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RESEARCH ESTABLISHMENT
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CRYSTALLOGRAPHIC TECHNIQUES AND DATA FOR TRANSMISSION ELECTRON MICROSCOPY OF ZIRCONIUM

by

A. JOSTSONS
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February 1970

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ABSTRACT

The crystallography of hexagonal close packed metals is discussed briefly in terms of the four-axis hexagonal reference basis and the Miller-Bravais notation which are used throughout the report. Electron diffraction problems are treated with reference to the four-axis hexagonal reciprocal lattice rather than the more usual three-axis hexagonal system. Using these concepts, analysis of electron diffraction spot and Kikuchi patterns is illustrated and applied to orientation and dislocation Burgers vector determinations. Computed values of interplanar spacings, interplanar angles, angles between directions, and extinction distances for zirconium are listed.

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1. INTRODUCTION

Zirconium and most of its alloys of interest in nuclear power technology have hexagonal close-packed (h.c.p.) structure. Many aspects of physical metallurgical studies of these alloys require a knowledge of their crystallography which, in contrast to that of cubic metals, is not considered in any detail in the many available texts on physical metallurgy. Most of the scattered information is in original papers in the literature and unfortunately some of them contain errors because of confusion over different crystallographic systems and notation used to describe h.c.p. structures. An exception is the review prepared by Partridge (1967) but he does not list specific crystallographic information such as values of interplanar angles and angles between directions for zirconium.

The aim of this report is to describe briefly the fundamentals of crystallography of h.c.p. metals and their application to problems in transmission electron microscopy of zirconium. The specific crystallographic formulae and data for zirconium required for quantitative electron microscopy have been calculated and are tabulated in the Appendices. Some of these data have more general application in X-ray diffraction studies and analyses of twinning and deformation modes in bulk materials.

2. CRYSTALLOGRAPHY OF h.c.p. METALS

2.1 Crystal Structure

The arrangement of atoms in a h.c.p. metal can be shown in terms of a hexagonal prism, Figure 1, where the filled circles represent atom centres. The primitive unit cell, (heavy lines) which does not immediately reveal the hexagonal symmetry, has axes $\underline{a}_1 = \underline{a}_2 \neq \underline{c}$ with the angle between $\underline{a}_1$ and $\underline{a}_2$ equal to 120° and $\underline{c}$ perpendicular to both $\underline{a}_1$ and $\underline{a}_2$. The unit cell contains two atoms with positions given by the coordinates 000 and $\frac{2}{3}\,\frac{1}{3}\,\frac{1}{2}$. Since the surroundings of the interior atom differ from those at the cell corners the atom positions in the h.c.p. structure do not constitute a space lattice. The actual space lattice remains primitive with points at cell corners only if two atoms are considered to be associated with each lattice point.

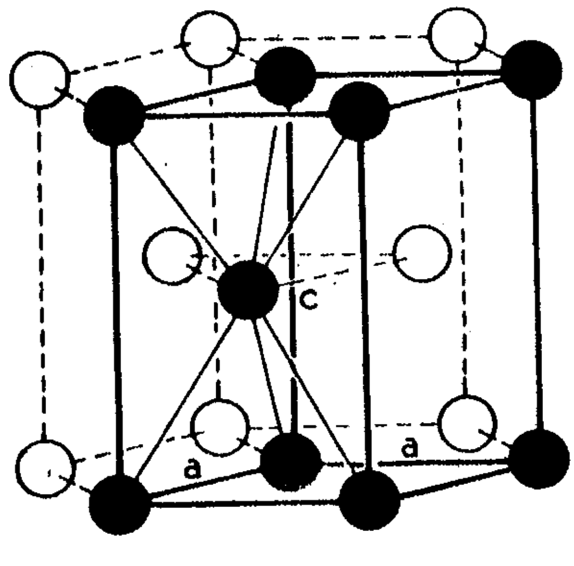
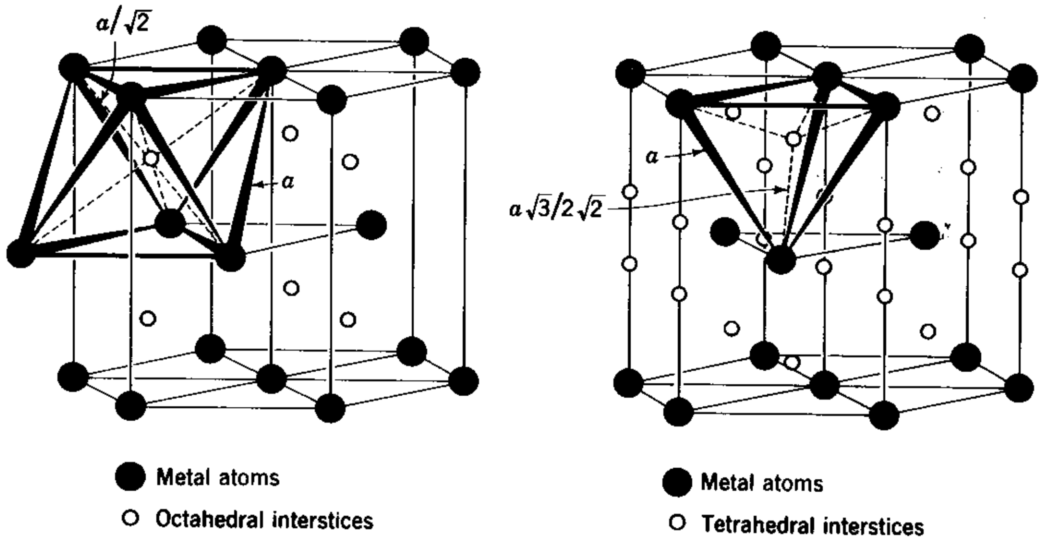


Figure 1. Model of hexagonal close-packed structure with unit cell shown in heavy outline.

If the atoms are considered as hard spheres, the plane containing both $\underline{a}_1$ and $\underline{a}_2$ is a close-packed plane. The hexagonal close-packed structure is characterised by an ABABABA. . . . stacking sequence of close-packed planes. In an ideal close-packed structure the axial ratio c/a is equal to

$$\left(\frac{8}{3}\right)^{\frac{1}{2}} = 1.633 \ .$$

Zirconium and its alloys have a strong affinity for hydrogen, oxygen, nitrogen and carbon which in solid solution occupy the interstitial holes. The locations of the octahedral and tetrahedral interstices in a h.c.p. structure are shown in Figure 2. There are 2 octahedral and 4 tetrahedral interstices per unit cell centred on coordinates $\frac{1}{3}\,\frac{2}{3}\,\frac{1}{4}$, $\frac{1}{3}\,\frac{2}{3}\,\frac{3}{4}$ and $\frac{2}{3}\,\frac{1}{3}\,\frac{1}{8}$, $\frac{2}{3}\,\frac{1}{3}\,\frac{7}{8}$, $00\frac{3}{8}$, $00\frac{5}{8}$ respectively. In an ideal h.c.p. structure of rigid spheres of radius r, the maximum radius of a sphere that can be accommodated in an octahedral or tetrahedral interstice is 0.41 r and 0.22 r respectively. The geometry of interstitial sites in a h.c.p. structure is revealed more clearly by the model shown in Figure 3.



(a) Octahedral voids (b) Tetrahedral voids

Figure 2. Interstitial voids in the h.c.p. structure with ideal axial ratio $c/a = \sqrt{8/3}$

(Barrett and Massalski 1966)

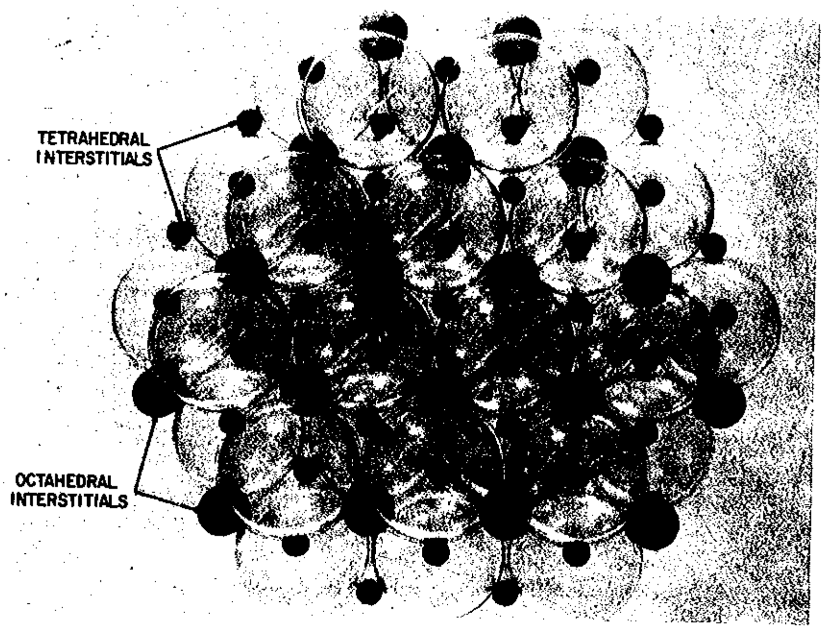


Figure 3. Model of hexagonal close-packing of spheres with octahedral and tetrahedral interstitial sites, looking along the c-axis.

(Gehman 1960)

2.2 Crystallographic Indices

Crystallographic indices constitute a convenient system of notation of crystal planes and directions. The different axial systems commonly used to define the indices of directions and planes in hexagonal crystals are three-axis hexagonal (Miller indices), four-axis hexagonal (Miller-Bravais indices) and orthohexagonal. Only the four-axis hexagonal system gives rise to similar indices for crystallographically equivalent directions and planes. This feature of the Miller-Bravais system is an asset when dealing with typical problems in physical metallurgy, for example, description of slip modes and Burgers vectors of dislocations, and accounts for its general acceptance. The main arguments against universal use of the Miller-Bravais notation are the alleged greater complexity of crystallographic formulae and claims that the four-axis hexagonal reciprocal lattice is physically meaningless (Partridge and Gardiner 1967 a). The simplifications introduced by Nicholas (1966) into Miller-Bravais formulae, however, lead to expressions of equal simplicity to those based on the alternative systems while still retaining the virtue of symmetry. Frank (1965), and more recently, Okamoto and Thomas (1967, 1968) have demonstrated that the four-axis hexagonal reciprocal lattice, though not strictly a reciprocal lattice according to the usual mathematical definitions, permits the analysis of the necessary crystallographic relationships directly in terms of Miller-Bravais indices using elementary vector analysis. Consequently, the four-axis hexagonal system with Miller-Bravais indices does not introduce any additional limitations for normal electron microscopy, compared with alternative systems of indexing, and is used throughout this report.

The nomenclature of planes in the Miller-Bravais system is shown in Figure 4 in which the primitive hexagonal unit cell with the three axes, $\underline{a}_1$, $\underline{a}_2$ and $\underline{a}_3$ at 120° to each other in the basal plane, is shown within the hexagonal prism. Thus, a three dimensional crystal is represented in four dimensional space. From the symmetry of the axes in the basal plane it is noted that

$$\underline{a}_3 = -(\underline{a}_1 + \underline{a}_2)\ . \qquad (1)$$

Consequently, when reciprocal intercepts of a plane on all four axes are determined and reduced to the smallest integers, the indices of a plane will be of the type (hkiℓ), with the restriction that

$$h + k + i = 0\ . \qquad (2)$$

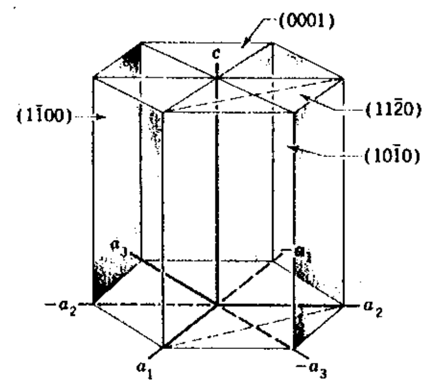


Figure 4. Indices of planes in hexagonal crystals.
(Barrett and Massalski 1966)

Similarly, an arbitrary crystal vector $\underline{r}$ can be written in the form

$$\underline{r} = u\underline{a}_1 + v\underline{a}_2 + t\underline{a}_3 + w\underline{c}\ , \qquad (3)$$

with the restriction that

$$u + v + t = 0\ . \qquad (4)$$

The indexing of directions in the Miller-Bravais notation is illustrated in Figure 5 where the translations leading from the origin to several lattice points are indicated. The comparatively unusual translations used in deriving the indices of a direction are necessitated by the restrictions imposed by Equation 4. The interplanar spacings in h.c.p. metals are given by

$$d_{hki\ell} = a\left\{\frac{2}{3}(h^2 + k^2 + i^2 + \ell^2\lambda^{-2})\right\}^{-\frac{1}{2}}, \qquad (5)$$

where $\lambda^2 = \frac{2}{3}\left(\frac{c}{a}\right)^2$.

The symbol λ has been retained in the crystallographic formulae to conform with the literature but it is used later to denote the wavelength of electrons also. The meaning of these symbols will be clear from the context.

Values of interplanar spacings of zirconium are tabulated in Appendix 1.

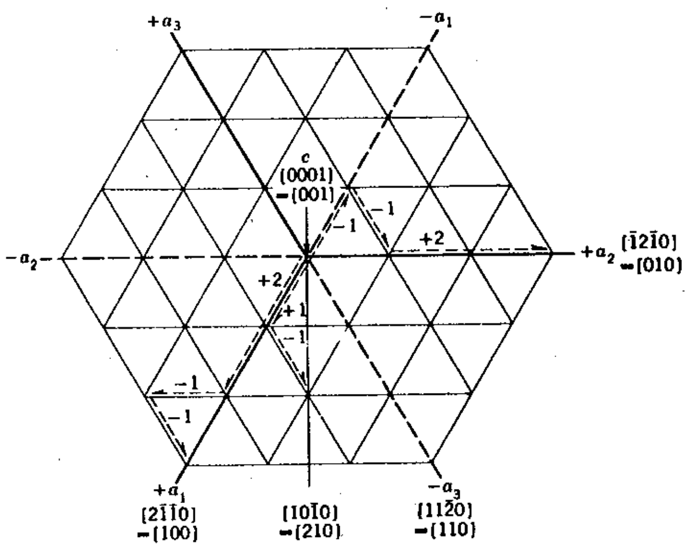


Figure 5. Indices of directions in the hexagonal system with both three– and four–digit indices. The c-axis is normal to the plane of the drawing; a_1, a_2 and a_3 in h.c.p. crystals are along the close-packed rows of atoms.
(Barrett and Massalski 1966)

In the h.c.p. system, in contrast to the cubic system, a direction is not normal to a plane with the same Miller-Bravais indices, except for directions of the form $\langle 000\ell \rangle$ and $\langle hki0 \rangle$. The indices of the normal to a plane $(hki\ell)$ are $[\,hki\ (\lambda^{-2}\ \ell)]$.

Interplanar angles and angles between directions in h.c.p. metals depend on the c/a ratio and are tabulated for zirconium in Appendices 2 and 3 respectively.

2.3 Stereographic Projections

Although electron micrographs can be readily interpreted analytically, it is often more convenient to use stereographic methods. Since directions in h.c.p. metals are not necessarily normal to planes of the same indices, two stereographic projections are required; one for plane normals (poles) and one for crystal directions. Packer and Miller (1967) have demonstrated the application of two standard projections printed on transparent film; one for poles and the other for directions; which when superimposed correctly permit the necessary operations. However Rarey et al. (1966) advocate the use of a 'double stereogram' on which are plotted both the poles and directions with rational indices.

A stereogram in the standard (0001) projection is shown in Appendix 4.

2.4 Dislocations in h.c.p. Metals

Since much of transmission electron microscopy is concerned with the study of dislocations, their geometry in h.c.p. metals will be considered. A notation based on the bipyramid shown in Figure 6, devised by Berghezan et al. (1961), is often used to describe dislocations in h.c.p. metals. Because the h.c.p. structure is a double lattice structure, not all nearest neighbour atomic translations are possible Burgers vectors of perfect dislocations. The types of dislocations which are likely to be stable have been discussed by Frank and Nicholas (1953) and Berghezan et al. (1961) and are summarised in Table 1.

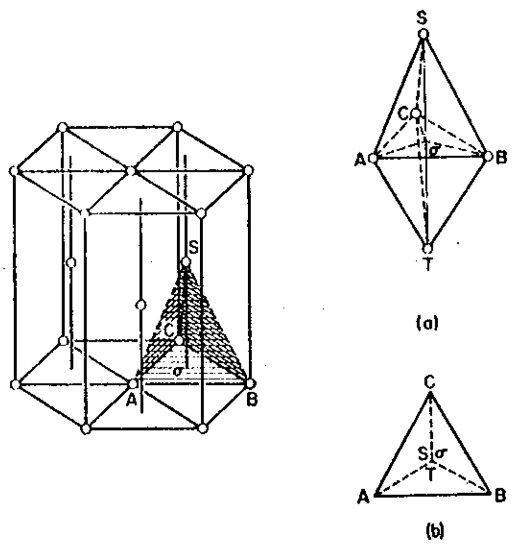


Figure 6. Burgers vectors in the hexagonal close-packed lattice.

(Berghezan, Fourdeux and Amelinckx, 1961)

A comprehensive review of deformation modes in h.c.p. metals has been made by Dorn and Mitchell (1965). In zirconium the primary slip mode is $\{10\bar{1}0\}$ $\langle 1\bar{2}10 \rangle$. There is very little unambiguous evidence for other slip systems in zirconium, although basal plane slip of dislocations with $\frac{1}{3}\langle 11\bar{2}0 \rangle$ Burgers vectors has been proposed to explain the formation of kink bands in zirconium deformed above 500°C (Reed-Hill 1964). Martin and Reed-Hill (1964) have reported slip markings which could not be explained in terms of the above slip systems in grains of deformed polycrystalline zirconium. Howe et al. (1962) suggest the operation of slip on $\{10\bar{1}3\}$ and $\{11\bar{2}1\}$ planes from observation of slip traces in thin foils but the Burgers vectors of these dislocations were not identified. There is no unambiguously proven case in zirconium of slip systems that will produce strain in a direction not contained in the basal plane, that is, dislocations with a Burgers vector with a non-zero fourth index. However dislocations with a Burgers vector $\frac{1}{3}\langle 11\bar{2}3 \rangle$ have been proposed by Rosenbaum (1964) to explain atom movements associated with $\{11\bar{2}2\}$ deformation twinning. The possibility of slip in $\langle 11\bar{2}3 \rangle$ directions is also suggested by analyses of deformation systems required to produce the observed rolling textures in zirconium (Picklesimer 1966).

TABLE 1

BURGERS VECTORS OF DISLOCATIONS IN h.c.p. METALS

Type of Dislocations (Figure 6)	Total No. of Dislocations	Vector	Direction Indices of Vector	Magnitude of Vector in Terms of Lattice Parameters	Relative Energies of Dislocations for c/a = 1.633
Perfect dislocations					
(1) AB, BC, CA, BA, CB, AC	6	$\pm \underline{a}_i$	$\frac{1}{3}\langle 11\bar{2}0\rangle$	$\lvert\underline{a}\rvert$	a^2
(2) ST, TS	2	$\pm \underline{c}$	$\langle 0001\rangle$	$\lvert\underline{c}\rvert$	$c^2 = \frac{8}{3}\ a^2$
(3) ±ST ±AB, ±ST ±BC, ±ST ±CA	12	$\pm\underline{c} + \underline{a}_i$	$\frac{1}{3}\langle 11\bar{2}3\rangle$	$(a^2 + c^2)^{\frac{1}{2}}$	$\frac{11}{3}\ a^2$
Partial dislocations					
(4) $\pm A\sigma$, $\pm B\sigma$, $\pm C\sigma$	6	$\pm\left(\frac{2}{3}\underline{a}_i + \frac{1}{3}\underline{a}_j\right)$	$\frac{1}{3}\langle 10\bar{1}0\rangle$	$\frac{\lvert a\rvert}{\sqrt{3}}$	$\frac{1}{3}\ a^2$
(5) $\pm\sigma S$, $\pm\sigma T$	4	$\pm\frac{1}{2}\ \underline{c}$	$\frac{1}{2}\langle 0001\rangle$	$\frac{\lvert c\rvert}{2}$	$\frac{2}{3}\ a^2$
(6) ±AS, ±BS, ±CS, ±AT, ±BT, ±CT	12	(4) + (5) above	$\frac{1}{6}\langle 20\bar{2}3\rangle$	$\left(\frac{a^2}{3} + \frac{c^2}{4}\right)^{\frac{1}{2}}$	a^2

Generally, the stability of various dislocations varies as the square of the Burgers vector as has been assumed in values given in Table 1. This, however, is only approximately true in an isotropic medium. Theoretical analyses of dislocation stability after including effects of elastic anisotropy in zirconium have been made by Roy (1967), Fisher and Alfred (1968) and Yoo (1968). Apparently, basal and prismatic slip of $\frac{1}{3}\langle 11\bar{2}0\rangle$ dislocations is energetically equally probable and Roy (1967) suggests that the predominant prismatic slip in zirconium may be due to the presence of interstitial impurities. Fisher and Alfred's calculations suggest that with increasing temperature, effects of elastic anisotropy render edge dislocations with $\underline{b} = \frac{1}{3}\langle 11\bar{2}0\rangle$ energetically unstable in the basal plane.

The stacking fault energy of zirconium is high and it is expected that partial dislocations would not be resolvable in the electron microscope.

3. ELECTRON MICROSCOPY OF h.c.p. METALS

3.1 The Reciprocal Lattice

For the interpretation of electron diffraction patterns the reciprocal lattice concept is the most convenient but some confusion in the literature is due to there being two alternative ways of defining the reciprocal lattice and the four index labelling of points (Frank 1965).

Mathematically the reciprocal lattice is defined by the following relationship between the reciprocal lattice axes $\underline{a}_1^*$, $\underline{a}_2^*$, $\underline{a}_3^*$ and the direct lattice axes $\underline{a}_1$, $\underline{a}_2$, $\underline{a}_3$:

$$\left.\begin{aligned} \underline{a}_1^* &= \frac{1}{V}(\underline{a}_2 \times \underline{a}_3) \\ \underline{a}_2^* &= \frac{1}{V}(\underline{a}_3 \times \underline{a}_1) \\ \underline{a}_3^* &= \frac{1}{V}(\underline{a}_1 \times \underline{a}_2) \end{aligned}\right\} , \qquad (6)$$

where V is the volume of the unit cell in the direct lattice, that is, $\underline{a}_1 \cdot \underline{a}_2 \times \underline{a}_3$. This leads to the general relation $\underline{a}_i^* \cdot \underline{a}_j = \delta_{ij}$, with $\delta_{ij} = 1$ when $i = j$ and otherwise zero. (7)
The reciprocal lattice is built up by repeated translations of the unit cell by the vectors $\underline{a}_1^*$, $\underline{a}_2^*$ and $\underline{a}_3^*$.

Frank (1965), Otte and Crocker (1965), Partridge and Gardiner (1967) and Okamoto and Thomas (1967, 1968) have shown that the reciprocal lattice defined by Equation 6 leads to difficulties in the four-axis hexagonal system using Miller-Bravais indices. The normal reciprocal lattice in hexagonal crystals has axes $\underline{a}_1^*$, $\underline{a}_2^*$, and $\underline{c}^*$ which are related by Equations 6 and 7 to the axes $\underline{a}_1$, $\underline{a}_2$ and $\underline{c}$ in the three-axis hexagonal system with Miller indices. The angle between $\underline{a}_1^*$ and $\underline{a}_2^*$ in this reciprocal lattice is 60° and thus, because they do not define a conventional hexagonal unit cell, the introduction of a fourth axis $\underline{a}_3^* = -(\underline{a}_1^* + \underline{a}_2^*)$ will not have the same geometrical significance as its analogue in the direct lattice. It is for this reason that the use of the three-axis hexagonal system with the Miller notation has been advocated for diffraction studies of hexagonal metals (Partridge and Gardiner 1967). This procedure is unattractive because the symmetry of the hexagonal system is not fully indicated and frequent transformation of indices is required if the end product of the calculation is to be Miller-Bravais indices.

The alternative approach, suggested by Frank (1965), Otte and Crocker (1965) and developed in greater detail by Okamoto and Thomas (1967, 1968) is to use a four-axis reciprocal lattice in which the introduction of the fourth axis has the same significance as in the direct lattice. The four-axis hexagonal reciprocal lattice is constructed according to the alternative definition of a reciprocal lattice, that is, a reciprocal lattice point hkiℓ lies at a distance from the origin which is the inverse of the spacing between (hkiℓ) planes in the direct lattice, in a direction normal to these planes. This definition of the reciprocal lattice is a corollary of the usual mathematical definition of a reciprocal lattice. The reciprocal axes $\underline{a}_1^*$, $\underline{a}_2^*$, $\underline{a}_3^*$, $\underline{c}^*$ are not related to the direct lattice axes in the customary way as defined by Equation 6 and do not satisfy the usual conditions of Equation 7. The reciprocal lattice axes are parallel to the respective axes in the direct lattice. The mathematical relationships between the basis vectors in the four-axis hexagonal reciprocal lattice and the axes in the direct lattice are given by:

$$\left.\begin{aligned} \underline{a}_1^* &= \left(\frac{2}{3}\, a^2\right) \underline{a}_1 \\ \underline{a}_2^* &= \left(\frac{2}{3}\, a^2\right) \underline{a}_2 \\ \underline{a}_3^* &= \left(\frac{2}{3}\, a^2\right) \underline{a}_3 \\ \underline{c}^* &= \left(\frac{1}{c^2}\right) \underline{c} \end{aligned}\right\} . \qquad (8)$$

It is obvious, that in the four-axis hexagonal reciprocal lattice the following restrictions apply:

$$\left.\begin{aligned} &\underline{a}_3^* = -(\underline{a}_1^* + \underline{a}_2^*) \\ \text{and} \\ &h + k + i = 0 \end{aligned}\right\} . \qquad (9)$$

Okamoto and Thomas (1967, 1968) have shown that only one out of every three reciprocal lattice points in the four-axis reciprocal lattice coincides with a conventional (three-axis hexagonal) reciprocal lattice point. However the additional points in the four-axis hexagonal reciprocal lattice have non-integral indices and cannot represent real crystal planes. They can be considered to represent reciprocal lattice points for which the structure factor is zero and hence will not be present in actual diffraction patterns.

Although the four-axis hexagonal reciprocal lattice is not a true reciprocal lattice in the usual mathematical sense, the concept is most useful because it permits the calculation of necessary crystallographic relationships directly in terms of Miller-Bravais indices using elementary vector analysis. For example, the scalar product of a reciprocal lattice vector $\underline{g}_{hki\ell}$ and a direct lattice vector $\underline{r}_{uvtw}$ is given by

$$\underline{g} \cdot \underline{r} = hu + kv + it + \ell w \quad . \tag{10}$$

The direct lattice vector $\underline{r}$ is normal to the reciprocal lattice vector $\underline{g}$ when $\underline{g} \cdot \underline{r} = 0$. This permits the determination of zonal relationships and invisibility criteria for dislocations directly in the Miller-Bravais notation.

Crystallographic relationships between crystal vectors and reciprocal lattice vectors in the Miller-Bravais notation have been derived by Otte and Crocker (1965, 1966), Nicholas (1966), Okamoto and Thomas (1968), Neumann (1966) and Schwartzkopff (1968). A summary of relationships useful in transmission electron microscopy is given in Appendix 5.

3.2 Electron Diffraction Patterns

Electron diffraction is interpreted most readily in terms of the reciprocal lattice and the Ewald sphere construction (Hirsch et al. 1965). In Figure 7, consider the sphere of radius $1/\lambda$ (the reciprocal of the electron wavelength) that intersects the origin of the reciprocal lattice at 0, where crystal planes spaced $d_{hki\ell}$ are inclined at an angle θ to the incident electron beam along E0. The reciprocal lattice vector $\underline{g}_{hki\ell}$ equal to 0P will be on the sphere if

$$\sin\theta = \frac{0P}{0E} = \frac{1/d_{hki\ell}}{2/\lambda} = \frac{\lambda}{2\,d_{hki\ell}} \quad . \tag{11}$$

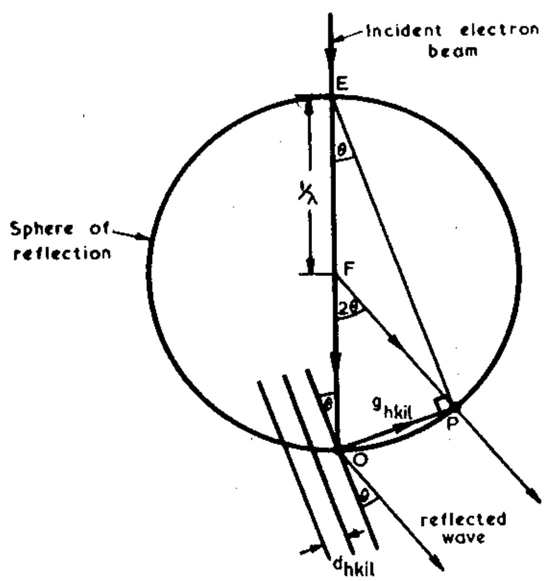


Figure 7. Ewald sphere construction in reciprocal lattice

This relation is equivalent to Bragg's law. Thus, depending on the structure factor, a diffracted beam FP will arise whenever the reflecting sphere intersects a reciprocal lattice point.

For electrons accelerated by 100 kV in the electron microscope, $\lambda \sim 0.037$ Å, the radius of the reflecting sphere is ~ 27 Å^{-1} which is large compared with the lattice spacing of metals. The Bragg angles are also very small (about 1°) and thus the reflecting sphere can be approximated to a plane over the visible area of the diffraction pattern. Consequently, if a plane in the reciprocal lattice is tangential to the reflecting sphere, a simple cross-grating spot pattern will be formed. The normal to the reciprocal lattice plane is a direct lattice vector $\underline{r}$ = [uvtw], the crystal zone axis, which is parallel to the direction of the incident electron beam. The indices of $\underline{r}$ can be determined from any two non-collinear spots $\underline{g}_1 = [h_1k_1i_1\ell_1]$ and $\underline{g}_2 = [h_2k_2i_2\ell_2]$ on the diffraction pattern since $\underline{g}_1 \cdot \underline{r} = \underline{g}_2 \cdot \underline{r} = 0$.

Okamoto and Thomas (1968) have shown that in the four-axis hexagonal notation this leads to three simultaneous equations:

$$\left.\begin{aligned} \underline{g}_1 \cdot \underline{r} &= h_1 u + k_1 v + i_1 t + \ell_1 w = 0 \\ \underline{g}_2 \cdot \underline{r} &= h_2 u + k_2 v + i_2 t + \ell_2 w = 0 \\ & \quad\; u + v + t = 0 \end{aligned}\right\} , \tag{12}$$

which can be rearranged to give:

$$\left.\begin{aligned} h_1 \frac{u}{w} + k_1 \frac{v}{w} + i_1 \frac{t}{w} &= -\ell_1 \\ h_2 \frac{u}{w} + k_2 \frac{v}{w} + i_2 \frac{t}{w} &= -\ell_2 \\ \frac{u}{w} + \frac{v}{w} + \frac{t}{w} &= 0 \end{aligned}\right\} , \tag{13}$$

and using Cramer's rule the ratio u:v:t:w: can be solved to give:

$$\underline{r} = [uvtw] = \left[\begin{vmatrix} \bar{\ell}_1 & k_1 & i_1 \\ \bar{\ell}_2 & k_2 & i_2 \\ 0 & 1 & 1 \end{vmatrix} , \begin{vmatrix} h_1 & \bar{\ell}_1 & i_1 \\ h_2 & \bar{\ell}_2 & i_2 \\ 1 & 0 & 1 \end{vmatrix} , \begin{vmatrix} h_1 & k_1 & \bar{\ell}_1 \\ h_2 & k_2 & \bar{\ell}_2 \\ 1 & 1 & 0 \end{vmatrix} , \begin{vmatrix} h_1 & k_1 & i_1 \\ h_2 & k_2 & i_2 \\ 1 & 1 & 1 \end{vmatrix} \right] . \tag{14}$$

Equation 14 contains a built-in symmetry check since the sum of the first three determinants must vanish. The indices of $\underline{r}$ represent the upward drawn normal to the diffraction pattern if $\underline{g}_1$ and $\underline{g}_2$ are chosen so that an anticlockwise rotation of $\underline{g}_1$ through an angle less than 180° would bring the direction of $\underline{g}_1$ into the direction of $\underline{g}_2$.

Consider the steps in indexing the electron diffraction pattern shown in Figure 8.

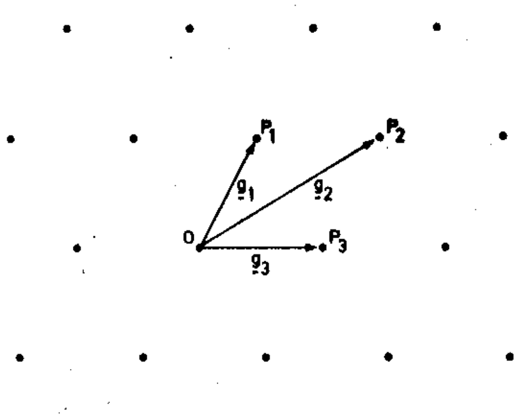


Figure 8. Indexing an unknown diffraction pattern

(i) Determine the interplanar spacings (d) of planes giving rise to the spots P_1, P_2, P_3 on the diffraction pattern using the relationship

$$d_{hki\ell} = \frac{\lambda L}{R} , \tag{15}$$

where R is the distance of the spots from the origin and λL is the camera constant obtained by calibration of the electron microscope with a material of known lattice spacing.

(ii) Usually a set of indices $h_1 k_1 i_1 \ell_1$ giving the correct value of d, can be assigned to $\underline{g}_1$ provided that not more than one crystallographically distinct set of such indices exists.

(iii) After assigning a specific set of indices to $\underline{g}_1$, there will still be many equivalent ways of indexing $\underline{g}_3$, corresponding to the multiplicity factor for the set of planes $h_3 k_3 i_3 \ell_3$. The multiplicity factors for planes in the h.c.p. system are listed below:

{hkiℓ} = 24, {hhiℓ} = 12, {okiℓ} = 12, {hkio} = 12,

{hhio} = 6, {okio} = 6, {oooℓ} = 2.

Many of the permutations of indices of $h_3k_3i_3\ell_3$ may lead to results which are not distinguishable crystallographically and a correct set can be chosen by measuring the angle between $\underline{g}_1$ and $\underline{g}_3$ and comparing the results with tables of interplanar angles in conjunction with a standard projection of plane normals.

(iv) Once two points on the diffraction pattern have been indexed consistently, all other points can be indexed by simple vector addition, for example,

$$[h_2\,k_2\,i_2\,\ell_2] = [h_1\,k_1\,i_1\,\ell_1] + [h_3\,k_3\,i_3\,\ell_3]\ .$$

(v) When the indices of two reciprocal lattice vectors $\underline{g}_1$ and $\underline{g}_2$ are known, the normal to the cross grating pattern, that is, the zone axis of the crystal parallel to the directions of the incident electron beam, can be obtained using Equations 12 and 14. The sense of the direction of the normal to the diffraction pattern in the usual right-handed system of axes can be obtained using the convention stated previously or by inspection using a double stereogram.

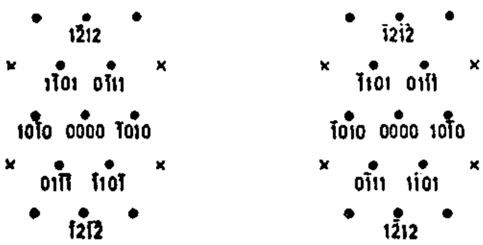
(vi) It must be noted that even if the sense of the normal to the diffraction pattern is known, there is still an ambiguity in indexing corresponding to a rotation of 180° about the normal to the foil. This ambiguity arises from the arbitrary assignment of the sign of the diffraction vector $\underline{g}_1$ in step (ii) above and can lead to erroneous results in some studies. This ambiguity can be removed by analysing the Kikuchi pattern (to be considered in the next section) or by noting the effect of an applied tilt as illustrated in the following example. Consider a diffraction pattern with the upward drawn normal given by $[\bar{1}2\bar{1}3]$. Such a pattern can be indexed in two ways as shown in Figures 9(a) and 9(b). The corresponding stereographic projections of plane normals giving the crystal orientation are shown in Figures 10(a) and 10(b) respectively. The correct indexing can be obtained by distinguishing the effects of tilting the crystal about $[10\bar{1}0]$ in the sense indicated on Figures 10(a) and 10(b). A tilt of 19° would bring the crystal into an orientation such that spots corresponding to $0\bar{1}12$, $1\bar{1}02$ and $1\bar{2}14$ would appear on the diffraction pattern if the crystal orientation assumed in Figure 10(a) is correct. Otherwise, a crystal tilt of 14° would have caused reflections $02\bar{2}\bar{1}$, $\bar{2}20\bar{1}$ and $\bar{1}2\bar{1}\bar{1}$ to appear as implied by Figure 10(b) which corresponds to the indexing assumed in Figure 9(b). It is important to realise that the alternative methods of indexing can lead to a deduction of opposite sense of inclination of planes in the [0001] zone for example.

It is often convenient to have available examples of indexed single crystal electron diffraction patterns to aid the identification of unknown patterns on the microscope screen by simple inspection. A number of the most commonly occurring diffraction patterns from h.c.p. crystals, indexed systematically in terms of Miller-Bravais indices, have been published by Partridge (1967) and Partridge and Gardiner (1967 b) and are reproduced with some variations in Appendix 6.

In h.c.p. crystals, the structure factor becomes zero and there is no diffracted beam when (h + 2k) is a multiple of 3 and ℓ is odd, for example $11\bar{2}1$, $303\bar{3}$ etc. These conditions, however, are relaxed for electron diffraction where extra spots may arise as a result of double diffraction, for example "forbidden" spots 0001 may arise from double diffraction involving reflections $10\bar{1}1$ and $\bar{1}010$. Spots due to double diffraction have been differentiated from the normal spots with non-zero structure factors in the diffraction patterns shown in Appendix 6.

3.3 Kikuchi Patterns

A most useful method of determining the crystal orientation involves the analysis of Kikuchi patterns associated with the spots in electron diffraction patterns. A Kikuchi pattern is formed as a result of Bragg diffraction of electrons which have suffered inelastic collisions involving only a small energy loss. The inelastic scattering provides the main component of the background intensity of the diffraction pattern. The Bragg scattered inelastic electrons, which form cones with the normal to the reflecting planes as axis, intersect the photographic plate in hyperbolae which under the geometrical



(a) (b)

Figure 9. Alternative ways of indexing a diffraction pattern from a thin foil with the upward drawn normal $[\bar{1}2\bar{1}3]$ parallel to the electron beam

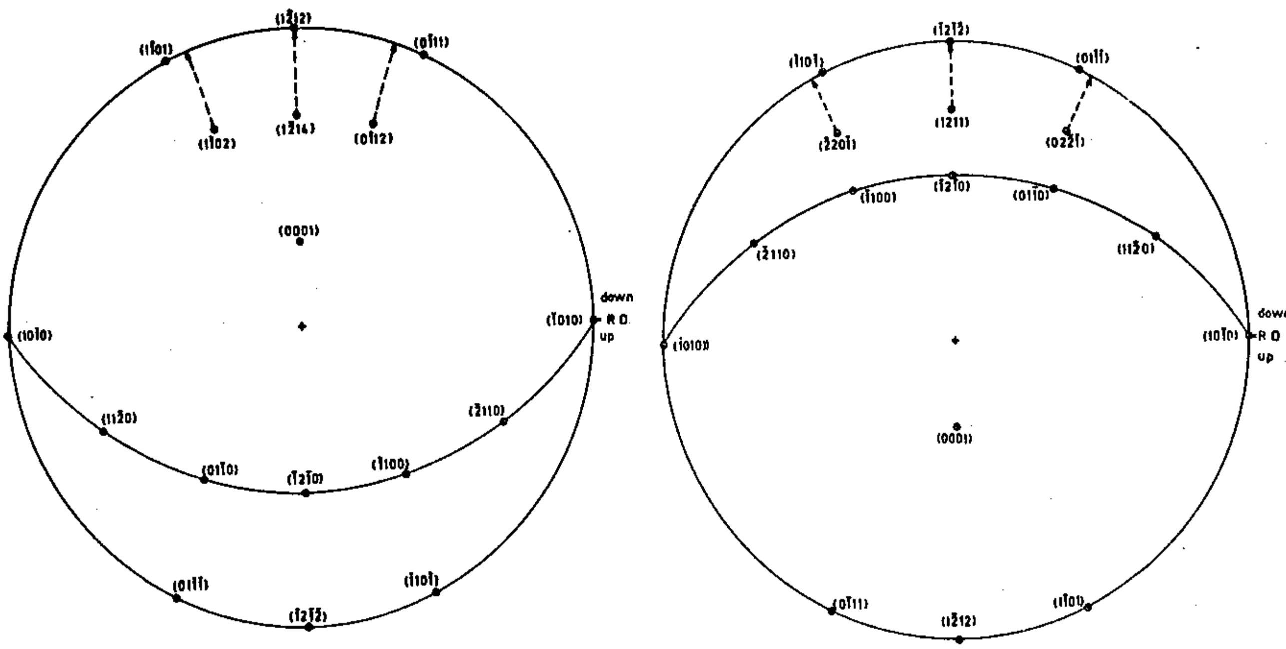


Figure 10. Stereographic projections of plane normals; (a) and (b) correspond to diffraction patterns in Figures 9(a) and 9(b) respectively. The broken lines show reflections which would appear on the diffraction pattern after tilting about $[10\bar{1}0]$ in the sense given on the projections

conditions in the electron microscope can be considered as nearly straight lines. Each set of planes gives rise to two parallel lines in a direction perpendicular to the projection of $\underline{g}$; the line separation R is given by

$$|g_{hki\ell}| = \frac{1}{d_{hki\ell}} = \frac{R}{\lambda L} , \quad (16)$$

where λL is the camera constant. The angular separation of the lines is thus 2θ and the lines are situated symmetrically on either side of the trace of the reflecting plane. This property leads to an important difference in behaviour between the Kikuchi lines and the normal spot pattern when the specimen is tilted. The spot pattern does not move on tilting over a small angular range, but the individual spots change in intensity; some spots disappear and others appear in different positions. On the other hand, the Kikuchi lines move as though rigidly fixed to the crystal, so that their direction and magnitude of movement reveals the orientation change with high accuracy. The determination of accurate orientations from Kikuchi patterns will be considered in this section.

Let us consider first, however, the limitations inherent in electron diffraction spot patterns for accurate orientation determinations.

(i) Because of the extension of reciprocal lattice points into relrods, reflections can remain unchanged over a large angular range of tilt ($\pm 5°$) leading to errors of at least this order of magnitude in the determined orientations. In very thin foils, spot patterns become complicated because second-Laue-layer relrods intersect the reflecting sphere.

(ii) A symmetrical diffraction pattern may be obtained from foils in non-symmetrical orientations as a result of foil buckling.

(iii) Perhaps the most important limitation in diffraction studies is the need to use two-beam conditions. It is not possible, of course, to determine an accurate orientation from one diffracted beam.

The indexing and application of Kikuchi patterns in orientation determinations is well covered in the text by Hirsch et al. (1965) and the papers by von Heimendahl et al. (1964) and Otte et al. (1964). The application of Kikuchi methods to h.c.p. metals has been discussed by Okamoto et al. (1967) and Okamoto and Thomas (1968). The essential features are discussed here.

The initial step in orientation determination is the identification of Kikuchi lines from measurements of the pair separation, whereby $d_{hki\ell}$ derived using Equation 16 is compared with values of interplanar spacings for the material studied. Ambiguities in indexing Kikuchi pairs with very similar separation can be avoided by comparison of measured angles between different sets of Kikuchi pairs with interplanar angles between the sets of planes responsible for the Kikuchi lines.

The intersection of the centre lines of different pairs of Kikuchi lines, the Kikuchi pole, corresponds to a crystal zone axis containing the planes giving rise to the Kikuchi lines. The indices of the zone axis can be obtained by substitution in Equation 12 and 14 of indices of the intersecting set of Kikuchi pairs. Thus, when a Kikuchi pole is located at the origin of the diffraction pattern the orientation is determined accurately.

The most common situation, however, is one where the Kikuchi poles are not located on the origin of the diffraction pattern. Then the orientation is determined by indexing three non-parallel Kikuchi pairs and the Kikuchi poles of their intersections (von Heimendahl et al. 1964). The orientation is then obtained from angles subtended between these poles and the origin of the diffraction pattern, either by calculation or stereographic projection. Since Kikuchi poles are zone axes in the crystal, that is, directions, a standard stereographic projection of directions must be employed for hexagonal metals. The method of orientation determination using two Kikuchi poles has been discussed by Otte et al. (1964).

Okamoto et al. (1967) and Okamoto and Thomas (1968) have constructed composite Kikuchi maps which cover a significant region of reciprocal space. These maps enable unknown Kikuchi patterns to be solved quickly even when they do not contain two Kikuchi poles. The detailed geometry of these maps for h.c.p. metals depends on the c/a ratio. Okamoto and Thomas (1968) have constructed a Kikuchi map for titanium (c/a = 1.588) which is reproduced in Appendix 7 and should be applicable to zirconium where c/a = 1.593.

Although Kikuchi patterns can be constructed for various orientations, the curvature of Kikuchi lines gives rise to difficulties if large angular ranges are required. The most satisfactory solution, according to Hirsch et al. (1965), is to present a large angle plot as a stereographic projection of plane traces, that is, the centre lines of Kikuchi pairs.

In conclusion, it is emphasised that because a Kikuchi pattern is uniquely representative of crystal symmetry the 180° ambiguity in orientation determination, which arises from the arbitrary assignment of the sign of the diffraction vector $\underline{g}$ in electron diffraction patterns, is not present in orientations determined from Kikuchi patterns.

3.4 Determination of Dislocation Burgers Vectors

Contrast in transmission electron microscope images of crystalline materials arises mainly from Bragg scattering of the incident electron beam. In bright field microscopy a small aperture is inserted in the objective lens so that diffracted electrons are blocked out and only the undeviated electrons are permitted to form the image. Thus, dark regions in the image correspond to regions of the thin foil which are favourably oriented to give strong diffracted beams. The interpretation of bright field contrast is facilitated if only one diffracted beam is operating, that is, the 'two-beam' case of one undeviated beam and a diffracted beam. The two-beam theory of dislocation image contrast is presented in detail in the text by Hirsch et al. (1965).

The usual method of Burgers vector determination makes use of the invisibility criteria. Thus a perfect screw dislocation, which produces atomic displacements in the direction of the Burgers vector, will be invisible, in an elastically isotropic material, if the condition $\underline{g} \cdot \underline{b} = 0$ is satisfied, where $\underline{g}$ is the diffracting vector and $\underline{b}$ is the Burgers vector of the dislocation. This, of course, is a general principle stating that displacements or lattice distortions parallel to the reflecting plane produce no contrast. Similarly, an edge dislocation will be nearly invisible if $\underline{g} \cdot \underline{b} = 0$ although some residual contrast may be observed because a pure edge dislocation produces minor atomic displacements perpendicular to $\underline{b}$. The condition for exact invisibility for a perfect edge dislocation is given by $\underline{g} \cdot \underline{b} = 0$ together with $\underline{g} \cdot \underline{b} \times \underline{u} = 0$, where $\underline{u}$ is a unit vector along the dislocation line. For mixed dislocations some contrast is expected under all two-beam conditions, but it is found (Hirsch et al. 1965) that dislocations are effectively invisible if $\underline{g} \cdot \underline{b} = 0$, and if $\underline{g} \cdot \underline{b} \times \underline{u} \leqslant 0.64$. At higher values of $\underline{g} \cdot \underline{b} \times \underline{u}$ some contrast is expected.

The above account of the invisibility criteria is adequate for materials which are elastically isotropic. Head et al. (1967) and Humble (1967) have demonstrated that as the degree of elastic anisotropy is increased, the residual contrast associated with the condition $\underline{g} \cdot \underline{b} = 0$ and $\underline{g} \cdot \underline{b} \times \underline{u} = 0$ generally, tends to increase. In practice, thus, 'near invisibility' is generally accepted as corresponding to the condition $\underline{g} \cdot \underline{b} = 0$.

The degree of elastic anisotropy in h.c.p. crystals cannot be described by a single parameter as in cubic crystals. For a h.c.p. metal to be elastically isotropic, the following three ratios must be simultaneously equal to unity:

$$A = 2c_{44}/(c_{11} - c_{12}) = c_{44}/c_{66}$$

$$B = c_{33}/c_{11}$$

$$C = c_{12}/c_{13} \ .$$

Fisher and Alfred (1968) showed that for zirconium at 300°K, A = 0.907, B = 1.149, C = 1.115 whereas at 1133°K the ratios are 2.00, 1.292 and 1.281 respectively. Thus, at room temperature elastic isotropy appears to be a reasonable approximation for zirconium and should permit the determination of the condition $\underline{g} \cdot \underline{b} = 0$ from observations of 'near-invisibility'.

The unambiguous determination of b requires two values of g which correspond to the condition g . b = 0. The Burgers vector is then parallel to the direction which is common to these two reflecting planes, that is, $g_1 \times g_2$. The indices of b in the Miller-Bravais notation are obtained by solving Equations 12 and 14.

Values of g . b for perfect dislocations in h.c.p. metals with various low order reflections have been tabulated by Partridge (1967) and are reproduced, with the addition of the $11\bar{2}4$ reflection, in Appendix 8. Using these values of g . b, the fraction of perfect dislocations visible for any of the low-order reflections included in Appendix 8 can be deduced and are shown in Table 2. The values shown in Table 2 are based on the assumptions:

(a) all Burgers vectors of similar form are equally represented,

(b) dislocations are visible only when g . b $\neq$ 0.

TABLE 2

THE FRACTION OF PERFECT DISLOCATIONS VISIBLE IN H.C.P. METALS

(ASSUMING ALL BURGERS VECTORS OF A FORM ARE EQUALLY REPRESENTED)

Reflection	Fraction of dislocation visible		
	$\frac{1}{3}\langle 11\bar{2}0\rangle$	$\frac{1}{3}\langle 11\bar{2}3\rangle$	$\langle 0001\rangle$
$10\bar{1}0$	2/3	2/3	0
0002	0	1	1
$10\bar{1}1$	2/3	2/3	1
$10\bar{1}2$	2/3	1	1
$10\bar{1}3$	2/3	1	1
$11\bar{2}0$	1	1	0
$11\bar{2}2$	1	5/6	1
$11\bar{2}4$	1	1	1

In addition to uncertainties introduced in bright-field image analysis of Burgers vectors in materials exhibiting elastic anisotropy, France and Loretto (1968) have demonstrated that under certain two-beam conditions dislocations are predicted to be invisible when g . b $\neq$ 0. These calculations have been supported by observation. This behaviour is connected with large values of w, the deviation parameter in the dynamical theory (Hirsch et al. 1965) which are usually associated with large diffraction vectors.

To avoid errors in the determination of Burgers vectors of dislocations, the observed bright field images should be compared with calculated images which take into full account the foil, dislocation and diffraction geometry under which the observations were made. Head (1967) has developed a method of computer calculation of theoretical images as pictures taking into account elastic anisotropy. This technique has been generalised by Humble (1968) for the case of a dislocation in a tilted foil. An important parameter in these calculations is the extinction distance corresponding to the various diffraction vectors used. Extinction distances for the common reflections in zirconium have been computed and are listed in Appendix 9.

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APPENDIX 1

INTERPLANAR SPACINGS IN α–ZIRCONIUM

Interplanar spacings for α–zirconium, shown in the table below, were calculated from lattice parameters determined with powder X-ray methods on zone refined zirconium supplied by Materials Research Corporation. The lattice parameters at 21 °C are:

$$a = 3.2330\ \text{Å}$$

$$c = 5.1497\ \text{Å}$$

$$c/a = 1.5927$$

$\{hki\ell\}$	d in Å	$\{hki\ell\}$	d in Å
$10\bar{1}0$	2.7999	$11\bar{2}4$	1.0071
0002	2.5749	$12\bar{3}2$	0.9788
$10\bar{1}1$	2.4598	$20\bar{2}4$	0.9476
$10\bar{1}2$	1.8953	$30\bar{3}0$	0.9333
$11\bar{2}0$	1.6165	$12\bar{3}3$	0.9008
$10\bar{1}3$	1.4634	$30\bar{3}2$	0.8774
$20\bar{2}0$	1.3999	$12\bar{3}4$	0.8175
$11\bar{2}2$	1.3691	$22\bar{4}0$	0.8083
$20\bar{2}1$	1.3509		
0004	1.2874		
$20\bar{2}2$	1.2299		
$20\bar{2}3$	1.0849		
$12\bar{3}0$	1.0583		
$12\bar{3}1$	1.0366		

APPENDIX 2 PAGE 1

ANGLES BETWEEN CRYSTALLOGRAPHIC PLANES IN ALPHA ZIRCONIUM

C/A = 1.5927

H K I L	H K I L	ANGLES IN DEGREES					
0 0 0 1	0 0 0 1	0.0					
	1 0 -1 0	90.00					
	1 0 -1 1	61.46					
	1 0 -1 2	42.60					
	1 0 -1 3	31.51					
	1 0 -1 4	24.69					
	2 0 -2 1	74.79					
	2 0 -2 3	50.80					
	3 0 -3 1	79.73					
	3 0 -3 2	70.07					
	1 1 -2 0	90.00					
	1 1 -2 1	72.57					
	1 1 -2 2	57.88					
	1 1 -2 3	46.72					
	1 1 -2 4	38.53					
	1 2 -3 0	90.00					
	1 2 -3 1	78.39					
	1 2 -3 2	67.66					
	1 2 -3 3	58.34					
1 0 -1 0	1 0 -1 0	0.0	60.00				
	1 0 -1 1	28.53	63.94				
	1 0 -1 2	47.40	70.22				
	1 0 -1 3	58.49	74.85				
	1 0 -1 4	65.31	77.94				
	2 0 -2 1	15.21	61.15				
	2 0 -2 3	39.20	67.20				
	3 0 -3 1	10.27	60.53				
	3 0 -3 2	19.93	61.96				
	1 1 -2 0	30.00	90.00				
	1 1 -2 1	34.28	90.00				
	1 1 -2 2	42.82	90.00				
	1 1 -2 3	50.92	90.00				
	1 1 -2 4	57.35	90.00				
	1 2 -3 0	19.11	40.89	79.11			
	1 2 -3 1	22.25	42.23	79.33			
	1 2 -3 2	29.08	45.64	79.93			
	1 2 -3 3	36.46	49.95	80.74			
1 0 -1 1	1 0 -1 1	0.0	52.11	57.07	80.93		
	1 0 -1 2	18.86	49.54	75.94	86.89		
	1 0 -1 3	29.96	50.44	79.77	87.03		
	1 0 -1 4	36.77	51.87	75.49	86.16		
	2 0 -2 1	13.33	43.74	56.69	72.63		
	2 0 -2 3	10.67	50.03	67.74	87.80		
	3 0 -3 1	18.26	38.81	58.84	69.69		
	3 0 -3 2	8.61	48.46	54.85	75.51		
	1 1 -2 0	40.46	90.00				
	1 1 -2 1	29.66	54.35	81.77			
	1 1 -2 2	26.06	67.02	75.28			
	1 1 -2 3	28.19	70.88	76.92			
	1 1 -2 4	32.04	68.06	84.24			
	1 2 -3 0	33.89	48.39	80.44			
	1 2 -3 1	24.59	41.70	44.20	56.33	75.00	86.19

ANGLES BETWEEN CRYSTALLOGRAPHIC PLANES IN ALPHA ZIRCONIUM

C/A = 1.5927

H K I L	H K I L	ANGLES IN DEGREES					
1 0 -1 1	1 2 -3 2	18.30	37.26	54.11	64.37	70.42	88.39
	1 2 -3 3	16.80	35.31	62.88	66.92	71.66	83.72
1 0 -1 2	1 0 -1 2	0.0	39.56	71.77	85.20		
	1 0 -1 3	11.09	36.44	63.21	74.11		
	1 0 -1 4	17.91	35.89	58.17	67.29		
	2 0 -2 1	32.19	58.69	62.61	82.33		
	2 0 -2 3	8.20	43.32	78.29	86.60		
	3 0 -3 1	37.13	57.67	62.34	78.36		
	3 0 -3 2	27.47	55.32	67.33	86.14		
	1 1 -2 0	54.11	90.00				
	1 1 -2 1	38.76	70.20	77.26			
	1 1 -2 2	27.39	66.96	83.97			
	1 1 -2 3	21.35	59.69	85.53			
	1 1 -2 4	19.78	54.84	77.84			
	1 2 -3 0	50.24	59.22	82.65			
	1 2 -3 1	39.22	49.51	61.42	69.33	74.13	88.69
	1 2 -3 2	29.38	41.14	66.54	71.84	78.85	80.70
	1 2 -3 3	21.45	34.73	60.32	73.89	80.90	87.18
1 0 -1 3	1 0 -1 3	0.0	30.30	53.82	63.02		
	1 0 -1 4	6.82	27.90	48.28	56.20		
	2 0 -2 1	43.28	61.59	73.70	88.37		
	2 0 -2 3	19.29	42.15	70.35	82.31		
	3 0 -3 1	48.22	65.85	68.76	83.97		
	3 0 -3 2	38.56	57.57	78.42	87.43		
	1 1 -2 0	63.09	90.00				
	1 1 -2 1	46.59	75.21	79.83			
	1 1 -2 2	33.21	63.04	85.99			
	1 1 -2 3	23.93	54.23	75.23			
	1 1 -2 4	18.40	48.17	67.36			
	1 2 -3 0	60.41	66.73	84.33			
	1 2 -3 1	49.05	56.04	71.81	74.43	77.56	85.71
	1 2 -3 2	38.66	46.41	65.45	76.54	82.38	87.63
	1 2 -3 3	29.80	38.40	57.89	68.69	83.62	88.45
1 0 -1 4	1 0 -1 4	0.0	24.11	42.42	49.38		
	2 0 -2 1	50.10	63.90	80.52	87.89		
	2 0 -2 3	26.11	42.60	65.64	75.49		
	3 0 -3 1	55.04	68.43	75.58	87.51		
	3 0 -3 2	45.38	59.60	83.50	85.23		
	1 1 -2 0	68.79	90.00				
	1 1 -2 1	51.88	74.21	85.81			
	1 1 -2 2	37.86	61.11	79.82			
	1 1 -2 3	27.59	51.47	68.93			
	1 1 -2 4	20.59	44.71	60.96			
	1 2 -3 0	66.75	71.59	85.47			
	1 2 -3 1	55.28	60.51	74.92	78.24	82.74	83.94
	1 2 -3 2	44.72	50.40	65.26	74.19	86.94	88.87
	1 2 -3 3	35.63	41.79	57.04	65.82	77.99	81.90
2 0 -2 1	2 0 -2 1	0.0	30.42	57.70	66.62		
	2 0 -2 3	23.99	54.41	57.34	77.99		
	3 0 -3 1	4.94	25.48	58.56	64.66		

ANGLES BETWEEN CRYSTALLOGRAPHIC PLANES IN ALPHA ZIRCONIUM

C/A = 1.5927

H K I L	H K I L	ANGLES IN DEGREES					
2 0 -2 1	3 0 -3 2	4.72	35.14	57.11	68.64		
	1 1 -2 0	33.31	90.00				
	1 1 -2 1	28.85	44.05	85.49			
	1 1 -2 2	32.09	55.37	81.98			
	1 1 -2 3	37.98	64.63	79.64			
	1 1 -2 4	43.46	71.62	78.16			
	1 2 -3 0	24.24	43.16	79.49			
	1 2 -3 1	18.92	32.82	39.89	48.57	76.62	82.77
	1 2 -3 2	19.42	39.25	41.96	54.90	74.43	86.05
	1 2 -3 3	23.96	40.66	50.32	61.10	72.97	88.99
2 0 -2 3	2 0 -2 3	0.0	45.59	78.40	[illegible]		
	3 0 -3 1	28.93	49.47	60.40	74.42		
	3 0 -3 2	19.28	54.57	59.13	81.44		
	1 1 -2 0	47.85	90.00				
	1 1 -2 1	33.94	63.19	79.09			
	1 1 -2 2	25.25	70.36	76.57			
	1 1 -2 3	22.80	64.32	86.83			
	1 1 -2 4	24.15	60.37	85.62			
	1 2 -3 0	42.93	54.14	81.58			
	1 2 -3 1	32.38	45.49	53.84	63.48	74.29	89.07
	1 2 -3 2	23.43	38.55	64.09	67.93	72.45	83.98
	1 2 -3 3	17.25	33.87	62.85	73.05	78.05	80.39
3 0 -3 1	3 0 -3 1	0.0	20.55	58.94	63.11		
	3 0 -3 2	9.65	30.20	58.45	66.31		
	1 1 -2 0	31.55	90.00				
	1 1 -2 1	29.95	40.57	86.94			
	1 1 -2 2	35.26	51.18	84.56			
	1 1 -2 3	42.05	60.13	82.98			
	1 1 -2 4	47.91	66.96	81.98			
	1 2 -3 0	21.60	41.94	79.28			
	1 2 -3 1	18.80	28.98	40.14	46.16	77.41	81.59
	1 2 -3 2	21.91	37.61	40.91	51.67	76.13	84.02
	1 2 -3 3	27.75	43.39	45.75	57.35	75.41	86.29
3 0 -3 2	3 0 -3 2	0.0	39.85	56.08	70.99		
	1 1 -2 0	35.49	90.00				
	1 1 -2 1	28.49	47.57	84.14			
	1 1 -2 2	29.45	59.45	79.56			
	1 1 -2 3	34.27	68.96	76.49			
	1 1 -2 4	39.30	74.54	76.08			
	1 2 -3 0	27.33	44.71	79.77			
	1 2 -3 1	20.15	36.72	40.12	51.13	75.96	83.95
	1 2 -3 2	17.97	38.11	46.20	58.15	72.91	88.01
	1 2 -3 3	20.77	38.39	54.74	64.78	70.73	88.42
1 1 -2 0	1 1 -2 0	0.0	60.00				
	1 1 -2 1	17.43	61.51				
	1 1 -2 2	32.12	64.95				
	1 1 -2 3	43.28	68.65				
	1 1 -2 4	51.47	71.85				
	1 2 -3 0	10.89	49.11	70.89			
	1 2 -3 1	15.87	50.11	71.30			

ANGLES BETWEEN CRYSTALLOGRAPHIC PLANES IN ALPHA ZIRCONIUM

C/A = 1.5927

H K I L	H K I L	ANGLES IN DEGREES					
1 1 -2 0	1 2 -3 2	24.74	52.74	72.38			
	1 2 -3 3	33.29	56.13	73.82			
1 1 -2 1	1 1 -2 1	0.0	34.86	56.99	68.57		
	1 1 -2 2	14.69	49.55	55.72	75.83		
	1 1 -2 3	25.85	56.45	60.71	81.84		
	1 1 -2 4	34.04	57.89	68.90	86.40		
	1 2 -3 0	20.46	51.35	71.80			
	1 2 -3 1	12.04	30.97	47.77	56.53	68.52	75.78
	1 2 -3 2	11.36	41.18	46.25	62.37	66.25	79.92
	1 2 -3 3	17.31	46.46	50.19	64.97	68.01	83.76
1 1 -2 2	1 1 -2 2	0.0	50.11	64.25	85.65		
	1 1 -2 3	11.16	47.71	75.41	86.77		
	1 1 -2 4	19.34	47.18	81.25	83.59		
	1 2 -3 0	33.73	56.33	73.91			
	1 2 -3 1	22.83	44.96	49.45	64.15	67.75	80.53
	1 2 -3 2	13.75	44.36	55.46	62.71	71.90	86.89
	1 2 -3 3	9.26	41.32	59.00	64.61	78.88	87.53
1 1 -2 3	1 1 -2 3	0.0	42.69	78.17	86.57		
	1 1 -2 4	8.18	40.26	71.97	85.25		
	1 2 -3 0	44.37	61.54	76.21			
	1 2 -3 1	33.05	52.78	55.79	68.20	70.80	84.53
	1 2 -3 2	22.80	45.46	61.25	66.39	79.62	87.69
	1 2 -3 3	14.46	40.05	55.76	75.60	80.97	87.37
1 1 -2 4	1 1 -2 4	0.0	36.30	65.30	77.06		
	1 2 -3 0	52.29	65.93	78.23			
	1 2 -3 1	40.83	56.16	63.79	69.07	76.00	87.58
	1 2 -3 2	30.32	47.58	60.92	74.43	83.75	85.42
	1 2 -3 3	21.37	40.74	54.26	76.29	83.68	86.36
1 2 -3 0	1 2 -3 0	0.0	21.79	38.21	60.00	81.79	
	1 2 -3 1	11.61	24.56	39.68	60.67	81.96	
	1 2 -3 2	22.34	30.81	43.39	62.45	82.41	
	1 2 -3 3	31.66	37.78	48.02	64.81	83.02	
1 2 -3 1	1 2 -3 1	0.0	21.34	23.23	31.74	37.40	44.49
		58.65	63.95	79.77	84.46		
	1 2 -3 2	10.73	23.39	33.96	37.97	40.12	50.56
		58.03	67.89	78.11	86.97		
	1 2 -3 3	20.04	28.37	40.47	43.27	48.04	56.67
		58.50	71.87	77.01	89.23		
1 2 -3 2	1 2 -3 2	0.0	20.13	35.25	44.69	49.47	55.09
		58.15	73.55	74.53	88.72		
	1 2 -3 3	9.31	21.47	35.10	53.62	54.00	57.89
		65.22	71.82	78.81	85.01		
1 2 -3 3	1 2 -3 3	0.0	18.51	32.36	50.38	63.31	66.59
		67.73	72.91	80.10	85.02		

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ANGLES BETWEEN DIRECTIONS IN ALPHA ZIRCONIUM

C/A = 1.5927

U V T W	U V T W	ANGLES IN DEGREES	
0 0 0 1	0 0 0 1	0.0	
	1 0 -1 0	90.000	
	1 0 -1 1	47.400	
	1 0 -1 2	28.535	
	1 0 -1 3	19.925	
	1 0 -1 4	15.210	
	1 0 -1 5	12.271	
	1 0 -1 6	10.273	
	2 0 -2 1	65.308	
	2 0 -2 3	35.942	
	2 0 -2 5	23.509	
	3 0 -3 1	72.959	
	3 0 -3 2	58.490	
	3 0 -3 4	39.201	
	3 0 -3 5	33.124	
	4 0 -4 1	77.053	
	4 0 -4 3	55.408	
	1 1 -2 0	90.000	
	1 1 -2 1	62.036	
	1 1 -2 2	43.283	
	1 1 -2 3	32.123	
	1 1 -2 4	25.216	
	1 1 -2 6	17.429	
	1 1 -2 9	11.821	
	2 2 -4 1	75.134	
	2 2 -4 3	51.468	
	2 2 -4 5	36.996	
	2 2 -4 9	22.713	
	1 2 -3 0	90.000	
	1 2 -3 1	70.835	
	1 2 -3 2	55.196	
	1 2 -3 3	43.803	
	1 2 -3 4	35.728	
	1 2 -3 5	29.918	
	1 2 -3 6	25.619	
	1 3 -4 0	90.000	
	1 3 -4 1	75.692	
	1 3 -4 2	62.975	
	1 3 -4 3	52.580	
	1 4 -5 0	90.000	
	1 4 -5 1	78.654	
	1 4 -5 2	68.133	
	1 4 -5 3	58.953	
	2 3 -5 0	90.000	
	2 3 -5 1	78.088	
	2 3 -5 2	67.124	
	2 3 -5 3	57.671	
1 0 -1 0	1 0 -1 0	0.0	60.000
	1 0 -1 1	42.600	68.405
	1 0 -1 2	61.465	76.181
	1 0 -1 3	70.075	80.189
	1 0 -1 4	74.790	82.462
	1 0 -1 5	77.729	83.900

ANGLES BETWEEN DIRECTIONS IN ALPHA ZIRCONIUM
C/A = 1.5927

U V T W	U V T W	ANGLES IN DEGREES			
1 0 -1 0	1 0 -1 6	79.727	84.884		
	2 0 -2 1	24.692	62.981		
	2 0 -2 3	54.058	72.933		
	2 0 -2 5	66.491	78.495		
	3 0 -3 1	17.041	61.442		
	3 0 -3 2	31.510	64.768		
	3 0 -3 4	50.799	71.577		
	3 0 -3 5	56.876	74.143		
	4 0 -4 1	12.947	60.837		
	4 0 -4 3	34.592	65.694		
	1 1 -2 0	30.000	90.000		
	1 1 -2 1	40.101	90.000		
	1 1 -2 2	53.576	90.000		
	1 1 -2 3	62.580	90.000		
	1 1 -2 4	68.349	90.000		
	1 1 -2 6	74.966	90.000		
	1 1 -2 9	79.781	90.000		
	2 2 -4 1	33.171	90.000		
	2 2 -4 3	47.355	90.000		
	2 2 -4 5	58.591	90.000		
	2 2 -4 9	70.465	90.000		
	1 2 -3 0	19.107	40.893	79.107	
	1 2 -3 1	26.806	44.436	79.717	
	1 2 -3 2	39.115	51.632	81.073	
	1 2 -3 3	49.152	58.450	82.484	
	1 2 -3 4	56.512	63.806	83.664	
	1 2 -3 5	61.882	67.850	84.591	
	1 2 -3 6	65.885	70.922	85.313	
	1 3 -4 0	13.898	46.102	73.898	
	1 3 -4 1	19.845	47.788	74.410	
	1 3 -4 2	30.148	51.854	75.696	
	1 3 -4 3	39.560	56.586	77.275	
	1 4 -5 0	10.893	49.107	70.893	
	1 4 -5 1	15.679	50.069	71.281	
	1 4 -5 2	24.310	52.587	72.315	
	1 4 -5 3	32.722	55.884	73.714	
	2 3 -5 0	23.413	36.587	83.413	
	2 3 -5 1	26.117	38.218	83.556	
	2 3 -5 2	32.276	42.285	83.933	
	2 3 -5 3	39.157	47.274	84.438	
1 0 -1 1	1 0 -1 1	0.0	43.191	79.208	85.200
	1 0 -1 2	18.865	39.604	65.239	75.935
	1 0 -1 3	27.475	40.378	59.275	67.325
	1 0 -1 4	32.190	41.434	56.178	62.610
	1 0 -1 5	35.129	42.300	54.325	59.671
	1 0 -1 6	37.127	42.974	53.103	57.673
	2 0 -2 1	17.908	51.892	67.292	87.040
	2 0 -2 3	11.458	40.178	70.611	83.342
	2 0 -2 5	23.891	39.870	61.713	70.909
	3 0 -3 1	25.559	56.616	59.641	81.169
	3 0 -3 2	11.090	48.122	74.109	87.709
	3 0 -3 4	8.198	40.786	73.028	86.601
	3 0 -3 5	14.276	39.826	68.546	80.524

APPENDIX [illegible]

ANGLES BETWEEN DIRECTIONS IN ALPHA ZIRCONIUM

C/A = 1.5927

ANGLES IN DEGREES

U V T W	U V T W						
1 0 -1 1	4 0 -4 1	29.653	55.846	59.313	78.051		
	4 0 -4 3	8.008	46.586	77.192	85.336		
	1 1 -2 0	50.396	90.000				
	1 1 -2 1	28.304	71.494	75.780			
	1 1 -2 2	21.595	60.479	66.808			
	1 1 -2 3	24.185	55.023	76.451			
	1 1 -2 4	27.876	52.238	70.075			
	1 1 -2 6	33.203	49.774	62.944			
	1 1 -2 9	37.523	48.508	57.864			
	2 2 -4 1	37.833	63.738	79.999			
	2 2 -4 3	23.025	65.060	85.583			
	2 2 -4 5	22.450	57.275	80.967			
	2 2 -4 9	29.480	51.363	67.775			
	1 2 -3 0	45.929	56.190	82.004			
	1 2 -3 1	28.453	41.599	64.228	69.292	72.339	84.790
	1 2 -3 2	16.772	32.517	59.963	74.210	79.351	85.954
	1 2 -3 3	14.079	29.112	54.211	66.907	84.068	89.595
	1 2 -3 4	17.129	29.025	50.897	62.079	77.023	81.759
	1 2 -3 5	20.999	30.208	49.000	58.849	71.992	76.128
	1 2 -3 6	24.345	31.687	47.896	56.621	68.301	71.966
	1 3 -4 0	44.394	59.310	78.220			
	1 3 -4 1	30.722	48.560	58.324	68.586	70.896	88.249
	1 3 -4 2	19.251	40.340	60.697	70.794	81.541	82.779
	1 3 -4 3	11.818	35.248	55.009	75.572	81.014	89.659
	1 4 -5 0	43.711	61.191	76.058			
	1 4 -5 1	32.661	52.725	54.863	68.321	70.166	84.084
	1 4 -5 2	22.642	45.628	61.594	65.246	78.748	88.367
	1 4 -5 3	14.447	40.364	56.253	74.325	81.798	86.344
	2 3 -5 0	47.508	53.768	85.156			
	2 3 -5 1	36.807	44.107	58.585	63.985	77.154	86.727
	2 3 -5 2	27.689	36.129	68.946	70.067	73.654	79.320
	2 3 -5 3	21.131	30.524	64.321	73.104	77.947	82.099
1 0 -1 2	1 0 -1 2	0.028	27.637	48.874	57.070		
	1 0 -1 3	8.609	24.861	41.881	48.460		
	1 0 -1 4	13.325	24.437	38.271	43.745		
	1 0 -1 5	16.264	24.603	36.129	40.806		
	1 0 -1 6	18.262	24.901	34.730	38.808		
	2 0 -2 1	36.773	54.268	81.374	86.157		
	2 0 -2 3	7.407	31.629	55.175	64.477		
	2 0 -2 5	5.026	25.726	44.738	52.044		
	3 0 -3 1	44.424	60.934	78.506	88.333		
	3 0 -3 2	29.955	48.487	75.195	87.025		
	3 0 -3 4	10.666	33.720	58.006	67.736		
	3 0 -3 5	4.589	29.971	52.754	61.659		
	4 0 -4 1	48.518	64.558	74.412	87.940		
	4 0 -4 3	26.872	45.941	72.413	83.943		
	1 1 -2 0	65.563	90.000				
	1 1 -2 1	38.982	65.672	87.331			
	1 1 -2 2	22.605	50.242	69.151			
	1 1 -2 3	15.419	41.924	58.396			
	1 1 -2 4	13.819	37.363	51.789			
	1 1 -2 6	15.824	33.050	44.416			
	1 1 -2 9	19.154	30.695	39.181			

ANGLES BETWEEN DIRECTIONS IN ALPHA ZIRCONIUM

C/A = 1.5927

U V T W	U V T W	ANGLES IN DEGREES					
1 0 -1 2	2 2 -4 1	51.300	76.974	79.953			
	2 2 -4 3	29.437	56.819	77.076			
	2 2 -4 5	18.083	45.440	63.082			
	2 2 -4 9	14.039	35.865	49.409			
	1 2 -3 0	63.168	68.832	84.820			
	1 2 -3 1	44.375	50.986	68.057	78.279	82.071	86.980
	1 2 -3 2	29.300	37.066	54.861	64.703	78.175	82.484
	1 2 -3 3	18.829	27.871	45.850	55.141	67.412	71.240
	1 2 -3 4	12.376	22.475	40.013	48.664	59.846	63.281
	1 2 -3 5	9.396	19.685	36.247	44.240	54.455	57.566
	1 2 -3 6	9.131	18.506	33.779	41.139	50.505	53.346
	1 3 -4 0	62.373	70.657	82.387			
	1 3 -4 1	48.207	57.449	69.789	76.572	84.039	84.910
	1 3 -4 2	35.683	46.033	58.855	73.671	84.023	89.204
	1 3 -4 3	25.562	37.165	50.278	64.620	74.289	80.470
	1 4 -5 0	62.025	71.776	81.004			
	1 4 -5 1	50.746	61.350	70.965	73.317	82.312	88.881
	1 4 -5 2	40.311	51.871	61.815	79.508	83.792	87.881
	1 4 -5 3	31.242	43.861	54.051	71.390	79.329	87.065
	2 3 -5 0	64.000	67.445	86.859			
	2 3 -5 1	52.391	56.176	75.665	76.411	78.816	82.662
	2 3 -5 2	41.807	45.980	66.921	73.081	86.424	89.319
	2 3 -5 3	32.836	37.446	58.928	64.943	81.622	84.295
1 0 -1 3	1 0 -1 3	0.0	19.622	34.332	39.851		
	1 0 -1 4	4.716	17.841	30.401	35.135		
	1 0 -1 5	7.655	17.278	28.061	32.196		
	1 0 -1 6	9.652	17.166	26.533	30.199		
	2 0 -2 1	45.383	56.801	76.237	85.234		
	2 0 -2 3	16.016	30.552	48.614	55.867		
	2 0 -2 5	3.583	21.554	37.427	43.434		
	3 0 -3 1	53.033	63.996	83.535	87.116		
	3 0 -3 2	38.565	50.459	69.752	78.416		
	3 0 -3 4	19.276	33.255	51.623	59.127		
	3 0 -3 5	13.199	28.301	46.034	53.050		
	4 0 -4 1	57.128	67.871	83.021	87.446		
	4 0 -4 3	35.482	47.622	66.829	75.333		
	1 1 -2 0	72.834	90.000				
	1 1 -2 1	45.451	63.842	79.621			
	1 1 -2 2	27.533	46.812	61.181			
	1 1 -2 3	17.608	37.231	50.263			
	1 1 -2 4	12.502	31.728	43.547			
	1 1 -2 6	9.811	26.237	36.043			
	1 1 -2 9	11.287	23.045	30.713			
	2 2 -4 1	58.233	76.042	87.475			
	2 2 -4 3	35.260	54.150	69.220			
	2 2 -4 5	21.802	41.334	55.022			
	2 2 -4 9	11.131	29.862	41.125			
	1 2 -3 0	71.215	75.071	86.307			
	1 2 -3 1	52.207	56.497	68.317	75.652	86.256	89.744
	1 2 -3 2	36.773	41.571	53.880	61.072	71.031	74.206
	1 2 -3 3	25.655	31.037	43.689	50.659	59.987	62.895
	1 2 -3 4	17.965	23.987	36.792	43.482	52.210	54.889
	1 2 -3 5	12.718	19.379	32.116	38.489	46.657	49.138

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ANGLES BETWEEN DIRECTIONS IN ALPHA ZIRCONIUM

C/A = 1.5927

U V T W	U V T W	ANGLES IN DEGREES					
1 0 -1 3	1 2 -3 6	9.267	16.443	28.889	34.929	42.582	44.890
	1 3 -4 0	70.681	76.332	84.576			
	1 3 -4 1	56.434	62.529	71.100	81.909	84.938	89.807
	1 3 -4 2	43.790	50.381	59.244	69.942	77.486	82.387
	1 3 -4 3	33.487	40.629	49.733	60.251	67.443	72.029
	1 4 -5 0	70.448	77.109	83.595			
	1 4 -5 1	59.131	66.190	72.882	81.770	85.665	88.064
	1 4 -5 2	48.644	56.137	63.020	75.722	81.773	87.732
	1 4 -5 3	39.504	47.467	54.518	67.089	72.919	78.571
	2 3 -5 0	71.776	74.119	87.760			
	2 3 -5 1	59.996	62.496	76.567	81.036	83.573	85.774
	2 3 -5 2	49.186	51.860	66.329	70.765	83.492	85.565
	2 3 -5 3	39.913	42.778	57.602	61.983	74.244	76.202
1 0 -1 4	1 0 -1 4	0.0	15.075	26.265	30.419		
	1 0 -1 5	2.939	13.879	23.787	27.480		
	1 0 -1 6	4.936	13.370	22.164	25.483		
	2 0 -2 1	50.099	58.514	73.506	80.518		
	2 0 -2 3	20.732	30.879	45.230	51.152		
	2 0 -2 5	8.299	20.413	33.638	38.718		
	3 0 -3 1	57.749	65.908	80.945	88.168		
	3 0 -3 2	43.281	51.963	66.890	73.700		
	3 0 -3 4	23.992	33.830	48.327	54.411		
	3 0 -3 5	17.915	28.378	42.568	48.334		
	4 0 -4 1	61.844	69.877	84.931	87.737		
	4 0 -4 3	40.198	49.017	63.905	70.617		
	1 1 -2 0	76.868	90.000				
	1 1 -2 1	49.219	63.096	75.415			
	1 1 -2 2	30.880	45.374	56.859			
	1 1 -2 3	20.273	35.190	45.859			
	1 1 -2 4	14.113	29.188	39.084			
	1 1 -2 6	8.613	22.976	31.502			
	1 1 -2 9	7.671	19.177	26.108			
	2 2 -4 1	62.149	75.666	88.397			
	2 2 -4 3	38.843	53.049	64.951			
	2 2 -4 5	24.848	39.583	50.655			
	2 2 -4 9	12.077	27.109	36.639			
	1 2 -3 0	75.647	78.561	87.158			
	1 2 -3 1	56.568	59.727	68.677	74.338	82.561	85.260
	1 2 -3 2	41.034	44.470	53.738	59.332	67.174	69.683
	1 2 -3 3	29.769	33.518	43.050	48.538	56.002	58.342
	1 2 -3 4	21.856	25.951	35.677	41.026	48.121	50.312
	1 2 -3 5	16.256	20.724	30.559	35.742	42.484	44.542
	1 2 -3 6	12.234	17.088	26.933	31.933	38.339	40.278
	1 3 -4 0	75.246	79.519	85.827			
	1 3 -4 1	60.972	65.497	72.002	80.331	86.434	89.524
	1 3 -4 2	48.293	53.094	59.783	68.059	73.954	77.784
	1 3 -4 3	37.943	43.043	49.898	58.090	63.775	67.412
	1 4 -5 0	75.071	80.110	85.074			
	1 4 -5 1	63.741	69.008	74.095	83.935	86.403	88.771
	1 4 -5 2	53.238	58.749	63.954	73.757	78.462	83.090
	1 4 -5 3	44.078	49.848	55.162	64.906	69.480	73.921
	2 3 -5 0	76.069	77.839	88.275			
	2 3 -5 1	64.231	66.090	76.783	80.227	87.915	89.603

ANGLES BETWEEN DIRECTIONS IN ALPHA ZIRCONIUM
C/A = 1.5927

U V T W	U V T W	ANGLES IN DEGREES					
1 0 -1 4	2 3 -5 2	53.350	55.305	66.244	69.672	79.570	81.182
	2 3 -5 3	43.989	46.049	57.216	60.619	70.242	71.783
1 0 -1 5	1 0 -1 5	0.0	12.200	21.212	24.541		
	1 0 -1 6	1.997	11.351	19.518	22.544		
	2 0 -2 1	53.038	59.686	71.842	77.579		
	2 0 -2 3	23.671	31.407	43.219	48.213		
	2 0 -2 5	11.238	20.209	31.388	35.780		
	3 0 -3 1	60.688	67.172	79.353	85.230		
	3 0 -3 2	46.220	53.037	65.159	70.761		
	3 0 -3 4	26.931	34.473	46.365	51.472		
	3 0 -3 5	20.854	28.786	40.511	45.395		
	1 1 -2 0	79.394	90.000				
	1 1 -2 1	51.628	62.729	72.804			
	1 1 -2 2	33.119	44.656	54.186			
	1 1 -2 3	22.267	34.151	43.140			
	1 1 -2 4	15.750	27.866	36.329			
	1 2 -3 0	78.415	80.755	87.698			
	1 2 -3 1	59.304	61.800	68.978	73.569	80.268	82.467
	1 2 -3 2	43.729	46.398	53.793	58.349	64.798	66.869
	1 2 -3 3	32.410	35.270	42.859	47.357	53.557	55.513
1 0 -1 6	1 0 -1 6	0.0	10.232	17.770	20.547		
	2 0 -2 1	55.035	60.524	70.730	75.582		
	2 0 -2 3	25.669	31.900	41.902	46.215		
	2 0 -2 5	13.236	20.304	29.919	33.782		
	3 0 -3 1	62.686	68.061	78.281	83.232		
	3 0 -3 2	48.217	53.823	64.008	68.764		
	3 0 -3 4	28.928	35.028	45.078	49.475		
	3 0 -3 5	22.851	29.216	39.165	43.398		
	1 1 -2 0	81.115	90.000				
	1 1 -2 1	53.287	62.523	71.036			
	1 1 -2 2	34.695	44.250	52.381			
	1 1 -2 3	23.729	33.558	41.305			
	1 1 -2 4	17.058	27.101	34.472			
	1 2 -3 0	80.298	82.252	88.068			
	1 2 -3 1	61.170	63.232	69.215	73.071	78.715	80.570
	1 2 -3 2	45.574	47.754	53.893	57.728	63.197	64.961
	1 2 -3 3	34.229	36.538	42.821	46.621	51.915	53.594
2 0 -2 1	2 0 -2 1	0.0	49.383	54.038	76.217		
	2 0 -2 3	29.366	52.782	78.750	85.897		
	2 0 -2 5	41.799	55.648	78.355	88.817		
	3 0 -3 1	7.651	41.733	56.168	71.825		
	3 0 -3 2	6.818	52.726	56.201	80.272		
	3 0 -3 4	26.107	52.349	75.490	87.903		
	3 0 -3 5	32.184	53.266	81.567	84.169		
	4 0 -4 1	11.745	37.638	57.566	69.565		
	4 0 -4 3	9.901	52.328	59.284	82.137		
	1 1 -2 0	38.108	90.000				
	1 1 -2 1	27.019	60.060	78.704			
	1 1 -2 2	32.482	72.296	76.387			
	1 1 -2 3	39.450	69.281	86.295			
	1 1 -2 4	44.509	67.794	87.552			

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ANGLES BETWEEN DIRECTIONS IN ALPHA ZIRCONIUM

C/A = 1.5927

U V T W	U V T W	ANGLES IN DEGREES					
2 0 -2 1	1 1 -2 6	50.637	66.512	80.626			
	1 1 -2 9	55.245	65.866	75.659			
	2 2 -4 1	29.810	49.207	83.847			
	2 2 -4 3	28.867	69.189	74.916			
	2 2 -4 5	36.184	70.510	81.961			
	2 2 -4 9	46.437	67.335	85.324			
	1 2 -3 0	30.849	46.622	80.113			
	1 2 -3 1	18.545	38.197	47.639	59.229	72.583	88.565
	1 2 -3 2	19.375	36.642	62.192	67.702	71.003	84.408
	1 2 -3 3	26.396	39.023	65.144	72.976	79.477	79.984
	1 2 -3 4	32.814	42.254	63.936	76.181	80.665	86.449
	1 2 -3 5	37.790	45.201	63.403	73.953	86.208	88.882
	1 2 -3 6	41.593	47.651	63.198	72.397	85.429	89.688
	1 3 -4 0	28.119	50.951	75.405			
	1 3 -4 1	16.695	41.290	44.465	59.522	69.671	81.898
	1 3 -4 2	12.715	41.323	53.426	65.526	68.199	88.013
	1 3 -4 3	17.388	41.047	63.001	63.472	75.730	86.921
	1 4 -5 0	26.849	53.502	72.699			
	1 4 -5 1	16.874	37.573	48.290	59.935	68.052	77.913
	1 4 -5 2	10.394	44.961	47.746	64.432	66.645	83.084
	1 4 -5 3	11.528	43.528	56.706	61.950	72.894	87.745
	2 3 -5 0	33.513	43.152	84.018			
	2 3 -5 1	25.574	36.865	43.149	51.127	79.152	89.098
	2 3 -5 2	21.476	33.431	52.714	59.351	75.024	86.195
	2 3 -5 3	21.888	32.876	61.241	66.855	71.853	82.222
2 0 -2 3	2 0 -2 3	0.0	34.133	61.105	71.884		
	2 0 -2 5	12.433	30.742	51.293	59.451		
	3 0 -3 1	37.017	58.811	71.099	87.516		
	3 0 -3 2	22.548	47.674	80.042	85.568		
	3 0 -3 4	3.259	35.621	63.775	75.143		
	3 0 -3 5	2.818	33.027	58.824	69.066		
	4 0 -4 1	41.111	62.134	67.005	83.994		
	4 0 -4 3	19.466	45.473	77.406	88.650		
	1 1 -2 0	59.447	90.000				
	1 1 -2 1	34.043	67.689	86.024			
	1 1 -2 2	20.300	53.887	76.062			
	1 1 -2 3	17.066	46.712	65.458			
	1 1 -2 4	18.373	42.907	58.942			
	1 1 -2 6	22.377	39.426	51.670			
	1 1 -2 9	26.289	37.586	46.503			
	2 2 -4 1	45.651	73.525	78.011			
	2 2 -4 3	25.579	59.712	83.875			
	2 2 -4 5	17.729	49.713	70.078			
	2 2 -4 9	19.421	41.684	56.595			
	1 2 -3 0	56.315	63.659	83.631			
	1 2 -3 1	37.844	46.772	68.249	75.043	80.734	81.180
	1 2 -3 2	23.435	34.266	56.414	68.222	84.389	89.617
	1 2 -3 3	14.483	26.945	48.617	59.500	73.908	78.439
	1 2 -3 4	11.154	23.604	43.779	53.668	66.538	70.526
	1 2 -3 5	11.944	22.628	40.795	49.730	61.287	64.843
	1 2 -3 6	14.109	22.799	38.924	46.996	57.441	60.646
	1 3 -4 0	55.265	65.984	80.631			
	1 3 -4 1	41.220	53.527	69.033	69.388	78.797	87.574

ANGLES BETWEEN DIRECTIONS IN ALPHA ZIRCONIUM
C/A = 1.5927

U V T W	U V T W	ANGLES IN DEGREES					
2 0 -2 3	1 3 -4 2	28.903	43.079	59.144	77.123	81.970	89.695
	1 3 -4 3	19.181	35.394	51.592	68.736	80.286	87.740
	1 4 -5 0	54.803	67.402	78.923			
	1 4 -5 1	43.580	57.586	66.056	69.656	77.440	88.333
	1 4 -5 2	33.232	48.841	61.325	76.504	82.921	86.843
	1 4 -5 3	24.305	41.689	54.397	75.348	84.932	85.626
	2 3 -5 0	57.409	61.881	86.139			
	2 3 -5 1	46.040	51.077	68.905	72.900	76.526	84.190
	2 3 -5 2	35.807	41.499	67.867	75.363	79.540	83.136
	2 3 -5 3	27.364	33.777	60.669	67.910	88.011	88.729
2 0 -2 5	2 0 -2 5	0.0	23.009	40.419	47.018		
	3 0 -3 1	49.450	62.650	83.532	85.524		
	3 0 -3 2	34.981	49.511	71.988	81.999		
	3 0 -3 4	15.692	33.210	54.229	62.710		
	3 0 -3 5	9.615	28.722	48.777	56.633		
	1 1 -2 0	69.791	90.000				
	1 1 -2 1	42.683	64.533	82.826			
	1 1 -2 2	25.258	48.122	64.487			
	1 1 -2 3	16.198	39.049	53.635			
	1 1 -2 4	12.369	33.941	46.965			
	1 2 -3 0	67.857	72.450	85.677			
	1 2 -3 1	48.923	54.136	68.146	76.712	86.848	89.070
	1 2 -3 2	33.604	39.558	54.177	62.517	73.990	77.649
	1 2 -3 3	22.674	29.479	44.439	52.437	63.057	66.364
3 0 -3 1	3 0 -3 1	0.0	34.082	57.116	68.212		
	3 0 -3 2	14.468	48.551	55.894	75.262		
	3 0 -3 4	33.757	58.045	67.840	85.696		
	3 0 -3 5	39.835	59.558	73.917	89.095		
	4 0 -4 1	4.094	29.988	57.890	66.407		
	4 0 -4 3	17.551	51.633	55.950	76.870		
	1 1 -2 0	34.106	90.000				
	1 1 -2 1	29.687	53.565	82.101			
	1 1 -2 2	38.646	69.247	77.682			
	1 1 -2 3	46.490	75.629	78.925			
	1 1 -2 4	51.838	74.625	84.973			
	1 1 -2 6	58.156	73.763	88.189			
	1 1 -2 9	62.841	73.331	83.268			
	2 2 -4 1	28.899	43.523	85.688			
	2 2 -4 3	33.873	62.280	79.481			
	2 2 -4 5	42.919	74.681	76.464			
	2 2 -4 9	53.840	74.316	87.170			
	1 2 -3 0	25.388	43.719	79.590			
	1 2 -3 1	18.275	38.841	40.787	54.093	74.521	85.730
	1 2 -3 2	24.621	40.472	54.932	64.774	71.601	88.917
	1 2 -3 3	33.192	44.620	65.554	70.332	73.216	85.041
	1 2 -3 4	40.053	48.705	69.915	73.164	79.390	82.392
	1 2 -3 5	45.203	52.086	69.872	78.662	80.567	83.888
	1 2 -3 6	49.089	54.777	69.979	79.274	82.739	87.234
	1 3 -4 0	21.859	48.476	74.623			
	1 3 -4 1	13.653	34.219	44.374	55.253	70.769	79.366
	1 3 -4 2	16.276	43.639	46.084	62.781	68.323	84.085
	1 3 -4 3	23.768	45.204	56.011	67.128	69.609	88.136

ANGLES BETWEEN DIRECTIONS IN ALPHA ZIRCONIUM

C/A = 1.5927

U V T W	U V T W	ANGLES IN DEGREES					
3 0 -3 1	1 4 -5 0	20.138	51.251	71.763			
	1 4 -5 1	11.993	30.361	47.830	56.219	68.623	75.571
	1 4 -5 2	11.343	40.344	46.368	61.853	66.448	79.555
	1 4 -5 3	17.155	46.576	49.215	65.212	67.350	83.282
	2 3 -5 0	28.673	39.852	83.704			
	2 3 -5 1	23.224	35.742	37.062	46.316	80.340	87.316
	2 3 -5 2	22.737	34.791	46.017	53.601	77.586	89.262
	2 3 -5 3	26.091	36.349	54.221	60.529	75.558	86.328
3 0 -3 2	3 0 -3 2	0.0	50.463	63.019	84.821		
	3 0 -3 4	19.289	47.590	82.208	82.308		
	3 0 -3 5	25.366	47.883	78.184	88.385		
	4 0 -4 1	18.563	44.456	57.823	72.642		
	4 0 -4 3	3.083	49.636	66.102	86.893		
	1 1 -2 0	42.410	90.000				
	1 1 -2 1	26.207	65.980	75.814			
	1 1 -2 2	27.542	67.637	82.777			
	1 1 -2 3	33.360	63.728	87.133			
	1 1 -2 4	38.058	61.781	80.892			
	1 1 -2 6	43.963	60.089	73.889			
	1 1 -2 9	48.486	59.232	68.881			
	2 2 -4 1	32.037	54.583	82.294			
	2 2 -4 3	25.425	70.999	75.405			
	2 2 -4 5	30.490	65.328	88.460			
	2 2 -4 9	39.898	61.177	78.637			
	1 2 -3 0	36.333	49.874	80.728			
	1 2 -3 1	21.169	38.709	53.888	64.076	71.109	88.889
	1 2 -3 2	16.304	34.158	64.494	68.705	76.651	80.444
	1 2 -3 3	20.802	34.584	60.743	74.593	79.606	86.050
	1 2 -3 4	26.531	36.812	58.778	70.719	87.251	87.356
	1 2 -3 5	31.263	39.247	57.768	68.122	82.440	87.065
	1 2 -3 6	34.956	41.416	57.253	66.322	78.896	82.939
	1 3 -4 0	34.147	53.762	76.323			
	1 3 -4 1	21.396	45.415	47.719	63.663	69.005	84.263
	1 3 -4 2	12.914	40.176	60.016	63.377	73.195	88.462
	1 3 -4 3	12.874	38.088	59.644	70.142	81.263	82.543
	1 4 -5 0	33.155	56.074	73.796			
	1 4 -5 1	22.534	44.110	49.455	63.615	67.887	80.166
	1 4 -5 2	13.684	44.551	54.387	63.022	71.136	86.312
	1 4 -5 3	9.318	41.607	59.427	63.403	77.958	88.254
	2 3 -5 0	38.523	46.799	84.388			
	2 3 -5 1	29.145	38.949	48.881	55.810	78.254	89.301
	2 3 -5 2	22.483	33.500	58.825	64.688	72.946	83.508
	2 3 -5 3	19.850	30.913	67.568	68.769	72.605	78.647
3 0 -3 4	3 0 -3 4	0.0	36.845	66.373	78.403		
	3 0 -3 5	6.077	34.746	61.556	72.326		
	1 1 -2 0	56.813	90.000				
	1 1 -2 1	32.131	68.692	83.103			
	1 1 -2 2	20.048	55.658	79.114			
	1 1 -2 3	18.675	48.982	68.578			
	1 1 -2 4	20.888	45.486	62.102			
	1 2 -3 0	53.328	61.459	83.140			
	1 2 -3 1	35.062	45.113	68.455	71.957	78.644	81.861

ANGLES BETWEEN DIRECTIONS IN ALPHA ZIRCONIUM
C/A = 1.5927

U V T W	U V T W	ANGLES IN DEGREES					
3 0 -3 4	1 2 -3 2	21.141	33.424	57.290	69.866	87.135	87.244
	1 2 -3 3	13.425	27.127	50.062	61.536	76.787	81.611
3 0 -3 5	3 0 -3 5	0.0	31.713	56.490	66.249		
	1 1 -2 0	61.755	90.000				
	1 1 -2 1	35.836	66.877	88.551			
	1 1 -2 2	20.912	52.434	73.429			
	1 1 -2 3	16.071	44.824	62.766			
	1 1 -2 4	16.403	40.740	56.216			
	1 2 -3 0	58.912	65.601	84.072			
	1 2 -3 1	40.299	48.308	68.131	77.714	79.782	83.382
	1 2 -3 2	25.579	35.195	55.750	66.846	82.020	86.903
	1 2 -3 3	15.879	27.081	47.475	57.795	71.428	75.699
4 0 -4 1	1 1 -2 0	32.434	90.000				
	1 1 -2 1	31.731	50.178	83.970			
	1 1 -2 2	42.119	65.445	80.613			
	1 1 -2 3	50.317	74.986	79.062			
	1 1 -2 4	55.787	78.305	80.974			
	1 2 -3 0	22.943	42.548	79.387			
	1 2 -3 1	19.369	37.222	39.697	51.514	75.669	84.237
	1 2 -3 2	27.867	42.878	51.077	61.507	73.793	88.662
	1 2 -3 3	36.953	47.806	61.593	69.620	73.191	88.039
4 0 -4 3	1 1 -2 0	44.527	90.000				
	1 1 -2 1	26.375	68.687	74.561			
	1 1 -2 2	25.567	65.588	85.671			
	1 1 -2 3	30.694	61.261	84.161			
	1 1 -2 4	35.179	59.094	77.883			
	1 2 -3 0	38.935	51.516	81.050			
	1 2 -3 1	22.908	39.269	56.745	66.333	70.529	87.740
	1 2 -3 2	15.686	33.383	63.142	71.659	78.679	79.226
	1 2 -3 3	18.528	32.809	58.840	72.419	82.607	88.797
1 1 -2 0	1 1 -2 0	0.0	60.000				
	1 1 -2 1	27.964	63.793				
	1 1 -2 2	46.717	69.952				
	1 1 -2 3	57.877	74.581				
	1 1 -2 4	64.784	77.701				
	1 1 -2 6	72.571	81.387				
	1 1 -2 9	78.179	84.121				
	2 2 -4 1	14.866	61.101				
	2 2 -4 3	38.532	66.975				
	2 2 -4 5	53.004	72.490				
	2 2 -4 9	67.287	78.869				
	1 2 -3 0	10.893	49.107	70.893			
	1 2 -3 1	21.943	51.803	71.990			
	1 2 -3 2	36.262	57.483	74.409			
	1 2 -3 3	47.179	63.055	76.905			
	1 2 -3 4	55.011	67.525	78.981			
	1 2 -3 5	60.674	70.942	80.604			
	1 2 -3 6	64.874	73.557	81.863			
	1 3 -4 0	16.102	43.898	76.102			
	1 3 -4 1	21.413	45.715	76.541			

ANGLES BETWEEN DIRECTIONS IN ALPHA ZIRCONIUM

C/A = 1.5927

U V T W	U V T W	ANGLES IN DEGREES					
1 1 -2 0	1 3 -4 2	31.145	50.067	77.645			
	1 3 -4 3	40.266	55.090	79.003			
	1 4 -5 0	19.107	40.893	79.107			
	1 4 -5 1	22.113	42.170	79.322			
	1 4 -5 2	28.726	45.449	79.899			
	1 4 -5 3	35.948	49.636	80.682			
	2 3 -5 0	6.587	53.413	66.587			
	2 3 -5 1	13.589	54.324	67.120			
	2 3 -5 2	23.756	56.691	68.524			
	2 3 -5 3	32.921	59.758	70.381			
1 1 -2 1	1 1 -2 1	0.0	52.415	55.928	80.202		
	1 1 -2 2	18.753	49.899	74.681	87.789		
	1 1 -2 3	29.913	50.805	80.660	85.840		
	1 1 -2 4	36.820	52.239	76.344	87.252		
	1 1 -2 6	44.607	54.573	71.632	79.465		
	1 1 -2 9	50.215	56.672	68.377	73.857		
	2 2 -4 1	13.097	42.830	56.829	72.150		
	2 2 -4 3	10.568	50.389	66.496	86.942		
	2 2 -4 5	25.040	50.189	80.968	83.756		
	2 2 -4 9	39.323	52.910	74.809	84.749		
	1 2 -3 0	29.850	54.675	73.195			
	1 2 -3 1	13.295	45.564	48.293	64.721	66.907	83.157
	1 2 -3 2	11.532	42.062	59.667	63.606	78.045	88.267
	1 2 -3 3	20.153	42.383	57.416	74.815	82.050	86.456
	1 2 -3 4	27.485	44.085	56.668	77.770	82.773	87.534
	1 2 -3 5	32.964	45.987	56.590	74.798	83.221	88.501
	1 2 -3 6	37.076	47.714	56.782	72.674	80.050	87.261
	1 3 -4 0	31.941	50.473	77.752			
	1 3 -4 1	20.256	42.896	45.058	59.945	71.249	84.854
	1 3 -4 2	14.304	38.738	57.121	66.294	69.275	88.620
	1 3 -4 3	16.485	37.777	63.036	67.106	77.260	83.313
	1 4 -5 0	33.427	48.113	80.391			
	1 4 -5 1	24.421	41.680	43.446	55.780	75.173	85.905
	1 4 -5 2	18.343	37.413	53.138	63.577	70.758	88.869
	1 4 -5 3	16.890	35.525	61.759	67.366	70.720	84.328
	2 3 -5 0	28.668	58.234	69.454			
	2 3 -5 1	17.194	40.383	52.273	63.883	65.271	75.723
	2 3 -5 2	7.825	48.139	51.235	59.626	72.376	81.890
	2 3 -5 3	7.173	45.924	56.816	60.617	78.809	87.375
1 1 -2 2	1 1 -2 2	0.0	40.095	72.847	86.566		
	1 1 -2 3	11.160	36.984	64.263	75.406		
	1 1 -2 4	18.067	36.424	59.165	68.499		
	1 1 -2 6	25.854	37.134	53.710	60.712		
	1 1 -2 9	31.462	38.486	50.035	55.104		
	2 2 -4 1	31.850	58.795	61.583	81.688		
	2 2 -4 3	8.185	43.809	79.319	85.249		
	2 2 -4 5	6.287	38.029	67.967	80.279		
	2 2 -4 9	20.570	36.498	57.374	65.996		
	1 2 -3 0	47.682	63.331	77.031			
	1 2 -3 1	28.964	48.475	63.194	66.612	79.341	88.452
	1 2 -3 2	14.461	38.367	53.146	76.630	82.108	87.308
	1 2 -3 3	7.516	33.273	47.099	68.281	77.601	86.596

ANGLES BETWEEN DIRECTIONS IN ALPHA ZIRCONIUM
C/A = 1.5927

U V T W	U V T W	ANGLES IN DEGREES					
1 1 -2 2	1 2 -3 4	10.229	31.454	43.779	62.618	70.799	78.590
	1 2 -3 5	14.815	31.260	42.021	58.733	65.977	72.832
	1 2 -3 6	18.646	31.736	41.111	55.988	62.462	68.574
	1 3 -4 0	48.799	60.394	80.522			
	1 3 -4 1	35.097	48.806	62.718	70.155	72.614	88.835
	1 3 -4 2	23.428	39.569	61.480	75.167	79.393	83.724
	1 3 -4 3	15.093	33.414	55.030	71.846	85.366	87.134
	1 4 -5 0	49.621	58.784	82.555			
	1 4 -5 1	38.886	49.356	60.531	68.597	74.320	89.072
	1 4 -5 2	29.266	41.226	66.960	70.726	77.887	81.322
	1 4 -5 3	21.491	34.968	60.892	74.666	79.655	86.068
	2 3 -5 0	47.072	65.880	74.191			
	2 3 -5 1	35.247	56.625	58.926	65.365	75.547	83.321
	2 3 -5 2	24.425	48.739	57.724	69.847	84.634	88.167
	2 3 -5 3	15.245	42.726	51.720	79.269	80.845	87.478
1 1 -2 3	1 1 -2 3	0.0	30.837	54.839	64.247		
	1 1 -2 4	6.907	28.421	49.237	57.339		
	1 1 -2 6	14.694	27.420	43.248	49.552		
	1 1 -2 9	20.302	27.944	39.242	43.944		
	2 2 -4 1	43.010	61.689	72.743	87.726		
	2 2 -4 3	19.345	42.645	71.361	83.591		
	2 2 -4 5	4.872	33.238	58.908	69.119		
	2 2 -4 9	9.410	27.885	47.268	54.836		
	1 2 -3 0	58.523	69.628	79.976			
	1 2 -3 1	39.533	52.638	63.740	77.573	83.476	87.089
	1 2 -3 2	24.197	39.716	51.222	70.095	78.606	86.868
	1 2 -3 3	13.428	31.550	42.970	60.610	68.268	75.535
	1 2 -3 4	7.056	27.027	37.893	54.135	61.037	67.505
	1 2 -3 5	6.023	24.815	34.829	49.667	55.915	61.731
	1 2 -3 6	8.341	23.913	32.976	46.498	52.184	57.462
	1 3 -4 0	59.277	67.470	82.662			
	1 3 -4 1	45.225	54.509	70.546	73.397	80.678	85.093
	1 3 -4 2	32.869	43.436	60.093	74.274	85.970	87.507
	1 3 -4 3	23.020	35.022	51.971	65.595	77.859	83.749
	1 4 -5 0	59.838	66.299	84.233			
	1 4 -5 1	48.757	55.894	70.973	74.624	76.851	86.095
	1 4 -5 2	38.581	46.491	65.878	77.164	81.323	86.697
	1 4 -5 3	29.859	38.633	58.474	69.470	84.698	89.638
	2 3 -5 0	58.114	71.522	77.802			
	2 3 -5 1	46.237	60.992	67.570	69.999	82.224	88.170
	2 3 -5 2	35.323	51.594	58.406	80.940	82.268	87.868
	2 3 -5 3	25.939	43.886	50.843	74.076	79.333	89.625
1 1 -2 4	1 1 -2 4	0.0	24.598	43.302	50.431		
	1 1 -2 6	7.787	22.032	36.930	42.644		
	1 1 -2 9	13.395	21.696	32.660	37.036		
	2 2 -4 1	49.918	64.024	79.651	88.497		
	2 2 -4 3	26.252	43.095	66.612	76.684		
	2 2 -4 5	11.780	31.706	53.531	62.211		
	2 2 -4 9	2.502	23.538	41.210	47.929		
	1 2 -3 0	65.270	73.805	81.984			
	1 2 -3 1	46.198	55.913	64.613	80.486	84.367	88.076
	1 2 -3 2	30.696	41.808	50.885	66.305	73.299	80.046

ANGLES BETWEEN DIRECTIONS IN ALPHA ZIRCONIUM

C/A = 1.5927

U V T W	U V T W	ANGLES IN DEGREES					
1 1 -2 4	1 2 -3 3	19.520	32.221	41.455	56.191	62.620	68.693
	1 2 -3 4	11.837	26.194	35.326	49.231	55.139	60.649
	1 2 -3 5	6.877	22.594	31.384	44.390	49.831	54.866
	1 2 -3 6	4.687	20.551	28.829	40.934	45.959	50.590
	1 3 -4 0	65.838	72.122	84.127			
	1 3 -4 1	51.670	58.598	71.172	80.035	82.853	85.763
	1 3 -4 2	39.131	46.800	59.852	71.342	82.090	87.337
	1 3 -4 3	28.975	37.481	50.875	62.064	72.185	77.016
	1 4 -5 0	66.262	71.213	85.382			
	1 4 -5 1	55.063	60.413	75.112	77.485	82.082	84.315
	1 4 -5 2	44.720	50.518	65.690	74.797	87.817	87.900
	1 4 -5 3	35.758	42.054	57.617	66.570	79.007	83.009
	2 3 -5 0	64.962	75.290	80.254			
	2 3 -5 1	53.069	64.202	69.367	76.859	86.462	88.791
	2 3 -5 2	42.130	54.151	59.492	78.713	83.239	87.809
	2 3 -5 3	32.708	45.702	51.181	70.076	74.380	82.750
1 1 -2 6	1 2 -3 0	72.895	78.692	84.374			
	1 2 -3 1	53.769	60.103	66.057	77.255	82.646	87.971
	1 2 -3 2	38.180	45.125	51.313	62.351	67.446	72.359
	1 2 -3 3	26.852	34.481	40.848	51.631	56.437	60.988
1 1 -2 9	1 2 -3 0	78.395	82.293	86.155			
	1 2 -3 1	59.247	63.384	67.377	75.049	78.775	82.454
	1 2 -3 2	43.628	48.027	52.141	59.761	63.350	66.828
	1 2 -3 3	32.258	36.942	41.164	48.700	52.151	55.447
2 2 -4 1	1 2 -3 0	18.357	50.747	71.556			
	1 2 -3 1	11.264	35.681	47.008	59.106	67.476	77.607
	1 2 -3 2	22.216	48.242	50.736	66.033	68.092	83.492
	1 2 -3 3	32.634	51.454	61.849	66.162	75.356	88.062
2 2 -4 3	1 2 -3 0	39.811	59.195	75.164			
	1 2 -3 1	21.550	46.509	58.595	63.489	73.787	87.859
	1 2 -3 2	9.492	39.099	55.542	74.027	81.645	86.277
	1 2 -3 3	11.095	36.479	51.183	74.195	84.542	85.290
2 2 -4 5	1 2 -3 0	53.778	66.800	78.640			
	1 2 -3 1	34.879	50.631	63.368	72.785	83.690	85.633
	1 2 -3 2	19.769	38.801	51.859	72.895	82.392	88.318
	1 2 -3 3	9.788	31.886	44.540	63.891	72.317	80.363
2 2 -4 9	1 2 -3 0	67.718	75.358	82.739			
	1 2 -3 1	48.625	57.208	65.025	79.429	86.326	86.829
	1 2 -3 2	33.088	42.772	50.929	64.993	71.401	77.574
	1 2 -3 3	21.844	32.784	41.129	54.671	60.608	66.215
1 2 -3 0	1 2 -3 0	0.0	21.787	38.213	60.000	81.787	
	1 2 -3 1	19.165	28.705	42.084	61.817	82.245	
	1 2 -3 2	34.804	40.318	49.822	65.760	83.264	
	1 2 -3 3	46.197	50.003	57.053	69.751	84.325	
	1 2 -3 4	54.272	57.165	62.690	73.024	85.215	
	1 2 -3 5	60.082	62.410	66.928	75.559	85.914	
	1 2 -3 6	64.380	66.327	70.139	77.514	86.459	

ANGLES BETWEEN DIRECTIONS IN ALPHA ZIRCONIUM

C/A = 1.5927

U V T W	U V T W	ANGLES IN DEGREES					
1 2 -3 0	1 3 -4 0	5.208	26.995	33.004	54.791	65.209	86.995
	1 3 -4 1	15.207	30.299	35.648	56.036	66.027	87.089
	1 3 -4 2	27.485	37.463	41.664	59.096	68.067	87.324
	1 3 -4 3	37.728	44.954	48.238	62.748	70.548	87.614
	1 4 -5 0	8.213	30.000	51.787	68.213	90.000	
	1 4 -5 1	13.975	31.886	52.663	68.660	90.000	
	1 4 -5 2	23.287	36.513	54.964	69.852	90.000	
	1 4 -5 3	32.010	42.101	57.996	71.459	90.000	
	2 3 -5 0	4.307	17.480	42.520	55.693	64.307	77.480
	2 3 -5 1	12.656	21.048	43.849	56.531	64.899	77.754
	2 3 -5 2	23.256	28.502	47.229	58.715	66.456	78.479
	2 3 -5 3	32.583	36.296	51.479	61.559	68.509	79.445
1 2 -3 1	1 2 -3 1	0.0	20.566	36.020	38.330	43.886	53.611
		56.366	70.224	76.394	88.872		
	1 2 -3 2	15.638	24.827	37.176	53.969	54.888	57.803
		65.037	72.652	78.438	85.608		
	1 2 -3 3	27.031	32.429	41.353	55.678	65.362	68.273
		70.711	73.932	81.748	84.838		
	1 2 -3 4	35.107	38.860	45.582	57.160	69.800	73.437
		75.779	79.181	80.394	89.468		
	1 2 -3 5	40.917	43.779	49.102	58.661	69.400	77.453
		79.247	81.204	85.088	87.192		
	1 2 -3 6	45.215	47.525	51.908	59.985	69.246	76.251
		83.546	84.733	85.225	88.574		
	1 3 -4 0	19.835	32.684	37.613	57.003	66.667	87.162
	1 3 -4 1	6.961	26.275	31.929	33.863	42.742	46.650
		52.494	62.295	63.475	72.383	82.582	88.100
	1 3 -4 2	9.202	25.983	31.260	46.465	50.632	53.088
		56.187	59.868	70.369	78.249	78.856	83.969
	1 3 -4 3	18.813	29.780	34.043	50.801	56.797	59.066
		62.033	64.555	76.184	76.524	80.783	83.392
	1 4 -5 0	20.790	35.112	54.246	69.477	90.000	
	1 4 -5 1	11.126	29.931	31.567	42.485	50.396	59.449
		65.901	73.791	86.297			
	1 4 -5 2	8.151	28.183	41.810	48.353	50.439	63.408
		65.166	78.282	82.977			
	1 4 -5 3	14.005	29.524	47.940	50.829	57.892	61.987
		70.653	80.252	82.470			
	2 3 -5 0	19.625	25.716	45.878	57.833	65.825	78.184
	2 3 -5 1	8.354	18.319	31.366	35.532	41.499	52.162
		53.936	62.065	63.054	70.553	74.448	82.381
	2 3 -5 2	5.470	16.721	39.731	42.251	45.374	51.820
		59.081	59.673	68.722	71.562	75.540	86.500
	2 3 -5 3	13.719	20.468	40.196	51.287	51.659	54.145
		58.560	65.624	69.599	74.080	80.186	89.854
1 2 -3 2	1 2 -3 2	0.0	17.854	31.182	48.480	65.033	69.607
		72.525	76.735	78.230	89.350		
	1 2 -3 3	11.393	19.999	30.851	45.884	60.454	70.686
		81.000	82.660	83.348	88.015		
	1 2 -3 4	19.469	24.691	32.852	45.325	57.870	66.743
		77.079	85.031	88.962	89.076		

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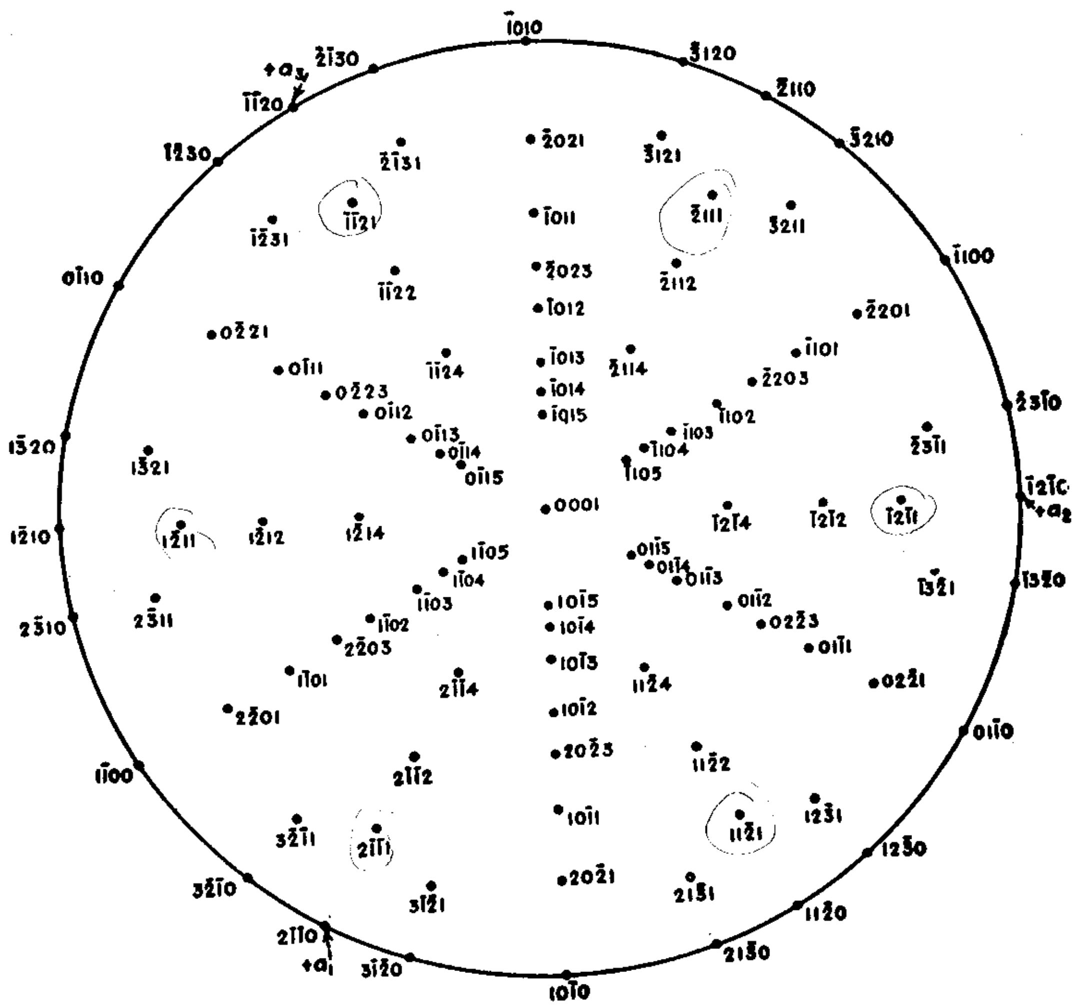
ANGLES BETWEEN DIRECTIONS IN ALPHA ZIRCONIUM

C/A = 1.5927

U V T W	U V T W	ANGLES IN DEGREES					
1 2 -3 2	1 2 -3 5	25.278	28.956	35.265	45.615	56.412	64.138
		73.146	80.042	83.430	85.115		
	1 2 -3 6	29.577	32.399	37.475	46.198	55.572	62.359
		70.298	76.368	79.341	80.816		
	1 3 -4 0	35.142	42.975	46.480	61.744	69.861	87.533
	1 3 -4 1	21.027	31.788	36.070	49.360	53.145	55.396
		58.251	61.662	71.477	78.897	79.470	84.298
	1 3 -4 2	8.967	24.342	29.219	47.073	55.525	62.025
		66.495	69.263	72.682	77.232	80.655	87.285
	1 3 -4 3	4.954	21.690	26.695	43.712	51.664	67.604
		72.?6?	72.365	76.453	78.459	85.792	88.329
	1 4 -5 0	35.640	44.675	59.474	72.256	90.000	
	1 4 -5 1	24.619	35.953	46.802	52.389	54.203	65.726
		67.312	79.251	83.552			
	1 4 -5 2	14.805	29.247	46.845	57.205	60.303	63.425
		75.001	77.726	85.971			
	1 4 -5 3	7.847	25.364	43.152	56.297	66.303	71.647
		72.880	81.906	88.093			
	2 3 -5 0	35.036	38.446	52.757	62.432	69.145	79.747
	2 3 -5 1	23.223	27.854	44.147	46.894	49.570	55.205
		61.683	62.216	70.420	73.023	75.672	86.770
	2 3 -5 2	12.506	19.357	38.788	49.518	56.642	57.824
		60.018	67.302	70.383	78.198	83.908	86.682
	2 3 -5 3	4.358	14.754	35.252	45.870	52.696	62.894
		67.254	69.111	78.103	81.093	85.076	89.747
1 2 -3 3	1 2 -3 3	0.0	15.033	26.191	40.497	53.891	63.100
		73.661	81.696	85.642	87.607		
	1 2 -3 4	8.075	16.010	25.382	38.002	49.937	58.119
		67.431	74.436	77.844	79.531		
	1 2 -3 5	13.885	18.894	26.260	37.045	47.557	54.815
		63.065	65.250	72.244	73.722		
	1 2 -3 6	18.184	24.770	27.634	36.831	46.091	52.554
		59.926	65.443	68.109	69.423		
	1 3 -4 0	46.423	51.920	54.515	66.479	73.127	87.921
	1 3 -4 1	32.108	39.105	42.197	55.593	60.686	62.639
		65.211	67.410	77.672	77.974	81.767	84.095
	1 3 -4 2	19.611	28.675	32.324	46.887	54.092	68.884
		72.906	73.374	77.203	79.096	86.022	88.420
	1 3 -4 3	9.592	21.814	25.897	40.930	48.005	62.136
		65.811	77.992	83.016	83.747	87.060	88.712
	1 4 -5 0	46.758	53.169	64.648	75.113	90.000	
	1 4 -5 1	35.542	43.137	55.820	58.014	63.529	66.804
		73.870	81.837	83.691			
	1 4 -5 2	25.231	34.399	48.227	59.521	68.470	73.290
		74.407	82.613	88.259			
	1 4 -5 3	16.430	27.650	42.348	53.678	68.147	77.601
		81.250	81.874	89.692			
	2 3 -5 0	46.352	48.683	59.325	67.037	72.536	81.370
	2 3 -5 1	34.478	37.347	49.597	57.947	58.238	60.196
		63.729	69.500	72.795	76.541	81.682	89.877
	2 3 -5 2	23.580	27.271	41.357	50.208	56.148	65.241
		69.183	70.868	79.077	81.819	85.475	89.767
	2 3 -5 3	14.257	19.293	35.210	44.306	50.242	59.153
		74.979	78.622	80.100	82.393	86.773	87.413

APPENDIX 4

STANDARD (0001) PROJECTION FOR H.C.P. ZIRCONIUM



APPENDIX 5

CRYSTALLOGRAPHIC FORMULAE FOR H.C.P. METALS

Errors exist in crystallographic formulae listed in the literature for h.c.p. metals due to confusion between Miller and Miller-Bravais indices. Correct formulae in the Miller-Bravais notation have been listed by Otte and Crocker (1966), Nicholas (1966) and Okamoto and Thomas (1968) and are reproduced below. These formulae are simplified by using a parameter λ defined as

$$\lambda^2 = \frac{2}{3}\left(\frac{c}{a}\right)^2 .$$

$$= 1.6914 \quad \text{for } \alpha\text{–Zr.}$$

1. The direction [uvtw] normal to the plane $(hki\ell)$ is given by

$$[uvtw] = [hki\ell\lambda^{-2}] . \tag{A5.1}$$

2. Conversely the plane $(hki\ell)$ normal to the direction [uvtw] is given by

$$(hki\ell) = (uvtw\,\lambda^2) . \tag{A5.2}$$

3. The interplanar spacing of planes $(hki\ell)$ is given by

$$d_{(hki\ell)} = a\left\{\frac{2}{3}(h^2 + k^2 + i^2 + \ell^2\lambda^{-2})\right\}^{-1/2} . \tag{A5.3}$$

4. The identity distance d^o along a direction [uvtw] is given by

$$d^o_{uvtw} = a\left\{\frac{3}{2}(u^2 + v^2 + t^2 + \lambda^2 w^2)\right\}^{1/2} . \tag{A5.4}$$

5. The angle between two directions $[u_1v_1t_1w_1]$ and $[u_2v_2t_2w_2]$ is given by

$$\cos\theta = \frac{u_1u_2 + v_1v_2 + t_1t_2 + \lambda^2 w_1w_2}{(u_1^2 + v_1^2 + t_1^2 + \lambda^2 w_1^2)^{1/2}(u_2^2 + v_2^2 + t_2^2 + \lambda^2 w_2^2)^{1/2}} . \tag{A5.5}$$

6. The angle between two planes $(h_1k_1i_1\ell_1)$ and $(h_2k_2i_2\ell_2)$ is given by

$$\cos\theta = \frac{h_1h_2 + k_1k_2 + i_1i_2 + \lambda^{-2}\ell_1\ell_2}{(h_1^2 + k_1^2 + i_1^2 + \lambda^{-2}\ell_1^2)^{1/2}(h_2^2 + k_2^2 + i_2^2 + \lambda^{-2}\ell_2^2)^{1/2}} . \tag{A5.6}$$

7. Direction [uvtw] common to planes $(h_1k_1i_1\ell_1)$ and $(h_2k_2i_2\ell_2)$ is given by the solution of the three simultaneous equations

$$h_1u + k_1u + i_1t + \ell_1w = 0$$

$$h_2u + k_2v + i_2t + \ell_2w = 0$$

$$u + v + t = 0$$

which gives

$$[uvtw] = \left[\begin{vmatrix}\bar{\ell}_1 & k_1 & i_1\\ \bar{\ell}_2 & k_2 & i_2\\ 0 & 1 & 1\end{vmatrix}, \begin{vmatrix}h_1 & \bar{\ell}_1 & i_1\\ h_2 & \bar{\ell}_2 & i_2\\ 1 & 0 & 1\end{vmatrix}, \begin{vmatrix}h_1 & k_1 & \bar{\ell}_1\\ h_2 & k_2 & \bar{\ell}_2\\ 1 & 1 & 0\end{vmatrix}, \begin{vmatrix}h_1 & k_1 & i_1\\ h_2 & k_2 & i_2\\ 1 & 1 & 1\end{vmatrix}\right] . \tag{A5.7}$$

APPENDIX 5 (continued)

8. Plane $(hki\ell)$ containing directions $[u_1v_1t_1w_1]$ and $[u_2v_2t_2w_2]$ is given by the solution of the three simultaneous equations

$$u_1h + v_1k + t_1i + w_1\ell = 0$$

$$u_2h + v_2k + t_2i + w_2\ell = 0$$

$$h + k + i = 0$$

which gives

$$(hki\ell) = \left[\begin{vmatrix} \bar{w}_1 & v_1 & t_1 \\ \bar{w}_2 & v_2 & t_2 \\ 0 & 1 & 1 \end{vmatrix}, \begin{vmatrix} u_1 & \bar{w}_1 & t_1 \\ u_2 & \bar{w}_2 & t_2 \\ 1 & 0 & 1 \end{vmatrix}, \begin{vmatrix} u_1 & v_1 & \bar{w}_1 \\ u_2 & v_2 & \bar{w}_2 \\ 1 & 1 & 0 \end{vmatrix}, \begin{vmatrix} u_1 & v_1 & t_1 \\ u_2 & v_2 & t_2 \\ 1 & 1 & 1 \end{vmatrix} \right] . \quad \text{(A5.8)}$$

9. Direction $[uvtw]$ perpendicular to direction $[u_1v_1t_1w_1]$ and lying in plane $(hki\ell)$ is given by the solution of the following simultaneous equations

$$hu + kv + it + \ell w = 0$$

$$u_1u + v_1v + t_1t + \lambda^2 w_1w = 0$$

$$u + v + t = 0$$

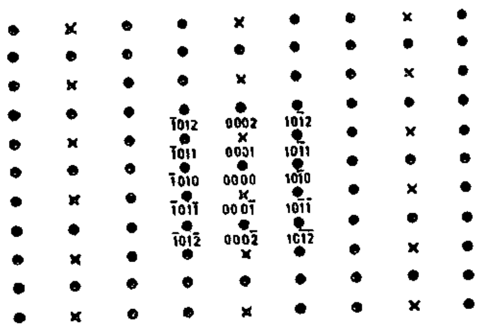
which gives

$$uvtw = \left[\begin{vmatrix} \bar{\ell} & k & i \\ -\lambda^2 w_1 & v_1 & t_1 \\ 0 & 1 & 1 \end{vmatrix}, \begin{vmatrix} h & \bar{\ell} & i \\ u_1 & -\lambda^2 w_1 & t_1 \\ 1 & 0 & 1 \end{vmatrix}, \begin{vmatrix} h & k & \bar{\ell} \\ u_1 & v_1 & -\lambda^2 w_1 \\ 1 & 1 & 0 \end{vmatrix}, \begin{vmatrix} h & k & i \\ u_1 & v_1 & t_1 \\ 1 & 1 & 1 \end{vmatrix} \right] . \quad \text{(A5.9)}$$

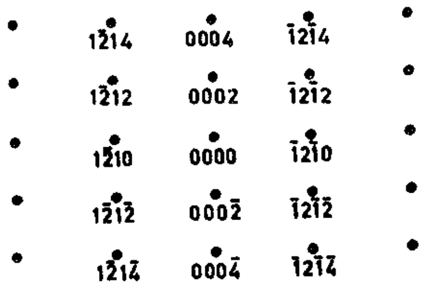
APPENDIX 6

ELECTRON DIFFRACTION PATTERNS FREQUENTLY OBTAINED FROM H.C.P. METALS

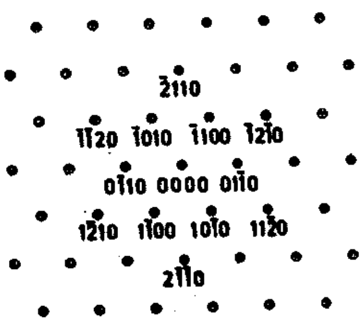
(Zone axes are given in terms of Miller-Bravais indices. The structure factor is zero for reflections marked x).



[1$\bar{2}$10]

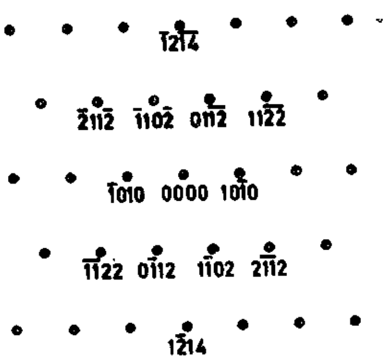


[10$\bar{1}$0]

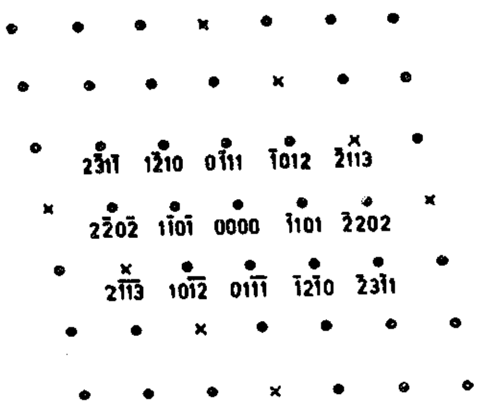


[0001]

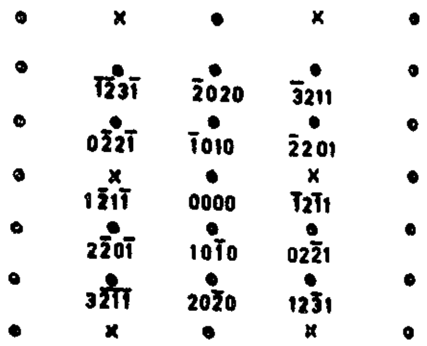
[1$\bar{2}$13]



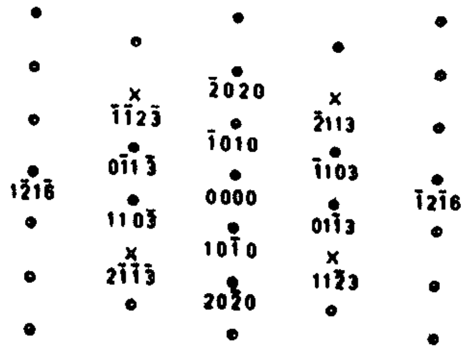
[$\bar{2}$4$\bar{2}$3]



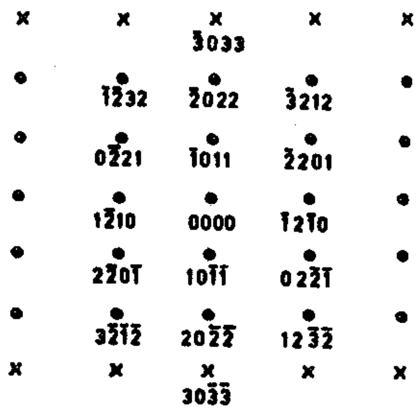
[10$\bar{1}$1]



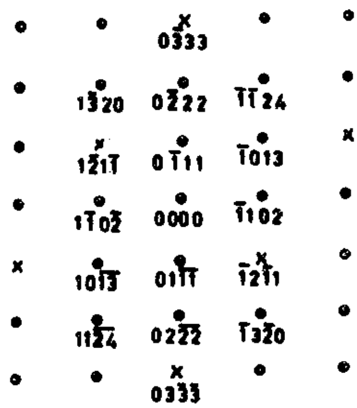
[$1\bar{2}16$]



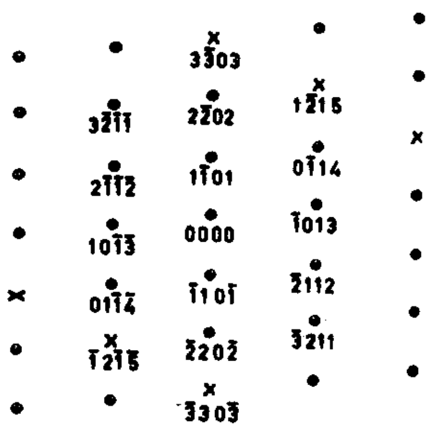
[$1\bar{2}11$]



[$10\bar{1}2$]



[$5\bar{1}\bar{4}3$]



[$25\bar{7}3$]

APPENDIX 7

KIKUCHI PATTERNS

The Kikuchi map of titanium, constructed by Okamoto and Thomas (1968) is reproduced in Figure A7.1

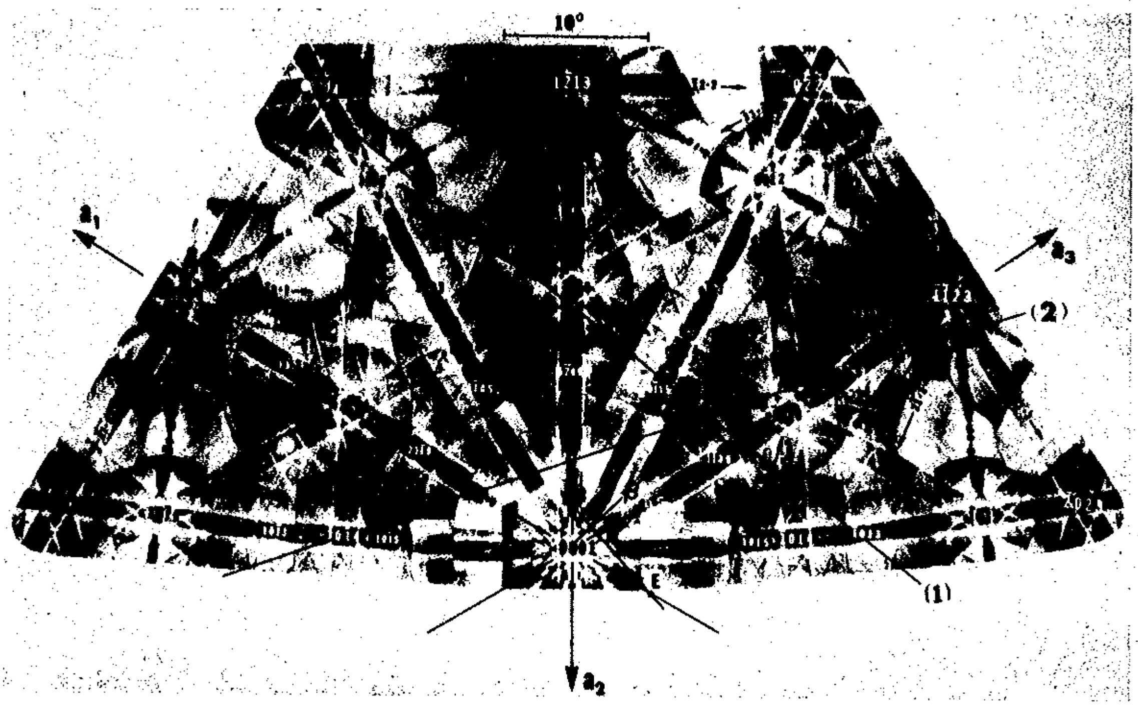


Figure A7.1 The [0001] Kikuchi map for h.c.p. titanium (c/a = 1.588). All poles are indexed in terms of directional indices.

Determination of the Exact Foil Orientation from Kikuchi Patterns

The unknown Kikuchi pattern is shown in Figure A7.2(a) together with a tracing of some of the centre lines of the Kikuchi pairs (traces of crystal planes) in Figure A7.2(b). The Kikuchi poles B and C are obtained by extrapolation of the relevant traces. The traces of planes giving rise to the Kikuchi pairs were indexed using Equation 16 and the Kikuchi map in Figure A7.1. Thus the poles A, B, C and D can be shown to be $[0\bar{1}14]$, $[0\bar{1}12]$, $[\bar{1}\bar{1}26]$ and $[0\bar{1}13]$ respectively. From the scale factor of the pattern it can be shown that the origin of the pattern, and hence the direction of the electron beam, is 4.3° from $[0\bar{1}14]$, 9.7° from $[0\bar{1}12]$ and 8.2° from $[\bar{1}\bar{1}26]$. The beam direction is shown on a section of the standard projection in Figure A7.3.

The beam orientation can also be given analytically because it is obvious that the origin of the Kikuchi pattern lies almost on the intersection of the $(11\bar{2}1)$ trace and the trace of a plane containing poles A and H.

Pole H lies on the intersection of the $(\bar{2}4\bar{2}2)$ trace and the trace of the plane containing poles B and G. Pole G, which lies on the intersection of traces $(\bar{2}3\bar{1}1)$ and $(22\bar{4}2)$, has indices $[\bar{1}\ \bar{4}\ 5\ 15]$ (use Equation A5.7). Hence the trace containing poles B and G is given by $(4\bar{1}\bar{3}1)$ (use Equation A5.8). Thus pole H, lying at the intersection of traces $(\bar{2}4\bar{2}2)$ and $(4\bar{1}\bar{3}1)$, is given by $[\bar{1}\ \bar{7}\ 8\ 21]$.

The plane containing $[0\bar{1}14]$ and $[\bar{1}\ \bar{7}\ 8\ 21]$ is $(\bar{6}511)$. Therefore, the beam orientation, given by the intersection of planes $(22\bar{4}2)$ and $(\bar{6}511)$, is $[\bar{1}\ \overline{10}\ 11\ 33]$.

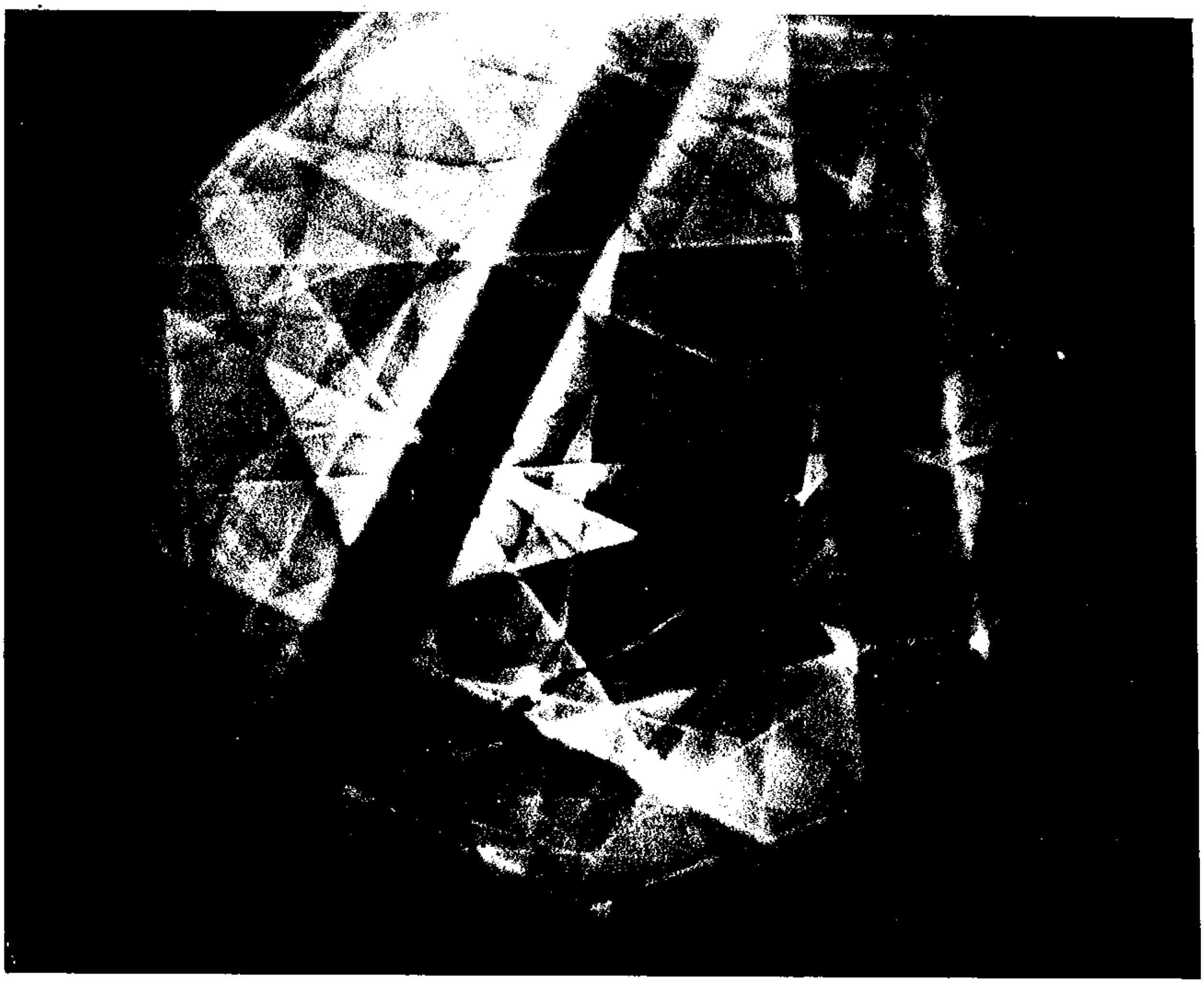


Figure A7.2. (a) A Kikuchi pattern from zirconium.

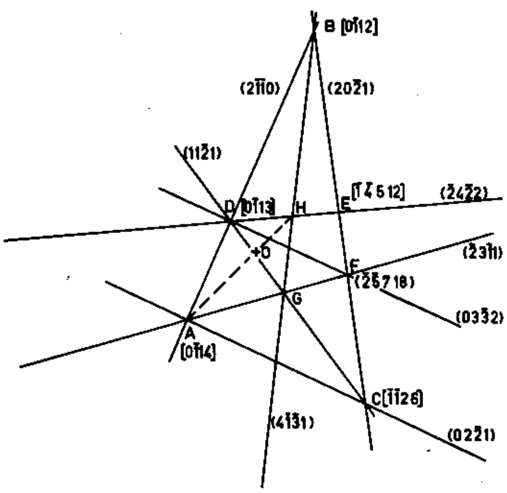


Figure A7.2 (b) Schematic drawing of centre lines of indexed Kikuchi pairs in (a).

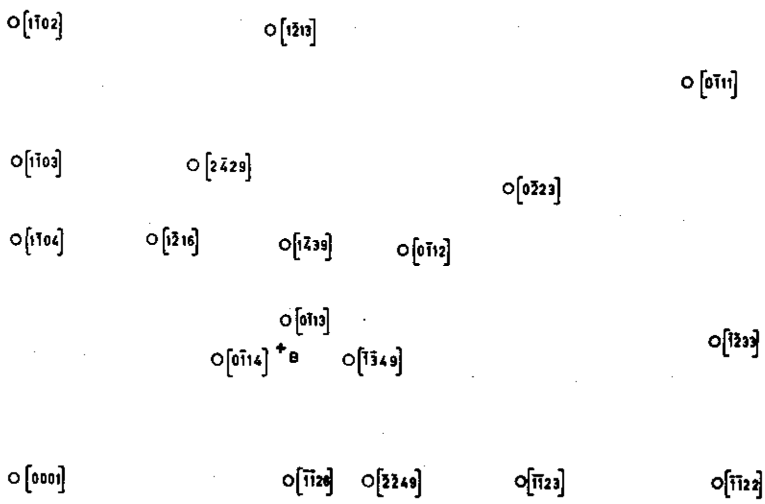


Figure A7.3 An enlarged section of the [0001] standard projection for directions for h.c.p. zirconium. The foil orientation in Figure A7.1 is shown at B.

APPENDIX 8

VALUES OF g.b FOR EIGHT REFLECTIONS AND $\frac{1}{3}$ <11$\bar{2}$0> –, $\frac{1}{3}$ <11$\bar{2}$3> – AND <0001> – TYPE BURGERS VECTORS

Reflection		Burgers Vectors of Perfect Dislocations (x$\frac{1}{3}$)									
		±[11$\bar{2}$0]	±[$\bar{1}$2$\bar{1}$0]	±[$\bar{2}$110]	±[11$\bar{2}$3]	±[$\bar{1}$2$\bar{1}$3]	±[$\bar{2}$113]	±[11$\bar{2}\bar{3}$]	±[$\bar{1}$2$\bar{1}\bar{3}$]	±[$\bar{2}$11$\bar{3}$]	±[0003]
1	10$\bar{1}$0	±1	0	∓1	±1	0	∓1	±1	0	∓1	0
	01$\bar{1}$0	±1	±1	0	±1	±1	0	±1	±1	0	0
	$\bar{1}$100	0	±1	±1	0	±1	±1	0	±1	±1	0
2	0002	0	0	0	±2	±2	±2	∓2	∓2	∓2	±2
3	10$\bar{1}$1	±1	0	∓1	±2	±1	0	0	∓1	∓2	±1
	10$\bar{1}\bar{1}$	±1	0	∓1	0	∓1	∓2	±2	±1	0	∓1
	01$\bar{1}$1	±1	±1	0	±2	±2	±1	0	0	∓1	±1
	01$\bar{1}\bar{1}$	±1	±1	0	0	0	∓1	±2	±2	±1	∓1
	$\bar{1}$101	0	±1	±1	±1	±2	±2	∓1	0	0	±1
	$\bar{1}$10$\bar{1}$	0	±1	±1	∓1	0	0	±1	±2	±2	∓1
4	10$\bar{1}$2	±1	0	±1	±3	±2	±1	∓1	∓2	∓3	±2
	10$\bar{1}\bar{2}$	±1	0	±1	∓1	∓2	∓3	±3	±2	±1	∓2
	01$\bar{1}$2	±1	±1	0	±3	±3	±2	∓1	∓1	∓2	±2
	01$\bar{1}\bar{2}$	±1	±1	0	∓1	∓1	∓2	±3	±3	±2	∓2
	$\bar{1}$102	0	±1	±1	±2	±3	±3	∓2	∓1	∓1	±2
	$\bar{1}$10$\bar{2}$	0	±1	±1	∓2	∓1	∓1	±2	±3	±3	∓2
5	11$\bar{2}$0	±2	±1	∓1	±2	±1	∓1	±2	±1	∓1	0
	$\bar{1}$2$\bar{1}$0	±1	±2	±1	±1	±2	±1	±1	±2	±1	0
	$\bar{2}$110	∓1	±1	±2	∓1	±1	±2	∓1	±1	±2	0
6	10$\bar{1}$3	±1	0	∓1	±4	±3	±2	∓2	∓3	∓4	±3
	10$\bar{1}\bar{3}$	±1	0	∓1	∓2	∓3	∓4	±4	∓3	±2	∓3
	01$\bar{1}$3	±1	±1	0	±4	±4	±3	∓2	±4	∓3	±3
	01$\bar{1}\bar{3}$	±1	±1	0	∓2	∓2	∓3	±4	∓2	±3	∓3
	$\bar{1}$103	0	±1	±1	±3	±4	±4	∓3	±4	∓2	±3
	$\bar{1}$10$\bar{3}$	0	±1	±1	∓3	∓2	∓2	±3	∓2	±4	∓3
7	11$\bar{2}$2	±2	±1	∓1	±4	±3	±1	0	∓1	∓3	±2
	11$\bar{2}\bar{2}$	±2	∓1	∓1	0	∓1	∓3	±4	±3	±1	∓2
	$\bar{1}$2$\bar{1}$2	±1	±2	±1	±3	±4	±3	∓1	0	∓1	±2
	$\bar{1}$2$\bar{1}\bar{2}$	±1	±2	±1	∓1	0	∓1	±3	0	±3	∓2
	$\bar{2}$112	∓1	±1	±2	±1	±3	±4	∓3	±3	0	±2
	$\bar{2}$11$\bar{2}$	∓1	±1	±2	∓3	∓1	0	±1	±1	±4	∓2
8	11$\bar{2}$4	±2	±1	±1	±6	±5	±3	∓2	∓3	∓5	±4
	11$\bar{2}\bar{4}$	±2	±1	±1	∓2	∓3	∓5	±6	±5	±3	∓4
	$\bar{1}$2$\bar{1}$4	±1	±2	±1	±5	±6	±5	∓3	∓2	∓3	±4
	$\bar{1}$2$\bar{1}\bar{4}$	±1	±2	±1	∓3	∓2	∓3	±5	±6	±5	∓4
	$\bar{2}$114	±1	±1	±2	±3	±5	±6	∓5	∓3	∓2	±4
	$\bar{2}$11$\bar{4}$	±1	±1	±2	∓5	∓3	∓2	±3	±5	±6	∓4

APPENDIX 9

EXTINCTION DISTANCES FOR VARIOUS REFLECTIONS FOR 100 kV ELECTRONS IN α–ZIRCONIUM

The extinction distance ξ_g is an important parameter in the dynamical theory of contrast and is given by the relationship

$$\xi_g = \frac{\pi V \cos\theta}{\lambda F_g} ,$$

where V is the volume of the unit cell and F_g is the structure factor for reflection **g**. This relationship is only valid when $s = 0$, that is, when the crystal is at the exact Bragg position and when only one reflection is operating. For 100 kV electrons it is necessary to use relativistically corrected values of F_g and λ.

In the dynamical theory of contrast, the deviation from the exact Bragg reflection position is denoted by a dimensionless parameter $\omega = s\xi_g$. The effective extinction distance ξ_g^{ω} when the crystal is tilted from the exact Bragg position is given by

$$\xi_g^{\omega} = \xi_g/(1 + \omega^2)^{\frac{1}{2}} .$$

Thus, when using observation of thickness fringes to determine the thickness of the foil it is necessary to note the deviation from the Bragg position, ω, of the operating reflection and to use values of ξ_g^{ω}.

The values of ξ_g for α-zirconium for 100 kV electrons, given in Table A9.1 have been calculated using the analytic representation of atomic scattering amplitude given by Smith and Burge (1962). Relativistic effects have been taken into account using correction factors listed in Appendix 4 of the text by Hirsch et al. (1965).

TABLE A9.1

EXTINCTION DISTANCES ξ_g(Å) IN α–ZIRCONIUM FOR 100 kV ELECTRONS

hkiℓ	ξ_g	hkiℓ	ξ_g
0002	315	$30\bar{3}0$	927
0004	622	$30\bar{3}2$	1012
$10\bar{1}0$	592	$11\bar{2}0$	488
$10\bar{1}1$	378	$11\bar{2}2$	582
$10\bar{1}2$	830	$11\bar{2}4$	836
$10\bar{1}3$	625	$12\bar{3}0$	1567
$10\bar{1}4$	1387	$12\bar{3}1$	930
$20\bar{2}0$	1136	$12\bar{3}2$	1736
$20\bar{2}1$	681	$12\bar{3}3$	1125
$20\bar{2}2$	1310	$12\bar{3}4$	2244
$20\bar{2}3$	877	$22\bar{4}0$	1982