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RESEARCH ESTABLISHMENT

LUCAS HEIGHTS

FUEL ELEMENT TRANSIENT TEMPERATURE STUDIES

by

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* University of New South Wales

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ABSTRACT

A method is presented for the analysis of transient temperatures in a homogeneous circular cylindrical fuel element in a coolant channel with no axial conduction and no heat loss to the channel wall. In addition, some results are obtained for mean fuel element temperatures in power transients for a simpler model, but accounting for details of the axial coolant temperature distribution in the unsteady state.

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CONTENTS

1. INTRODUCTION

In reactor dynamics, the interaction between nuclear and thermal characteristics via temperature coefficients of reactivity, involves a consideration of the transient temperature fluctuations in fuel elements. It is usual to make certain assumptions to simplify this problem e.g. O'Neill (1958). These assumptions are that the mean coolant temperature in a channel is the arithmetic mean of the inlet and outlet temperatures, and that heat transfer from the element to the coolant is proportional to the difference between the mean temperatures of fuel element and coolant, throughout the transient.

To investigate the validity of these assumptions and to suggest modifications if necessary, a more detailed analysis of the unsteady heat flow problem has been attempted. This investigation has also been prompted by the need for fuel element temperature distributions in thermal stress analysis. A complete analysis would take into account fuel element heterogeneity and shape, and the question of heat transfer coefficients under transient conditions.

In the following analysis, a circular, cylindrical, homogeneous, uncanned fuel element is considered. Heat is exchanged between this element and the coolant only. Application to particular systems has not yet been attempted, though further progress will depend on arguments based on orders of magnitude.

2. THEORY

2.1 Fuel Element Temperatures

With temperatures and power density referring to departures from the steady state ($-\infty < t < 0$) the equations to be considered are as follows:

$$\rho S \frac{\partial}{\partial t} T(r,z,t) = \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}T(r,z,t)\right) + H(r,z,t) \tag{1}$$

$$-k\frac{\partial}{\partial r}\Big(T(r,z,t)\Big)_R = h\Big(T(R,z,t) - T^*(z,t)\Big) \tag{2}$$

$$\rho_c S_c A_c\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial z}\right)T^*(z,t) = hP\Big(T(R,z,t) - T^*(z,t)\Big) \tag{3}$$

$$\left(\frac{\partial}{\partial r}T(r,z,t)\right)_{r=o} = 0 \tag{4}$$

$$H(r,z,t) = h(r)f(z)g(t) \tag{5}$$

The homogeneous equation and boundary condition

$$\frac{1}{r}\frac{d}{dr}\left(r\,\frac{dG(r)}{dr}\right) + \mu\frac{\rho S}{k}G(r) = 0 \tag{6}$$

$$k\left(\frac{d}{dr}G(r)\right)_R + h\,G(R) = 0 \tag{7}$$

has solutions

$$G_n(r) = J_0(\sqrt{\mu_n \rho S/k}\; r)$$

with the eigenvalues μ_n determined from (7) i.e.,

$$J_0(\sqrt{\mu_n \rho S/k}\; R) - (k\sqrt{\mu_n \rho S/k}/h)\; J_1(\sqrt{\mu_n \rho S/k}\; R) = 0$$

More conveniently:

$$G_n(r) = J_0(\alpha_n r/R) \tag{8}$$

$$J_0(\alpha_n) = \alpha_n C\, J_1(\alpha_n) \tag{9}$$

where

$$\alpha_n = (\mu_n \rho S R^2/k)^{1/2} = (\mu_n W)^{1/2} \tag{10}$$

$$C = k/hR \tag{11}$$

It is easily shown that these eigenfunctions are orthogonal, with weighting function r, in the range from O to R.

$$\int_0^R r\, G_n(r) G_m(r) dr = R^2 \left(J_0^2(\alpha_n) + J_1^2(\alpha_n) \right) / 2 \quad \text{for } m = n$$
$$= 0 \quad \text{for } m \neq n$$

Multiplying (1) by $rG_n(r)$ and integrating from O to R,

$$\rho S \int_0^R r\, G_n(r) \frac{\partial}{\partial t}\left(T(r,z,t) \right) dr = k \int_0^R G_n(r) \frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r} T(r,z,t) \right) dr$$
$$+ f(z)\, g(t) \int_0^R r\, G_n(r)\, h(r)\, dr \tag{12}$$

The first integral on the right hand side of this equation can be integrated several times by parts.

$$\int_0^R G_n(r) \frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r} T(r,z,t) \right) dr = G_n(R) R \left(\frac{\partial}{\partial r} T(r,z,t) \right)_R$$
$$- \int_0^R r \frac{\partial}{\partial r}\left(T(r,z,t) \right) \frac{d}{dr}\left(G_n(r) \right) dr$$

$$\int_0^R r \frac{\partial}{\partial r}\left(T(r,z,t) \right) \frac{d}{dr}\left(G_n(r) \right) dr = R\, T(R,z,t) \left(\frac{d}{dr}\left(G_n(r) \right) \right)_R$$
$$- \int_0^R T(r,z,t) \frac{d}{dr}\left(r \frac{d}{dr} G_n(r) \right) dr$$

Using (6) and (7) this original integral becomes

$$R\, G_n(R) \left[\left(\frac{\partial}{\partial r} T(r,z,t) \right)_R + \frac{h}{k} T(R,z,t) \right] - \frac{\alpha_n^2}{R^2} \int_0^R G_n(r)\, T(r,z,t)\, r\, dr$$

If now the following transforms are defined,

$$T_n(z,t) = \int_o^R r\, G_n(r)\, T(r,z,t)\, dr \tag{13}$$

$$h_n = \int_o^R r\, G_n(r)\, h(r)\, dr \tag{14}$$

equation (12) becomes, using boundary condition (2),

$$\rho S \frac{\partial}{\partial t} T_n(z,t) = R\, G_n(R)\, h\, T^*(z,t) - (k\, \alpha_n^2 T_n(z,t)/R^2)$$

$$+ f(z)\, g(t)\, h_n$$

This can be written as

$$\rho S\left(\frac{\partial}{\partial t} + \mu_n\right) T_n(z,t) = R\, G_n(R)\, h\, T^*(z,t) + f(z)\, g(t)\, h_n \tag{15}$$

The radial temperature distribution can be expanded as a series of the eigenfunctions $G_n(r)$, i.e.,

$$T(r,z,t) = \frac{2}{R^2} \sum_1^\infty \frac{T_n(z,t)\, G_n(r)}{J_o^2(\alpha_n) + J_1^2(\alpha_n)} \tag{16}$$

The problem has now been reduced to the solution of equation (15) for each T_n in conjunction with equation (3), which can be written as

$$\left(\frac{\partial}{\partial t} + V \frac{\partial}{\partial z}\right) T^*(z,t) = \beta_c \left\{ \sum_1^\infty \gamma_n T_n(z,t) - T^*(z,t) \right\} \tag{17}$$

where $\beta_c = h\, P/\rho_c\, S_c A_c$

and $\gamma_n = 2\, G_n(R)/R^2 \left(J_o^2(\alpha_n) + J_1^2(\alpha_n)\right)$

$$= 2\, \alpha_n^2\, C^2/R^2\, G_n(R)\, (1 + \alpha_n^2 C^2) \tag{18}$$

One approach to (15) and (17) is to use Laplace transforms. Using bars to denote transforms with respect to time,

$$\rho S(p + \mu_n)\, \bar{T}_n(z) = R\, G_n(R)\, h\, \bar{T}^*(z) + f(z)\, \bar{g}\, h_n \tag{19}$$

$$\left(p + V \frac{d}{dz}\right) \bar{T}^*(z) = \beta_c \left\{ \sum_1^\infty \gamma_n \bar{T}_n(z) - \bar{T}^*(z) \right\} \tag{20}$$

$$\bar{T}_n(z) = \left(R\, G_n(R)\, h\, \bar{T}^*(z) + f(z)\, \bar{g}\, h_n\right)/\rho S(p + \mu_n) \tag{21}$$

i.e.

$$\frac{\beta_c f(z) \bar{g}}{\rho S} \sum_1^\infty \frac{\gamma_n h_n}{p + \mu_n} = \left(V \frac{d}{dz} + \left\{p + \beta_c - \frac{\beta_c}{\rho S} \sum_1^\infty \frac{\gamma_n R\, G_n(R)\, h}{p + \mu_n}\right\}\right) \bar{T}^*(z) \tag{22}$$

Collecting results, the equations for the Laplace transforms of the fuel element and coolant temperatures are as follows, assuming that the inlet temperature remains constant.

$$\overline{T}(r,z) = \frac{\overline{T}^*(z)Rh}{\rho S}\sum_1^{\infty}\frac{\gamma_n G_n(r)}{p+\mu_n} + \frac{f(z)\bar{g}}{\rho S}\sum_1^{\infty}\frac{h_n\gamma_n G_n(r)}{G_n(R)(p+\mu_n)} \tag{23}$$

$$\overline{T}^*(z) = X(p)\int_0^z f(\zeta)\exp\{-\Omega(p)(z-\zeta)\}\,d\zeta \tag{24}$$

$$X(p) = \frac{\beta_c\bar{g}}{\rho S}\sum_1^{\infty}\frac{\gamma_n h_n}{p+\mu_n} \tag{25}$$

$$V\Omega(p) = p + \beta_c\left(1-\frac{Rh}{\rho S}\sum_1^{\infty}\frac{\gamma_n G_n(R)}{p+\mu_n}\right) \tag{26}$$

Some of these series can be summed by contour integration. Thus, if z is a complex variable, then the function

$$q(z) = -\left\{\frac{Cz\,J_0(z)+J_1(z)}{J_0(z)-Cz\,J_1(z)}\right\}$$

has simple poles at $z = \pm\alpha_n$ each with residue 1. Hence, it can be shown that for suitably restricted functions F(z), such that $|z\,F(z)q(z)| \to 0$ uniformly as $|z| \to \infty$

$$-\sum_{-\infty}^{+\infty} F(\alpha_n) = \text{sum of the residues of } q(z)\,F(z) \text{ at the poles of } F(z)$$

Now

$$\sum_1^{\infty}\frac{\gamma_n G_n(R)}{p+\mu_n} = \sum_1^{\infty}\frac{2C^2\alpha_n^2}{R^2(p+\mu_n)(1+\alpha_n^2C^2)}$$

$$= \frac{2W}{R^2}\sum_1^{\infty}\frac{\alpha_n^2}{(\alpha_n^2+\frac{1}{C^2})(\alpha_n^2+Wp)} \tag{27}$$

To sum this series, we require the residues of

$$-\left\{\frac{CzJ_0(z)+J_1(z)}{J_0(z)-CzJ_1(z)}\right\}\frac{z^2}{(z^2+\frac{1}{C^2})(z^2+Wp)}$$

at the poles $z = \pm i/C$; $z = \pm i\sqrt{Wp}$

Note that $\alpha_{-n} = -\alpha_n$ and $F(\alpha_{-n}) = F(\alpha_n)$

This calculation presents no difficulty and the final result is

$$\sum_1^{\infty}\frac{\alpha_n G_n(R)}{p+\mu_n} = \frac{W}{R^2(Wp-\frac{1}{C^2})}\left[\sqrt{Wp}\left\{\frac{I_1(\sqrt{Wp})+C\sqrt{Wp}\,I_0(\sqrt{Wp})}{I_0(\sqrt{Wp})+C\sqrt{Wp}\,I_1(\sqrt{Wp})}\right\}-\frac{1}{C}\right] \tag{28}$$

The series in (25) depends on the nature of h_n .

A reasonable assumption for preliminary investigations would be that h(r) is constant over a cross section and can be taken as 1.

In this case, from (14)

$$h_n = R^2 J_1(\alpha_n)/\alpha_n$$

$$= R^2 G_n(R)/C\,\alpha_n^2$$

$$\therefore \sum_1^\infty \frac{\gamma_n h_n}{p+\mu_n} = \frac{2W}{C}\sum_1^\infty \frac{1}{(\alpha_n^2+\frac{1}{C^2})(\alpha_n^2+Wp)} \tag{29}$$

The summation proceeds as before and the result is

$$\sum_1^\infty \frac{\gamma_n h_n}{p+\mu_n} = \frac{W}{C(Wp-\frac{1}{C^2})}\left[C-\frac{1}{\sqrt{Wp}}\left\{\frac{I_1(\sqrt{Wp})+C\sqrt{Wp}\,I_0(\sqrt{Wp})}{I_0(\sqrt{Wp})+C\sqrt{Wp}\,I_1(\sqrt{Wp})}\right\}\right] \tag{30}$$

Thus, for the case of uniform heat generation across a fuel element, the functions $X(p)$ and $\Omega(p)$ appearing in (25) and (26) are given analytically as functions of p by the use of (28) and (30).

The solution of the problem as given in equations (23) to (26) is not yet in a convenient form for computation. The inversion of the Laplace transforms is a great difficulty. However, the frequency response functions

$$\bar{T}(r,z,iw)/\bar{g}(iw) \quad \text{and} \quad \bar{T}^*(z,iw)/\bar{g}(iw)$$

can be readily evaluated from these equations by the substitution $p = iw$, if the integration in (24) can be performed. The response to realistic time variations of power can therefore be calculated by use of Fourier Series or Fourier Integrals. There is no difficulty in choosing an appropriate series for the expansion of f(z) in order to apply equation (24).

The analysis described above will give a fairly complete description of the transient temperatures in fuel element and coolant. However, a great deal of numerical work will be required, and it is worth considering a model in which the fuel element temperature distribution is simplified to elucidate some points in connection with the transient coolant temperature along a channel.

2.2 Coolant Temperatures With Lumped Fuel Element Model

The starting point in this investigation is the set of equations, referring to departures from the steady state,

$$\rho SA \frac{\partial}{\partial t} T_m(z,t) = H_m A - P\epsilon\left(T_m(z,t) - T^*(z,t)\right) \tag{31}$$

$$\rho_c S_c A_c\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial t}\right)T^*(z,t) = P\epsilon\left(T_m(z,t) - T^*(z,t)\right) \tag{32}$$

T_m denotes the mean temperature over a fuel element cross section, H_m is the mean heat release density and ϵ is an overall heat transfer coefficient, assumed constant. In the steady state, with a uniform heat release, no cladding and a circular cross section,

$$\epsilon^{-1} = (2/3k) + (1/h) \tag{33}$$

In this model, fuel element geometry enters only through P, A, A_c and ϵ. It is convenient to write, as before

$$H_m(z,t) = f_m(z)\,g(t) \tag{34}$$

The usual treatment of these equations is as follows:

Let

$$\beta = P\epsilon/\rho S A \quad ; \quad \beta_c = P\epsilon/\rho_c S_c A_c$$

and $\tilde{X} = \frac{1}{L}\int_0^L X(z)\,dz$ i.e., average over L.

Integrating (31) and (32) over the length of the fuel element L,

$$\frac{d}{dt}\tilde{T}_m(t) = \tilde{f}_m \frac{g(t)}{\rho S} - \beta\left(\tilde{T}_m(t) - \tilde{T}^*(t)\right) \tag{35}$$

$$\frac{d}{dt}\tilde{T}^*(t) + \frac{V}{L}\left(T^*(t)\right)_{z=L} = \beta_c\left(\tilde{T}_m(t) - \tilde{T}^*(t)\right) \tag{36}$$

It has again been assumed that the inlet temperature does not vary.

The assumption is now made that

$$\left(T^*(t)\right)_{z=L} = 2\,\tilde{T}^*(t) \tag{37}$$

so that (36) becomes

$$\left(\frac{d}{dt} + \frac{2V}{L}\right)\tilde{T}^*(t) = \beta_c\left(\tilde{T}_m(t) - \tilde{T}^*(t)\right) \tag{38}$$

Equations (35) and (38) can now be used to obtain the transfer function relating mean temperatures and the power variations. Taking Laplace transforms,

$$\bar{\tilde{T}}_m = \{(\tilde{f}_m \bar{g}/\rho S) + \beta\,\bar{\tilde{T}}^*\}/(p + \beta)$$

$$\bar{\tilde{T}}^* = \beta_c \bar{\tilde{T}}_m / (p + \beta_c + \frac{2V}{L})$$

and therefore

$$\rho S\,\bar{\tilde{T}}_m/\bar{g} = \tilde{f}_m(p + \beta_c + \frac{2V}{L}) / \{p(p + \beta + \beta_c + \frac{2V}{L}) + \frac{2V\beta}{L}\} \tag{39}$$

$$\rho S\,\bar{\tilde{T}}^*/\bar{g} = \tilde{f}_m \beta_c / \{p(p + \beta + \beta_c + \frac{2V}{L} + \frac{2V\beta}{L}\} \tag{40}$$

These expressions are comparatively simple for use in the overall reactor transfer function via the temperature coefficients of reactivity based on mean temperatures. In a realistic analysis, heat exchange with the channel walls would be included, but the approach is identical.

The validity of (37) can be checked by solving equations (31) and (32). Again taking transforms, these become

$$(p+\beta)\,\overline{T}_m(z) = \beta\,\overline{T}^*(z) + \left(f_m(z)\,\overline{g}/\rho S\right) \tag{41}$$

$$(p + V\tfrac{d}{dz})\,\overline{T}^*(z) = \beta_c\left(\overline{T}_m(z) - \overline{T}^*(z)\right) \tag{42}$$

The solutions are

$$\overline{T}_m(z) = \frac{\beta}{p+\beta}\,\overline{T}^*(z) + \frac{f_m(z)\,\overline{g}}{\rho S(p+\beta)} \tag{43}$$

$$\overline{T}^*(z) = \frac{\beta_c\overline{g}}{V\rho S(p+\beta)} \int_0^z f_m(\zeta)\exp\left\{-\frac{p(p+\beta+\beta_c)}{V(p+\beta)}(z-\zeta)\right\} d\zeta \tag{44}$$

These solutions are sufficient for calculating frequency response functions and hence temperature response. However, it is possible to proceed further.

Let

$$G(p,z) = \frac{1}{V(p+\beta)} \int_0^z \exp\{-\psi(p)(z-\zeta)\}\, d\zeta \tag{45}$$

where $\psi(p) = \frac{p}{V}\cdot\left(\frac{p+\beta+\beta_c}{p+\beta}\right)$ (46)

Then

$$\overline{T}^*(z) = \frac{\beta_c}{\rho S}\,\overline{g}\left[f(o)G(p,z) + \int_0^z \frac{d f(\zeta)}{d\zeta} G(p,z-\zeta)\, d\zeta\right]$$

If now G(p, z) is the Laplace transform of the function $\Omega(z,t)$ it follows that

$$T^*(z,t) = \frac{\beta_c}{\rho S} \int_0^t g(t-\tau)\left\{f(o)\,\Omega(z,\tau) + \int_0^z \frac{d f(\zeta)}{d\zeta}\,\Omega(z-\zeta,\tau)\, d\zeta\right\} d\tau \tag{47}$$

Also, from (43)

$$T_m(z,t) = \int_0^f \left\{\beta\, T^*(z,\tau) + \frac{f_m(\zeta)}{\rho S}\, g(\tau)\right\} e^{-\beta(t-\tau)} d\tau \tag{48}$$

Now from (45) and (46)

$$G(p,z) = \frac{1}{p(p+\beta+\beta_c)}\left[1 - \exp\left\{\frac{-pz}{V}\cdot\frac{p+\beta+\beta_c}{p+\beta}\right\}\right]$$

$$= \frac{1}{\beta+\beta_c}\left[\frac{1}{p} - \frac{1}{p+\beta+\beta_c} - \left(\frac{\beta}{p} + \frac{\beta_c}{p+\beta+\beta_c}\right)\left(\frac{1}{p+\beta}\exp\left\{-\frac{\beta_c z}{V} - \frac{zp}{V} + \frac{\beta\beta_c z}{V(p+\beta)}\right\}\right)\right] \tag{49}$$

Using the fact that

$$\mathcal{L}^{-1}\,\frac{1}{p}\,e^{a/p} = I_0(2\sqrt{at}) \tag{50}$$

it can be shown that

$$\Omega(z,t) = \frac{1}{\beta+\beta_c}\left[1 - e^{-(\beta+\beta_c)t} - e^{-\beta_c z/V}\left\{\beta\int_0^{t-\frac{z}{V}} e^{-\beta\tau}\, I_0\left(2\sqrt{\beta\beta_c z\,\tau/V}\right)d\tau\right.\right.$$

$$\left.\left. + \beta_c e^{-(\beta+\beta_c)(t-\frac{z}{V})}\int_0^{t-\frac{z}{V}} e^{\beta_c\tau}\, I_0\left(2\sqrt{\beta\beta_c z\,\tau/V}\right)d\tau\right\}\right] \qquad (51)$$

where the function in { } is defined to be zero for $t < \frac{z}{V}$ Equations (47), (48), and (51) thus represent the formal solution of the problem.

3. CONCLUSION

The equations derived above can be used in computer programs to explore the nature of transient temperature distributions for the simple models investigated.

It is important to note that the theory of Section 2.1 is not restricted to circular homogeneous fuel elements. If the thermal conductivity k is variable over the cross section it is still possible to obtain a series of eigenfunctions analogous to G_n, though for elements that depart significantly from simple shapes it is probably not worth while.

Although a direct numerical approach is possible, the methods outlined, by reducing the number of dimensions, should prove useful on small capacity computers.

4. REFERENCES

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5. NOTATION

A	Fuel element cross sectional area
A_c	Coolant channel flow area
G_n	n'th eigenfunction
h	Heat transfer coefficient (surface to coolant)
$H = h(r)f(z)g(t)$	Heat release density per unit volume
I_0, I_1	Modified Bessel functions of first kind
J_0, J_1	Bessel functions of first kind
k	Thermal conductivity of fuel element material
L	Length of fuel element
P	Perimeter of fuel element
r	Radial co-ordinate
R	Radius of circular fuel element

S	Heat capacity of fuel element material
S_c	Specific heat of coolant
t	Time
T	Fuel element temperature
T^*	Mixed mean coolant temperature
T_m	Average fuel element temperature over a cross section
V	Velocity of coolant
z	Axial co–ordinate measured from coolant entry
ϵ	Overall heat transfer coefficient (fuel element to coolant)
μ_n	Eigenvalue
ρ	Density of fuel element material
ρ_c	Density of coolant
$\sim$	Denotes average over the length L
—	Denotes Laplace transform w.r.t. time