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ENERGY AND LETHARGY DISTRIBUTION OF NEUTRONS SLOWING DOWN IN GRAPHITE

by

K. C. HINES

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Abstract

For neutrons slowing down in an infinite homogeneous moderator consisting of a single element the energy dependent flux of neutrons satisfies a simple integral equation when the sources of fast fission neutrons are uniformly distributed in space. This equation is readily soluble for neutrons slowing down in hydrogen and in heavier moderators for the energy range $\alpha E_o \leqslant E \leqslant E_o$ (where αE_o is the lowest energy that a neutron of initial energy E_o can have after a single collision in the moderator). For $E < \alpha E_o$, however, an analytical solution of the integral equation is not possible for moderators heavier than hydrogen.

In the present work it is shown how a solution for lower energies may be obtained using a step–by–step procedure based on the simple solution of the problem for energies close to the initial energy.

The method lends itself to programming for a digital computer and, for graphite, numerical results have been obtained using the I.B.M. 650 Data Processing Machine.

The results presented here take no account of absorption in the graphite during slowing down or of inelastic scattering by nuclei of the moderator and it is assumed that, for all energies, scattering is spherically symmetric in the centre of mass system.

CONTENTS

1. SYMBOLS

E = neutron energy

$\Sigma_s(E)$ = macroscopic scattering cross section at energy E.

$\phi(E)$ = neutron flux at energy E.

$S(E)$ = fission spectrum = $0.45270 \exp(-E/0.965)\sinh\sqrt{2.29E}$
(see Argonne National Laboratory – Reactor Physics Constants: ANL–5800).
In this formula E is in Mev and the function is so normalized that $\int_0^\infty S(E)dE = 1$.

Q = total number of neutrons (of all energy) produced in the moderator by fission per cubic centimetre per second (the number of neutrons produced by fission / cm^3 / sec of energy between E and E + dE will be $QS(E)\,dE$).

E_o = maximum energy of neutrons considered in the calculation; it is assumed that $S(E_o) \approx 0$.

A = mass number of the moderator nuclei.

α = $(A-1)^2 / (A+1)^2$; the minimum possible energy of a neutron after a single collision with a moderator nucleus before which the neutron energy was E_o is αE_o.

$F(E)$ = collision density = $\phi(E)\,\Sigma_s(E)/Q$

$q(E)$ = slowing down density at energy E = $Q \int_E^\infty S(E')\,dE'$

E_n = $\alpha^n E_o$

u = lethargy = $\ln(E_o/E)$

ξ = average increase of lethargy per elastic collision;

$$= 1 + \frac{\alpha}{1-\alpha} \ln\alpha$$

2. INTRODUCTION

It is well known that, in an infinite homogeneous hydrogen moderator, a monoenergetic source of fast neutrons with a uniform spatial distribution will give rise to a slowing down spectrum of neutrons following a 1/E law. If the monoenergetic source of neutrons is replaced by, say a source of fission neutrons displaying a wide spread in neutron energies, the slowing down spectrum, although again readily calculable, is no longer of the form 1/E. At lower energies, however, the slowing down spectrum approximates more and more closely to a 1/E distribution and becomes indistinguishable from it at energies such that almost all the source neutrons are emitted at higher energies.

For moderators heavier than hydrogen the situation is more complicated. Whereas in colliding with a proton a neutron may lose virtually all its energy in a single collision, the maximum amount of energy which can be lost by a neutron of initial energy E_o in colliding with a nucleus of a moderator of mass number A is $E_o (1 - \alpha)$. In the energy range $\alpha E_o \leqslant E \leqslant E_o$, the equation expressing the conservation of neutrons can be solved as for hydrogen moderator. For energies $< \alpha E_o$ this is no longer possible and in this report a method of solving the problem by a step–by–step method is presented.

The solution must be such that it gives a collision density at an energy $E = \alpha E_o$ in agreement with that from the previously mentioned simple analytical procedure. Further it must give the correct form for the slowing down density at any energy and go over into the simple asymptotic (1/E) form at low energy.

Placzek [1] has given a solution to the problem in the case of monoenergetic source neutrons but, in the present case, with source neutrons of variable energy this method is inapplicable. It should be noted that the sharp discontinuities in the collision density at $E_n = \alpha^n E_o$ $(n = 1, 2, \ldots)$ which occur when the source is monoenergetic, are not present with a source distribution which is a continuous function of energy.

The method of solution which will be elaborated in the following sections is based on the fact that the collision density can be easily determined in the range $\alpha E_o \leqslant E \leqslant E_o$ and may therefore be taken as a known quantity in calculating the collision density in the next lowest energy range $(\alpha^2 E_o = \alpha E_1 \leqslant E \leqslant E_1)$. The collision density is now a known function over the first two ranges and may be used in the calculation of the collision density in the next lowest energy range and so on.

The expression which is obtained as a result of the analytical work would be extremely arduous to evaluate on a hand calculating machine. It lends itself, however, to evaluation on a digital computer, and a programme has been prepared which enables the calculations to be done on the I.B.M. 650.

One must be careful about applying the results of such a calculation to any given moderating material in practice. In the balance equation for conservation of neutrons, absorption during slowing down has been neglected. While the absorption cross section of most moderators is small, the effects of absorption on the slowing down spectrum may not be entirely negligible.

Further it has been assumed that the scattering is spherically symmetric in the centre of mass coordinate system and this assumption, together with the neglect of inelastic scattering, could have a considerable effect on the results obtained. It is true that the effects of both inelastic scattering and p–wave elastic scattering are negligible in most moderators at energies below 1 Mev. Also, around 10 Mev where the effects are becoming more marked, the yield of fission neutrons is very low. Nevertheless, the value of the collision density at any given energy depends directly on the values which it had at higher energies and, in consequence, a considerable error in the (very small) collision density near the top of the energy range considered could be felt cumulatively at lower energies. This would introduce considerable errors into the results for the collision density at all energies.

The method described here is only applicable in its present form to moderating materials consisting of pure elements. Above all, it is clear that the present calculations will be inaccurate if they are used to determine the slowing down spectrum in beryllium or beryllium oxide. For these materials neutrons of energy above about 2 Mev are capable of inducing the beryllium (n, 2n) reaction. Among the products of this reaction are further neutrons exhibiting an energy distribution different from that of the fission neutrons considered as the primary source.

Apart from interest in the slowing down spectrum itself, the main object of calculations to determine the distribution in energy of neutrons slowing down in a moderating material is to enable the satisfactory evaluation of group constants used in reactor criticality calculations. In two-group calculations, for example, such quantities as the transfer cross sections and the fast diffusion coefficients represent averages over the epithermal spectrum. Usually it is assumed that this spectrum is strictly of the 1/E form, but for more precise work the accurate form of the slowing down spectrum must be used to determine these average quantities.

3. SLOWING DOWN IN HYDROGEN MODERATOR

In this section we consider the simple case of slowing down in an infinite homogeneous hydrogen moderator with a source of fission neutrons uniformly distributed in space. Under these conditions the neutron flux and collision density are independent of position.

Consider the balance of neutrons in an infinitesimal energy region dE around the energy E.

The number of neutrons lost by scattering in the region per cm^3 per sec will be

$$\phi(E)\,\Sigma_s(E)\,dE.$$

The number of neutrons arising from fission in the region per cm^3 per sec is

$$QS(E)\,dE,$$

while the number of neutrons per cm^3 per sec which enter the region as a result of scattering from higher energies is given by

$$dE \int_E^{E_o} \phi(E')\ \Sigma_s(E')\ \frac{dE'}{E'} . \qquad (1)$$

This follows since the number of scattering events in dE' about E' ($E' > E$) is

$$\phi(E')\ \Sigma_s(E')\,dE' ,$$

while the probability that a neutron scattered in dE' about E' will have an energy after scattering in dE about E is

$$dE/E' . \qquad (2)$$

Clearly equation (1) follows by integration over all values of E' from E to E_o, the maximum energy of source neutrons considered in the calculations.

Since at equilibrium as many neutrons must enter dE as leave it, the balance equation is found to be (after cancellation of a factor dE):

$$\phi(E)\,\Sigma_s(E) = QS(E) + \int_E^{E_o} \phi(E')\ \Sigma_s(E')\ \frac{dE'}{E'} \qquad (3)$$

or, more succinctly,

$$F(E) = S(E) + \int_E^{E_o} F(E')\ dE'/E' . \qquad (3a)$$

Equation (3a) can be reduced to a differential equation by differentiating with respect to E:

$$F'(E) = S'(E) - (F(E)/E).$$

This equation is linear with the integrating factor E and the solution is

$$F(E) = -(1/E) \int_E^{E_o} E'S'(E')\,dE' + C/E$$

which, after integrating the first term on the R.H.S. by parts and noting that at $E = E_o$, $S(E) = F(E) \approx 0$, gives

$$F(E) = S(E) + \frac{1}{E} \int_E^{E_o} S(E')\,dE' . \qquad (4)$$

That the solution (4) satisfies the original equation (3a) can be checked immediately by substitution.

It can also be seen that if the energy E is taken low enough so that $S(E') = 0$ for all $E' \leq E$ then

$$F(E) = \frac{1}{E} \int_o^{\infty} S(E')\,dE' . \qquad (4a)$$

If now one writes

$$Q = Q \int_o^{\infty} S(E')\,dE'$$

equation (4a) can be written in the equivalent form:

$$\phi(E) = Q/(E\,\Sigma_s) \qquad (4b)$$

which is the well known expression for the flux per unit energy in a hydrogen moderator2.

Consider now the slowing down density at energy E. By analogy with expression (2) the fraction of collisions in hydrogen taking place in dE' about E' which scatter neutrons past the energy E is E/E'. But the number of collisions in dE' is $Q\,F(E')\,dE'$; thus the number of neutrons per cm^3 per sec slowed down past E after scattering in dE' is

$$Q\,F(E')\,(E/E')dE' .$$

It is clear that the slowing down density q(E) is given by

$$q(E) = QE \int_E^{E_o} F(E') \frac{dE'}{E'} . \qquad (5)$$

The expression (4) for F(E) is now substituted into (5) and, after a partial integration the result is

$$q(E) = Q \int_E^{E_o} S(E')\,dE' \qquad (6)$$

which is correct if absorption in the moderator has been neglected (all neutrons emitted from the source at energies above E must sooner or later slow down past E in the absence of absorption.)

4. SLOWING DOWN IN A SINGLE ELEMENT HEAVIER THAN HYDROGEN IN THE ENERGY RANGE $\alpha E_o \leq E \leq E_o$

The neutron balance equation in this case is set up in the same way as in the preceding section with the one difference that equation (2) for the probability that a neutron scattered in dE' about E' will have an energy in dE about E after scattering is replaced by

$$\frac{dE}{E'(1-\alpha)} \quad . \tag{7}$$

The balance equation (3a) is then replaced by

$$F(E) = S(E) + \frac{1}{(1-\alpha)} \int_E^{E_o} F(E') \frac{dE'}{E'} \quad . \tag{8}$$

The solution of this equation can be carried out in the same way as for equation (3a). After differentiating with respect to E the integrating factor for the resulting linear differential equation is $E^{1/(1-\alpha)}$ and the solution becomes

$$F(E) = -(1/E^{\frac{1}{1-\alpha}}) \int_E^{E_o} S'(E)\ E'^{\frac{1}{1-\alpha}}\, dE' + C/E^{\frac{1}{1-\alpha}}$$

Integrating the first term on the R.H.S. by parts and making use of the initial conditions the final result is

$$F(E) = S(E) + \frac{1}{(1-\alpha)E^{\frac{1}{1-\alpha}}} \int_E^{E_o} S(E')\, E'^{\frac{\alpha}{1-\alpha}} dE' \quad . \tag{9}$$

Equation (9) can be readily evaluated numerically but it is of little use, as the energy range to which it is applicable is too high to be of any practical interest in neutron slowing down problems. Let us consider beryllium as an example and assume that most of the source fission neutrons (actually $\sim 99\%$) are included if we choose E_o as 7 Mev. With $\alpha \approx 0.64$ for beryllium, it is clear that the expression (9) is only valid in the energy range from 7 Mev down to 4.48 Mev, i.e. well above the energies considered in determining average fast group constants for reactor calculations. It will be shown in the next section, however, that the solution (9) is important, since it forms the basis for the step–by–step solution which can be extended down to as low an energy as required.

That the expression (9) is the correct solution of equation (8) can be checked as before by evaluating the slowing down density in the range $\alpha E_o \leq E \leq E_o$. The contribution to the slowing down density at E from neutrons scattered in dE' is

$$QF(E') \frac{E - \alpha E'}{E'(1-\alpha)} dE' \quad ,$$

and we can then write instead of (5):

$$q(E) = \frac{Q}{(1-\alpha)} \int_E^{E_o} \frac{E - \alpha E'}{E'} F(E')\, dE' \quad . \tag{10}$$

Expression (9) for F (E) is substituted into (10) and one obtains, after some analysis

$$q(E) = Q\left(\frac{1}{1-\alpha} - \frac{\alpha}{1-\alpha}\right)\int_E^{E_o} S(E')\,dE'$$

$$= Q\int_E^{E_o} S(E')\,dE',$$

which is correct (see equation (6)). This establishes that the expression (9) is the correct solution to equation (8).

5. SLOWING DOWN IN A SINGLE ELEMENT HEAVIER THAN HYDROGEN IN THE ENERGY RANGE $E < \alpha E_o$

(i) Analytical Solution by a Step-by-step Method

The integral equation in this case is obtained in the same way as in section 4. It is only necessary to note that no neutrons of energy $E' > E/\alpha$ can enter the range E about dE as a result of scattering in dE', and so equation (8) is replaced by

$$F(E) = S(E) + \frac{1}{1-\alpha}\int_E^{E/\alpha} F(E')\,\frac{dE'}{E'} . \qquad (11)$$

Since the variable E occurs in both upper and lower limits of the integral, this equation cannot be solved in the same simple way as equation (8).

If we call the solutions of equations (8) and (11), $F_1(E)$ and $F_2(E)$ respectively, then it is clear by inspection of the equations that

$$F_1(\alpha E_o) = F_2(\alpha E_o) \qquad (12)$$

since, for the distributed fission source, there should be no discontinuity at αE_o as there would be for a monoenergetic neutron source of energy E_o.

The first point to note is that, subsequent to the solution of equation (8), the function F(E) is known in the range $E_1 \leqslant E \leqslant E_o$ and therefore, in the next lowest energy range $\alpha E_1 \leqslant E \leqslant E_1$ (i.e. $E/\alpha > E_1$), equation (11) may be written in the form:

$$F(E) = S(E) + \frac{1}{1-\alpha}\int_E^{E_1} F(E')\,\frac{dE'}{E'} + \frac{1}{1-\alpha}\int_{E_1}^{E/\alpha} F(E')\,\frac{dE'}{E'} , \qquad (13)$$

where now the integrand of the last term is a known function of E. Using equation (13) F(E) can now be found in the second energy range (over which (13) is valid) and the function F(E) will then be known from the initial energy E_o down to an energy $E_2 = \alpha E_1 = \alpha^2 E_o$. The process can be extended to the next lowest energy range by replacing E_1 in equation (13) by E_2; the resulting equation is now valid for $E_3 \leqslant E \leqslant E_2$. In this way the solution, in principle, may be extended down to as low energies as required.

It is now necessary to see how this procedure can be applied in detail to the series of equations of the type (13). Differentiate (13) with respect to E:

$$F'(E) = S'(E) - \frac{1}{1-\alpha}\frac{F(E)}{E} + \frac{1}{1-\alpha}\frac{F(E/\alpha)}{E} .$$

The integrating factor is again $E^{\frac{1}{1-\alpha}}$ and, using the relation

$$\alpha^{\frac{1}{1-\alpha}} \int_{E_1}^{E/\alpha} E'^{\frac{1}{1-\alpha}} F(E') \frac{dE'}{E'} = \int_{E_1}^{E} \frac{F(E'/\alpha)}{E'} E'^{\frac{1}{1-\alpha}} dE' ,$$

one finds,

$$F(E) = -\frac{1}{E^{\frac{1}{1-\alpha}}} \int_{E}^{E_1} E'^{\frac{1}{1-\alpha}} S'(E')dE' + \frac{\alpha^{\frac{1}{1-\alpha}}}{(1-\alpha) E^{\frac{1}{1-\alpha}}} \int_{E_1}^{E/\alpha} E'^{\frac{\alpha}{1-\alpha}} F(E')dE' +$$

$$+ \frac{C}{E^{\frac{1}{1-\alpha}}}$$

The first term on the R.H.S. can be integrated by parts and the integration constant C is determined by the requirement that, at $E = E_1$, the function F(E) shall satisfy equation (12). The result is

$$F(E) = S(E) + \frac{1}{(1-\alpha)E^{\frac{1}{1-\alpha}}} \int_{E}^{E_o} E'^{\frac{\alpha}{1-\alpha}} S(E')dE'$$

$$- \frac{\alpha^{\frac{1}{1-\alpha}}}{(1-\alpha)E^{\frac{\alpha}{1-\alpha}}} \int_{E/\alpha}^{E_o} E'^{\frac{\alpha}{1-\alpha}} F(E')dE' . \qquad (14)$$

Equation (14) represents the solution to the problem in the energy range $E_2 \leq E \leq E_1$. Although it is true that the last term involves the unknown function F(E), this is not a difficulty since, as E ranges from E_2 to E_1 the value of E/α ranges from E_1 to E_o. In other words, for E within the range specified, the region of integration in the last term is exclusively within the next highest energy range, throughout which F(E) is a known function (equation (9)).

The fact that the separation energy E_1 between the top two energy ranges occurs nowhere in equation (14) suggests that this form for the solution may have general validity. To prove that this is indeed the case consider the next lowest energy range, $E_3 \leq E \leq E_2$. The relation (13) is still true if E_1 is replaced by E_2 and the procedure is identical to that outlined for the previous range with E_1 everywhere replaced by E_2. In the determination of the constant of integration C it is no longer possible to use (12). The boundary condition in the present case is that F(E) in the range under consideration shall, for $E = E_2$, take on the value $F(E_2)$ determined by equation (14). When this is done it is found that the expression for F(E) in the range under consideration is identical to (14). Thus (14) represents the complete solution to the slowing down problem we are considering, down to any required energy.

From what has been said above, the solution can only be carried out in successive stages, starting from the highest energy range and working down in energy range by range.

It is worth noting that (9) is a special case of (14). For the first energy range $E_1 \leqslant E \leqslant E_o$, E/α is always greater than E_o; since $F(E) = S(E) \approx 0$ for $E > E_o$, the last term on the R.H.S. of (14) is zero for the whole of the first range and (14) reduces to (9). Thus equation (14) is quite general and represents the solution of the problem for all energies.

As a final check on the validity of (14) we wish to show that it gives the correct form (6) for the slowing down density for any energy within the second range ($E_2 \leqslant E \leqslant E_1$). For an energy in this range only neutrons of energy below E/α contribute directly to the slowing down density. Thus instead of (10) we have

$$q(E) = \frac{Q}{(1-\alpha)} \int_E^{E/\alpha} \frac{E-\alpha E'}{E'} F(E')\, dE' . \qquad (15)$$

As before, the integral in (15) can be split into two parts and (15) resolves itself into two terms. The first involves an integral from E to E_1 over which range the appropriate form of F(E) is taken from equation (14); in the second, containing an integral from E_1 to E/α, F(E) has the form (9). If the appropriate forms for F(E) are substituted into the two terms of q(E), a rather complex expression results. By a series of partial integrations it is possible to show that this expression reduces to the correct form (6) for the slowing down density q(E). It is obvious that (14) will satisfy this requirement also for the successively lower energy ranges.

Continuity of the function F(E) and its derivatives at the energies E_1, E_2 . . . may be established by examining the limits of F(E), F'(E) . . . as E approaches the dividing energy in question from above and below.

It is interesting to investigate the asymptotic solution to equation (11). If $E \ll E_o$ the first term on the R.H.S. of (11) disappears and it can be readily checked by substitution that the resulting equation is satisfied by

$$F(E) = C/E . \qquad (16)$$

The determination of the constant C must yield expression (6) for the slowing down density. Substitution for F(E) from (16) into relation (15) for the slowing down density gives

$$q(E) = QC\xi .$$

Thus, since for any energy,

$$q(E) = Q \text{ (no absorption)},$$

$$C = 1/\xi$$

and finally

$$F(E) = 1/(\xi E) . \qquad (17)$$

(ii) 'For Transit' Programme for Evaluating the Solution in (i)

The labour involved in evaluating the solution (14) numerically using a hand machine would be prohibitive, but the calculation can be readily adapted for a digital computer. The programme for evaluating (14) using the I.B.M. 650 Data Processing Machine is given below:

(No. stands for Statement Number).

No.	Statement
	READ, N, $E\bar{O}$, ALPHA
	DIMENSION FINT (35)
	BETA = 1 – ALPHA
	GAMMA = 1./BETA
	DELTA = ALPHA * GAMMA
	EPSI = ALPHA * * GAMMA
	DE = $E\bar{O}$ * BETA / N
	GINT = O
	I = 1
	FINT(I) = O
	K = 1
	E = $E\bar{O}$
	SE = 0.45270 * EXPF(–E/O.965) * SINHF (SQRTF (2.29* E))
	CE = E ** DELTA
	BE = GAMMA/E * * GAMMA
	FE = SE
2	FU = E* FE
	SU = E* SE
5	PUNCH, E, SE, FE, SU, FU, GINT, FINT (I)
	SE1 = SE
	CE1 = CE
	FE1 = FE
	IF(E–DE/10 – ALPHA * * K* $E\bar{O}$) 9,9,4
9	K = K + 1
	DE = ALPHA * DE
4	E = E – DE
	IF(I – N – 1) 10,3,10

No.	Statement
3	FINT(1) = FINT(N + 1)
	I = I – N
10	I = I + 1
	SE = 0.45270*EXPF(–E/0.965)*SINHF(SQRTF(2.29*E))
	CE = E**DELTA
	BE = GAMMA/E** GAMMA
	TERM = (SE1* CE1 + SE * CE) * DE/2
	GINT = GINT + TERM
	FE = SE + BE * GINT
	IF(E + DE/10 – ALPHA * $E\bar{O}$)7,6,6
7	FE = FE – BE* EPSI * FINT(I)
6	PERM = (FE1 * CE1 + FE* CE) * DE/2
	FINT(I) = FINT(I – 1) + PERM
	IF(E – 0.02)8,8,2
8	END

NOTES ON THE PROGRAMME

(a) For details of the statements reference should be made to the I.B.M. For Transit Manual.

(b) In statement number 5, GINT and FINT(I) are not required as results of the calculation. For detecting faults in the programme, however, it was useful to have values of these quantities available.

(c) The term DE/10, which occurs in the IF statement immediately preceding statement number 9, is inserted to prevent the accumulation of rounding errors from making the expression in brackets positive when it is expected to be negative. The same device is used in the IF statement preceding statement number 7.

(d) The statement immediately preceding statement number 4 is inserted so that, for the purposes of the numerical integration, the number of intervals in all energy ranges is the same. This means that a particular energy E in one range is identical to the energy E'/α, for E' in the range below, provided both E and E' are the same (integral) number of intervals from the top of their respective ranges. Thus the $F(E')$ which is required for evaluating the integrand in the last term of equation (14) has already been calculated at the correct energy when determining F(E) for the previous range. Values of the integral in the last term of (14) are stored in the machine as FINT(I) and used to evaluate F(E) N steps later in the programme.

(e) The number N represents the (constant) number of energy values in each range at which the integrands of the two integrals in equation (14) are evaluated for the purposes of numerical integration. N is arbitrary, the only condition being that the subdivisions shall be small enough to yield accurate values of the integrals. In the calculations for graphite we have made two runs with N = 34 and with N = 10. The results for F(E) in the two cases are in good agreement showing that the accuracy obtained with N = 10 is quite adequate. In fact there is evidence that this choice is preferable to a larger value of N. With the larger choice of N, rounding errors are introduced for small E which are not present if N = 10.

(iii) Results for Graphite

The programme outlined in the previous section has been applied first to the calculation of the collision density in graphite. Essentially no fission neutrons are produced with energies above 12 Mev and accordingly we take $E_o = 12$.

As mentioned in Section 5 (ii) (e) a good choice for N, the number of intervals of integration per energy range, is 10.

The isotopic composition of carbon is:

C^{12} 98.89%; C^{13} 1.11%.

and, using the definition of α in Section 1, we find:

$$\alpha = 0.71618410.$$

The values of E_o, N and α constitute the data which must be fed into the machine with the programme.

The results for graphite are presented in Fig. 1. For convenience we have plotted the collision density per unit lethargy as a function of both energy and lethargy where

$$F(u) = E\,F(E)\,. \qquad (18)$$

From equation (17) it can be seen that the function F(u) will have the asymptotic value (for large u)

$$F(u) = 1/\xi\,,$$

and this is shown as a straight line parallel to the E(u) axis in Fig. 1.

(iv) Discussion of Results for Graphite

It can be seen from Fig. 1 that, for E = 0.2 Mev, F(u) is already within a few percent of its asymptotic value. This means that the calculation of the collision density from, say E = 0.1 Mev, to E = 0.02 Mev is unnecessary. For future calculations, therefore, the last statement but one in the programme of section 5 (ii) should be modified to read:

IF (E − 0.1) 8, 8, 2.

At an energy E = 0.5 Mev the function F(E) deviates from the asymptotic value by about 10% and for higher energies this deviation increases rapidly. It is clear that any attempt to estimate the effect of threshold reactions, such as inelastic scattering, on the slowing down spectrum cannot be based on the asymptotic 1/E law for the neutron energy dependent flux, but must be based on the rigorous calculation of F(E). This follows since the threshold energies for all such reactions in light elements are expected to lie in the region above 1 Mev, i.e. the region for which the asymptotic form for the collision density cannot be used.

For the evaluation of group constants the results given in this report for F(E) will be much superior to the usual $1/E$ distribution. However, it must be emphasised again that the validity of the present work cannot be accurately assessed until the effects of absorption, p–wave elastic scattering and inelastic scattering have been investigated.

6. ACKNOWLEDGMENT

Both the writing of the programme and the elimination of errors at the I.B.M. 650 were the results of joint effort. The author is grateful to M.E. Duncan and J.L. Symonds who shared with him equally in this part of the work.

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7. REFERENCES

1. G. Placzek, Phys. Rev., 69, 423 (1946)

2. Glasstone & Edlund – "Elements of Nuclear Reactor Theory". (D. Van Nostrand Co., Inc. 1952).

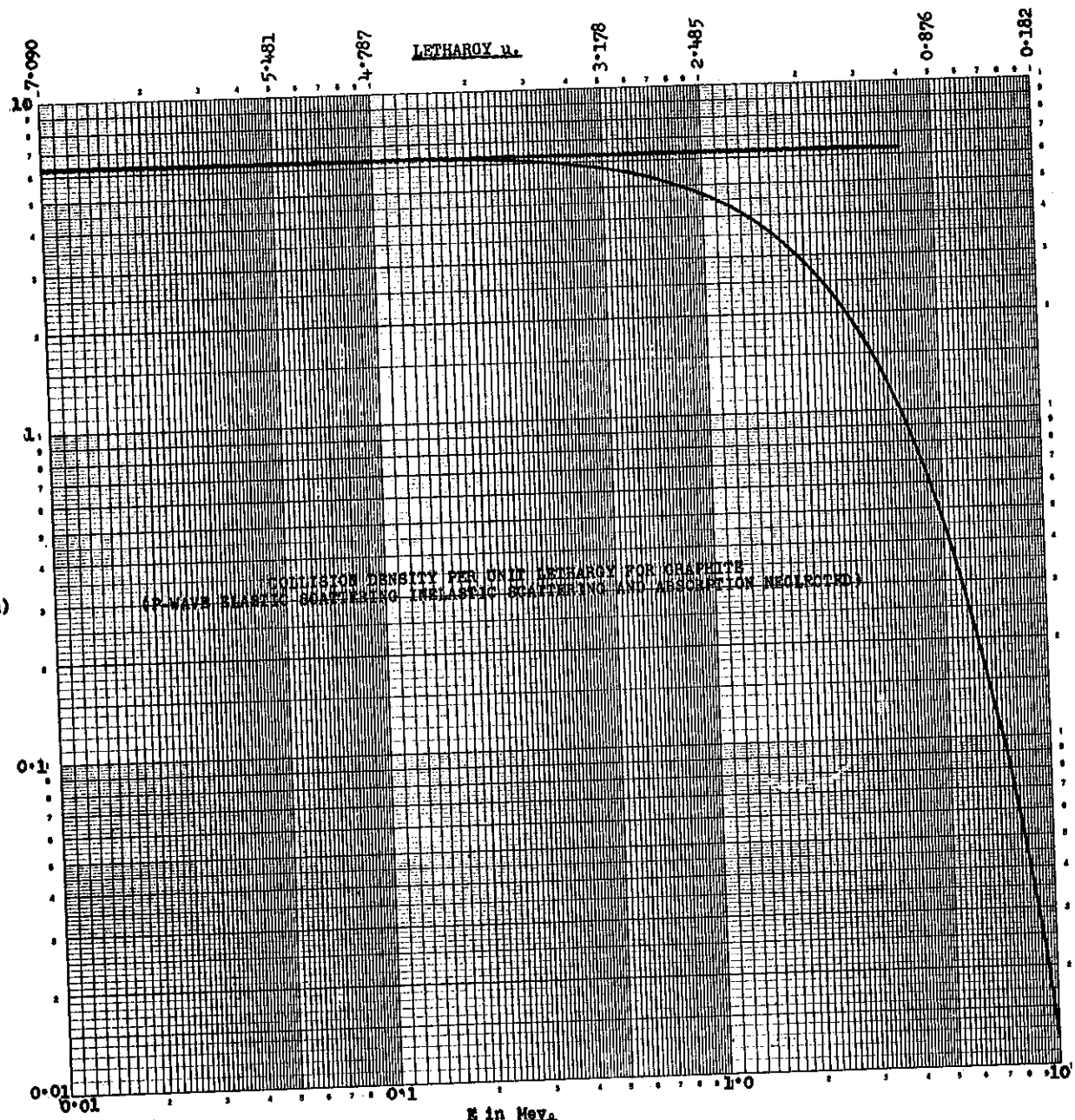
LETHARGY u.
7·090
5·481
4·787
3·178
2·485
0·876
0·182
10
1·
0·1
0·01
F(u)
COLLISION DENSITY PER UNIT LETHARGY FOR GRAPHITE
(P-WAVE ELASTIC SCATTERING INELASTIC SCATTERING AND ABSORPTION NEGLECTED)
0·01
0·1
1·0
10
E in Mev.

Fig. 1