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RADIAL HEAT TRANSFER AT THE WALL IN PACKED BEDS

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ABSTRACT

A literature survey indicates that adequate mathematical models are available for the study of heat transfer in packed beds. However, experimental data have been obtained at low Reynolds numbers only, below 3500.

Experimental results are reported for a 7 in. diameter bed packed with 0.489 in. and 0.658 in. diameter glass spheres, with preheated air as the fluid flowing in the bed. Results indicate that the bed-to-wall heat transfer coefficient h_w, based on the wall temperature and a bulk gas temperature, may be best correlated by the relation $Nu = 2.35\ Re^{0.63}$ to a standard deviation of 11.4 per cent for values between 2,000 and 15,000.

CONTENTS

1. INTRODUCTION

The study of radial heat transfer in packed beds is part of a heat transfer research programme associated with the design of a pebble bed nuclear reactor for a high temperature gas-cooled reactor system. Heat transfer through the container walls of a pebble bed and through control rod sheaths or other surfaces in the core has been investigated.

Radial heat transfer in a packed bed can be expressed in terms of an effective mean radial conductivity. At the wall the heat transfer can be expressed in terms of effective heat transfer coefficients, h_w, such that:

$$- q_w = h_w (T_w - T) \quad , \qquad .....(1.1)$$

where q_w = heat flux, T_w = wall temperature, and T = some temperature in the bed. The corresponding non-dimensional parameter the Nusselt number ($Nu = h_w D_p/k_g$), is then used for empirical correlations of the experimental data in terms of the other relevant non-dimensional parameters, such as Reynolds and Prandtl numbers:

$$Nu = f\left(Re, Pr, \frac{D_t}{D_p}, \epsilon, \text{etc.}\right) \quad . \qquad .....(1.2)$$

The use of the Stanton number, St = Nu/RePr, may appear to provide a better correlation, but as St is a derived group (Rowe 1963, 1964; Rowe et al. 1965) this should be avoided until a satisfactory relationship has been found between primary groups such as Nu, Re, Pr.

Since the average Reynolds number, based on ball diameter and superficial mass velocity of the gas flow in the bed is 45,300 (Hayes, private communication) for the 200 MW (e) Upflow Pebble-Bed Reactor Reference Design, values of Nusselt number are required corresponding to Reynolds numbers up to about 50,000.

1.1 Theoretical Analysis

The partial differential equation for the temperature at any position in a bed through which a fluid is flowing may be written as follows:

$$\rho C_p \frac{\partial T}{\partial t} = \nabla . k \nabla T - \rho C_p (u . \nabla T) + Q \quad , \qquad .....(1.3)$$

assuming negligible radiative transfer.

The left-hand side of the above equation represents the transient rate of heat content variation caused by changes in temperature, and the three terms on the right-hand side represent rates of conductive and convective heat transfer, and rate of heat generation respectively. In cylindrical coordinates, $T = T(r,z,\theta)$:

$$\nabla.k\nabla T = k_r\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \frac{\partial k_r}{\partial r}\frac{\partial T}{\partial r} +$$

$$+ k_z\frac{\partial^2 T}{\partial z^2} + \frac{\partial k_z}{\partial z}\frac{\partial T}{\partial z} +$$

$$+ \frac{k_\theta}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{1}{r^2}\frac{\partial k_\theta}{\partial \theta}\frac{\partial T}{\partial \theta} \quad . \qquad .....(1.4)$$

$$\rho C_p (u.\nabla T) = \rho \dot{C}_p \left(U_r \frac{\partial T}{\partial r} + U_z \frac{\partial T}{\partial z} + \frac{U_\theta}{r}\frac{\partial T}{\partial \theta}\right) \quad . \qquad .....(1.5)$$

Assuming that:

(i) the system is at a steady state, $\frac{\partial}{\partial t} = 0$;

(ii) there is no internal heat source, $Q = 0$;

(iii) radial convective transfer is negligible, $U_r \frac{\partial T}{\partial r} \ll U_z \frac{\partial T}{\partial z}$,

(iv) axial conductive transfer is negligible, $k_z \frac{\partial^2 T}{\partial z^2} \ll \rho C_p U_z \frac{\partial T}{\partial z}$, and

(v) a constant effective thermal conductivity k_e is applicable, that is $\frac{\partial k_e}{\partial r} = \frac{\partial k_e}{\partial z} = 0$,

then at the inner surfaces of the reactor wall,

$$- k_e \left[\frac{\partial T}{\partial r}\right]_{r=R} = q_w \quad , \qquad .....(1.6)$$

where q_w is given by Equation 1.1.

Equation 1.3 now reduces to:

$$\rho C_p U_z \frac{\partial T}{\partial z} = k_e \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \qquad .....(1.7)$$

and thus a knowledge of the effective heat transfer coefficient at the wall would provide one of the boundary conditions for the solution of the temperature distribution equation for the bed.

2. REVIEW OF THE LITERATURE

Most of the studies on heat transfer in packed beds have been related to packed bed chemical reactors. Others considered the packed beds as a limit of the fluidized bed system which has been extensively investigated. This survey is

presented in terms of (i) mathematical models proposed by various workers, and (ii) experimental investigations to obtain data for design purposes.

2.1 Mathematical Models

Ranz (1952) suggested a simple model for radial heat transfer in packed beds, and this model, modified by Yagi and Kunii (1957), has formed the basis of many subsequent studies. It assumes that the mechanisms of heat transfer consist of:

(a) Heat transfer mechanisms which are independent of fluid flow, namely:

(i) conduction through the solid,

(ii) conduction through contact surfaces of adjacent packing elements,

(iii) radiation between surfaces of packing elements, and

(iv) radiation between neighbouring voids.

(b) Heat transfer mechanisms which depend on fluid flow:

(i) conduction through the fluid film near contact surfaces of packing elements,

(ii) convection in the fluid transferring heat from one solid to another,

(iii) heat transfer by radial mixing of fluid.

Introducing the Peclet number (Pe) the model equation is:

$$\frac{k_e}{k_g} = \frac{k_{eo}}{k_g} + \alpha\beta\, Pe \quad . \qquad .....(2.1)$$

Hanratty (1954), investigating wall to fluid heat transfer in a packed bed, proposed a mechanism based upon the surface renewal theory of mass transfer (Danckwert 1951). According to this mechanism a mass of fluid impinges on the wall, the fluid at the wall assumes the wall temperature, and this mass of fluid moves away from the wall and is replaced by a new mass of fluid. The corresponding relation is:

$$Nu = 1.1\, Pe^{\frac{1}{2}} \quad . \qquad .....(2.2)$$

Smith (1956), outlining a design method for heat and mass transfer in packed bed reactors, proposed a model for estimating k_e in Equation 1.7 by assuming that:

$$q = - k_e \frac{\partial T}{\partial r} = q_{solid} + q_{void} \quad , \qquad .....(2.3)$$

where q_{solid} = heat transfer across the solid particles by (i) conduction within the solid, and (ii) convection and (iii) radiation at the surfaces;

q_{void} = heat transfer across the void spaces by (i) molecular conduction and (ii) turbulent diffusion in the fluid, and (iii) radiation.

Using simplifying assumptions (Argo and Smith 1953) the final result was:

$$k_e = \delta \left(k_e^1 + \frac{D_p C_p G}{Pe_m \delta} + 4 \frac{\epsilon}{2-\epsilon} D_p \times 0.173 \frac{Ta^3}{100^4} \right)$$

$$+ (1-\delta) \frac{hk_s D_p}{(2k_s + h D_p)} \quad . \qquad .....(2.4)$$

Yagi et al. (1957, 1959, 1960a, 1960b, 1961a, 1961b, 1964), Kunii and Smith (1960, 1961), Willhite et al. (1962) published a series of papers on heat transfer in cylindrical beds of consolidated and unconsolidated particles. The term (k_{eo}/k_g) in Equation 2.1 was estimated by means of a model based upon that of Ranz (1952) and gave:

$$\frac{k_{eo}}{k_g} = \delta \frac{k_s}{k_g} + \frac{(1 - \epsilon - \delta)\beta}{\gamma \frac{k_g}{k_s} + \frac{1}{\frac{1}{\phi} + \frac{D_p h_{rs}}{k_g}}} + \epsilon \beta \frac{D_p h_{rv}}{k_g} \quad . \qquad .....(2.5)$$

For heat transfer near the wall, a relation analogous to Equation 2.1 was proposed (Yagi and Kunii 1960a):

$$Nu = Nu_o + \alpha_w Pe \quad . \qquad .....(2.6)$$

Using a laminar boundary layer model, an equation for the film coefficient at the reactor wall was derived (Yagi and Kunii 1961b):

$$Nu = a_1 Pr^{1/3} Re^{\frac{1}{2}} \quad , \qquad .....(2.7)$$

where a_1 = 2.6 for liquid-solid systems, and a_1 = 4.0 for gas-solid systems.

Recently, Crider and Foss (1965) proposed a one-dimensional model of heat transfer in packed beds:

$$\frac{1}{H_1} = R + \frac{1}{H_2} \quad , \qquad .....(2.8)$$

in which the effective bed resistance, R, may be estimated by R^*, given by two models (with an error of less than 7 per cent.):

$$R^* = \frac{D_R Pe_r}{6.133} \qquad .....(2.9a)$$

from a partial differential model, and

$$R^* = 1.304 D_R - 2.734 \qquad .....(2.9b)$$

from a finite stage model, where $D_R = D_t/D_p$.

2.2 Experimental Investigations

Colburn (1931) studied wall-to-bed heat transfer in two steam-jacketed packed tubes of diameters 1¼ in. and 3 in. (L/D_t = 15), for packings ranging from ⅛ in. diameter pellets to 1 inch diameter zinc balls. Pressurised air at 1 to 6 atm was used as the moving fluid. The inner surface temperature of the tube was constant at 180°C, and the air inlet temperature was approximately 35°C. The heat transfer coefficient h was based on the log of the mean temperature difference between values at the ends of the tube. Within the Reynolds number ($Re = D_p G/\mu$) range of 180 to 15,000, it was found that h could be correlated by:

$$h_1 = a_1 G^{0.83} \qquad .....(2.10)$$

(in lb. ft. h units), where a_1 is a constant depending on D_t/D_p, and has a maximum value of 0.00862 at D_t/D_p = 6.67. It was found that h_1 was approximately eight times the corresponding coefficient for empty tubes.

Leva and co-workers (Leva 1947, 1950, 1948a, 1948b), following the pioneering work of Colburn (1931) studied wall-to-bed as well as bed-to-wall heat transfer in small packed tubes. For wall-to-bed heat transfer, the tube-side film coefficient h_1 was obtained from two tubes (½ in. and 2 in. diameters) packed with glass beads to a height of 1 ft and porcelain balls to a height of 3 ft. (D_t/D_p = 1.6 to 20.0). The tubes were steam-jacketed, and air and carbon dioxide were the fluids passed through the tubes. The overall heat transfer coefficient, based again on the log of the mean temperature difference between values at the ends of the tube was measured and this was assumed approximately equal to h_1, neglecting wall and steam-side resistances. For $(D_t/D_p) > 3.0$, and for Re = 40 to 3500, h_1 was correlated by:

$$h_1 = 0.813 \frac{k_g}{D_t} e^{-6 \frac{D_p}{D_t}} Re^{0.90} \quad , \qquad .....(2.11)$$

while for $1.6 < (D_t/D_p) < 3.0$, it was found that:

$$Nu = \frac{h_1 D_p}{k_g} = 0.125 Re^{0.75} \quad . \qquad .....(2.12)$$

Equation 2.11 is applicable to packings of other shapes provided that a suitable value of D_p is chosen. For packings of high thermal conductivity, the coefficient

h_1 was correlated by modifying Equation 2.10:

$$h_1' = f a_1 G^{0.83} \quad , \qquad .....(2.13)$$

where f is a numerical factor varying from 1.0 for glass to 1.5 for copper. Surface roughness of the packings appeared to increase f slightly.

For bed-to-wall heat transfer, the steam jackets were replaced by water jackets and the overall heat transfer coefficient for water cooling was measured for steam passing through the bed.

This overall heat transfer coefficient was found to be 400 Btu h^{-1} ft^{-2} $(°F)^{-1}$. The water-side coefficient was assumed to reach this value when gas was flowing in the tube and the tube-side film coefficient, h_2, was found by calculation from the measured overall coefficient when gas was flowing and the wall resistance was considered negligible. Glass and porcelain beads were used (D_t/D_p = 3.7 to 12.5) with air and carbon dioxide as the moving fluids.

For Re = 250 to 3000, h_2 was correlated by:

$$h_2 = 3.50 \frac{k_g}{D_t} \exp\left(-4.6 \frac{D_p}{D_t}\right) Re^{0.70} \quad , \qquad .....(2.14)$$

or, less accurately, by:

$$h_2 = 0.40 \frac{k_g}{D_t} Re^{0.70} \quad , \qquad .....(2.15)$$

and h_2 was found to be independent of the tube length for ratios of tube length to tube diameter greater than 10.

Values of h_2 were approximately 15 per cent. higher than those of h_1 for the same values of Re. The randomly packed tubes had voidages between 37 and 43 per cent. but the effect of voidage was not included in the correlation. By differentiating Equations 2.11 and 2.14 with respect to the (D_t/D_p) ratio and setting the derivations to zero, h_1 and h_2 were found to have maximum values at (D_t/D_p) of 6.67 and 6.54 respectively. These conclusions were supported by the experimental data obtained.

Coberly and Marshall (1951), using a 5 in. diameter cylindrical bed packed with cylinders of various sizes ($\frac{1}{8}$ in. diameter by $\frac{1}{8}$ in. to $\frac{3}{8}$ in. diameter by $\frac{1}{2}$ in), obtained the effective thermal conductivity and wall film coeffient (in lb. ft. h. units):

$$k_e = 0.18 + 0.00098 \left(G \sqrt{a_p}/\mu\right) \quad \text{and} \qquad .....(2.16)$$

$$h_{w1} = 2.95\, G^{0.33} \quad . \qquad .....(2.17)$$

This was done by measuring the temperature gradients in heated air stream flowing through the bed at mass velocities from 175 to 1215 lb h^{-1} ft^{-2}, corresponding to Re = 140 to 1000 approximately.

Irven et al. (1951) using a 2 in. diameter tube packed with $\frac{1}{8}$ in. alumina pellets found that:

$$k_e \propto Re^{0.49} \qquad .....(2.18)$$

for sulphur dioxide gas at mass velocities from 147 to 517 lb h^{-1} ft^{-2}, which correspond approximately to Re = 40 to 150.

Hougen and Piret (1951), using 1.37 in. to 3.75 in. diameter tubes packed with spheres (from $\frac{1}{16}$ in. diameter) and cylinders (up to $\frac{3}{8}$ in. diameter by $\frac{1}{2}$ in.) found that the effective thermal conductivity, k_e, may be correlated by:

$$k_e/C_p \mu = \frac{3.7}{\epsilon} Re^{0.33} \qquad .....(2.19)$$

for cooling air at mass velocities from 75 to 3700 lb h^{-1} ft^{-2} (Re = 60 to 3000) at bed temperatures up to 800°F. The values of k_e were found to be from 20 to 100 times that of the fluid in the static state, and from 2 to 15 times that of the solid.

Campbell and Huntingdon (1952), using 2 in. to 6 in. diameter tubes packed with cylinders and spheres of alumina, metal, and glass (equivalent diameters 0.2 to 1.0 inch) obtained the overall heat transfer coefficients h, effective thermal conductivities, and wall heat transfer film coefficients:

$$\frac{h}{C_p G} = 0.76 \exp(-0.0225\, a\, D_t) \left(\frac{G}{a\mu}\right)^{-0.42} \quad , \qquad .....(2.20)$$

$$\frac{k_e}{k_g} = 10.0 + 0.267 \left(\frac{G}{a\mu}\right) \quad , \quad \text{and} \qquad .....(2.21)$$

$$h_{w1} = 0.42 \left(\frac{G}{a_p \mu}\right)^{0.47} \quad , \qquad .....(2.22)$$

using natural gas (specific gravity 0.7) and air for flow rates corresponding to values for $\frac{G}{a_p\mu}$ up to 500 and hence Re up to 3000.

Hanratty correlated the previous experimental data of Plautz and Felix (1951, 1953, quoted by Hanratty, 1954) on wall film heat transfer coefficient for glass spheres with air as flowing fluid:

$$Nu_1 = 0.12 \left(\frac{Re}{\epsilon}\right)^{0.77} \qquad .....(2.23a)$$

$$= 0.243\, Re^{0.77} \qquad .....(2.23b)$$

for Re = 100 to 1000 and G = 0.40 .

Plautz and Johnstone (1955) measured effective thermal conductivities and wall film heat transfer coefficients in an 8 in. diameter steam-jacketed tube packed with $\frac{1}{2}$ in. and $\frac{3}{4}$ in. diameter glass spheres. For mass velocities of 110 to 1640 lb h^{-1} ft^{-2} of air, corresponding to Re = 100 to 2000, the results were correlated by:

$$k_e = 0.439 + 0.00129\ \mathrm{Re},\ \mathrm{Btu\ h^{-1}\ ft^{-2}\ (^\circ F)^{-1}} \qquad \text{.....(2.24)}$$

and

$$h_w = 0.090\ G^{0.75},\ \mathrm{Btu\ h^{-1}\ ft^{-2}\ (^\circ F)^{-1}} \ . \qquad \text{.....(2.25)}$$

Quinton and Storrow (1956), using a 1.625 in. diameter tube packed with 0.173 in. diameter glass spheres, obtained the effective thermal conductivities and wall film heat transfer coefficients for air through the bed. For Re = 30 to 1100, the results were correlated by:

$$k_e = 0.24 + 0.000109\ \mathrm{Re},\ \mathrm{Btu\ h^{-1}\ ft^{-1}\ (^\circ F)^{-1}} \qquad \text{.....(2.26)}$$

$$h_w = 0.04G,\ \mathrm{Btu\ h^{-1}\ ft^{-2}\ (^\circ F)^{-1}} \ . \qquad \text{.....(2.27)}$$

Gopalarathnam et al. (1961) extended Equation 2.1 to include natural convection:

$$\frac{k_e}{k_g} = \delta' \frac{k_s}{k_g} + A\,(\mathrm{Gr.Pr})^{B} + \alpha\,\beta\,\mathrm{Pe}\ , \qquad \text{.....(2.28)}$$

and found that this equation was adequate for correlating liquid systems such as nitrobenzene, aqueous glycerol, and toluene for $3 < (D_t/D_p) < 14$, and for $50 < \mathrm{Re} < 2100$.

Baddour and Yoon (1961) reported the measurement of local effective conductivity measurements carried out on 6 in. diameter annular beds packed with a total of six different sets of spherical pellets, diameters varying from 0.165 to 0.282 in. and made of any of four different materials; alumina, steel, aluminium, and glass. Air was used as the fluid and the particle Reynolds number (D_pG/μ) was varied between 50 and 1300. The solid-fluid-solid series conduction mechanism was shown to be essentially independent of Reynolds number. The radial effective conductivity within a $\frac{1}{2}$-particle-diameter distance from the column wall was found to be significantly different from that in the interior of the bed, giving rise to the so-called wall effect. The radiation mechanism was affected by the solid conductivities. For the interior of the bed, the fluid-phase effective conductivity, K_g Btu h^{-1} ft^{-1} $(^\circ F)^{-1}$ is given by:

$$\frac{K_g}{k_g} = \left(\frac{1}{11}\right)\left(\frac{C_p\mu}{k_g}\right)\left(\frac{D_pG}{\mu}\right) + \epsilon\ 1.3\ . \qquad \text{.....(2.29)}$$

For the $\frac{1}{2}$-particle-diameter intervals from the wall, the fluid-phase effective conductivity, K_g' Btu h^{-1} ft^{-1} $(^\circ F)^{-1}$, is given by:

$$\frac{K_g'}{k_g} = \left(\frac{1}{100}\right)\left(\frac{C_p}{k_g}\right)\left(\frac{D_pG}{\mu}\right) + \epsilon\ 1.3\ , \qquad \text{.....(2.30)}$$

so that, for gas flows corresponding to relatively high Reynolds numbers (>1000) : $(K_g/K_g') \approx (100/11) = 9.1$.

Burnell et al. (1949), Irvin et al. (1951), Morales et al. (1951), and Masamune and Smith (1963) reported several studies of effective thermal conductivities in packed beds at low fluid velocities. Using a 2 in. diameter tube packed with $\frac{1}{8}$ in. alumina cylinders, to heights up to 16 in. it was found that the effective thermal conductivity K_g, varied from 0.1 to 0.4 Btu h^{-1} ft^{-1} $(^\circ F)^{-1}$ within the air mass velocity interval of 150 to 500 lb h^{-1} ft^{-2}, corresponding to Re = 15 to 50:

$$\frac{K_g}{k_{air}} = 5.0 + 0.061\ \mathrm{Re}\ . \qquad \text{.....(2.31)}$$

There appeared to be an appreciable but inconsistent variation of (K_g/k_{air}) with bed depth.

Using hot wire anemometers, it was found that the fluid velocity in packed beds decreases near the wall of the containing vessel and also near the centre. Similar variations with radial position were observed in the group (k/C_pG). A model of heat transfer was proposed based on combinations of conduction, convection, and radiation.

Yagi and co-workers, using Equation 2.1, correlated previous data as well as their own to obtain the parameters (k_{eo}/k_g), $\alpha\beta$, for $2.7 < (D_t/D_p) < 50$, and $0 < \mathrm{Re} < 3500$:

Using glass spheres,

$$\frac{k_e}{k_g} = 6.0 + 0.11\ \mathrm{Pe},\ \text{for } D_t/D_p = 14 \text{ to } 48\ , \qquad \text{.....(2.32a)}$$

and

$$\frac{k_e}{k_g} = 6.0 + 0.09\ \mathrm{Pe},\ \text{for } D_t/D_p = 6 \text{ to } 8\ . \qquad \text{.....(2.32b)}$$

Using metal spheres,

$$\frac{k_e}{k_g} = 13.0 + 0.11\ Pe, \text{ for } D_t/D_p = 12 \text{ to } 48 \qquad .....(2.32c)$$

$$\text{Also } j_H = St.\ Pr^{0.67},$$

$$= 0.20\ Re^{-0.20}, \text{ for } Re < 20 \qquad .....(2.33)$$

Using an annular bed of radii 0.87 in. and 2.76 in. with steam in an outer jacket and water flowing in the inner tube, the wall film coefficients, obtained with air flowing axially through the bed, were correlated, for $Re < 600$, by Equation 2.6. The value of Nu_o varied with packing materials, and $\alpha_w = 0.054$ and 0.041 for cylindrical and annular packed beds respectively.

Yagi et al. (1964) have shown that Equation 2.1 is applicable when water is flowing through a bed at Re = 7.45 to 125. Table 1 summarises all relevant available data.

2.3 Discussion

The foregoing literature survey indicates that, apart from the pressurised system used by Colburn (1931), no experimental data have been obtained on radial heat transfer in packed beds at Reynolds numbers above 3500, and that the Nusselt number at the wall, $Nu = h_w D_p/k_g$, is a strong function of Re but a weak function of (D_t/D_p). For gases, the Prandtl number, $Pr = C_p\mu/k_g$, does not vary much (approximately 0.7), so that it is difficult to evaluate the dependence of Nu upon Pr unless liquids are used as the flowing fluid.

Inside the bed the effective thermal conductivity k_e, for solid packings of low conductivity with gas flow, may be taken as that given by the correlation of Equation 2.32a:

$$\frac{k_e}{k_g} = 6.0 + 0.08\ Re, \qquad .....(2.34)$$

obtained by putting Pr = 0.7, and substituting Pe = Pr.Re. For solids of high conductivity, Equation 2.32c gives:

$$\frac{k_e}{k_g} = 13.0 + 0.08\ Re. \qquad .....(2.35)$$

The contribution from the solid conductivity, k_s, is incorporated in the constant term, (see Equation 2.1). At high Reynolds numbers, the effects of k_s

on k_e is small (less than 2 per cent. at Re = 10,000).

To serve as a basis for comparing the various correlations, heat transfer at the wall may be reduced to a common form by selecting suitable reference conditions. The reference conditions arbitrarily chosen are: 3/16 in. spherical particles, bed-to-particle diameter ratio of 10 and air flow at 100°C and 1 atm. The corresponding values of fluid viscosity and thermal conductivity are respectively $\mu = 0.05082$ lb $ft^{-1}\ h^{-1}$ (Perry 1950) and $k_g = 0.01848$ Btu $h^{-1}\ ft^{-1}\ (°F)^{-1}$ (de Vahl Davis 1958). These conditions are in general within the ranges of experimental data covered by the correlations.

The common form chosen is:

$$Nu = A\ Re^{B}. \qquad .....(2.36)$$

The correlations may then be compared with the well known correlation for pipe flow (McAdams 1954) :

$$Nu = 0.021\ Pr^{0.4}\ Re^{0.8}. \qquad .....(2.37)$$

Table C gives the results of these calculations and correlations are shown in Figure 1. The Nusselt number, $Nu = h_w D_p/k_g$ in these correlations was always calculated using the temperature-dependent gas conductivity.

Examination of Table 2 indicates that values of the exponent of the Reynolds number, Re, vary over the range 0.33 to 1. Yagi et al. showed that according to the laminar boundary layer model the exponent should be 0.5 but most of the correlations show that it lies between 0.75 and 1.

Equation 1.2 for the correlation of data on the wall heat transfer coefficient for gases, may be adequately represented by:

$$Nu = f\ (Re), \qquad .....(2.38)$$

$$\text{or} \quad Nu = A_o + A\ Re^{B}. \qquad .....(2.39)$$

For high Re (≃ 10,000 and above) the constant A_o will be small in comparison with the second term, and Equation 2.36 is suitable.

Some previous workers defined the wall heat transfer coefficient, h_w, Equation 1.1, in terms of the difference between the wall temperature, T_w, and the film temperature T. This implies a knowledge of the radial temperature gradient near the wall. For practical purposes it would be more convenient to use the bulk temperature of the fluid flowing in the bed instead of a film temperature.

3. EXPERIMENT

3.1 Experimental Assembly

The test assembly was set up on a 35 hp air rig (Appendix 2) in the Engineering Research Laboratory. The packed section was cylindrical (10.2 in. high and 7 in. diameter), and a 72 kW electrical air-preheater was located upstream of the test section. A schematic diagram of the test rig is shown in Figure 2.

The test section was constructed of mild steel except for the test wall which consisted of a ring of stainless steel, $2\frac{1}{2}$ in. high and 3/16 in. thick. Thermocouples were inserted diametrically opposite each other in the wall and two each at the inlet and outlet of the bed. The bed was surrounded by a jacket of mild steel through which passed a separate coolant air stream from a small blower. Inlet and outlet temperatures of this secondary circuit were also measured with thermocouples. Orifice plates and water manometers were used to meter the air rates through both the bed and the jacket. The former air rate is used in evaluating the average bed Reynolds number, and the latter (in conjunction with the corresponding temperature rise) provides a measure of the heat flux through the test wall.

Two ball sizes were used : 12.5 mm and 17 mm nominal diameter glass spheres. Wire gauze was used to hold the balls in the packed section.

3.2 Experimental Procedure

Ball diameter was taken as the arithmetic mean of single measurements on 10 balls selected at random and measured with a micrometer. Average bed voidages were determined from the bed dimensions and the number of balls in the bed. To obtain the number of balls all the balls in the bed were weighed together and then 30 balls taken from the bed at random were weighed. Thus the number of balls was calculated and assuming the balls to be perfect spheres the total volume of the balls was found. Hence the bed voidage could be calculated.

The experiments were performed under steady state conditions. The air rate was set, the heater switched on and then the secondary air introduced. The thermocouple readings were taken every few minutes until the temperature changes were less than 1°C in 15 minutes. The average time for the system to reach a steady state was 75 minutes.

Readings were taken of:

(a) Hot-gas-side air rate as indicated on the water manometer air inlet and outlet temperatures T_1 and T_2, and wall temperatures T_w.

(b) Cold-gas-side air rate, and inlet and outlet temperatures T_3 and T_4.

Using published values of the thermodynamic properties of gases (de Vahl Davis 1958), digital computer programmes were used to process the experimental data and obtain the following parameters:

(a) Superficial mass velocities of the gas, G_1 and G_2 lb h^{-1} ft^{-2} .

(b) Reynolds number, $Re = D_p\, G_1/\mu$.

(c) Bed-to-wall heat transfer coefficient,

$$h_w = \left[C_p G_2 \ (T_4 - T_3) \right] / \left[A \left\{ \tfrac{1}{2} (T_1 + T_2) - T_w \right\} \right] \quad .$$

(d) Nusselt number, $Nu = h_w D_p / k_g$.

(e) $T_R = (T_w/T_B)$, where $T_B = \frac{1}{2}\ (T_1 + T_2)$.

(f) The constants A and B in a correlation of the form given by Equation 2.36, using the method of least squares.

(g) The standard deviation, SD per cent.

3.3 Experimental Results

Correlations were obtained for Re and Nu with gas properties (μ, k_g) evaluated at a mean film temperature:

$$T_f = \tfrac{1}{2}\ (T_w + T_B) \qquad .....(3.1)$$

and at the bulk temperature, T_B.

The values of T_R, h_w, Nu_F, Nu_B, Re_F and Re_B are shown in Table 3. The corresponding values of G_1, T_1, T_2, G_2, T_3, T_4 and T_w are given in the Appendix 3 and the values of A, B, and S.D. are shown in Table 4..

3.4 Discussion of Results

It appears that over the range of Reynolds numbers used in these experiments, $2000 < Re < 10{,}000$, Equation 2.36 is adequate for correlation of the experimental data. Use of the mean film temperature T_F, instead of the bulk temperature T_B, as a basis for calculating the non-dimensional groups did not improve the correlation. However, if the correlations were to be extended to conditions of higher temperature levels, such a correlation, using T_F, may show significant improvement.

The correlations based on data from a given bed seemed to indicate that the ball sizes, or the (D_t/D_p) ratios, have some small effects. The ranges of these variables however, were too small to bring them into the correlation ($10 < D_R < 14$). To increase the ranges of Re and other variables, further experiments will be required using a larger bed and a blower with large capacity as well as high delivery pressure head (See Appendix 1).

The data in Table 3 are plotted on Figure 1 for comparison with the existing data and correlations discussed in Section 2. The least squares line is drawn through the points. The correlation equation is:

$$Nu = 2.35 Re^{0.63} \quad .$$

It would appear that use of the wall heat transfer coefficient, based upon the temperature difference between the walls and the bulk temperature in the bed, is more practical than the mean film temperature approach. The correlation obtained here permits the coefficient to be estimated from the average Reynolds number in the bed without knowing the radial distribution of temperatures up to the wall.

Experimental errors caused by conduction losses were small as the heat transfer surface was well separated by a small gap filled with rubber cement, and insulated from the rest of the bed wall. Perhaps the main source of systematic error was the accuracy of the temperature readings. This error was reduced by using a small coolant flow rate G_2 to give a large temperature differential ($T_4 - T_3$) in the secondary circuit. The errors in T_3 and T_4 were $\pm$ 1°C, making a total error of about $\pm$ 2°C in temperature differentials of about 20°C, or $\pm$ 10 per cent. In the primary circuit, the temperature differentials were small, sometimes less than 1°C, so that a check of heat balances is not feasible. The values T_1 and T_2 were used only for estimating the bulk gas temperatures in the bed. Hence the heat fluxes and the wall coefficients would have errors of the order of $\pm$ 10 per cent.

The heat transfer surface was $2\frac{1}{2}$ in. high in the 10 in. high bed and both upstream and downstream bed lengths were provided. The experiments were performed at bed temperatures 60°C to 300°C, and it was found that the effect of temperature level is only of secondary importance.

Perhaps a more satisfactory model for studying wall heat transfer in packed beds would be to regard the main bulk of the packing as a cylinder (with effective conductivities), surrounded by an irregular turbulent 'annulus'. This turbulent annulus would comprise the fluid between the wall and the adjacent particles of packing and might be regarded as a very thin turbulent layer of constant temperature with the normal boundary layer film between it and the wall surface. This will be studied in further work. However, the simple practical model proposed here seems to correlate the data adequately. A further experiment has been designed

with a square section bed, cooled on one side only and is now under construction. This will enable the above results to be extended to Reynolds numbers up to 100,000.

4. CONCLUSIONS

A survey of the literature indicates that adequate mathematical models are available for the study of heat transfer in packed beds. However, experimental data have been mainly limited to low Reynodls numbers, below 3500.

Experimental data were obtained on radial heat transfer in a 7 inch diameter bed packed with 0.658 in. and 0.459 in. diameter glass spheres, with air as the fluid flowing through the bed. Results indicated that the bed-to-wall heat transfer coefficient, h_w, based upon the difference between the wall temperature and a bulk gas temperature, may be best correlated by:

$$Nu = (h_w D_p/k_g)$$
$$= 2.35 Re^{0.63} \quad ,$$

where Re = (D_p G/μ), for $2000 < Re < 10{,}000$, with a standard deviation of 11.4 per cent.

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6. BIBLIOGRAPHY

Argo, W. B., and Smith, J. M. (1953). - Chem. Eng. Prog., 49 : 443.

Baddour, R. F., and Yoon, C. Y. (1961). - Chem. Eng. Prog. Sym. Ser. No. 32, 57 : 35.

Baron, T. (1952). - Chem. Eng. Prog. 48 : 118.

Benenati, R. F., and Brosilow, C. B. (1962). - A.I.Ch.E.J., 8 : 359.

Bischoff, K. B. (1962). - Canad. J. Chem. E., 40 : 161.

Bowers, T. G., and Reintjes, H. (1961). - A.I.Ch.E. - C.E.P. Sym. Ser. No. 32, 57 : 69.

Burnell, D. G., Irvin, H. B., Wilson, R. W., and Smith, J. M. (1949). - Ind. Eng. Chem. 49 : 1977.

Campbell, J. M., and Huntington, R. L. (1952). - Pet. Ref., 31 (2) : 123.

Coberly, C. A., and Marshall, Jr., W. R. (1951). - Chem. Eng. Prog., 47 : 141.

Colburn, A. P. (1931). - Trans. A.I.Ch.E., 26 : 166.

Crider, J. E., and Foss, A. S. (1965). - A.I.Ch.E.J., 11 : 1012.

de Vahl Davis, G. (1958). - Unpublished data.

Dankwerts, P. V. (1951). - Ind. Eng. Chem., 43 : 1460.

Dzung, L. S. (1960). - Int. J. Heat Mass Transfer, 1 : 236.

Glaser, H. (1962). - Chem. Ing. Tech., 34 : 468.

Gopalarathnam, C. D., Chennakesavan, P., and Laddha, G. S. (1962). - J. Sci. Indust. Res., A21 : 186.

Gopalarathnam, C. D., Hoelscher, H. E., and Laddha, G. S. (1961). - A.I.Ch.E.J., 7 : 249.

Green, D. W., and Perry, R. H. (1961). - A.I.Ch.E. - C.E.P. Sym. Ser. No. 32, 57 : 61.

Hanratty, T. J. (1954). - Chem. Eng. Sci. 3 : 209.

Hart, J. A., Lawther, K. R., and Szomanski, E. (1965). - "Fluid Flow in Packed Beds", Proc. of Second Australasian Conf. on Hydraulics and Fluid Mechanics, University of Auckland, N. Z., Dec. 1965.

Hougen, J. O., and Piret, E. L. (1951). - Chem. Eng. Prog. 47 : 295.

Irvin, H. B., Olson, R. W., and Smith, J. M. (1951). - Chem. Eng. Prog., 47 : 287.

Klinkenberg, A. (1960). - Chem. Eng. Sci., 11 : 260.

Kunii, D., and Smith, J. M. (1960). - A.I.Ch.E.J., 6 : 71.

Kunii, D., and Smith, J. M. (1961). - A.I.Ch.E.J., 7 : 29.

Lancashire, R. B. et al. (1960). - Ind. Eng. Chem., 52 : 433.

Leva, M. (1947). - Ind. Eng. Chem., 39 : 857.

Leva, M. (1950). - Ind. Eng. Chem., 42 : 2498.

Leva, M., and Grummer, M. (1948a). - Ind. Eng. Chem. 40 : 914.

Leva, M., Weintraub, M., and Grummer, M. (1948b). - Ind. Eng. Chem., 40 : 747.

Levenspiel, O., and Walton, J. S. (1954). - A.I.Ch.E. - C.E.P. Sym. Ser. No. 9, 50 : 1

McAdams, W. H. (1954). - Heat Transmission, 3rd. ed., McGraw-Hill, N.Y. p.219.

Masamune, S., and Smith, J. M. (1963). - Ind. Eng. Chem. Fund., 2 : 136.

Morales, M., Spinn, C. W., and Smith, J. M. (1951). - Ind. Eng. Chem., 43 : 225.

Pasternak, I. S., and Gauvin, W. H. (1960). - Canad. J. Ch. E. 38 : 35.

Perry, J. H. (1950). - Chemical Engineers Handbook, 3rd. ed., McGraw-Hill, N.Y. p.371.

Plautz, D. A., and Johnstone, H. F. (1955). - A.I.Ch.E.J., 1 : 193.

Pratt, A. W. (1962). - Res. Appl. Indust., 15 : 214.

Quinton, J. H., and Storrow, J. A. (1956). - Chem. Eng. Sci., 5 : 245.

Ranz, W. E. (1952). - Chem. Eng. Prog., 48 : 247.

Rowe, P. N. (1963). - The Chemical Engineer, No. 166 : CE69.

Rowe, P. N. (1964). - Trans. Inst. Chem. Engrs., 42 : T22.

Rowe, P. N., Claxton, K. T., and Lewis, J. B. (1965). - Trans. Inst. Chem. Engrs., 43 : T14.

Schotte, W. (1960). - A.I.Ch. E.J., 6 : 63.

Schuler, R. W., Stallings, V. D., and Smith, J. M. (1952). - A.I.Ch.E. - C.E.P. Sym. Ser. No. 4, 48 : 19.

Schwartz, C. E., and Smith, J. M. (1953). - Ind. Eng. Chem., 45 : 1209.

Singer, E., and Wilhelm, R. H. (1950). - Ind. Eng. Chem., 46 : 343.

Smith, J. M. (1956). - Chemical Engineering Kinetics, p. 288, McGraw-Hill, N.Y.

Takeoka, S., and Nagasako, N. (1961). - Chem. Eng. (Jap), 25 : 588.

Thompson, J. J. (1964). - Atomic Energy in Australia, 7 : 6.

Toyama, S. (1962). - Chem. Eng. (Jap.), 26 : 976.

Verschoor, H., and Schmidt, G. C. A. (1952). - Appl. Sci. Res., 42 : 97.

Willhite, G. D., Kunii, D., and Smith, J. M. (1962). - A.I.Ch.E.J., 8 : 340.

Yagi, S., and Kunii, D. (1954). - Chem. Eng. (Jap.), 18 : 576.

Yagi, S., and Kunii, D. (1957). - A.I.Ch.E.J., 3 : 373.

Yagi, S., and Wakao, N. (1959). - A.I.Ch.E.J., 5 : 79.

Yagi, S., and Kunii, D. (1960a). - A.I.Ch.E.J., 6 : 97.

Yagi, S., Kunii, D., and Wakao, N. (1960b). - A.I.Ch.E.J., 6 : 543.

Yagi, S., Kunii, D., and Wakao, N. (1961a). - Int. Develop. Heat Transfer, p. 742, A.S.M.E., N. Y.

Yagi, S., and Kunii, D. (1961b). - Int. Develop. Heat Transfer, p. 750, A.S.M.E.,N.Y.

Yagi, S., Kunii, D., and Endo, K. (1964). - Int. J. Heat Mass Transfer, 7 : 333.

Zenz, F. A., and Othmer, D. F. (1960). - Fluidization and Fluid - Particle Systems, Reinhold, N. Y.

APPENDIX 1

NOTATION

A	=	area of test section
A	=	constant, Eqn. (2.28) and (2.36)
A_o	=	constant, Eqn. (2.39)
a_p	=	surface area per unit volume of packing material
a_1	=	constant, Eqn. (2.7) and (2.10)
B	=	constant, Eqn. (2.38) and (2.36)
C_p	=	heat capacity
D_R	=	diameter ratio (D_t/D_p)
D_e	=	equivalent diameter $(D_t - D_r)$
D_p	=	ball diameter
D_r	=	control rod diameter
D_t	=	bed diameter
f	=	factor, Eqn. (2.13)
G	=	gas mass velocity based upon empty bed
G_1, G_2,	=	value of G for hot and cold gases respectively
Gr	=	Grashof number = $(L^3 \rho g/^{\beta}e\ \Delta t/\mu^2)$
g	=	acceleration due to gravity
H_1	=	dimensionless heat transfer coefficient : effective wall coefficient, Eqn. (2.8)
H_2	=	dimensionless heat transfer coefficient : at wall, Eqn. (2.8)
h	=	heat transfer coefficient
h_{rs}	=	value of h for radiation (solid-to-solid)
h_{rv}	=	value of h for radiation (void-to-void)
h_w	=	value of heat reactor wall
$h_{w.1}$	=	value of h for wall-to-bed transfer based upon film temperature
h_1	=	value of h for wall-to-bed transfer based upon LMTD
h_2	=	value of h for bed-to-wall transfer based upon LMTD
j_H	=	dimensionless heat transfer j-factor = $St.Pr^{2/3}$
K_g	=	fluid-phase effective thermal conductivity
k'_g	=	k_g for the $\frac{1}{2}$-particle diameter internal from the wall
k	=	thermal conductivity
h'_c	=	value of k for molecular conduction
k_e	=	value of k, effective value
k_{eo}	=	value of k, effective value (for stationary fluid)
k_g	=	value of k, for flowing fluid in packed bed
k_r	=	value of k for radial transfer

Continued...

APPENDIX 1 (Continued)

k_s	=	value of k for solid packings
k_z	=	value of k for axial transfer
k_θ	=	value of k for azimuthal transfer
L	=	length
Nu	=	Nusselt number = hL/kg
Nu_B	=	value of Nu based upon bulk temperature
Nu_F	=	value of Nu based upon mean film temperature
Nu_o	=	value of Nu for stationary fluid
Nu_1	=	value of Nu for wall-to-bed transfer
Pe	=	Peclet number = Pr.Re
Pe_m	=	value of Pe, modified
Pe_r	=	value of Pe for radial transfer
Pr	=	Prandtl number = $C_p\mu/k$
Q	=	rate of heat generation, Eqn. (1.3)
q	=	heat flux
q_w	=	value of q at reactor wall
q_{solid}	=	value of q through solid
q_{void}	=	value of q through void space between packings
R	=	one dimensional effective bed resistance to heat transfer, Eqn. (2.8)
R^*	=	value of R from mathematical model
r	=	radius of packed bed
Re	=	Reynolds number = LG/μ ; L is usually D_p = ball diameter
Re_B	=	value of Re based on fluid properties at temperature T_B
Re_F	=	value of Re band on fluid properties at temperature T_F
SD	=	standard deviation
St	=	Stanton number = Nu/Pe
T	=	temperature
T_B	=	value of T, bulk
T_F	=	value for T, mean film
T_R	=	temperature ratio (T_w/T_B)
T_w	=	value of T at reactor wall
T_a	=	value of T_1 absolute
T_1	=	value of T for hot gas at inlet
T_2	=	value of T for hot gas at outlet
T_3	=	value of T for coolant gas at inlet
T_4	=	value of T for coolant gas at outlet
S_t	=	temperature difference between heat transfer surfaces

Continued...

APPENDIX 1 (Continued)

t	=	time
U	=	velocity
U_r	=	value of U, radial component
U_z	=	value of U, axial component
U_θ	=	value of U, azimuthal component
z	=	distance in axial direction of packed bed.

Greek Letters

α	=	Mass velocity of fluid flowing in direction of heat transfer/mass velocity of fluid based on sectional area of empty tubes in the direction of fluid flowing, Equation (2.1), α_w = at reactor wall.
β	=	Average length between centres of adjacent solids in the direction of heat transfer/mean diameter of packing.
β_e	=	Coefficient of thermal expansion.
γ	=	Length of solid affected by thermal conductivity/mean diameter of solid.
δ	=	Area of perfect contact surfaces as solid phase in a given section/ total area of the section.
δ'	=	$[\ \delta + \{\ \beta\ (1-\epsilon-\delta)\ /\ \gamma\ \}\ ]$
ϵ	=	Fractional voidage.
θ	=	Azimuthal angle.
μ	=	Absolute viscosity.
ρ	=	Density.
$\emptyset$	=	Effective thickness of fluid film in void in relation to thermal conduction/mean diameter of solid.

APPENDIX 2

DESIGN OF EXPERIMENTAL RIG FOR DETAIL STUDY OF RADIAL HEAT TRANSFER IN PACKED BEDS

In a randomly packed bed of spheres, wall effects on the structure inside the bed may be considered negligible if $D_R = (D_t/D_p) \geqq 30 : 1$. The ratio of control rod to fuel ball diameter (D_r/D_p) is assumed to be approximately 6 : 1 (Hayes 1965). To design a representative experiment for this geometry a pipe, representing the control rod, could be inserted in a bed of equivalent diameter, D_e, equal to 30 ball diameters. The value of D_e is taken as : $D_e = (D_t - D_r)$, where $D_r = D_p$. To achieve a stable velocity distribution within the bed free from entrance and exit effects, the bed height should be 2 to 3 times the bed diameter. To minimize the cost of pipe work, the blower must produce a Reynolds number of the order of 10,000, preferably without using a pressurised system. With these criteria the design parameters for the specification of the air compressor, which is the most expensive component in the rig, are then calculated as shown in Table 5. It is assumed that the inlet air from the blower is preheated to approximately 100°C.

TABLE

Design of Required Air Compressor for Experiment
(Re = 10,000, Inlet Air Temp. = 100°C)

Design No.	1	2	3	4	5
D_p (in)	1/4	1/2	3/4	3/4	1
D_t (in)	9	18	27	30	36
Flow Rate (CFM) (Re = 10,000)	3,530	7,500	12,230	14,000	15,280
ΔP/L (p.s.i./ft)	224.5*	28.1	8.32	8.32	3.51
G_1 (lb/sec)	3.29	6.91	10.36	12.90	13.84
ΔP for (a) $L = 1.5D_t$ (p.s.i.)	253	63	28	31	15.8
(b) $L = 2.2\ 3D_t$	449	112	51	55	28
ΔP for entrance loss (p.s.i.)	0.72	0.97	1.34	1.69	1.82
Pressure Ratio for (a)	18.2	5.36	3.00	3.24	2.2
(b)	31.6	8.69	4.53	4.89	3.04

* First approximation only

Continued...

APPENDIX 2 (Continued)

Inspection of the Table indicates that Designs 1 and 2 require excessive pressure ratios, and under such high pressure conditions, the method of calculation (based on the Ideal Gas Laws) would require modifications which would take into account effects such as compressibility factors. Design 5 requires a high capacity. Design 3 represents the minimum bed size, for $\frac{3}{4}$ in. dia. balls, which gives a ratio of $(D_e/D_p) = 30$. Thus a suitable choice would be Design No. 4 which shows that a 30 in. dia. bed would fulfil the design criteria. Technical enquiries of blower manufacturers indicated that an axial air compressor of the following specifications would be suitable:

Intake Pressure	14.7 p.s.i.a.
Discharge Pressure	66 p.s.i.a.
Pressure Ratio	4.5
Capacity	10,600 SCFM (13 lb. per sec.)
Power Requirement	1675 H.P.

The cost of this equipment, excluding the gear box and motor system, is estimated to be in excess of $A20,000. Hence one reason for the published experimental data being mainly limited to low Reynolds numbers (below 3,500) is probably the high cost of blower equipment for experiments at high Reynolds numbers (above 10,000). An alternative design to achieve the required Reynolds number would use a liquid, such as water, as the moving fluid, or use large size balls with gas fluid (at the expense of the desired bed-to-wall diameter ratio). Such a gas fluid experiment, using 3 in. diameter walls in a 13 in. square bed, is being designed and will be reported later.

APPENDIX 3

EXPERIMENTAL DATA

Radial Heat Transfer in Packed Beds (7 in. dia.)

Run	D_p (in)	Hot air through packed bed			Coolant Air			Wall Temp.
		G_1 lb h^{-1} ft^{-2}	T_1 (°C)	T_2	G_2	T_3	T_4	T_w
1	0.658	2025	202.0	194.3	552	50.8	84.0	150.0
2	"	2025	189.3	182.4	396	45.4	85.1	146.0
3	"	1795	211.0	202.0	552	47.0	85.2	152.0
4	"	1795	213.4	204.8	396	47.7	92.3	161.8
5	"	1485	250.3	237.5	552	50.0	93.0	171.0
6	"	1485	249.5	239.5	396	52.3	102.5	182.0
7	"	1214	295.6	282.3	552	53.4	104.0	194.1
8	"	1214	289.1	276.8	396	55.0	112.7	202.6
9	"	2335	70.3	69.0	552	46.0	51.5	62.0
10	"	2335	70.5	70.0	396	44.6	51.5	61.9
11	"	768	166.8	159.5	396	44.5	71.0	111.8
12	0.489	1960	68.1	68.0	396	42.8	48.8	60.0
13	"	1960	68.4	68.0	186	38.5	49.6	61.7
14	"	1795	67.7	67.1	396	39.8	47.9	59.3
15	"	1795	71.8	70.4	186	39.9	51.1	64.9
16	"	1485	74.7	72.1	396	40.8	50.3	62.4
17	"	1485	73.6	72.0	186	37.5	50.3	64.3
18	"	1214	72.5	72.4	396	39.8	48.9	61.4
19	"	1214	79.3	77.1	186	37.5	51.3	67.4
20	"	1644	227.0	218.8	396	59.8	105.5	172.3
21	"	1644	224.0	216.0	186	54.5	123.8	185.0
22	"	768	168.3	159.5	396	45.8	74.9	112.3
23	"	768	165.8	158.5	186	47.0	86.5	126.3

TABLE 1

SUMMARY OF AVAILABLE DATA

Investigator	Equivalent Diameter (in)	Material	Diameter (in)	Fluid	Reynolds Number	Comments on Correlations of h, k	Correlation Equation
Colburn	$\frac{1}{8}$ to 1	Zinc	1.25 to 3	Pressurized Air	180-15000	Wall-to-bed Transfer, LMTD used	2.10
Leva	0.10-0.31	Glass, clay porcelain	0.5 to 2	Air, carbon dioxide	40-3500	"	2.11
"	"	"	"	"	250-3000	Bed-to-wall transfer, LMTD	2.13
Coberly	$\frac{1}{8}$ to $\frac{3}{8}$	Iron oxide	5	Air	140-1000	Wall-to-bed transfer, film temp. used	2.16 2.17
Irvin	$\frac{1}{8}$	Alumina	2	Sulphur dioxide	40-150	"	2.18
Hougen	$\frac{1}{16}$ to $\frac{3}{8}$	Porcelain	1.37 to 3.75	Air	60-3000	"	2.19
Campbell	0.2-1.0	Alumina, metal, glass	2 to 6	Air, natural gas	to 3000	"	2.20 2.22
Plautz	$\frac{1}{2}$, $\frac{3}{4}$	Glass	8	Air	100-2000	"	2.24-5
Quinton	0.173	Glass	1.625	Air	30-1100	"	2.26-7
Baddour	0.165-0.282	Alumina, steel, glass aluminium	6	Air	50-1300	"	2.29 2.30
Smith	$\frac{1}{8}$ cyl.	Alumina	2	Air	10-50	"	2.31
Yagi	0.1-1.8	Glass, metal	5	Air, water	to 3500	"	2.32a-c 2.33

TABLE 2

COMPARISON OF CORRELATIONS FOR HEAT TRANSFER COEFFICIENT AT REACTOR WALL

	Conditions	Investigator	Equation 2.36		Re Range	Original Equation
			A	B		
	h_w based on LMTD					
1	Wall-to-bed transfer	Colburn (1931)	0.0175	0.83	180-15000	(2.10)
2	" "	Leva (1947, 1950)	0.0447	0.90	40-3500	(2.11)
3	Bed-to-wall transfer	Leva (1948a, b)	0.2209	0.70	250-3000	(2.14)
	h_w based on Film Temp.					
4	Wall-to-bed transfer	Coberly (1951)	3.6790	0.33	140-1000	(2.17)
5	" "	Campbell (1952)	0.1531	0.47	to 3000	(2.22)
6	" "	Hanratty (1954)	0.2430	0.77	100-2000	(2.23b)
7	" "	Plautz (1955)	0.1838	0.75	100-2000	(2.25)
8	" "	Quinton (1956)	0.1100	1.00	30-1100	(2.27)
9	" "	Yagi (Theor.) *	3.5539	0.50	-	(2.7)
10	" "	Yagi (Expt.) *	0.1800	0.80	to 3500	(2.30)
11	Pipe Flow	McAdams (1954)	0.0182	0.80	> 2100	(2.37)

* Refer to Yagi (1954, 1957, 1959, 1960a, 1960b 1961a, 1961b, 1964)

TABLE 3

Radial Heat Transfer in Packed Beds (D_t = 7 in.) - Data Based Upon Mean Film Temperatures, T_F, and Bulk Temperature, T_B.

Run	(T_w/T_B)	Btu h^{-1} $ft^{-2}(°F)^{-1}$	Nu_F	Nu_B	Re_F	Re_B
(i)	D_p = 0.658 in. (D_t/D_p) = 10.64, and ϵ = 45.0%					
1	0.899	240	616	591	6968	6702
2	0.913	249	648	626	7062	6835
3	0.886	244	621	592	6124	5864
4	0.902	236	593	570	6063	5841
5	0.859	206	498	469	4844	4586
6	0.879	201	482	458	4801	4582
7	0.831	186	426	397	3762	3520
8	0.856	180	411	387	3756	3549
9	0.970	250	794	787	9807	9730
10	0.976	206	654	648	9802	9718
11	0.882	129	354	337	2813	2691
(ii)	D_p = 0.489 in. (D_t/D_p) = 14.31, and ϵ = 36.9%					
12	0.976	186	440	437	6141	6090
13	0.981	200	473	470	6129	6088
14	0.976	250	592	586	5632	5585
15	0.982	212	497	494	5578	5542
16	0.968	216	506	500	4615	4564
17	0.975	177	414	410	4609	4569
18	0.968	206	483	478	3781	3738
19	0.969	150	348	344	3735	3695
20	0.898	226	414	397	4047	3892
21	0.929	233	423	411	4016	3910
22	0.882	141	287	274	2088	1997
23	0.918	130	260	252	2066	2003

TABLE 4

Radial Heat Transfer in Packed Beds (D_t = 7 in.)

Correlation of Data : Nu = A Re^B (S.D. = standard deviation)

	Data Based Upon Mean Film Temperature $T_F = \frac{1}{2}(T_W + T_B)$			Data Based Upon Bulk Temperature T_B		
	D_p=0.658 in	D_p=0.489 in	Overall	D_p=0.658 in	D_p=0.489 in	Overall
No. of runs	11	12	23	11	12	23
A	3.22	3.81	2.49	2.72	3.34	2.35
B	0.594	0.564	0.619	0.611	0.579	0.625
S.D.%	8.0	14.1	11.5	8.1	14.3	11.4

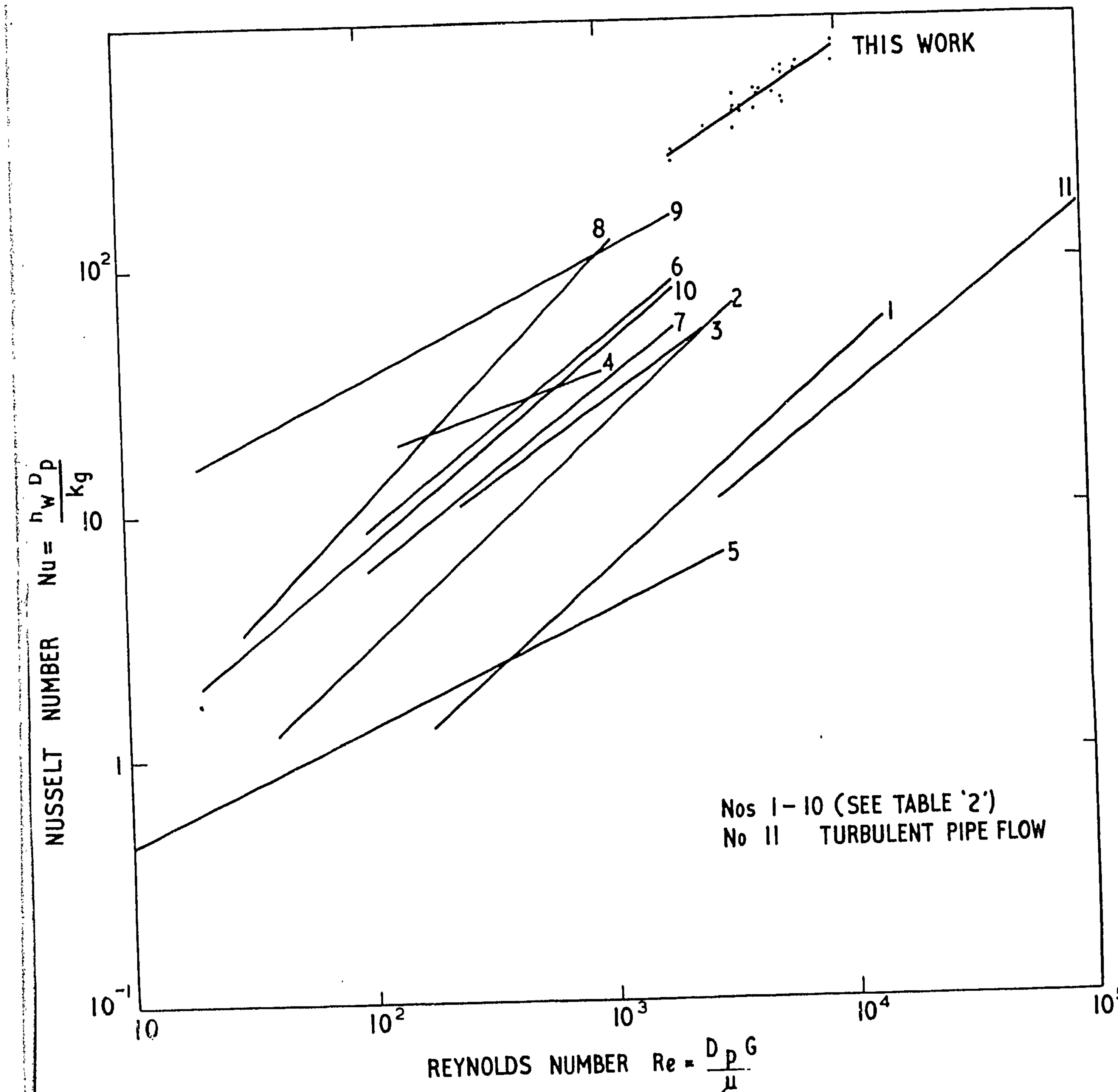

FIGURE 1. HEAT TRANSFER AT THE WALL FOR RANDOMLY PACKED BEDS
(Results Based on Mean Temperature)

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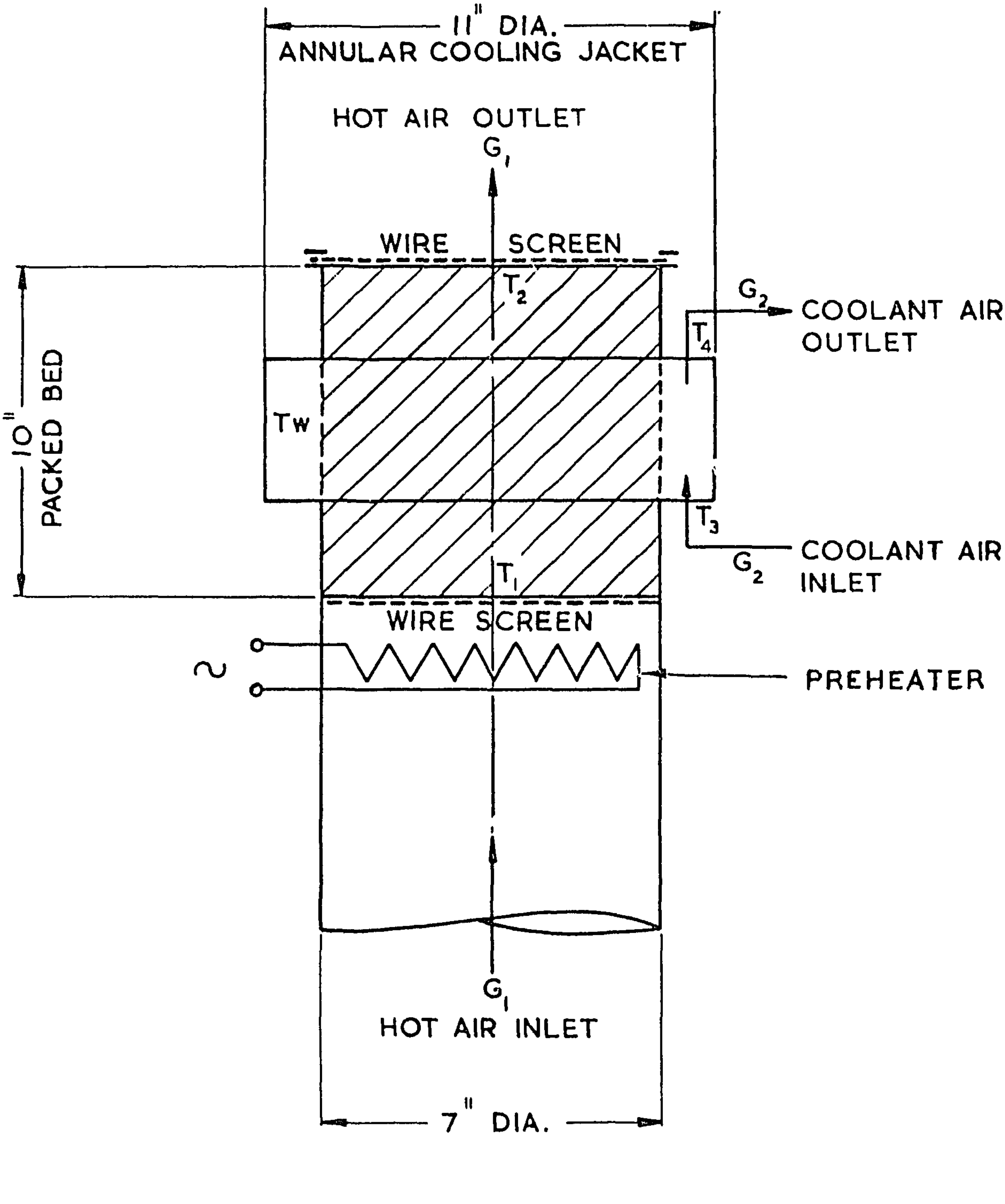

FIGURE 2. SCHEMATIC DIAGRAM OF EXPERIMENTAL RIG USED TO STUDY RADIAL HEAT TRANSFER IN PACKED BEDS

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