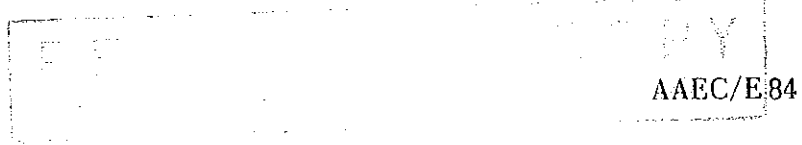


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AUSTRALIAN ATOMIC ENERGY COMMISSION  
RESEARCH ESTABLISHMENT  
LUCAS HEIGHTS

A TABULATED FUNCTION ASSOCIATED WITH DOPPLER BROADENING  
FOR NON-UNIFORM SPATIAL TEMPERATURE DISTRIBUTIONS

\*

by

A. REICHEL \*

\* Attached from the University of New South Wales

Issued Sydney, March 1962



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ABSTRACT

A table to five decimal places is given for the function

$$X(x, \xi, \tau) = \frac{\xi}{2} \sqrt{\tau+1} \int_0^{\infty} e^{-p^2} \left\{ \tan^{-1}\left(x + \frac{2p}{\xi}\right) - \tan^{-1}\left(x - \frac{2p}{\xi}\right) \right\} \operatorname{erf} \frac{p}{\sqrt{\tau}} dp$$

A function of this form arises in consideration of effective resonance line shapes, when, in heterogeneous reactor systems, a parabolic temperature distribution exists across the fuel lumps.

The parameter ranges are

$x = 0 (5) 60 (10) 200$ ;  $\xi = 0.05 (0.01) 0.1 (0.1) 0.5, 1$

and  $\tau = 1, 1.5, 2, 5, 10, 50, 500$ .

\* Attached from the University of New South Wales



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## 1. INTRODUCTION

Associated with Breit-Wigner resonances there is the Doppler-broadened line shape function

$$\psi(x, \xi) = \frac{\xi}{2\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-\frac{\xi^2}{4}(x-y)^2} \frac{dy}{1+y^2}$$

$$\text{where } \xi = \Gamma \left\{ \frac{M}{4E_r k T} \right\}^{1/2}$$

$$\text{and } x = \frac{E_r - E}{\Gamma/2}$$

employing the usual notation.

The average of  $\psi(x, \xi)$  over an undeflected neutron path in a fuel lump in which there is a parabolic temperature distribution is given by the function

$$X(x, \xi_m, \tau) = \frac{\xi_m}{2} \sqrt{\tau+1} \int_0^{\infty} e^{-p^2} \left\{ \tan^{-1} \left( x + \frac{2p}{\xi_m} \right) - \tan^{-1} \left( x - \frac{2p}{\xi_m} \right) \right\} \operatorname{erf} \frac{p}{\sqrt{\tau}} dp$$

where  $\xi_m$  is a special value of  $\xi$  and  $\tau$  is related to the temperature gradient in the fuel lump. The use of these parameters is explained in Section 3.

Some properties of the function  $X(x, \xi, \tau)$  have been discussed by Keane (1958) and the need for computation was suggested by Reichel and Keane (1961). The present tables have been used by Reichel (1961) in the study of the related function

$$G(\beta, \xi_m, \tau) = \int_0^{\alpha} \frac{X(x, \xi_m, \tau)}{X(x, \xi_m, \tau) + \beta} dx$$

The special case when  $\tau = \infty$ , reduces the function  $X(x, \xi, \tau)$  to the function  $\psi(x, \xi)$ . This latter function has been tabulated extensively by Rose et al. (1954).

## 2. THE COMPUTATION

A table to 5 decimal places is appended for the function  $X(x, \xi, \tau)$  for values of  $x$  in the range 0(5) 60(10) 200, for values of  $\xi$  in the range 0.05(0.01) 0.1(0.1) 0.5, 1, and for  $\tau = 1, 1.5, 2, 5, 10, 50, 500$ .

The computation was carried out on UTECOM, the English Electric Deuce computer at the University of New South Wales.

Simpson's Rule was used to evaluate the integral over the range 0 to 3.9375 using 169 ordinates at  $0 \left( \frac{3}{128} \right) \frac{168 \times 3}{128}$ . The truncation error is less than  $10^{-8}$ .

The various approximations used in computing the functions in the integrand are as follows:

$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du = 1 - \left[ \sum_{i=0}^6 a_i x^i \right]^{-16}$$

with

$$\begin{aligned} a_0 &= 1 \\ a_1 &= 0.07052\ 30784 & a_4 &= 0.00015\ 20143 \\ a_2 &= 0.04228\ 20123 & a_5 &= 0.00027\ 65672 \\ a_3 &= 0.00927\ 05272 & a_6 &= 0.00004\ 30638 \end{aligned}$$

$$e^{-x} = \sum_{i=0}^6 a_i x^i \quad \text{with}$$

$$\begin{aligned} a_0 &= 1 \\ a_1 &= 0.2499\ 86842 & a_4 &= 0.00017\ 15620 \\ a_2 &= 0.03125\ 75832 & a_5 &= 0.00000\ 54302 \\ a_3 &= 0.00259\ 13712 & a_6 &= 0.00000\ 06906 \end{aligned}$$

$$\arctan X = \frac{\pi}{4} + \sum_{i=0}^6 C_{2i+1} \left( \frac{X-1}{X+1} \right)^{2i+1} \quad \text{in } 0 \leq X \leq \infty$$

with

$$\begin{aligned} C_1 &= 0.99999\ 6115 & C_9 &= 0.07962\ 6318 \\ C_3 &= -0.33317\ 3758 & C_{11} &= -0.03360\ 6269 \\ C_5 &= 0.19807\ 8690 & C_{13} &= 0.00681\ 2411 \\ C_7 &= -0.13233\ 5096 \end{aligned}$$

In computing the difference of the two arctangents,

$$\text{let } \tan^{-1} \frac{4p}{1+x^2 - \frac{4p^2}{\xi^2}} = \tan^{-1} X,$$

$$\text{and } T = \tan^{-1} \left( x + \frac{2p}{\xi} \right) - \tan^{-1} \left( x - \frac{2p}{\xi} \right).$$

Then for

$$1+x^2 - \frac{4p^2}{\xi^2} = 0, \quad T = \pi/2$$

$$1+x^2 - \frac{4p^2}{\xi^2} < 0, \quad T = \pi - \tan^{-1} |X|$$

$$1+x^2 - \frac{4p^2}{\xi^2} > 0, \quad T = \tan^{-1} X.$$

In computing  $\tan^{-1} X$ , floating point arithmetic was used. The machine worked to an accuracy of more than 6 decimal digits. In the tables the fifth decimal digit is accurate. To this accuracy the function  $X(x, \xi, \tau)$  reaches its asymptotic value  $X = \frac{1}{1+x^2}$  for  $x$  in excess of 180.

### 3. USE OF THE TABLES IN REACTOR THEORY

The function can be used for the three fuel lump geometries; slabs, spheres, and cylinders. The quantity  $\xi_m$  is the value of  $\xi$  corresponding to the maximum temperature on an undeflected neutron path within the fuel lump and  $\tau$  is a function of the surface and central temperatures, the dimensions of the lump and the length of the neutron path. Details are as follows:

(a) Slabs. The temperature is assumed to vary parabolically along a line normal to the slab faces from  $T_s$  on the surface to  $T_c$  on the central plane of the slab. In this case, on any undeflected neutron path in the slab,

$$\tau = \frac{T_s}{T_c - T_s} \quad .$$

$$\xi_m = \frac{2C}{(\tau+1)^{1/2}} \quad ,$$

$$\text{where } C = \frac{A}{2(T_c - T_s)^{1/2}} \quad \text{and } A = \Gamma \left( \frac{M}{4 E_r k} \right)^{1/2} \quad .$$

(b) Spheres. The temperature is assumed to vary parabolically along a diameter, from  $T_s$  at the surface to  $T_c$  at the centre of the sphere. In this case

$$\tau = \frac{4a^2}{L^2} \cdot \frac{T_s}{T_c - T_s} \quad ,$$

where  $a$  is the radius of the sphere and  $L$  is the length of an undeflected neutron path.

$$\xi_m = \frac{2C}{(\tau+1)^{1/2}} \quad ,$$

$$\text{where } C = \frac{A a}{(T_c - T_s)^{1/2} L} \quad \text{and } A = \Gamma \left( \frac{M}{4 E_r k} \right)^{1/2} \quad .$$

(c) Cylinders. The temperature is assumed to vary parabolically along any normal to the side surface of the cylinder from  $T_s$  on the surface to  $T_c$  on the axis.

$$\tau = \frac{4a^2}{Y^2} \cdot \frac{T_s}{T_c - T_s} \quad ,$$

where  $a$  is the radius of the cylinder and  $Y$  is the length of the projection of an undeflected neutron path onto a plane normal to the axis of the cylinder.

$$\xi_m = \frac{2C}{(\tau+1)^{1/2}}$$

$$\text{where } C = \frac{A a}{(T_c - T_s)^{1/2} Y} \quad \text{and } A = \Gamma \left\{ \frac{M}{4 E_r k} \right\}^{1/2}$$

### 4. ACKNOWLEDGEMENT

Grateful acknowledgement is made to the Utecom staff, particularly to Miss Judith Campbell who devised an elegant machine programme, economical of machine time.

The project was suggested by Dr. A. Keane.

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Vol II.

APPENDIX

THE TABULATION

$\tau$	$x$	$\xi=0.05$	$\xi=0.06$
1.0	0	0.04769	0.05688
1.0	5	0.04678	0.05534
1.0	10	0.04417	0.05097
1.0	15	0.04014	0.04447
1.0	20	0.03514	0.03681
1.0	25	0.02967	0.02897
1.0	30	0.02417	0.02171
1.0	35	0.01902	0.01554
1.0	40	0.01451	0.01066
1.0	45	0.01071	0.00704
1.0	50	0.00769	0.00449
1.0	55	0.00538	0.00280
1.0	60	0.00367	0.00171
1.0	70	0.00162	0.00065
1.0	80	0.00070	0.00029
1.0	90	0.00032	0.00017
1.0	100	0.00017	0.00012
1.0	110	0.00011	0.00009
1.0	120	0.00008	0.00007
1.0	130	0.00006	0.00006
1.0	140	0.00005	0.00005
1.0	150	0.00004	0.00004
1.0	160	0.00004	0.00004
1.0	170	0.00003	0.00003
1.0	180	0.00003	0.00003
1.0	190	0.00002	0.00002
1.0	200	0.00002	0.00002

x	$\xi = 0.07$	$\xi = 0.08$	$\xi = 0.09$
0	0.06595	0.07491	0.08376
5	0.06354	0.07138	0.07883
10	0.05686	0.06182	0.06581
15	0.04733	0.04878	0.04895
20	0.03673	0.03522	0.03266
25	0.02668	0.02342	0.01975
30	0.01821	0.01444	0.01097
35	0.01174	0.00836	0.00572
40	0.00720	0.00461	0.00289
45	0.00425	0.00248	0.00149
50	0.00245	0.00134	0.00083
55	0.00140	0.00077	0.00052
60	0.00082	0.00049	0.00037
70	0.00034	0.00026	0.00024
80	0.00020	0.00018	0.00017
90	0.00014	0.00013	0.00013
100	0.00011	0.00010	0.00010
110	0.00009	0.00008	0.00008
120	0.00007	0.00007	0.00007
130	0.00006	0.00006	0.00006
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00004	0.00004	0.00004
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

x	$\xi=0.10$	$\xi=0.20$	$\xi=0.30$
0	0.09250	0.17424	0.24669
5	0.08587	0.13222	0.13939
10	0.06886	0.06024	0.03215
15	0.04801	0.01895	0.00706
20	0.02943	0.00557	0.00298
25	0.01612	0.00227	0.00176
30	0.00809	0.00133	0.00118
35	0.00387	0.00091	0.00085
40	0.00187	0.00068	0.00064
45	0.00099	0.00052	0.00050
50	0.00061	0.00042	0.00040
55	0.00043	0.00034	0.00033
60	0.00033	0.00028	0.00028
70	0.00023	0.00020	0.00020
80	0.00017	0.00015	0.00015
90	0.00013	0.00012	0.00012
100	0.00010	0.00010	0.00010
110	0.00008	0.00008	0.00008
120	0.00007	0.00007	0.00006
130	0.00006	0.00005	0.00005
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00003	0.00003	0.00003
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

$x$	$\xi=0.40$	$\xi=0.50$	$\xi=1.00$
0	0.31108	0.36846	0.57756
5	0.12380	0.10168	0.04882
10	0.01792	0.01326	0.01043
15	0.00527	0.00489	0.00452
20	0.00272	0.00262	0.00252
25	0.00168	0.00165	0.00161
30	0.00115	0.00113	0.00111
35	0.00083	0.00082	0.00081
40	0.00063	0.00063	0.00062
45	0.00050	0.00049	0.00049
50	0.00040	0.00040	0.00040
55	0.00033	0.00033	0.00033
60	0.00028	0.00027	0.00027
70	0.00020	0.00020	0.00020
80	0.00015	0.00015	0.00015
90	0.00012	0.00012	0.00012
100	0.00010	0.00010	0.00009
110	0.00008	0.00008	0.00008
120	0.00006	0.00006	0.00006
130	0.00005	0.00005	0.00005
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00003	0.00003	0.00003
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

$\tau$	$x$	$\xi=0.05$	$\xi=0.06$
1.5	0	0.04653	0.05550
1.5	5	0.04569	0.05409
1.5	10	0.04330	0.05006
1.5	15	0.03957	0.04403
1.5	20	0.03490	0.03682
1.5	25	0.02974	0.02932
1.5	30	0.02447	0.02226
1.5	35	0.01947	0.01613
1.5	40	0.01501	0.01120
1.5	45	0.01120	0.00747
1.5	50	0.00811	0.00481
1.5	55	0.00572	0.00301
1.5	60	0.00393	0.00185
1.5	70	0.00175	0.00070
1.5	80	0.00075	0.00030
1.5	90	0.00034	0.00017
1.5	100	0.00018	0.00012
1.5	110	0.00011	0.00009
1.5	120	0.00008	0.00007
1.5	130	0.00006	0.00006
1.5	140	0.00005	0.00005
1.5	150	0.00004	0.00004
1.5	160	0.00004	0.00004
1.5	170	0.00003	0.00003
1.5	180	0.00003	0.00003
1.5	190	0.00002	0.00002
1.5	200	0.00002	0.00002

x	$\xi=0.07$	$\xi=0.08$	$\xi=0.09$
0	0.06437	0.07312	0.08177
5	0.06215	0.06988	0.07724
10	0.05599	0.06103	0.06516
15	0.04709	0.04877	0.04921
20	0.03702	0.03578	0.03343
25	0.02727	0.02418	0.02057
30	0.01889	0.01514	0.01160
35	0.01234	0.00887	0.00611
40	0.00766	0.00493	0.00309
45	0.00455	0.00265	0.00158
50	0.00263	0.00143	0.00087
55	0.00150	0.00081	0.00054
60	0.00087	0.00050	0.00038
70	0.00036	0.00026	0.00024
80	0.00020	0.00018	0.00017
90	0.00014	0.00013	0.00013
100	0.00011	0.00010	0.00010
110	0.00009	0.00008	0.00008
120	0.00007	0.00007	0.00007
130	0.00006	0.00006	0.00006
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00004	0.00004	0.00004
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

x	$\xi=0.10$	$\xi=0.20$	$\xi=0.30$
0	0.09032	0.17042	0.24165
5	0.08422	0.13128	0.14033
10	0.06838	0.06181	0.03351
15	0.04854	0.01991	0.00725
20	0.03036	0.00582	0.00301
25	0.01693	0.00232	0.00177
30	0.00860	0.00134	0.00119
35	0.00412	0.00092	0.00085
40	0.00198	0.00068	0.00064
45	0.00104	0.00052	0.00050
50	0.00063	0.00042	0.00040
55	0.00044	0.00034	0.00033
60	0.00033	0.00028	0.00028
70	0.00023	0.00020	0.00020
80	0.00017	0.00015	0.00015
90	0.00013	0.00012	0.00012
100	0.00010	0.00010	0.00010
110	0.00008	0.00008	0.00008
120	0.00007	0.00007	0.00006
130	0.00006	0.00005	0.00005
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00003	0.00003	0.00003
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

x	$\xi=0.40$	$\xi=0.50$	$\xi=1.00$
0	0.30515	0.36189	0.57010
5	0.12608	0.10425	0.04935
10	0.01846	0.01345	0.01045
15	0.00532	0.00491	0.00453
20	0.00273	0.00263	0.00252
25	0.00168	0.00165	0.00161
30	0.00115	0.00113	0.00111
35	0.00083	0.00082	0.00081
40	0.00063	0.00063	0.00062
45	0.00050	0.00049	0.00049
50	0.00040	0.00040	0.00040
55	0.00033	0.00033	0.00033
60	0.00028	0.00027	0.00027
70	0.00020	0.00020	0.00020
80	0.00015	0.00015	0.00015
90	0.00012	0.00012	0.00012
100	0.00010	0.00010	0.00009
110	0.00008	0.00008	0.00008
120	0.00006	0.00006	0.00006
130	0.00005	0.00005	0.00005
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00003	0.00003	0.00003
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

$\tau$	$x$	$\xi=0.05$	$\xi=0.06$
2.0	0	0.04584	0.05469
2.0	5	0.04505	0.05334
2.0	10	0.04276	0.04850
2.0	15	0.03920	0.04373
2.0	20	0.03472	0.03678
2.0	25	0.02974	0.02949
2.0	30	0.02462	0.02256
2.0	35	0.01971	0.01649
2.0	40	0.01530	0.01153
2.0	45	0.01150	0.00775
2.0	50	0.00838	0.00502
2.0	55	0.00595	0.00316
2.0	60	0.00411	0.00194
2.0	70	0.00184	0.00073
2.0	80	0.00079	0.00031
2.0	90	0.00035	0.00017
2.0	100	0.00018	0.00012
2.0	110	0.00011	0.00009
2.0	120	0.00008	0.00007
2.0	130	0.00006	0.00006
2.0	140	0.00005	0.00005
2.0	150	0.00004	0.00004
2.0	160	0.00004	0.00004
2.0	170	0.00003	0.00003
2.0	180	0.00002	0.00002
2.0	190	0.00002	0.00002
2.0	200	0.00002	0.00002

x	$\xi=0.07$	$\xi=0.08$	$\xi=0.09$
0	0.06342	0.07206	0.08059
5	0.06132	0.06898	0.07628
10	0.05544	0.06052	0.06472
15	0.04689	0.04872	0.04931
20	0.03714	0.03606	0.03387
25	0.02760	0.02462	0.02108
30	0.01929	0.01558	0.01202
35	0.01272	0.00921	0.00637
40	0.00796	0.00515	0.00324
45	0.00476	0.00278	0.00165
50	0.00276	0.00150	0.00090
55	0.00158	0.00084	0.00055
60	0.00091	0.00052	0.00038
70	0.00037	0.00027	0.00024
80	0.00020	0.00018	0.00017
90	0.00014	0.00013	0.00013
100	0.00011	0.00010	0.00010
110	0.00009	0.00008	0.00008
120	0.00007	0.00007	0.00007
130	0.00006	0.00006	0.00006
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00004	0.00004	0.00004
170	0.00003	0.00003	0.00003
180	0.00002	0.00002	0.00002
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

x	$\xi=0.10$	$\xi=0.20$	$\xi=0.30$
0	0.08903	0.16814	0.23862
5	0.08322	0.13061	0.14077
10	0.06803	0.06274	0.03442
15	0.04880	0.02057	0.00738
20	0.03090	0.00600	0.00302
25	0.01745	0.00236	0.00178
30	0.00895	0.00135	0.00119
35	0.00431	0.00092	0.00085
40	0.00207	0.00068	0.00064
45	0.00107	0.00053	0.00050
50	0.00064	0.00042	0.00040
55	0.00044	0.00034	0.00033
60	0.00034	0.00028	0.00028
70	0.00023	0.00020	0.00020
80	0.00017	0.00015	0.00015
90	0.00013	0.00012	0.00012
100	0.00010	0.00010	0.00010
110	0.00008	0.00008	0.00008
120	0.00007	0.00007	0.00006
130	0.00006	0.00005	0.00005
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00003	0.00003	0.00003
170	0.00003	0.00003	0.00003
180	0.00002	0.00002	0.00002
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

$x$	$\xi=0.40$	$\xi=0.50$	$\xi=1.00$
0	0.30156	0.35791	0.56549
5	0.12741	0.10586	0.04973
10	0.01885	0.01359	0.01046
15	0.00535	0.00492	0.00453
20	0.00273	0.00263	0.00252
25	0.00169	0.00165	0.00161
30	0.00115	0.00113	0.00111
35	0.00083	0.00083	0.00081
40	0.00063	0.00063	0.00062
45	0.00050	0.00049	0.00049
50	0.00040	0.00040	0.00040
55	0.00033	0.00033	0.00033
60	0.00028	0.00027	0.00027
70	0.00020	0.00020	0.00020
80	0.00015	0.00015	0.00015
90	0.00012	0.00012	0.00012
100	0.00010	0.00010	0.00010
110	0.00008	0.00008	0.00008
120	0.00006	0.00006	0.00006
130	0.00005	0.00005	0.00005
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00003	0.00003	0.00003
170	0.00003	0.00003	0.00003
180	0.00002	0.00002	0.00002
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

$\tau$	$x$	$\xi=0.05$	$\xi=0.06$
5.0	0	0.04434	0.05291
5.0	5	0.04363	0.05170
5.0	10	0.04157	0.04824
5.0	15	0.03835	0.04299
5.0	20	0.03426	0.03659
5.0	25	0.02966	0.02977
5.0	30	0.02486	0.02316
5.0	35	0.02019	0.01725
5.0	40	0.01591	0.01230
5.0	45	0.01215	0.00843
5.0	50	0.00901	0.00556
5.0	55	0.00651	0.00356
5.0	60	0.00457	0.00221
5.0	70	0.00210	0.00084
5.0	80	0.00091	0.00035
5.0	90	0.00040	0.00018
5.0	100	0.00020	0.00012
5.0	110	0.00012	0.00009
5.0	120	0.00008	0.00007
5.0	130	0.00007	0.00006
5.0	140	0.00005	0.00005
5.0	150	0.00004	0.00004
5.0	160	0.00004	0.00004
5.0	170	0.00003	0.00003
5.0	180	0.00003	0.00003
5.0	190	0.00002	0.00002
5.0	200	0.00002	0.00002

x	$\xi=0.07$	$\xi=0.08$	$\xi=0.09$
0	0.06138	0.06975	0.07803
5	0.05949	0.06698	0.07414
10	0.05417	0.05931	0.06363
15	0.04636	0.04846	0.04938
20	0.03730	0.03658	0.03472
25	0.02824	0.02556	0.02220
30	0.02016	0.01658	0.01301
35	0.01360	0.01005	0.00707
40	0.00869	0.00574	0.00364
45	0.00531	0.00314	0.00186
50	0.00312	0.00170	0.00099
55	0.00180	0.00094	0.00059
60	0.00104	0.00056	0.00040
70	0.00040	0.00028	0.00024
80	0.00021	0.00018	0.00017
90	0.00015	0.00014	0.00013
100	0.00011	0.00011	0.00010
110	0.00009	0.00008	0.00008
120	0.00007	0.00007	0.00007
130	0.00006	0.00006	0.00006
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00004	0.00004	0.00004
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

x	$\xi=0.10$	$\xi=0.20$	$\xi=0.30$
0	0.08621	0.16314	0.23194
5	0.08097	0.12889	0.14138
10	0.06713	0.06466	0.03667
15	0.04922	0.02219	0.00778
20	0.03203	0.00652	0.00307
25	0.01863	0.00246	0.00179
30	0.00983	0.00137	0.00119
35	0.00483	0.00093	0.00086
40	0.00232	0.00069	0.00065
45	0.00118	0.00053	0.00050
50	0.00068	0.00042	0.00041
55	0.00046	0.00034	0.00033
60	0.00034	0.00028	0.00028
70	0.00023	0.00021	0.00020
80	0.00017	0.00015	0.00015
90	0.00013	0.00012	0.00012
100	0.00010	0.00010	0.00010
110	0.00008	0.00008	0.00008
120	0.00007	0.00007	0.00006
130	0.00006	0.00005	0.00005
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00003	0.00003	0.00003
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

x	$\xi=0.40$	$\xi=0.50$	$\xi=1.00$
0	0.29362	0.34903	0.55498
5	0.13017	0.10953	0.05072
10	0.01992	0.01399	0.01050
15	0.00544	0.00496	0.00453
20	0.00275	0.00264	0.00252
25	0.00169	0.00165	0.00161
30	0.00115	0.00113	0.00111
35	0.00084	0.00083	0.00081
40	0.00063	0.00063	0.00062
45	0.00050	0.00049	0.00049
50	0.00040	0.00040	0.00040
55	0.00033	0.00033	0.00033
60	0.00028	0.00027	0.00027
70	0.00020	0.00020	0.00020
80	0.00015	0.00015	0.00015
90	0.00012	0.00012	0.00012
100	0.00010	0.00010	0.00009
110	0.00008	0.00008	0.00008
120	0.00006	0.00006	0.00006
130	0.00005	0.00005	0.00005
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00003	0.00003	0.00003
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

$\tau$	$x$	$\xi=0.05$	$\xi=0.06$
10.0	0	0.04375	0.05220
10.0	5	0.04306	0.05104
10.0	10	0.04109	0.04772
10.0	15	0.03799	0.04266
10.0	20	0.03404	0.03648
10.0	25	0.02959	0.02984
10.0	30	0.02492	0.02338
10.0	35	0.02035	0.01754
10.0	40	0.01614	0.01262
10.0	45	0.01242	0.00872
10.0	50	0.00927	0.00581
10.0	55	0.00675	0.00374
10.0	60	0.00477	0.00235
10.0	70	0.00223	0.00089
10.0	80	0.00098	0.00037
10.0	90	0.00043	0.00019
10.0	100	0.00021	0.00012
10.0	110	0.00012	0.00009
10.0	120	0.00009	0.00007
10.0	130	0.00007	0.00006
10.0	140	0.00005	0.00005
10.0	150	0.00005	0.00004
10.0	160	0.00004	0.00004
10.0	170	0.00003	0.00003
10.0	180	0.00003	0.00003
10.0	190	0.00002	0.00002
10.0	200	0.00002	0.00002

x	$\xi=0.07$	$\xi=0.08$	$\xi=0.09$
0	0.06056	0.06883	0.07700
5	0.05875	0.06617	0.07327
10	0.05364	0.05879	0.06315
15	0.04610	0.04831	0.04935
20	0.03731	0.03674	0.03502
25	0.02847	0.02591	0.02265
30	0.02050	0.01699	0.01343
35	0.01396	0.01041	0.00738
40	0.00901	0.00601	0.00384
45	0.00556	0.00332	0.00197
50	0.00330	0.00180	0.00104
55	0.00191	0.00100	0.00061
60	0.00110	0.00059	0.00041
70	0.00042	0.00028	0.00024
80	0.00022	0.00018	0.00017
90	0.00015	0.00014	0.00013
100	0.00011	0.00011	0.00010
110	0.00009	0.00008	0.00008
120	0.00007	0.00007	0.00007
130	0.00006	0.00006	0.00006
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00004	0.00004	0.00004
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

x	$\xi=0.10$	$\xi=0.20$	$\xi=0.30$
0	0.08508	0.16113	0.22924
5	0.08005	0.12811	0.14150
10	0.06671	0.06538	0.03766
15	0.04932	0.02291	0.00799
20	0.03245	0.00679	0.00310
25	0.01912	0.00252	0.00180
30	0.01022	0.00139	0.00120
35	0.00507	0.00094	0.00086
40	0.00245	0.00069	0.00065
45	0.00124	0.00053	0.00051
50	0.00070	0.00042	0.00041
55	0.00047	0.00034	0.00033
60	0.00035	0.00028	0.00028
70	0.00023	0.00021	0.00020
80	0.00017	0.00015	0.00015
90	0.00013	0.00012	0.00012
100	0.00010	0.00010	0.00010
110	0.00008	0.00008	0.00008
120	0.00007	0.00007	0.00006
130	0.00006	0.00005	0.00005
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00003	0.00003	0.00003
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

x	$\xi=0.40$	$\xi=0.50$	$\xi=1.00$
0	0.29039	0.34541	0.55060
5	0.13120	0.11102	0.05119
10	0.02045	0.01420	0.01052
15	0.00548	0.00498	0.00454
20	0.00276	0.00265	0.00253
25	0.00170	0.00166	0.00161
30	0.00115	0.00113	0.00111
35	0.00084	0.00083	0.00081
40	0.00063	0.00063	0.00062
45	0.00050	0.00049	0.00049
50	0.00040	0.00040	0.00040
55	0.00033	0.00033	0.00033
60	0.00028	0.00027	0.00027
70	0.00020	0.00020	0.00020
80	0.00015	0.00015	0.00015
90	0.00012	0.00012	0.00012
100	0.00010	0.00010	0.00009
110	0.00008	0.00008	0.00008
120	0.00006	0.00006	0.00006
130	0.00005	0.00005	0.00005
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00003	0.00003	0.00003
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

$\tau$	$x$	$\xi=0.05$	$\xi=0.06$
50.0	0	0.04322	0.05158
50.0	5	0.04256	0.05046
50.0	10	0.04066	0.04726
50.0	15	0.03767	0.04236
50.0	20	0.03384	0.03636
50.0	25	0.02952	0.02989
50.0	30	0.02496	0.02355
50.0	35	0.02049	0.01779
50.0	40	0.01634	0.01289
50.0	45	0.01264	0.00898
50.0	50	0.00951	0.00603
50.0	55	0.00696	0.00392
50.0	60	0.00496	0.00248
50.0	70	0.00235	0.00095
50.0	80	0.00104	0.00039
50.0	90	0.00046	0.00020
50.0	100	0.00022	0.00013
50.0	110	0.00013	0.00009
50.0	120	0.00009	0.00007
50.0	130	0.00007	0.00006
50.0	140	0.00005	0.00005
50.0	150	0.00005	0.00004
50.0	160	0.00004	0.00004
50.0	170	0.00003	0.00003
50.0	180	0.00003	0.00003
50.0	190	0.00002	0.00002
50.0	200	0.00002	0.00002

x	$\xi=0.07$	$\xi=0.08$	$\xi=0.09$
0	0.05984	0.06802	0.07610
5	0.05810	0.06545	0.07250
10	0.05316	0.05832	0.06271
15	0.04586	0.04815	0.04930
20	0.03731	0.03686	0.03527
25	0.02864	0.02621	0.02303
30	0.02078	0.01735	0.01381
35	0.01427	0.01074	0.00768
40	0.00931	0.00627	0.00404
45	0.00579	0.00349	0.00208
50	0.00346	0.00191	0.00110
55	0.00202	0.00106	0.00064
60	0.00117	0.00062	0.00042
70	0.00044	0.00029	0.00025
80	0.00022	0.00019	0.00017
90	0.00015	0.00014	0.00013
100	0.00011	0.00011	0.00010
110	0.00009	0.00009	0.00008
120	0.00007	0.00007	0.00007
130	0.00006	0.00006	0.00006
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00004	0.00004	0.00004
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

x	$\xi=0.10$	$\xi=0.20$	$\xi=0.30$
0	0.08409	0.15936	0.22686
5	0.07924	0.12738	0.14153
10	0.06633	0.06598	0.03856
15	0.04939	0.02358	0.00820
20	0.03281	0.00705	0.00313
25	0.01956	0.00258	0.00180
30	0.01058	0.00140	0.00120
35	0.00531	0.00094	0.00086
40	0.00258	0.00069	0.00065
45	0.00130	0.00053	0.00051
50	0.00073	0.00042	0.00041
55	0.00048	0.00034	0.00033
60	0.00035	0.00029	0.00028
70	0.00023	0.00021	0.00020
80	0.00017	0.00016	0.00015
90	0.00013	0.00012	0.00012
100	0.00010	0.00010	0.00010
110	0.00008	0.00008	0.00008
120	0.00007	0.00007	0.00006
130	0.00006	0.00005	0.00005
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00004	0.00003	0.00003
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

x	$\xi=0.40$	$\xi=0.50$	$\xi=1.00$
0	0.28754	0.34220	0.54668
5	0.13206	0.11234	0.05165
10	0.02096	0.01440	0.01054
15	0.00552	0.00499	0.00454
20	0.00277	0.00265	0.00253
25	0.00170	0.00166	0.00161
30	0.00115	0.00114	0.00111
35	0.00084	0.00083	0.00081
40	0.00063	0.00063	0.00062
45	0.00050	0.00049	0.00049
50	0.00040	0.00040	0.00040
55	0.00033	0.00033	0.00033
60	0.00028	0.00027	0.00027
70	0.00020	0.00020	0.00020
80	0.00015	0.00015	0.00015
90	0.00012	0.00012	0.00012
100	0.00010	0.00010	0.00009
110	0.00008	0.00008	0.00008
120	0.00006	0.00006	0.00006
130	0.00005	0.00005	0.00005
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00003	0.00003	0.00003
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

$\tau$	$x$	$\xi=0.05$	$\xi=0.06$
500.0	0	0.04297	0.05128
500.0	5	0.04232	0.05018
500.0	10	0.04044	0.04700
500.0	15	0.03748	0.04216
500.0	20	0.03369	0.03621
500.0	25	0.02940	0.02980
500.0	30	0.02488	0.02350
500.0	35	0.02044	0.01777
500.0	40	0.01632	0.01290
500.0	45	0.01264	0.00900
500.0	50	0.00952	0.00606
500.0	55	0.00698	0.00394
500.0	60	0.00498	0.00250
500.0	70	0.00237	0.00096
500.0	80	0.00105	0.00039
500.0	90	0.00046	0.00020
500.0	100	0.00022	0.00013
500.0	110	0.00013	0.00009
500.0	120	0.00009	0.00007
500.0	130	0.00007	0.00006
500.0	140	0.00005	0.00005
500.0	150	0.00005	0.00004
500.0	160	0.00004	0.00004
500.0	170	0.00003	0.00003
500.0	180	0.00003	0.00003
500.0	190	0.00002	0.00002
500.0	200	0.00002	0.00002

x	$\xi = 0.07$	$\xi = 0.08$	$\xi = 0.09$
0	0.05950	0.06763	0.07566
5	0.05777	0.06509	0.07210
10	0.05288	0.05803	0.06241
15	0.04565	0.04795	0.04911
20	0.03717	0.03675	0.03519
25	0.02858	0.02617	0.02303
30	0.02077	0.01735	0.01383
35	0.01429	0.01076	0.00771
40	0.00933	0.00629	0.00406
45	0.00581	0.00352	0.00209
50	0.00349	0.00192	0.00111
55	0.00204	0.00106	0.00064
60	0.00118	0.00062	0.00042
70	0.00044	0.00029	0.00025
80	0.00022	0.00019	0.00017
90	0.00015	0.00014	0.00013
100	0.00011	0.00011	0.00010
110	0.00009	0.00008	0.00008
120	0.00007	0.00007	0.00007
130	0.00006	0.00006	0.00006
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00004	0.00004	0.00004
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

x	$\xi=0.10$	$\xi=0.20$	$\xi=0.30$
0	0.08361	0.15847	0.22562
5	0.07881	0.12680	0.14104
10	0.06602	0.06587	0.03861
15	0.04923	0.02362	0.00821
20	0.03276	0.00708	0.00312
25	0.01957	0.00259	0.00180
30	0.01062	0.00140	0.00119
35	0.00534	0.00094	0.00086
40	0.00260	0.00069	0.00065
45	0.00131	0.00053	0.00050
50	0.00073	0.00042	0.00040
55	0.00048	0.00034	0.00033
60	0.00035	0.00028	0.00028
70	0.00023	0.00020	0.00020
80	0.00017	0.00015	0.00015
90	0.00013	0.00012	0.00012
100	0.00010	0.00010	0.00010
110	0.00008	0.00008	0.00008
120	0.00007	0.00006	0.00006
130	0.00006	0.00005	0.00005
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00003	0.00003	0.00003
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002

$x$	$\xi=0.40$	$\xi=0.50$	$\xi=1.00$
0	0.28600	0.34040	0.54404
5	0.13176	0.11220	0.05157
10	0.02099	0.01439	0.01050
15	0.00551	0.00498	0.00453
20	0.00276	0.00265	0.00252
25	0.00169	0.00165	0.00160
30	0.00115	0.00113	0.00111
35	0.00083	0.00082	0.00081
40	0.00063	0.00063	0.00062
45	0.00050	0.00049	0.00049
50	0.00040	0.00040	0.00039
55	0.00033	0.00033	0.00032
60	0.00027	0.00027	0.00027
70	0.00020	0.00020	0.00020
80	0.00015	0.00015	0.00015
90	0.00012	0.00012	0.00012
100	0.00010	0.00009	0.00009
110	0.00008	0.00008	0.00008
120	0.00006	0.00006	0.00006
130	0.00005	0.00005	0.00005
140	0.00005	0.00005	0.00005
150	0.00004	0.00004	0.00004
160	0.00003	0.00003	0.00003
170	0.00003	0.00003	0.00003
180	0.00003	0.00003	0.00003
190	0.00002	0.00002	0.00002
200	0.00002	0.00002	0.00002