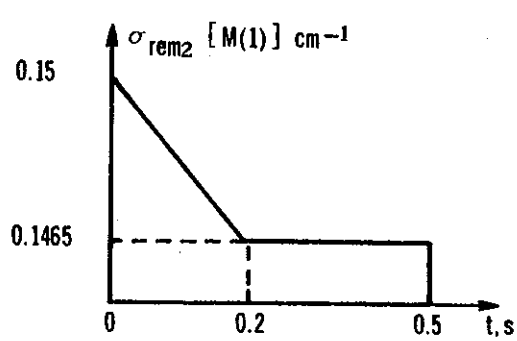
5.3 Steady State Calculation, $t \leq 0$

The results obtained for the steady state calculation are:

| | TWIGL | POW |
|---|---|---|
| multiplication,k | 0.914193 | 0.914194 |

5.4 Time Dependent Data

The thermal removal cross section of the driver material, M(1), is reduced following the ramp variation indicated.



The asymptotic reactivity corresponding to the full ramp is:

$\rho = 0.517\$$ (using equation (15)).

5.4.1 Temporal step length, $\delta t$

The temporal step length, $\delta t$, used is:

| | TWIGL | LUMAC | MITKIN |
|---|---|---|---|
| $\delta t$,s | $10^{-2}$ | $8 \times 10^{-4}$ | $10^{-3}$; $10^{-2}$, $t \geq 0.25$ |

| | SADI | POW |
|---|---|---|
| $\delta t$,s | $2.5 \times 10^{-4}$ | $10^{-2}$ |

POW convergence for the kinetics section corresponds to the condition;

$$S_{err}^{(n)}(t) \leq 10^{-3} \quad \text{(see equation (11))},$$

which is recommended by Pollard [1974] to save machine time, although the IBM360/50 computational time for the job of 22 CPU-minutes is hardly excessive.

TABLE 4

NORMALISED CENTRAL THERMAL FLUX

| Time, (s) | TWIGL | LUMAC | MITKIN | SADI | POW |
|---|---|---|---|---|---|
| 0. | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.02 | | | 1.030 | 1.033 | 1.037 |
| 0.05 | 1.120 | | 1.122 | 1.116 | 1.121 |
| 0.10 | 1.298 | 1.297 | 1.299 | 1.293 | 1.298 |
| 0.15 | 1.550 | | 1.549 | 1.543 | 1.550 |
| 0.20 | 1.933 | 1.940 | 1.929 | 1.919 | 1.931 |
| 0.25 | 2.033 | | 2.037 | 2.064 | 2.036 |
| 0.30 | 2.044 | 2.079 | 2.004 | 2.047 | 2.045 |
| 0.35 | 2.053 | | 1.984 | 2.056 | 2.052 |
| 0.40 | 2.062 | | 1.981 | 2.066 | 2.062 |
| 0.45 | 2.070* | | | | 2.069 |
| 0.50 | 2.079* | | | | 2.079 |

** estimated from published total thermal flux*

5.5 Kinetic Calculation, $t \geq 0$

Results of the calculation are presented for the normalised central thermal flux (Table 4)

$$\phi_2(0,0,t)/\phi_2(0,0,0) \quad ,$$

that is $\phi_{1,1,2}(t)/\phi_{1,1,2}(0)$ in terms of flux at mesh point (1,1), group 2.

5.6 Observation on Results

Unlike the problem given in Section 4, the time variation does not introduce asymmetric flux behaviour, and mostly the time variation at all points is similar. For the simpler problem given in the present section, POW again achieves acceptable accuracy whereas LUMAC, MITKIN and SADI do not.

## 6. 2D TWIGL TEST-POSITIVE STEP OF $\frac{1}{2}$\$

The present calculation is also for the reactor described in Section 5, but with the ramp of Section 5.4 changed to a step of the same size.

6.1 Time Dependent Data

The thermal removal cross section of the driver material, M(1), is reduced instantaneously from 0.15 $cm^{-1}$ to 0.1465 $cm^{-1}$ at t=0. The asymptotic reactivity is again $\rho$ = 0.517\$.

6.1.1 Temporal step length

The temporal step length, $\delta t$, is as indicated:

| | TWIGL | LUMAC | MITKIN | SADI |
|---|---|---|---|---|
| $\delta t$,s | $10^{-3}$ | $8 \times 10^{-5}$ | $2 \times 10^{-4}$ | $2.5 \times 10^{-4}$ |

| | POWa | POWb | POWc | POWd | POWe |
|---|---|---|---|---|---|
| $\delta t$,s | $10^{-2}$ | $10^{-2}$ | $10^{-3}$ | $10^{-3}$ | $10^{-4}$ |

POW convergence during the kinetics calculation is taken as:

$$S_{err}^{(n)}(t) \leq 10^{-3} \quad \text{POWa and POWc} \quad , \quad \text{and}$$

$$S_{err}^{(n)}(t) \leq 10^{-4} \quad \text{POWb, POWd and POWe} \quad ,$$

in order to verify the adequacy of the lower accuracy convergence limit.

6.2 Kinetics Calculation, $t \geq 0$

Results of the calculation are presented in Table 5 for the normalised central thermal flux, $\phi_2(0,0,t)/\phi_2(0,0,0)$.

TABLE 5

NORMALISED CENTRAL THERMAL FLUX

| Time (s) | TWIGL | LUMAC | MITKIN | SADI | POWa | POWb | POWc | POWd | POWe |
|---|---|---|---|---|---|---|---|---|---|
| 0. | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.01 | 1.594 | 1.629 | 1.631 | 1.621 | 1.627 | 1.627 | 1.623 | 1.624 | 1.623 |
| 0.02 | 1.837 | 1.879 | 1.881 | 1.983 | 1.923 | 1.924 | 1.870 | 1.872 | 1.872 |
| 0.03 | 1.934 | 1.973 | 1.978 | 2.026 | 1.953 | 1.954 | 1.964 | 1.966 | |
| 0.04 | 1.979 | | 2.028 | 2.024 | 2.029 | 2.031 | 2.000 | 2.002 | |
| 0.05 | 2.002 | | 2.067 | 2.023 | 2.000 | 2.000 | 2.015 | 2.017 | |
| 0.10 | 2.030 | | | 2.032 | | 2.044 | | 2.033 | |
| 0.20 | 2.048 | | | 2.050 | | 2.053 | | 2.051 | |

It is interesting to note that dropping the POW region rebalance scheme [Pollard 1975] makes little difference to the POWb computational time, whereas dropping group rebalance for the early time steps increases the computational time by more than a factor of 6 and convergence is not completely achieved. This result is in contrast with the asymmetric 1D calculations given in Section 4 where group rebalance is only of marginal help.

6.3 Observation on Results

As one would expect, the step change is harder to calculate than the equivalent ramp discussed in Section 5.

From the POW results, we note the insignificant loss of accuracy using the convergence condition

$$S_{err}^{(n)}(t) \leqslant 10^{-3}$$

and that a step length of a least $\delta t = 10^{-3}$ s is needed for early times as the $\delta t = 10^{-2}$ s results show a slight 'wobble'. Comparison of POW with the other codes suggests that the TWIGL step length of $\delta t = 10^{-3}$ s is inadequate, but otherwise reasonable agreement is obtained.

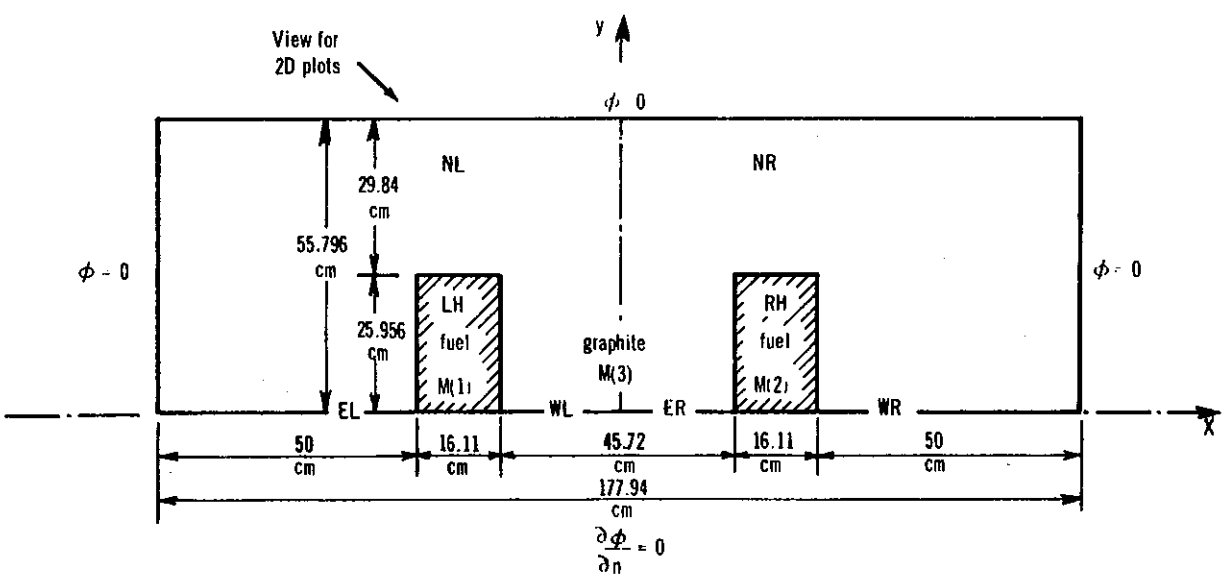
## 7. 2D MOATA - SPATIALLY ASYMMETRIC NEGATIVE STEP OF 1$

The present calculation concerns the estimation of neutron flux following the dropping of a safety rod in the AAEC research reactor Moata (a 100 kW Argonaut type reactor of highly enriched uranium in aluminium plates, cooled and moderated by light water with graphite

reflector). The drop in one tank only of the two-tank reactor causes flux asymmetries as calculated for the earlier 1D benchmark (Section 4). A comparison is made with an experiment conducted on Moata as part of a Summer School run at the Research Establishment for final year high school students [Barry *et al.* 1974]. The main purpose of the present calculation is to provide a possible 'benchmark' problem for use by others.

7.1 Problem Description

A 2D model of Moata, which is an approximation of that of J. Connolly (AAEC private communication), is subject to a negative reactivity step in the left hand fuel tank. The half reactor layout is as follows:




NL, *etc.*, are detector positions detailed later.

POW input data for the calculation is listed in Appendix A.

7.2 Initial Data Required

7.2.1 Mesh spacing

The following mesh spacing is used:

$\delta x$, left and right hand graphite = 50/7 = 7.1428571 cm,

left and right hand fuel = 16.11/5 = 3.222 cm, and

| | | | | |
|---|---|---|---|---|
| | central graphite | = | 45.72/7 | = 6.5314285 cm; |
| $\delta y$, | fuel | = | 25.956/5 | = 5.1912 cm, and |
| | top graphite | = | 29.84/5 | = 5.968 cm. |

The result of this choice of large mesh is a set of 32 x 11 solution points for POW. The boundary condition $\phi = 0$ is taken close to the actual boundary with $c = 10^{-6}$ for ease of calculation by others using codes having this requirement.

### 7.2.2 Initial nuclear data

The 2-group data for the problem are indicated below:

| | M(1) | M(2) | M(3) | |
|---|---|---|---|---|
| $\sigma_{tr}$ $cm^{-1}$, g=1 | 0.2436 | same | 0.279 | D=1.36836 1.19474 for M(1),M(3) |
| =2 | 1.558 | as | 0.3646 | 0.21395 0.914244 for M(1),M(3) |
| $\sigma_{rem}$ $cm^{-1}$, g=1 | 3.9655-2 | M(1) | 5.6669-3 | rem = absorption plus axial leakage |
| =2 | 5.5971-2 | ↓ | 2.081-3 | plus outscatters |
| $\nu\sigma_f$ $cm^{-1}$, g=1 | 2.13-3 | | 0 | |
| =2 | 8.074-2 | | 0 | |
| $\sigma_{sg'\to g}$ $cm^{-1}$, g'=1 | 0,3.532-2 | | 0,3.461-3 | |
| =2 | 6.302-4,0 | | 1.651-4,0 | note the slight upscatters |
| $\chi_p$, g=1 | 1 | | same | |
| =2 | 0 | | as | |
| v cm $s^{-1}$, g=1 | 9.+6 | | M(1) | |
| =2 | 2.3+5 | | ↓ | |

### 7.2.3 Delayed group data

Effective 6 delayed group data used are derived from data of Keepin [1965] and Tomlinson [1972].

| Family d | Effective delay fraction $\beta_d$ | Decay constant $\lambda_d$ $s^{-1}$ |
|---|---|---|
| 1 | 2.511-4 | 1.24-2 |
| 2 | 1.664-3 | 3.05-2 |
| 3 | 1.495-3 | 1.11-1 |
| 4 | 2.975-3 | 3.01-1 |
| 5 | 8.571-4 | 1.14 |
| 6 | 3.113-4 | 3.01 |
| | $\beta$ = 7.5535-3 | |

The delayed group fission spectra are taken to be the same as the prompt spectrum, $\chi_p$.

7.3 Steady State Calculation, $t \leq 0$

The POW steady state calculation flux results are displayed in Figures 2 and 8. The multiplication is k = 1.01227 and the normalisation adopted is

$$\sum_g \int_{\text{reactor}} \frac{\nu}{k} \sigma_{fg}(\underline{r})\ \phi_g(\underline{r}) d\underline{r} = 1 \quad . \tag{16}$$

7.4 Time Dependent Data

As a very approximate representation of the safety rod drop in the near vicinity of the left hand fuel tank, the thermal removal cross section of M(1) is instantaneously increased by $9.74887 \times 10^{-4}$ $cm^{-1}$. The asymptotic reactivity corresponding to this change is $\rho = -0.798\$$. The cross section change for M(1) was determined by a SEARCH calculation in POW to give $\rho \simeq -80¢$, which is the reactivity of the safety rod determined at an AAEC Summer School [Barry *et al.* 1974] and elsewhere [T. Wall, private communication].

7.4.1 Temporal step length $\delta t$

The temporal step length, $\delta t$, is chosen in a stepwise constant fashion as indicated

| t, (s) | $\delta t$, (s) | e-folding time, (s) |
|---|---|---|
| $0 \leq t < 10^{-2}$ | $10^{-3}$ | $-3.3 \times 10^{-2}$ |
| $10^{-2} \leq t < 10^{-1}$ | $9 \times 10^{-3}$ | $-9.9 \times 10^{-1}$ |
| $10^{-1} \leq t < 1$ | $9 \times 10^{-2}$ | -9.1 |
| $1 \leq t < 10$ | 0.9 | -17 |
| $10 \leq t < 100$ | 9 | -38 |
| $100 \leq t < 200$ | 10 | -60 |

This type of stepwise change was chosen so that $\delta t$ was always much less than the magnitude of the overall e-folding time observed during the calculation. The 0.9 factors were chosen so that every tenth output corresponded to $10^{-2}$, $10^{-1}$, 1, 10, 100 s as required for display of results given in Figures 2 to 9.

The convergence test used in the POW calculation is taken as:

$$S_{err}^{(n)}(t) \leqslant 10^{-3} \quad .$$

7.5 Kinetics Calculation, $t \geqslant 0$

In terms of the spatial mesh indices numbered from the bottom left hand corner,

i=1,2,...32; j=1,2,...,11,

thermal flux results are given for the following detector positions:

EL(6,1)

NL(10,9) NL is very approximately the detector position for experimental results given in Figure 9.

WL(15,1)

ER(18,1)

NR(23,9)

WR(27,1)

The flux results are recorded in Table 6 and, in addition, an asymmetry tilt indicator is given:

$$T = \frac{\phi_{NR,2}(t) - \phi_{NL,2}(t)}{\phi_{NR,2}(t) + \phi_{NL,2}(t)} \quad . \qquad (17)$$

For completeness Appendix B details some output of the POW run.

As the last time step length, $\delta t$ = 10 s, may be somewhat excessive, the calculation is repeated from t = 100 s with $\delta t$ = 5 s. Results are indicated in Table 7.

7.6 Observation on Results

The results beyond about 10 s show a 'wobble' indicative of an excessively large step length. A strict 'benchmark' calculation would need to employ a step length of probably no more than one second in order to give results to four significant figures. This can be left as an exercise for those with fast computing equipment, since the IBM360/50 took about 180 CPU minutes to produce the entries in Table 6.

8. FLUX TILT IN SYMMETRIC SYSTEMS ASYMMETRICALLY DISTURBED

From results given for the 1D benchmark calculation (Figure 1) and the 2D Moata calculation (Figure 8), we notice the extreme tilting that can result from an asymmetric disturbance to a symmetrical system. Wade & Rydin [1972], using perturbation analysis, produced a relationship connecting the tilt indicator, T (equation (17)), with the reactivity, $\rho$ (equation (15)) and the dominance ratio (ratio of k-eigenvalues). Here

TABLE 6

THERMAL FLUX AT DIFFERENT DETECTOR POSITIONS x $10^8$

| time (s) | EL | NL | WL | ER | NR | WR | T |
|---|---|---|---|---|---|---|---|
| 0 | 935800 | 355600 | 1432000 | 1432000 | 355700 | 935900 | 0 |
| $10^{-2}$ | 576800 | 228400 | 990600 | 1133000 | 299700 | 806200 | 0.135 |
| $10^{-1}$ | 417900 | 164600 | 713400 | 815100 | 215100 | 580800 | 0.133 |
| 1 | 370600 | 146100 | 635300 | 729000 | 192700 | 520900 | 0.138 |
| 10 | 213500 | 84540 | 370700 | 431200 | 114600 | 310500 | 0.151 |
| 55 | 65160 | 25900 | 114300 | 134500 | 35900 | 97410 | 0.162 |
| 100 | 29410 | 11700 | 51750 | 60890 | 16270 | 44190 | 0.163 |
| 150 | 11980 | 4772 | 21120 | 25070 | 6713 | 18220 | 0.169 |
| 200 | 5882 | 2341 | 10400 | 12190 | 3258 | 8865 | 0.164 |
| | | | | | | asymptotic | 0.182 |

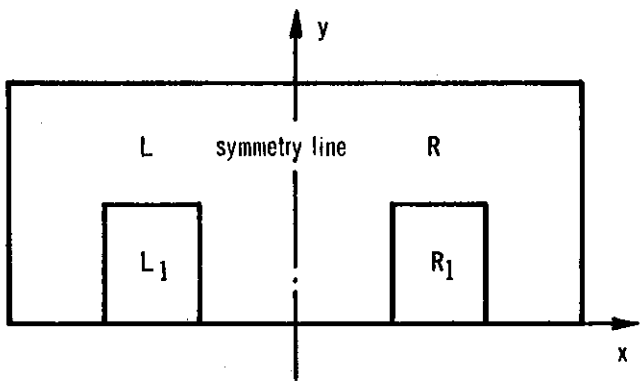
TABLE 7

THERMAL FLUX FOR DIFFERENT STEP LENGTHS x $10^8$

| time (s) | EL | NL | WL | ER | NR | WR | T |
|---|---|---|---|---|---|---|---|
| 150 ($\delta$t=10) | 11980 | 4772 | 21120 | 25070 | 6713 | 18220 | 0.169 |
| 150 ($\delta$t=5) | 12490 | 4972 | 22050 | 25950 | 6938 | 18860 | 0.165 |
| 200 ($\delta$t=10) | 5882 | 2341 | 10400 | 12190 | 3258 | 8865 | 0.164 |
| 200 ($\delta$t=5) | 5847 | 2326 | 10340 | 12120 | 3239 | 8815 | 0.164 |

we pursue a simpler approach to the asymmetric disturbance to produce a somewhat similar result.

Given a symmetric reactor, such as the one below




with boundary condition $\partial\phi/\partial n = 0$ or $\phi = 0$, the steady state equation is (compare with equation (5b))

$$\{-\nabla.D_g(\underline{r})\nabla + \sigma_{remg}(\underline{r})\}\,\phi_{g(i)}(\underline{r}) =$$

$$= \sum_{g'} [\sigma_{sg'\to g}(\underline{r}) + \chi_g \frac{\nu}{k_{(i)}} \sigma_{fg'}(\underline{r})]\phi_{g',(i)}(\underline{r}) \quad , \qquad (18)$$

where $k_{(1)} = k$, the normal multiplication, $\phi_{g(1)}$ the corresponding eigenfunction, and $k_{(2)} = dk_{(1)}$, the second highest magnitude eigenvalue, $\phi_{g(2)}$ the eigenfunction, with d = the dominance ratio.

Consider an asymmetric disturbance in $L_1$ corresponding to a pure absorption effect, $\sigma'_{remg} = \sigma_{remg} + \delta\sigma_{remg}$, then the steady state is retained by changing the multiplication to k', the first eigenvalue of the equation

$$\{-\nabla.D_g(\underline{r})\nabla + \sigma_{remg}(\underline{r}) + \delta\sigma_{remg}(\underline{r})\}\phi'_g(\underline{r}) =$$

$$= \sum_{g'} [\sigma_{sg'\to g}(\underline{r}) + \chi_g \frac{\nu}{k'} \sigma_{fg'}(\underline{r})]\phi'_{g'}(\underline{r}) \quad . \qquad (19)$$

Let us assume that the disturbance does not encourage high harmonics, so that

$$\phi'_g(\underline{r}) = a\phi_{g(1)}(\underline{r}) + b\phi_{g(2)}(\underline{r}) \qquad (20)$$

is a reasonable approximation with a and b yet to be determined. Introducing the half source terms

$$F^{(i)}(V) = \sum_g \int_V \frac{\nu}{k_{(i)}} \sigma_{fg}(\underline{r}) \phi_{g(i)}(\underline{r}) d\underline{r} \quad (V = L \text{ or } R) \qquad (21)$$

and $$F'(V) = \sum_g \int_V \frac{\nu}{k'} \sigma_{fg}(\underline{r}) \phi'_g(\underline{r}) d\underline{r} \quad , \qquad (22)$$

we adopt the arbitrary normalisations (as in equation (16))

$$F^{(1)}(V) = \tfrac{1}{2} \text{ since the 1st harmonic flux is symmetric,} \qquad (23)$$

$$F^{(2)}(L) = -\tfrac{1}{2} \ , \ F^{(2)}(R) = \tfrac{1}{2} \text{ since the 2nd harmonic flux is asymmetric, and} \qquad (24)$$

$$F'(L) + F'(R) = 1 \ ; \qquad (25)$$

equation (20) then gives

$$k' = ak_{(1)} \quad . \qquad (26)$$

If equation (20) held strictly, we would have for the right half of the reactor (R)

$$aF^{(1)}(R)\left(\frac{k_{(1)}}{k'} - 1\right) + bF^{(2)}(R)\left(\frac{k_{(2)}}{k'} - 1\right) = 0 \ , \qquad (27)$$

which is obtained by subtracting a and b proportions of equation (18) from equation (19) and integrating. Hence

$$\tfrac{1}{2}a\left(\frac{k_{(1)}}{k'} - 1\right) + \tfrac{1}{2}b\left(\frac{k_{(2)}}{k'} - 1\right) = 0 \quad . \qquad (28)$$

Now the reactivity, $\rho'$, ($=\beta\rho$ of equation (15)) is

$$\rho' = \frac{k' - k_{(1)}}{k'} \quad , \qquad (29)$$

therefore $a = 1/(1-\rho')$ (30)

and $$b = -\rho'/\{(1-\rho')[1-d(1-\rho')]\} \quad . \qquad (31)$$

An overall, normalisation independent, tilt indicator is taken similar to equation (17), namely

$$T' = \frac{F'(R) - F'(L)}{F'(R) + F'(L)} \quad , \qquad (32)$$

giving $T' = db/a$ (33)

and finally

$$T' = -\rho' d/[1-d(1-\rho')] \qquad (34)$$

which is the required result.

The connection given by equation (34) differs slightly from that of Wade & Rydin [1972] because of the appearance of the additional factor $(1-\rho')$ in the denominator, which is normally quite close to one. Wade & Rydin demonstrate the applicability of equation (34) over a wide range of variation of $\rho'$ and d by a series of 1D calculations.

Using the Moata calculations of Section 7 as an example, we determine $k_{(2)} = 0.97863 = k$ of an isolated right hand half of the reactor, hence $d = 0.9668$ which compares well with the result required for Chebyshev extrapolation and estimated during the full symmetric reactor calculation (starting from an asymmetric trial flux corresponding to the left hand safety rod inserted), $d = 0.9664$. We then have T' (equation (34)) $= 0.213$ (0.176 without the additional $(1-\rho')$ factor), which is in very good agreement with the directly estimated quantity T' (equation (32)) $= 0.214$.

It is perhaps not surprising that the thermal flux local tilt indicator at position NR compared with NL, T (equation (17)) $= 0.182$ is only in approximate agreement with the overall tilt indicator of equation (32). Accepting the estimates of a and b made available through equations (30) and (31), $a = 0.9940$ and $b = 0.2189$, with the local flux estimate of $\phi'_g$ given by equation (20), we calculate for the local tilt $T = 0.184$. Again very good agreement is obtained, suggesting the adequacy of the approximation (20).

Equation (34) ultimately breaks down since for

$$\rho'_L = -\frac{(1-d)}{d} \quad \text{(corresponding to completely removing L)}, \qquad (35)$$

we have $T' \to \infty$ instead of the actual limit $T' \to 1$. This suggests that results from equation (34) should be ignored if T' exceeds 1, that is if

$$-\rho' > -\rho'_L/2 \quad . \qquad (36)$$

As an example close to the limit expressed by equation (36), we return to the asymmetrically disturbed, extremely loosely coupled 1D system of Section 4. We have $\rho' = -0.000707$, $d = 0.99844$, $\rho'_L/2 = -0.000781$, therefore $a = 0.99929$ and $b = 0.82709$, giving an estimate for the thermal peak flux tilt (mesh position 99 compared with 23) of $T = 0.895$ (0.453 without the $(1-\rho')$ factor), which is within reasonable agreement

of the directly estimated quantity T = 0.870.

## 9. CONCLUSIONS

The checkout of POW with results of other multidimensional kinetic codes for benchmark type calculations indicates that POW is correctly solving the steady state and kinetic equations. It is to be hoped that further benchmark calculations of the type undertaken here will be tackled by laboratories with fast computing equipment as confidence in the results of running such a code needs to rest on adequate code checkout.

## 10. ACKNOWLEDGEMENT

Thanks are expressed to Mr Robert Knott, Physics Division, AAEC, for setting up the Moata experiments used by the Summer School to yield results reported in Figure 9.

## 11. REFERENCES

Adams, C.H. & Stacey, W.M. Jr. [1967] - RAUMZEIT - a program to solve coupled time-dependent neutron diffusion equations in one space dimension. KAPL-M-6728.

Argonne National Laboratory [1968] - Argonne Code Center. Benchmark problem book. ANL-7416.

Argonne National Laboratory [1972] - Argonne Code Center. Benchmark problem book. ANL-7416, Supplement 1.

Barry, J.M., Clancy, B.E., Gilbert, C.P., McCulloch, D.B., Pollard, J.P. & Sanger, P.L. [1974] - A mathematician's computer study of the reactor Moata. AAEC Summer School Lecture Notes. AAEC/S15.

Cadwell, W.R., Henry, A.F. & Vigilotti, A.J. [1964] - WIGLE - a program for the solution of the two-group space-time diffusion equation in slab geometry. WAPD-TM-416.

ENEA/EURATOM [1972] - Space dependent reactor dynamics. Proc. of ENEA/EURATOM meeting 'Reactivity effects in large power reactors'. Ispra, Italy, 28-31 October, 1970. EUR-4731 f-e.

Fuller, E.L. [1972] - One dimensional space-time kinetics benchmark calculations. Argonne National Laboratory, Applied Physics Div., Annual Report, 1 July 1970 to 30 June 1971. ANL-7910, pp. 497-502.

Hageman, L.A. & Yasinsky, J.B. [1969] - Comparison of alternating-direction time-differencing methods with other implicit methods for the solution of the neutron group-diffusion equations. *Nucl. Sci. Eng.*, 38 : 8-32.

Hassitt, A. [1962] - A computer program to solve the multigroup diffusion equations. TRG-229(R).

Hopkins, D.R. & Oakes, D.B. [1968] - The two-dimensional, multigroup diffusion code, GOG. AEEW-R532.

Keepin, G.R. [1965] - Physics of Nuclear Kinetics. Addison-Wesley, Reading, Mass.

McCormick, W.T. [1969] - Numerical solution of the two-dimensional multigroup kinetics equations. Ph.D thesis. Mass., MIT-NE-99.

Meneley, D.A., Ott, K.O. & Wiener, E.S. [1971] - Fast reactor kinetics - the QX1 code. ANL-7769.

Pollard, J.P. [1973] - Numerical methods used in neutronics calculations. Ph.D thesis, UNSW.

Pollard, J.P. [1974] - AUS module POW - a general purpose 0, 1 and 2D multigroup neutron diffusion code including feedback-free kinetics. AAEC/E269.

Pollard, J.P. [1975] - Solution of neutron diffusion equations. In 'Computational Methods in Mathematical Physics' (edited by Anderssen, R.S. & Watts, R.O.). University of Queensland Press, Brisbane, pp.138-157.

Reed, W.H. & Hansen, K.F. [1970] - Alternating direction methods for the reactor kinetics equations. *Nucl. Sci. Eng.*, 41 : 431-442.

Robinson, G.S. [1975] - AUS - the Australian modular scheme for reactor neutronic computations. AAEC/E369

Stacey, W.M. Jr. [1969] - Space-Time Nuclear Reactor Kinetics. Academic Press, N.Y.

Tomlinson, L. [1972] - Delayed neutrons from fission. AERE-R6993.

Wade, D.C. & Rydin, R.A. [1972] - An experimentally measurable relationship between asymptotic flux tilts and eigenvalue separation. In 'Dynamics of Nuclear Systems' (edited by Hetrick, D.L.). University of Arizona, pp.335-351.

Wight, A.L., Hansen, K.F. & Ferguson, D.R. [1971] - Application of alternating-direction implicit methods to the space-dependent kinetics equations. *Nucl. Sci. Eng.*, 44 : 239-251.

Yasinsky, J.B., Natelson, M. & Hageman, L.A. [1968] - TWIGL - a program to solve the two-dimensional, two-group space-time neutron diffusion kinetics equations with temperature feedback. WAPD-TM-743.

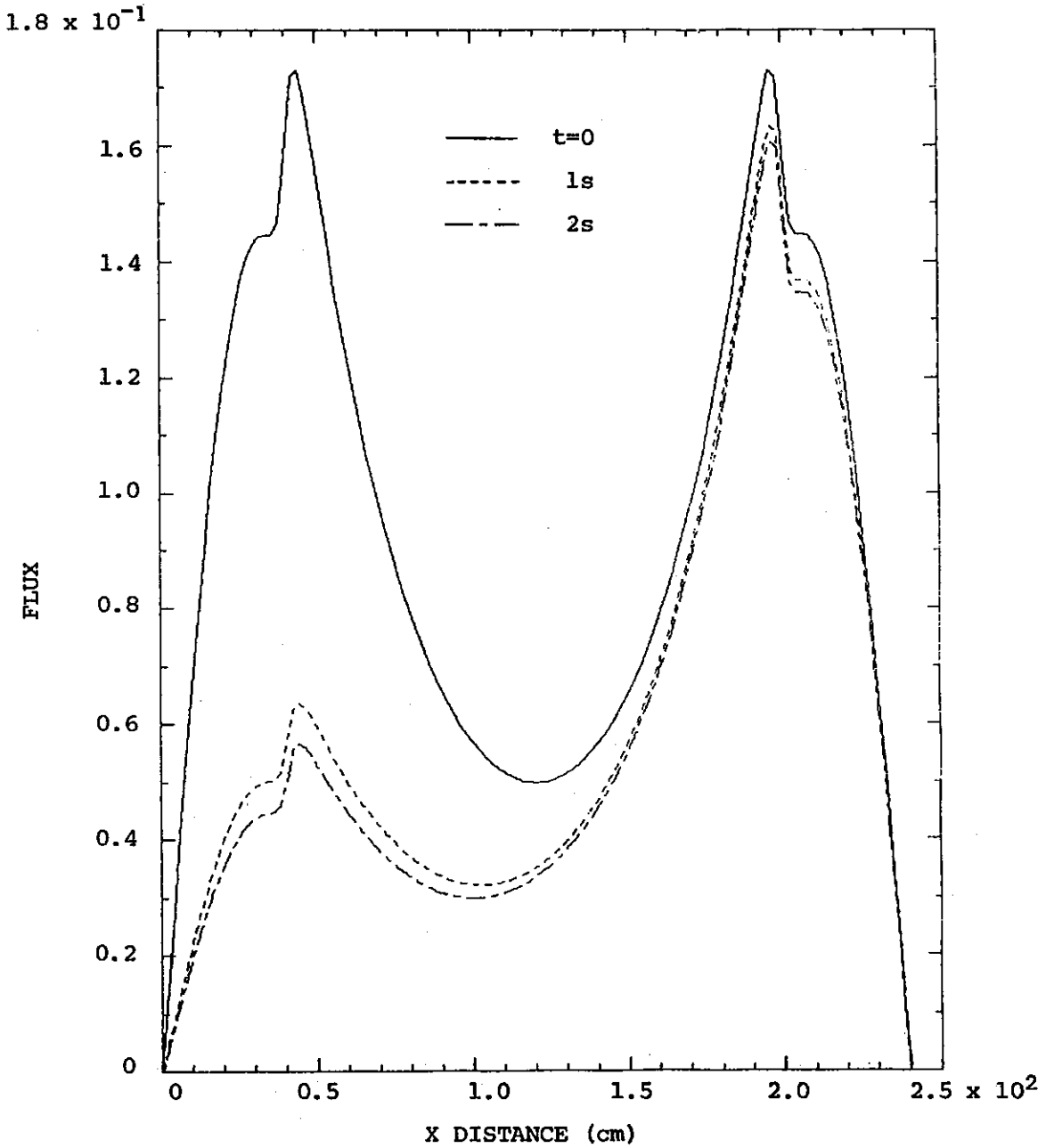



**FIGURE 1. 1D BENCHMARK FLUX PLOT**

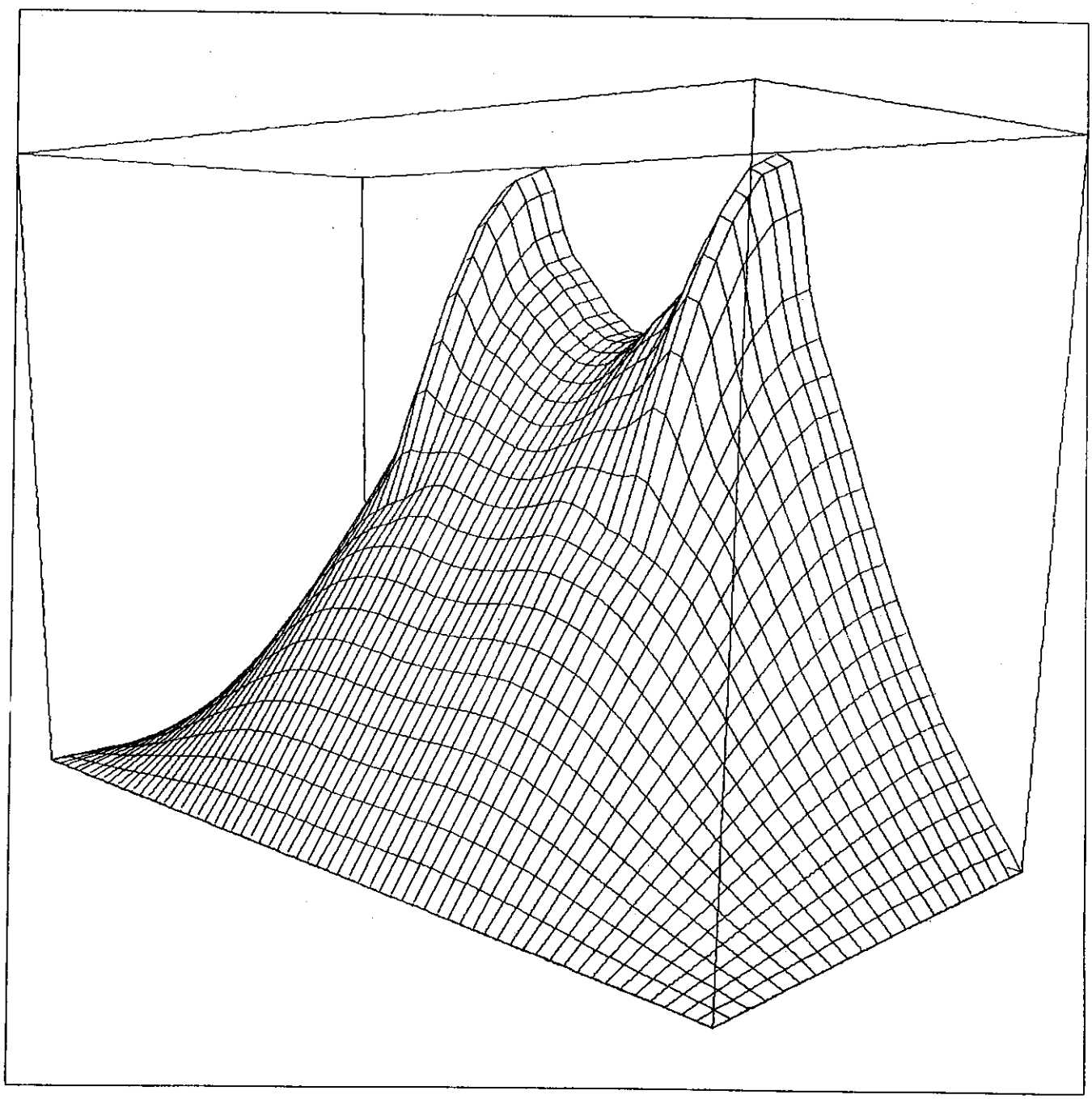


FIGURE 2. 2D MOATA FLUX PLOT, t = 0

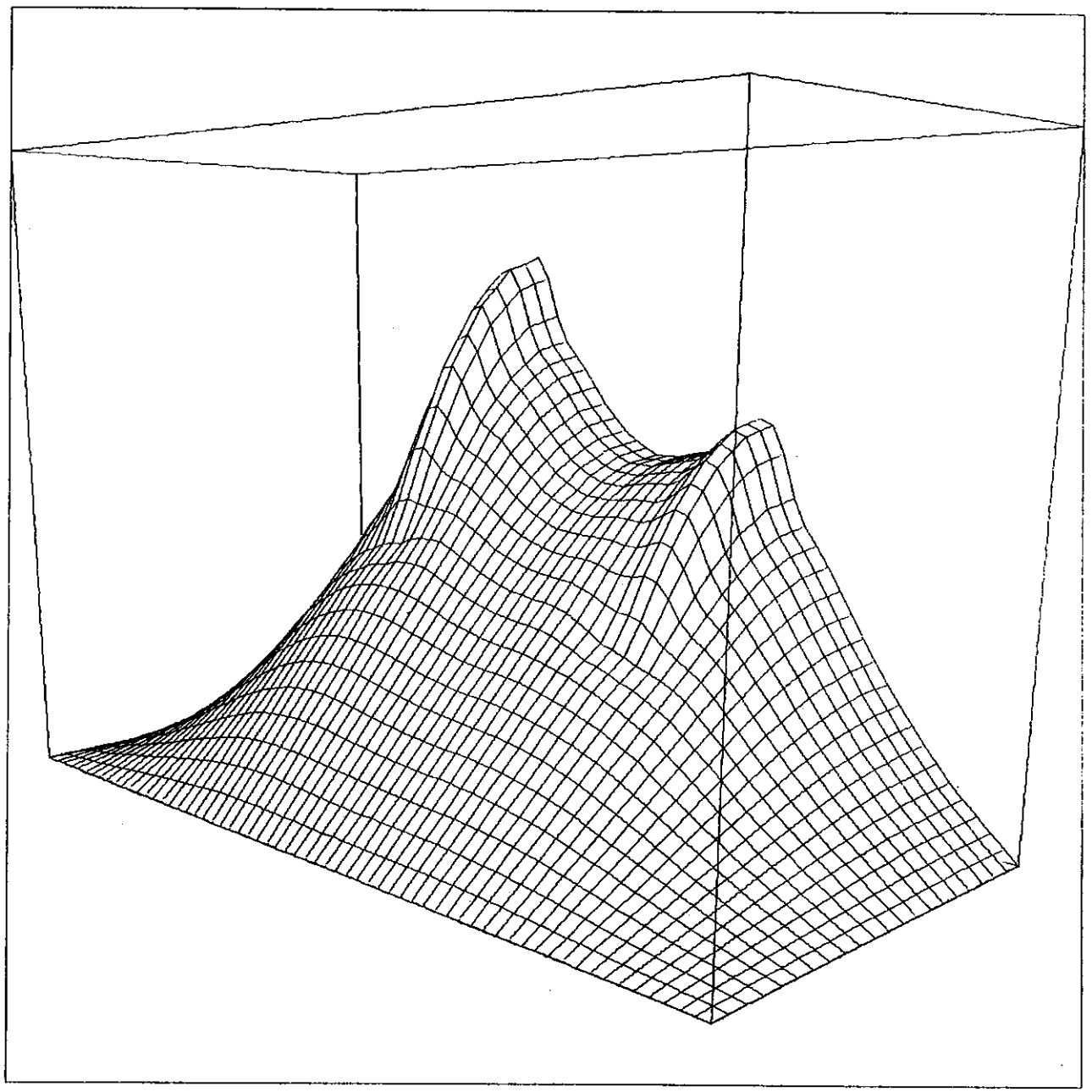


FIGURE 3. 2D MOATA FLUX PLOT, $t = 10^{-2}$ s

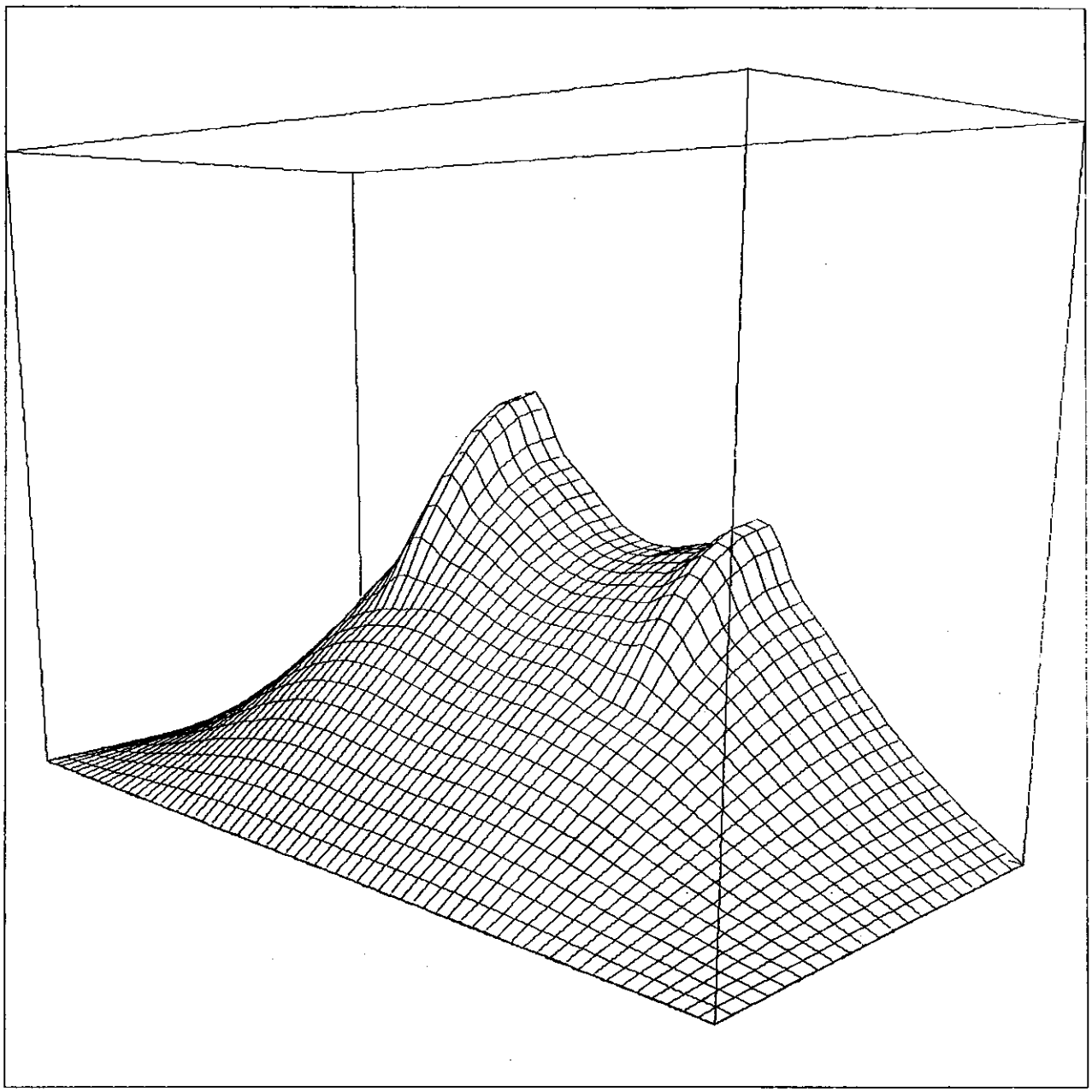


FIGURE 4. 2D MOATA FLUX PLOT, $t = 10^{-1}$ s

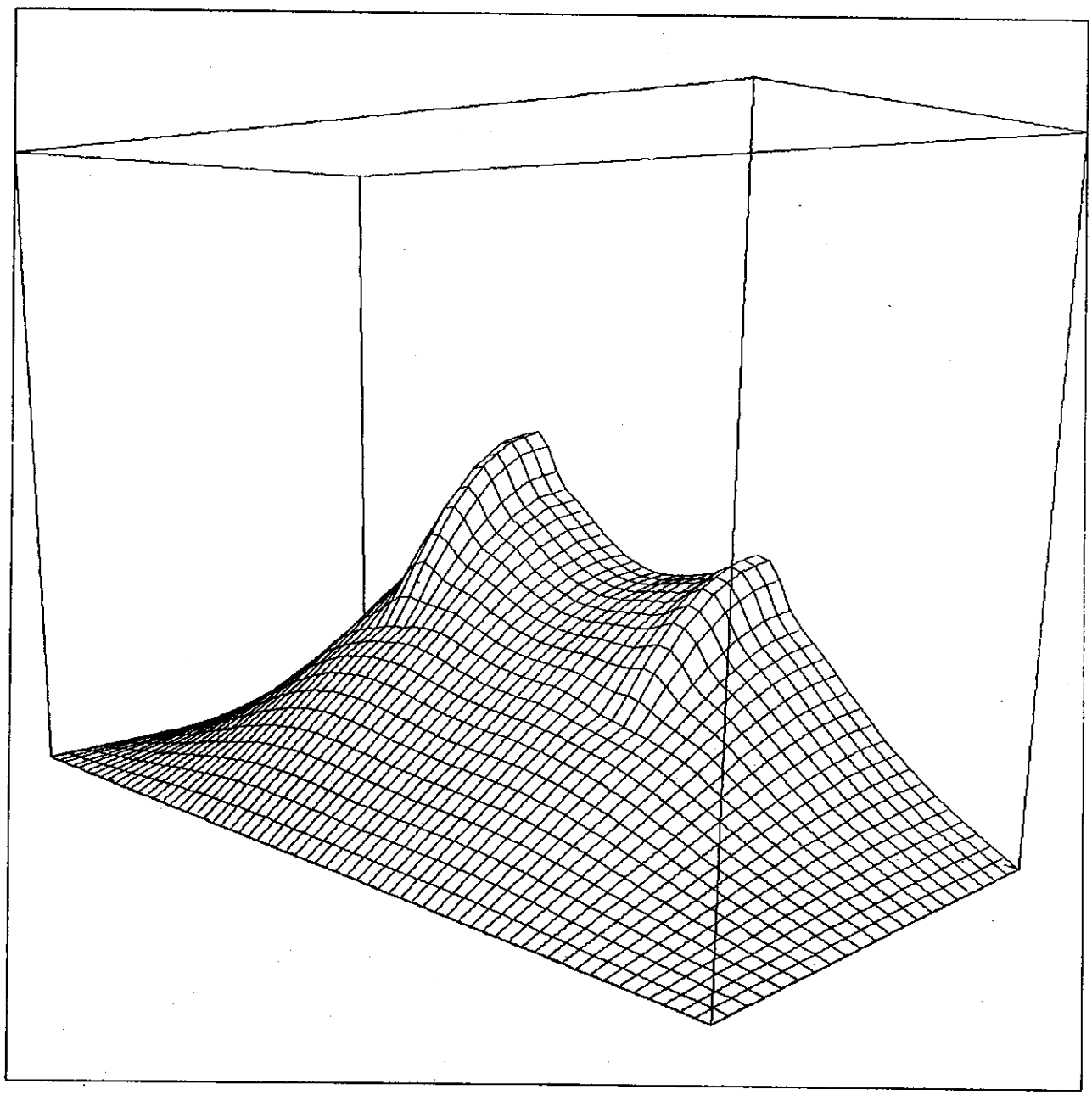


FIGURE 5. 2D MOATA FLUX PLOT, t = 1 s

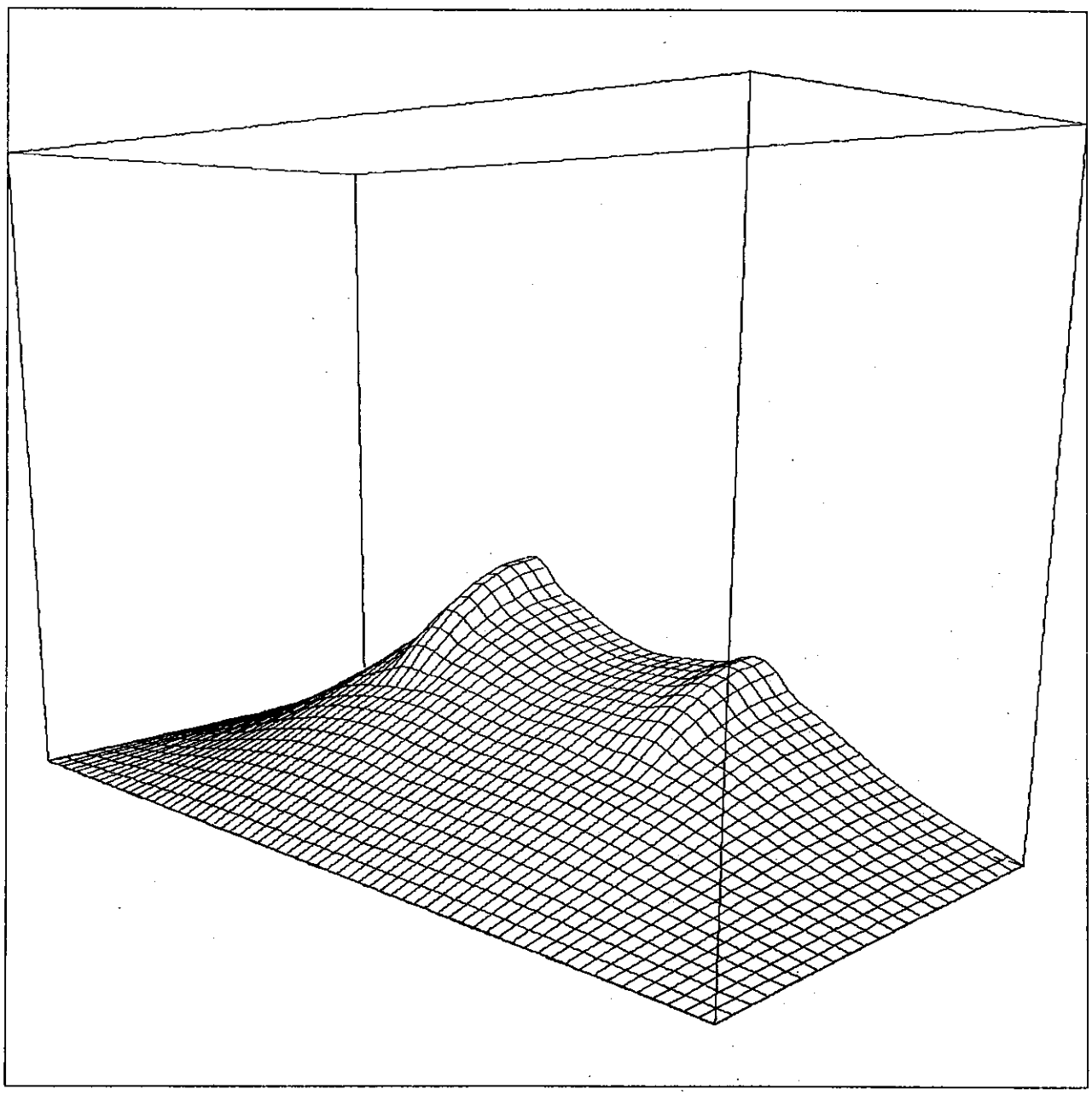


FIGURE 6. 2D MOATA FLUX PLOT, t = 10 s

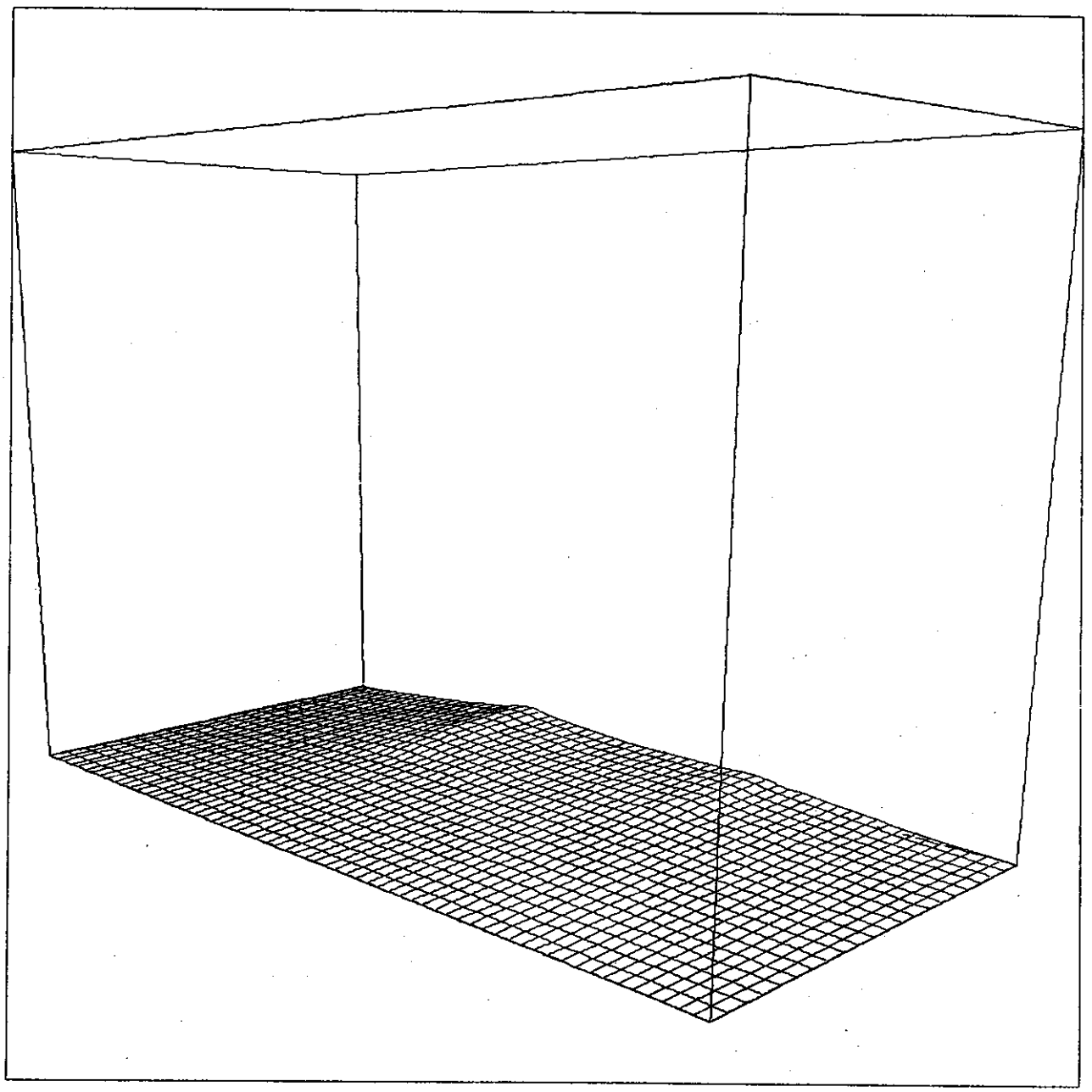


FIGURE 7. 2D MOATA FLUX PLOT, t = 100 s

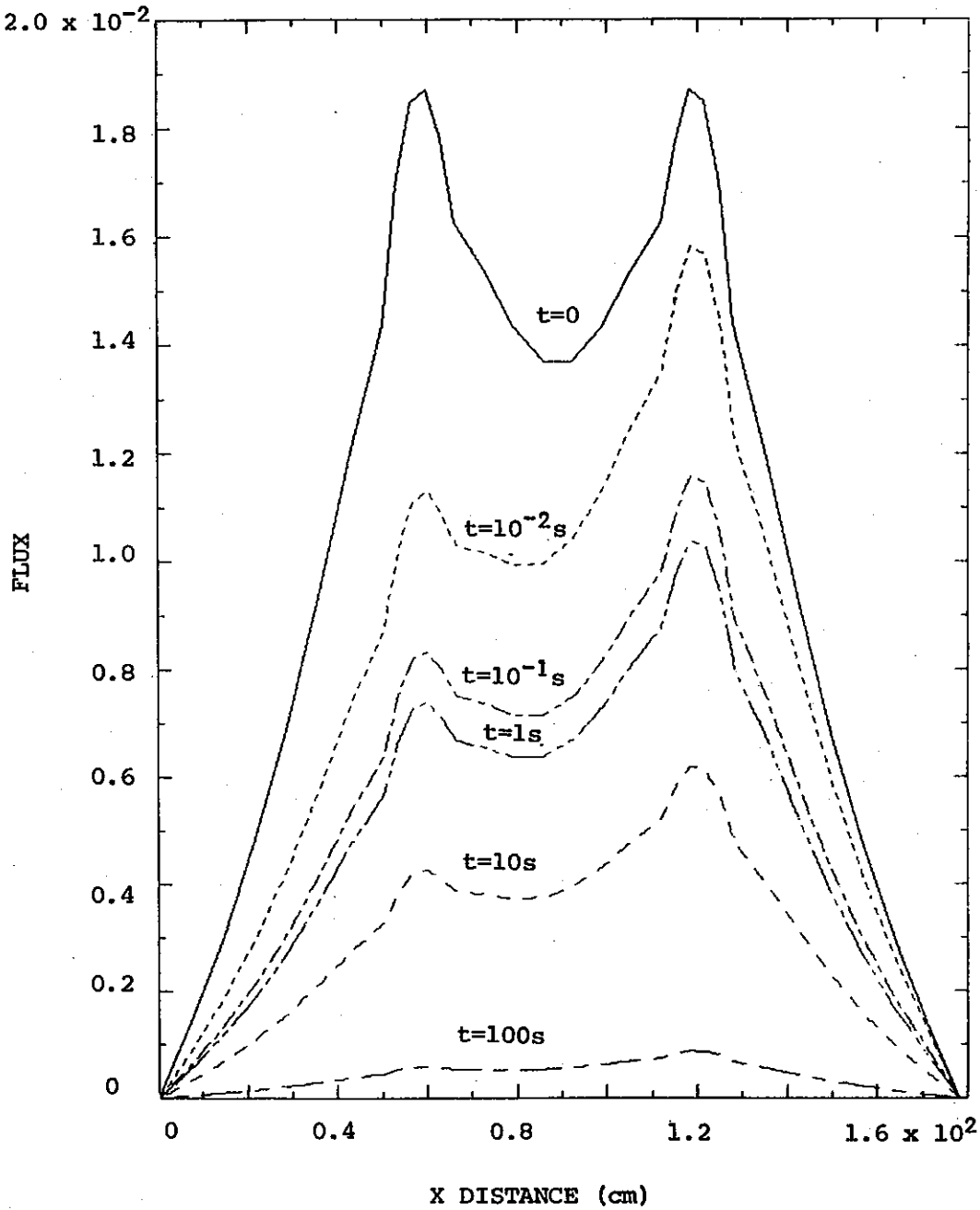



**FIGURE 8. MOATA FLUX ALONG X-AXIS**

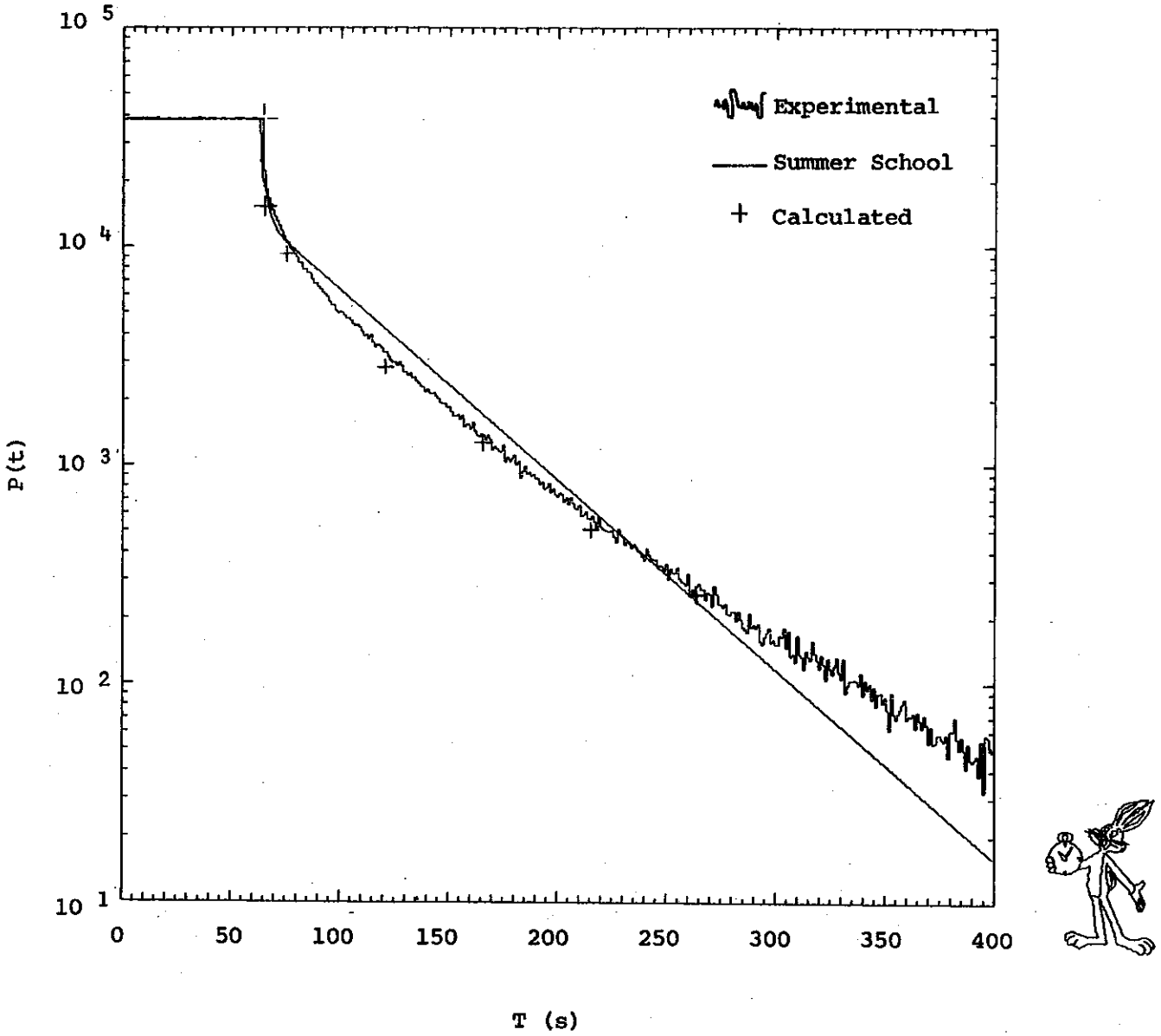



**FIGURE 9. MOATA FLUX AT POSITION N1 AGAINST TIME**

## APPENDIX A

## POW INPUT DATA LISTING FOR MOATA CALCULATION

```
//JPPMOAT2 JOB ('PH100930/P22PHNRA',N1),J.P.POLLARD,
//              CLASS=J,
//              TIME=180
//  EXEC  BUFFPROG,PRG=VPLOT
// EXEC AUS,FL1='AUS.JPPOWFLX',DISPFL1=OLD
//GO.SYSIN DD *
*DD1
STEP *
      LINK POW(1,2)
      END
STOP
*DD2
PRELUDE MAXX=32,MAXY=11,MAXG=2,MAXF=1,MAXGD=6,MAXM=5 END
POW MOATA2G  MOATA KINETICS CALC FOLLOWING ROD DROP IN M(1)
* 2GP XS DATA FOLLOWS
* COLLAPSED FROM 4GP DATA OF CONNOLLY (SEE POW REPORT,PG E3)
* REM INCLUDES LEAKAGE BASED ON EXPTL AXIAL BSQ OF 0.00184
NG=2
XSD FUELLT M(1)
2.436-1,1.558
3.9655-2,5.5971-2
2.130-3,8.074-2
0,3.532-2
6.302-4,0
DEFN FUELRT M(2)=M(1)
XSD C M(3)
2.790-1,3.646-1
5.6669-3,2.0810-3
2*0
0,3.461-3
1.651-4,0
* VEL ADJUSTED FROM FUEL CELL CALC TO ALLOW FOR C REFL
VEL 9.+6,2.3+5
SP 1,0
* 6GP EFFECTIVE DELAYED DATA FOLLOWS
* DERIVED FROM DATA OF KEEPIN (1965) AND TOMLINSON (1972)
IGD=6
SD
DLAMDA
1.24-2,3.05-2,1.11-1,3.01-1,1.14,3.01
BETAD
2.511-4,1.664-3,1.495-3,2.975-3,8.571-4,3.113-4
XM=1.-6,7*7.1428571,5*3.222,7*6.5314285,5*3.222,7*7.1428571,1.-6
YM=0,5*5.1912,5*5.968,1.-6
REG MX=1(1)31 MY=1(1)10 M(3)
REG MX=8(1)12 MY=1(1)5 M(1) MX=20(1)24 M(2)
OUTPUT DATA PRINT FLUX
RESTART
GROUPS 1 2,2
MREG -1=1,2,3
PLOT -3,135
PUREJY 0 3 3 10 10 0.7 0.8 3 1.-20  -1
EDIT -1,0,0,0
* ADJUST ABSORBER TO GIVE REQD REACTIVITY
* AKEFF=AKEFF*0.993934 IE -80 CENTS WITH BETA=7.5535-3
LIST AKEFF END
COMPUTE (0,1,0)=(0,1,0)(2,0,0.993934) 9
SEARCH(-0.01) #AKEFF 1.-4
DEFN M(5)=M(1) XSD MODIFY M(5)=3*1.-20,0.012,6*1.-20
WSEA 4*0,1 DEFN FUELLT WSEA M(1)=M(2) M(5)
```

```
OUTPUT DATA PRINT
START
* RESTORE SYMMETRIC T=Ø FLUX
SEARCH(Ø)
DEFN FUELLT M(1)=M(2)
RESTART
SEARCH(-Ø.Ø1)
WSEA 4*Ø,1 DEFN FUELLT WSEA M(1)=M(2) M(5)
CALC=REAL,KINETICS
OUTPUT DATA PRINT FLUX
IOUT Ø 5 1Ø
PULSE(21)=(Ø,1),(1ØØØ,1)
REACP -21,1,Ø,Ø
PULSE(22)=(Ø,1.-3),(Ø.99999-2,1.-3),(Ø.99999-2,9.-3),(Ø.99999-1,9.-3),
(Ø.99999-1,9.-2),(Ø.99999,9.-2),(Ø.99999,Ø.9),(9.9999,Ø.9),
(9.9999,9.),(99.999,9.),(99.999,1Ø.),(1ØØØ,1Ø.)
OTS -22,1,2ØØ
ACCFO=1.-3,ACCLAM=1.-3
RESTART
TLIM=Ø.9
NIL=Ø OUTPUT
AFTER 1 RESTART
PLOT -3 EDIT -1 Ø Ø Ø
AFTER 2 RESTART
PLOT -3 EDIT -1 Ø Ø Ø
AFTER 3 RESTART
PLOT -3 EDIT -1 Ø Ø Ø
AFTER  4 RESTART
PLOT -3 EDIT -1 Ø Ø Ø
AFTER 5  RESTART
PLOT -3 EDIT -1 Ø Ø Ø
* COLLECTION OF PLOTS OF THERMAL FLUX ALONG X-AXIS
XYPLT -6 -8
MSEA 2 1 1    Ø(1)5  CALL SUB6
STOP
/*
//
//
```

# APPENDIX B

## SOME USEFUL OUTPUT FROM MOATA CALCULATION

## LAYOUT INCLUDING DETECTOR POSITIONS

```
POW MOATA2G    MOATA KINETICS CALC FOLLOWING ROD DROP IN M(1)

LAYOUT OF XY REACTOR SEGMENT                          0.00
                                                      Y****
                EXTRAPOLATION DISTANCES 0.00 Y    * 0.00 IN TR.M.F.PATHS (0.0 IMPLIES ZERO CURRENT CONDITION)
                                                      *XXXX
                                                      0.0
                NOTE THAT 1=M(1),A=M(10),10=M(36) AND THAT THE FISSILE MATS ARE IN HEAVY TYPE
                                    1                 2                 3
     1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1

 10  3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
  9  3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
  8  3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
  7  3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
  6  3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
  5  3 3 3 3 3 3 3 1 1 1 1 1 3 3 3 3 3 3 3 2 2 2 2 2 3 3 3 3 3 3 3
  4  3 3 3 3 3 3 3 1 1 1 1 1 3 3 3 3 3 3 3 2 2 2 2 2 3 3 3 3 3 3 3
  3  3 3 3 3 3 3 3 1 1 1 1 1 3 3 3 3 3 3 3 2 2 2 2 2 3 3 3 3 3 3 3
  2  3 3 3 3 3 3 3 1 1 1 1 1 3 3 3 3 3 3 3 2 2 2 2 2 3 3 3 3 3 3 3
  1  3 3 3 3 3 3 3 1 1 1 1 1 3 3 3 3 3 3 3 2 2 2 2 2 3 3 3 3 3 3 3

SOLN METHOD TO BE USED FOR 2D DIFFUSION EQN
                        *****                                          *LX=  5 SEG BET LINES   1   8  13  20  25  32
SLOR WITH LINE= 2 , I.E. *   * ,GROUP AND REGION REBALANCE WITH *
                        LLLLL                                          *LY=  3 SEG BET LINES   1   3   6  11
```

POW MOATA2G  MOATA KINETICS CALC FOLLOWING ROD DROP IN M(1)

MESH POINT COORDINATES

*** DESIGNATES BOUNDARY CONDITION -TR.M.F.PATHS (0.0 IMPLIES ZERO CURRENT CONDITION)

| X MESH POINT | MESH WIDTH (CMS) | POSITION (CMS) | Y MESH POINT | MESH WIDTH (CMS) | POSITION (CMS) |
|---|---|---|---|---|---|
| *** | 1.00000E-06 | | *** | 0.0 | |
| 1 | 7.14286E+00 | 0.0 | 1 | 5.19120E+00 | 0.0 |
| 2 | 7.14286E+00 | 7.14286E+00 | 2 | 5.19120E+00 | 5.19120E+00 |
| 3 | 7.14286E+00 | 1.42857E+01 | 3 | 5.19120E+00 | 1.03824E+01 |
| 4 | 7.14286E+00 | 2.14286E+01 | 4 | 5.19120E+00 | 1.55736E+01 |
| 5 | 7.14286E+00 | 2.85714E+01 | 5 | 5.19120E+00 | 2.07648E+01 |
| 6 | 7.14286E+00 | 3.57143E+01 | 6 | 5.96800E+00 | 2.59560E+01 |
| 7 | 7.14286E+00 | 4.28571E+01 | 7 | 5.96800E+00 | 3.19240E+01 |
| 8 | 3.22200E+00 | 5.00000E+01 | 8 | 5.96800E+00 | 3.78920E+01 |
| 9 | 3.22200E+00 | 5.32220E+01 | 9 | 5.96800E+00 | 4.38600E+01 |
| 10 | 3.22200E+00 | 5.64440E+01 | 10 | 5.96800E+00 | 4.98280E+01 |
| 11 | 3.22200E+00 | 5.96660E+01 | 11 | 1.00000E-06 | 5.57960E+01 |
| 12 | 3.22200E+00 | 6.28880E+01 | *** | | |
| 13 | 6.53143E+00 | 6.61100E+01 | | | |
| 14 | 6.53143E+00 | 7.26414E+01 | | | |
| 15 | 6.53143E+00 | 7.91729E+01 | | | |
| 16 | 6.53143E+00 | 8.57043E+01 | | | |
| 17 | 6.53143E+00 | 9.22357E+01 | | | |
| 18 | 6.53143E+00 | 9.87671E+01 | | | |
| 19 | 6.53143E+00 | 1.05299E+02 | | | |
| 20 | 3.22200E+00 | 1.11830E+02 | | | |
| 21 | 3.22200E+00 | 1.15052E+02 | | | |
| 22 | 3.22200E+00 | 1.18274E+02 | | | |
| 23 | 3.22200E+00 | 1.21496E+02 | | | |
| 24 | 3.22200E+00 | 1.24718E+02 | | | |
| 25 | 7.14286E+00 | 1.27940E+02 | | | |
| 26 | 7.14286E+00 | 1.35083E+02 | | | |
| 27 | 7.14286E+00 | 1.42226E+02 | | | |
| 28 | 7.14286E+00 | 1.49369E+02 | | | |
| 29 | 7.14286E+00 | 1.56511E+02 | | | |
| 30 | 7.14286E+00 | 1.63654E+02 | | | |
| 31 | 7.14286E+00 | 1.70797E+02 | | | |
| 32 | 1.00000E-06 | 1.77940E+02 | | | |
| *** | | | | | |

```
OUTPUT DATA FOR PDW MOATA2G     MOATA KINETICS CALC FOLLOWING ROD DROP IN M(1)

XSC FUELLT    M(  1)
 2.43600E-01 1.55800E+00                                              SIGTR
 3.96550E-02 5.59710E-02                                              SIGREM
 2.13000E-03 8.07400E-02                                              NUSIGF
 0.0         3.53200E-02                                              OUTS  1
 6.30200E-04 0.0                                                      OUTS  2
DCX(  1)
 1.36836E+00 2.13950E-01
DCY(  1)
 1.36836E+00 2.13950E-01

XSC FUELRT    M(  2)
 2.43600E-01 1.55800E+00                                              SIGTR
 3.96550E-02 5.59710E-02                                              SIGREM
 2.13000E-03 8.07400E-02                                              NUSIGF
 0.0         3.53200E-02                                              OUTS  1
 6.30200E-04 0.0                                                      OUTS  2
DCX(  2)
 1.36836E+00 2.13950E-01
DCY(  2)
 1.36836E+00 2.13950E-01

XSC C         M(  3)
 2.79000E-01 3.64600E-01                                              SIGTR
 5.66690E-03 2.08100E-03                                              SIGREM
 0.0         0.0                                                      NUSIGF
 0.0         3.46100E-03                                              OUTS  1
 1.65100E-04 0.0                                                      OUTS  2
DCX(  3)
 1.19474E+00 9.14244E-01
DCY(  3)
 1.19474E+00 9.14244E-01

FER
 3.17200E-11 3.17200E-11 0.0          0.0          0.0
SP
 1.00000E+00 0.0

********************************* ****************************************
```

OUTPUT PRINT FOR PUw MOATA2G  MOATA KINETICS CALC FOLLOWING ROD DROP IN M(1)  CONVERGED

EDGE REAL  FLX (MULTIPLIED BY 1.E+08) FROM SOURCE OF IN/SEC FOLLOWS

GRCUP 1

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 7327 | 15957 | 27202 | 42246 | 61628 | 84135 | 105311 | 111907 | 116132 | 117687 | 116590 | 113190 | 101109 | 89004 |
| 9 | 0 | 14683 | 32576 | 56026 | 88156 | 130910 | 182716 | 233918 | 250018 | 260175 | 263463 | 259880 | 250391 | 218409 | 188038 |
| 8 | 0 | 22790 | 50266 | 87611 | 140648 | 214831 | 311175 | 415346 | 449065 | 469991 | 475531 | 465509 | 442311 | 369425 | 306249 |
| 7 | 0 | 30976 | 68939 | 122073 | 200785 | 317818 | 484595 | 692789 | 765300 | 809337 | 818602 | 792228 | 735081 | 571960 | 449483 |
| 6 | 0 | 39120 | 87846 | 158013 | 266282 | 437403 | 708069 | 1129627 | 1305084 | 1401172 | 1418620 | 1354677 | 1201162 | 833434 | 614509 |
| 5 | 0 | 45727 | 103367 | 188055 | 322318 | 542583 | 911097 | 1543146 | 1822745 | 1975538 | 1999515 | 1891502 | 1642631 | 1070500 | 758900 |
| 4 | 0 | 51465 | 116936 | 214558 | 372146 | 636174 | 1088097 | 1876105 | 2230271 | 2425976 | 2454309 | 2311477 | 1993427 | 1275578 | 886786 |
| 3 | 0 | 55914 | 127498 | 235264 | 411102 | 708854 | 1222891 | 2121390 | 2527492 | 2752770 | 2784030 | 2617018 | 2250614 | 1430990 | 985799 |
| 2 | 0 | 58733 | 134199 | 248415 | 435806 | 754650 | 1306773 | 2271781 | 2706854 | 2951592 | 2984562 | 2803248 | 2408009 | 1527462 | 1046086 |
| 1 | 0 | 59699 | 136495 | 252922 | 444260 | 770266 | 1335209 | 2322467 | 2769857 | 3018386 | 3051921 | 2865862 | 2461020 | 1560133 | 1069311 |

| | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 81822 | 81824 | 89010 | 101116 | 113202 | 116604 | 117701 | 116146 | 111921 | 105325 | 84146 | 61636 | 42252 | 27206 | 15959 |
| 9 | 170580 | 170584 | 188050 | 218429 | 250420 | 259911 | 263496 | 260208 | 250050 | 233948 | 182740 | 130928 | 88168 | 56034 | 32580 |
| 8 | 271792 | 271799 | 306270 | 369462 | 442362 | 465566 | 475590 | 470051 | 449123 | 415401 | 311217 | 214860 | 140667 | 87623 | 50273 |
| 7 | 387453 | 387464 | 449516 | 572019 | 735158 | 792325 | 818706 | 809442 | 765401 | 692882 | 484660 | 317851 | 200813 | 122090 | 68948 |
| 6 | 513166 | 513181 | 614557 | 833523 | 1201307 | 1354847 | 1418803 | 1401358 | 1305260 | 1129781 | 708167 | 437463 | 266319 | 158035 | 87858 |
| 5 | 620542 | 620561 | 758961 | 1070617 | 1642833 | 1891745 | 1999781 | 1975806 | 1822996 | 1543360 | 911224 | 542658 | 322363 | 188081 | 103381 |
| 4 | 715876 | 715898 | 886859 | 1275720 | 1993676 | 2311779 | 2454640 | 2426311 | 2230582 | 1876368 | 1088249 | 636263 | 372198 | 214588 | 116952 |
| 3 | 790314 | 790340 | 985881 | 1431151 | 2250897 | 2617362 | 2784408 | 2753153 | 2527848 | 2121689 | 1223063 | 708954 | 411160 | 235297 | 127516 |
| 2 | 837479 | 837506 | 1048174 | 1527635 | 2408314 | 2803619 | 2984970 | 2952004 | 2709236 | 2272103 | 1306957 | 754757 | 435868 | 248450 | 134218 |
| 1 | 853612 | 853640 | 1069401 | 1560310 | 2461332 | 2866242 | 3052338 | 3018808 | 2770248 | 2322796 | 1335398 | 770375 | 444323 | 252957 | 136515 |

| | 31 | 32 |
|---|---|---|
| 11 | 0 | 0 |
| 10 | 7328 | 0 |
| 9 | 14685 | 0 |
| 8 | 22793 | 0 |
| 7 | 30980 | 0 |
| 6 | 39125 | 0 |
| 5 | 45733 | 0 |
| 4 | 51472 | 0 |
| 3 | 55922 | 0 |
| 2 | 58742 | 0 |
| 1 | 59707 | 0 |

GROUP 2

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 20491 | 42021 | 65331 | 90527 | 116703 | 141743 | 162725 | 170063 | 175959 | 180368 | 183331 | 184969 | 184810 | 182384 |
| 9 | 0 | 40931 | 84053 | 130975 | 182014 | 235349 | 286465 | 328980 | 343772 | 355624 | 364452 | 370349 | 373561 | 372852 | 367234 |
| 8 | 0 | 61152 | 125853 | 196810 | 274766 | 356975 | 435778 | 499720 | 521729 | 539412 | 552615 | 561567 | 566737 | 565811 | 555706 |
| 7 | 0 | 80780 | 166712 | 261902 | 367861 | 480952 | 588838 | 669545 | 696278 | 718930 | 736050 | 748514 | 758978 | 762482 | 746550 |
| 6 | 0 | 99185 | 205328 | 324250 | 458726 | 604701 | 743271 | 820873 | 838082 | 866591 | 884752 | 896189 | 933436 | 960075 | 935631 |
| 5 | 0 | 113493 | 235568 | 373703 | 532281 | 708453 | 883577 | 1006175 | 1128877 | 1212250 | 1230030 | 1188272 | 1147835 | 1138546 | 1092819 |
| 4 | 0 | 125564 | 261231 | 416097 | 596311 | 800861 | 1012879 | 1183024 | 1371244 | 1485695 | 1506050 | 1439261 | 1345098 | 1300662 | 1231585 |
| 3 | 0 | 134751 | 280847 | 448737 | 646099 | 873548 | 1115424 | 1321073 | 1550260 | 1684772 | 1707114 | 1624701 | 1497349 | 1427995 | 1340002 |
| 2 | 0 | 140507 | 293171 | 469335 | 677688 | 919882 | 1180767 | 1407699 | 1659972 | 1806036 | 1829562 | 1738243 | 1592498 | 1508709 | 1408827 |
| 1 | 0 | 142468 | 297376 | 476376 | 688511 | 935782 | 1203157 | 1437149 | 1696932 | 1846791 | 1870710 | 1776478 | 1624802 | 1536305 | 1432400 |

| | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 180340 | 180343 | 182392 | 184823 | 184985 | 183349 | 180387 | 175978 | 170083 | 162745 | 141761 | 116718 | 90539 | 65340 | 42026 |
| 9 | 362559 | 362565 | 367251 | 372889 | 373596 | 370387 | 364492 | 355654 | 343813 | 329020 | 286501 | 235380 | 182038 | 130993 | 84064 |
| 8 | 547264 | 547273 | 555733 | 565853 | 566791 | 561625 | 552677 | 539475 | 521792 | 499782 | 435835 | 357023 | 274803 | 196837 | 125870 |
| 7 | 732632 | 732645 | 746587 | 762540 | 759052 | 748594 | 736134 | 719017 | 696365 | 669631 | 588917 | 481017 | 367911 | 261938 | 166735 |
| 6 | 913524 | 913540 | 935679 | 960151 | 933531 | 896291 | 884859 | 866701 | 838192 | 820982 | 743371 | 604783 | 458789 | 324294 | 205356 |
| 5 | 1059808 | 1059828 | 1092877 | 1138639 | 1147954 | 1188419 | 1230191 | 1212414 | 1129031 | 1006313 | 883699 | 708550 | 532354 | 373755 | 235601 |
| 4 | 1186929 | 1186951 | 1231652 | 1300771 | 1345242 | 1439442 | 1506251 | 1485899 | 1371434 | 1183189 | 1013020 | 800972 | 596393 | 416155 | 261267 |
| 3 | 1285595 | 1285624 | 1340077 | 1428117 | 1497512 | 1624908 | 1707344 | 1685005 | 1550477 | 1321258 | 1115580 | 873669 | 646189 | 448799 | 280886 |
| 2 | 1348118 | 1348144 | 1408906 | 1508839 | 1592672 | 1738466 | 1829810 | 1806287 | 1660205 | 1407897 | 1180932 | 920010 | 677782 | 469400 | 293212 |
| 1 | 1369524 | 1369551 | 1432480 | 1536438 | 1624980 | 1776707 | 1870963 | 1847048 | 1697171 | 1437352 | 1203326 | 935912 | 688607 | 476442 | 297417 |

| | 31 | 32 |
|---|---|---|
| 11 | 0 | 0 |
| 10 | 20494 | 0 |
| 9 | 40536 | 0 |
| 8 | 61160 | 0 |
| 7 | 80791 | 0 |
| 6 | 99198 | 0 |
| 5 | 113508 | 0 |
| 4 | 125581 | 0 |
| 3 | 134770 | 0 |
| 2 | 140526 | 0 |
| 1 | 142488 | 0 |

## APPENDIX C

## 2D FAST REACTOR BENCHMARK WITH FEEDBACK

Since the main study reported here, a further series of benchmark studies has come to the author's attention [Buffoni, *et al.* 1975]. Benchmark problem 1 of this series, with the descriptive title, 'Super-prompt critical transient with feedback, 2 group neutron diffusion problem in fast reactor', was undertaken with POW. The fast reactor benchmark is well documented, and affords POW the opportunity to tackle a simple single temperature feedback model. In addition, four laboratories have submitted four different sets of solutions, and so a further contribution is warranted.

Rather than specify all of the data used by POW, we will be content to indicate only those data which are not detailed as part of the benchmark problem, namely:

(1) r-mesh spacing in cm (5* means 5 of the numbers that follow)
5*3.4, 7*3., 10*3.5374, 10*4.,

(2) z-mesh spacing in cm
10*4., 10*3.85, 4*3.25, 10*3.85, 10*4.,

(3) number of spatial solution points is 1485, and

(4) temporal step length is $\delta t=2.5 \times 10^{-4}$ s for $t<20 \times 10^{-3}$ s, otherwise $\delta t=5 \times 10^{-3}$ s.

Results of the POW calculation are shown in Tables C1 and C2 and Figures C1 to C3. These results are considered acceptable compared with those of the other contributors, namely

UK-RIS, Fletcher using the code SPARK,

CRNL, McDonnell using the code ADEP,

CSN, Buffoni using the code NADYP-A, and

GfK, Väth using the code KINTIC-1.

POW gave a steady state multiplication of k=1.000093, with flux almost the same as the plotted GfK results and used 130 minutes of CPU time (now on an IBM360/65) to calculate the reactor behaviour to $t=100 \times 10^{-3}$ s.

REFERENCE

Buffoni, A., Fletcher, J.K., Galati, A., McDonnell, F.N., Musco, A. & Väth, L. [1975] - Review of kinetics benchmark calculations. In 'Proceedings of the Joint NEACRP/CSNI specialists' meeting on new developments in three-dimensional neutron kinetics and

review of kinetics benchmark calculations'. Technische University ät München, MRR 145, pp.505-551.

TABLE C1

CASE 1. 2-GROUP SPECTRA (%) AT z = 88.25 cm FOR START AND END OF TRANSIENT IN CENTRE OF REACTOR AND AT POSITION OF MOVABLE ABSORBER

| t (s) | Calculated by | r = 0 | | r = 20 cm | |
|---|---|---|---|---|---|
| | | Group 1 | Group 2 | Group 1 | Group 2 |
| 0 | UK-RIS | 86.0 | 14.0 | 86.1 | 13.9 |
| | CRNL | 86.0 | 14.0 | 86.0 | 14.0 |
| | CSN | 85.8 | 14.2 | 85.8 | 14.2 |
| | GfK | 86.1 | 13.9 | 86.0 | 14.0 |
| | AAEC | 86.0 | 14.0 | 85.9 | 14.1 |
| $100 \times 10^{-3}$ | UK-RIS | 86.2 | 13.8 | 84.5 | 15.5 |
| | CRNL | - | - | - | - |
| | CSN | 86.5 | 13.5 | 85.1 | 14.9 |
| | GfK | 86.2 | 13.8 | 84.5 | 15.5 |
| | AAEC | 86.2 | 13.8 | 84.5 | 15.5 |

TABLE C2

CASE 1. TIME AND GROUP DEPENDENT FLUX SHAPE INDICATOR, $\Phi_G$ (r = 20, z = 88.25, t)/$\Phi_G$(r = 38, z = 88.25, t)

| t (s) | Calculated by | Group 1 | Group 2 |
|---|---|---|---|
| 0 | UK-RIS | 0.87 | 0.85 |
| | CRNL | 0.90 | 0.90 |
| | CSN | 0.88 | 0.93 |
| | GfK | 0.88 | 0.89 |
| | AAEC | 0.87 | 0.90 |
| $100 \times 10^{-3}$ | UK-RIS | 0.96 | 1.12 |
| | CRNL | - | - |
| | CSN | 0.92 | 1.05 |
| | GfK | 0.93 | 1.12 |
| | AAEC | 0.93 | 1.11 |

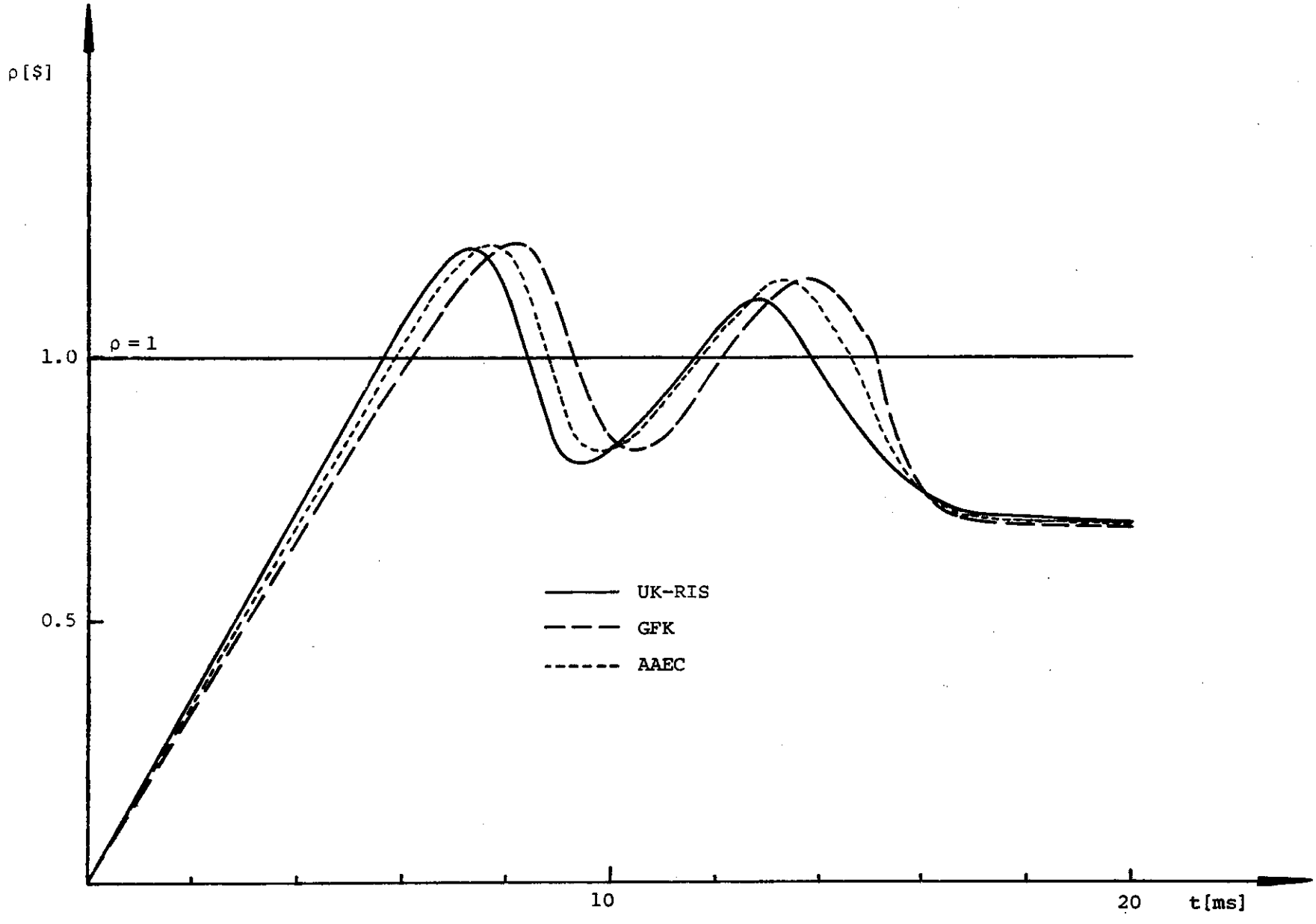



FIGURE C1. 2D FAST REACTOR BENCHMARK 1, ρ(t)

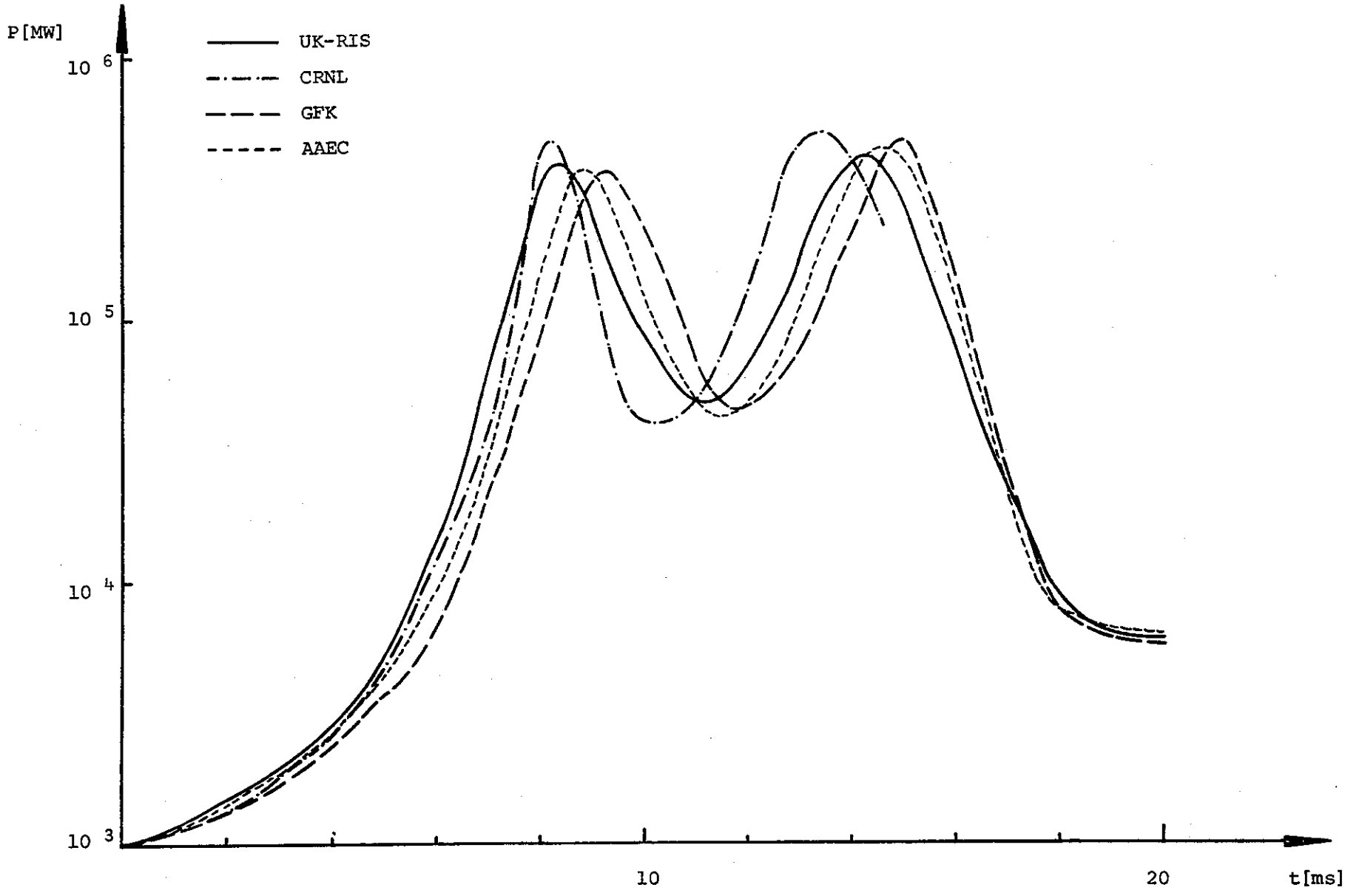



FIGURE C2. 2D FAST REACTOR BENCHMARK 1, POWER(t)

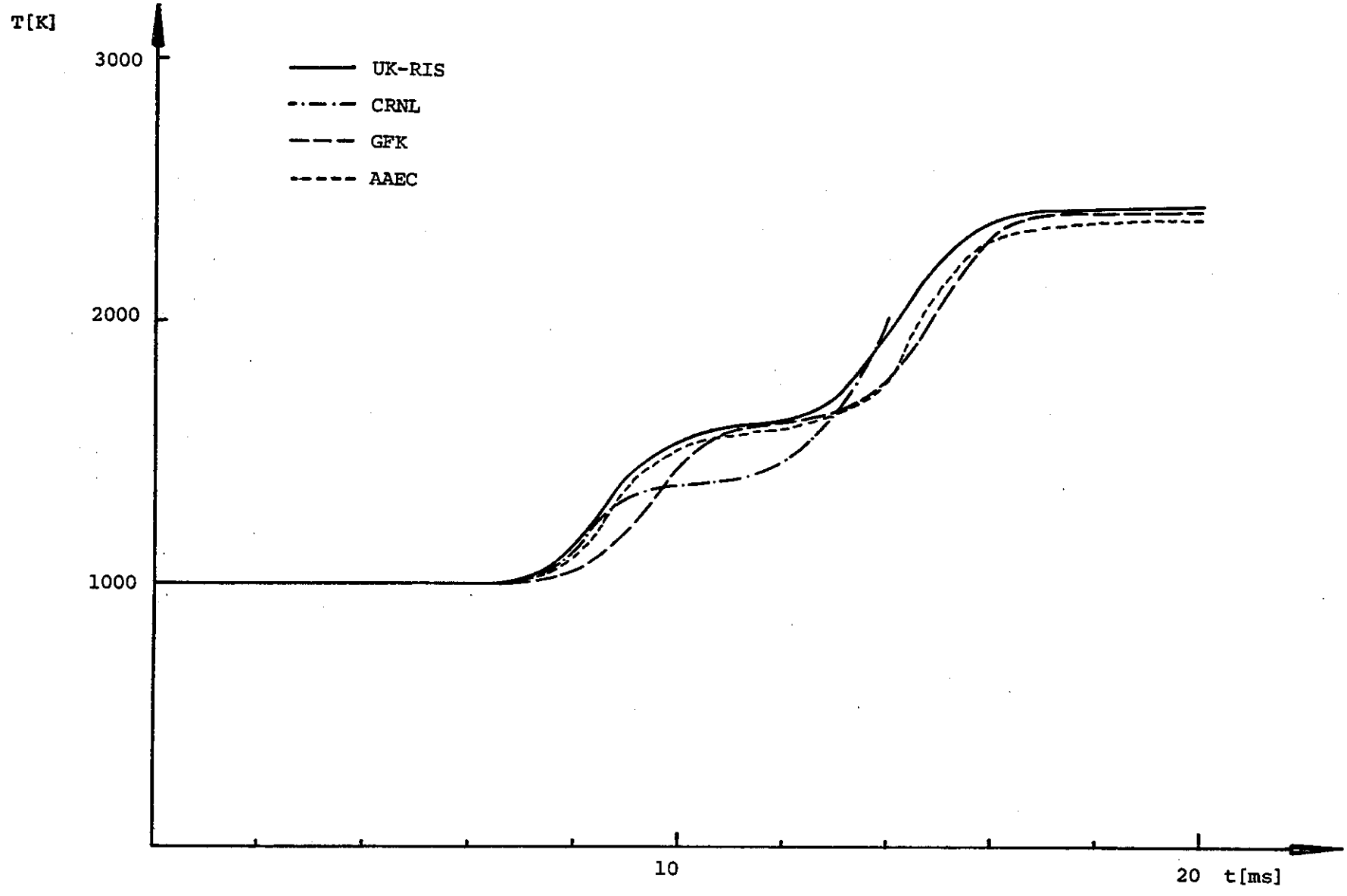



FIGURE C3. 2D FAST REACTOR BENCHMARK 1, TEMP.(t)