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LUCAS HEIGHTS

DIFF, A CODE TO PREPARE NEUTRON DIFFUSION THEORY PARAMETERS FROM THE RESULTS OF THE CODE WDSN

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ABSTRACT

With a solution of the multigroup neutron transport equation available for a problem in one dimensional geometry, diffusion theory parameters are determined so as to reproduce in diffusion theory the essential characteristics of the transport solution. Calculation of a more realistic reactor configuration can then be attempted in diffusion theory.

Diffusion coefficients, taken as constant within a uniform region, can be determined so that the diffusion flux is an optimum fit to the transport flux. As an option the conventional $1/3\Sigma_{t_r}$ can be used.

The homogeneous boundary condition is found from a diffusion theory calculation which uses the transport net current as an inhomogeneous boundary condition. The homogeneous boundary condition forces the diffusion theory to reproduce exactly the transport leakage and multiplication constant.

A discontinuity in the diffusion flux is permitted at interfaces between different materials. Interface conditions, if applied, guarantee that any reaction rate, integrated over the volume of a uniform region, is preserved by the diffusion theory.

The DIFF code uses the results of the WDSN transport code to calculate diffusion theory parameters for the CRAM diffusion code.

CONTENTS

1. INTRODUCTION

Numerical methods of solution of the neutron transport equation, for example the SN methods of Carlson et al. (1960), allow the computation of quite realistic reactor configurations though computing time can be a real problem when the reactor has to be described using more than one space dimension (1D).

For many such reactors it is possible to isolate, in a 1D model, that portion of the reactor configuration which requires solution at a high level of transport theory. If diffusion theory parameters could be obtained so as to reproduce, in diffusion theory, the essential characteristics of the 1D transport solution, then a calculation of the more realistic reactor geometry could be made at the diffusion theory level of approximation.

As an example, take the 3D problem of a reactor with an array of partially inserted control rods. The transport effects, occurring in and around the rod, which cannot be predicted by elementary diffusion theory, can be isolated in the 1D model of a control rod at the centre of a cylindrical cell. To calculate the more realistic geometry using diffusion theory we require, from the 1D transport solution, the conditions for the diffusion theory flux at the control rod surface.

Diffusion theory boundary conditions are obtained from analytical solutions of the transport equation by recognising, in the analytical solution, a portion which satisfies the diffusion theory relation:

$$J = - \text{constant} \cdot \nabla \phi ,$$

and which asymptotically approaches the total transport solution as the distance from the boundary is increased. The behaviour of this diffusion theory portion at the boundary in question determines the diffusion theory boundary conditions.

When numerical solutions of the transport equation are available the diffusion theory boundary conditions cannot be obtained in the same manner since one cannot extract directly the portion of the solution which satisfies diffusion theory.

We discuss below a method for determining the diffusion theory boundary conditions when numerical solutions of the transport equation are available. In fact the method has general application.

In the determination of the boundary conditions it is assumed that the diffusion coefficient is known. In many situations the well known expression:

$$D = \frac{1}{3(\Sigma_t - \bar{\mu} \Sigma_s)} ,$$

is quite accurate. However this result from analytical transport theory is based on a model which ignores the space dependence of neutron sources in the particular energy group under consideration. With a multigroup transport solution available it would be better to find D from this solution than to use the conventional formula.

Our approach, in general, is to retain in the realistic reactor calculation most of the flux fine scale that is produced in the 1D transport calculation. Some detail is lost in the method of determining the diffusion coefficient (see Section 2) since we restrain the diffusion coefficient to a constant value in each uniform region. Some fine scale is lost when we apply boundary conditions at internal surfaces, such as control rod surfaces. We lose the flux inside the internal boundary. Some energy fine scale is lost if the 1D transport solution is used in an energy condensation (see Section 4) for the following diffusion theory calculation. A deliberate reduction of the spatial fine scale is attempted in Section 5 but with limited success. Homogenisation of the Wigner–Seitz cell should be attacked by other methods.

Throughout we endeavour to preserve, in the diffusion theory, the group dependent reaction rates, for any reaction, integrated over the volume of each uniform region.

The DIFF code, which applies the methods to the results of the WDSN transport code of Francescon (1963) is described in Section 6.

The DIFF code was used in a theoretical analysis of the worth of a B^{10} rod inserted in the A.A.E.C.'s Argonaut type reactor MOATA. The calculated worth departed from the measured worth by the factor 0.10. A two-space-dimension transport calculation was used as a sophisticated method of analysing the experiment, though the energy variable was condensed to two groups for this calculation. The scalar fluxes from the transport calculation were compared to the fluxes determined by a diffusion calculation in the same geometry using the same two-group data but with diffusion coefficients and boundary conditions supplied by DIFF. The experiment and calculations are described fully by Spinks (1967 a).

2. DIFFUSION COEFFICIENTS FROM 1D TRANSPORT SOLUTIONS

In 1D geometry since net current and flux gradient must be antiparallel, or possibly parallel, we are at liberty to define D from

$$J(r) \equiv - D(r) \nabla \phi(r) \quad ,$$

J and ϕ being the transport net current and scalar flux. The resulting dependence of D on position, even in a uniform region, is not convenient. Instead we seek a single diffusion coefficient for each uniform region of the 1D problem.

Our approach relies on the approximation:

$$J_d(r) \approx J(r) \quad , \qquad 2.1$$

for the diffusion theory current, an approximation to be discussed below. In the 1D geometry we have:

$$J_d(r) = -D^{(n)} \frac{d\phi_d(r)}{dr} \quad , \quad n = 1 \text{ to } N \quad , \qquad 2.2$$

applying within the n^{th} uniform region. Diffusion coefficients $D^{(n)}$ are required for N adjacent uniform regions. Continuity of the diffusion flux ϕ_d is required throughout the N regions.

From Equations 2.1 and 2.2:

$$\phi_d(r) = \phi_d(r_1) - \sum_{n'=1}^{n-1} \left[\frac{1}{D^{(n')}} \int_{r_{n'}}^{r_{n'+1}} J(r') \, dr' \right] - \frac{1}{D^{(n)}} \int_{r_n}^{r} J(r') \, dr' \quad , \qquad 2.3$$

for $r_n \leq r \leq r_{n+1}$, $n = 1$ to N , r_n being the inside dimension of the n^{th} uniform region.

The N + 1 parameters $\phi_d(r_1)$ and $D^{(n)}$, n = 1 to N, are determined by fitting the diffusion flux $\phi_d(r)$ to the transport flux $\phi(r)$. The N + 1 parameters could be considered optimum when:

$$F = \sum_{n=1}^{N} \int_n \left[\phi_d(r) - \phi(r) \right]^2 dv \text{ is a minimum,} \qquad 2.4$$

dv being a volume element. It is shown in Section 3 that there is some advantage in applying the N restraints:

$$\int_n \phi_d(r) \, dv = \int_n \phi(r) \, dv \quad , \quad n = 1 \text{ to } N \quad , \qquad 2.5$$

while minimising F.

The diffusion flux could be estimated by substituting the optimum parameters $\phi_d(r_1)$ and $D^{(n)}$, n = 1 to N, into Equation 2.3. The result, in general, would not satisfy the diffusion equation since, in general, Equation 2.1 is not exact. Even so, the difference between $J_d(r)$ and $J(r)$ is normally sufficiently small to allow Equation 2.3 to be used as a good approximation to the dependence of the diffusion theory flux on the diffusion coefficient.

In regions where any diffusion theory would fail, owing for example to strong absorption or to extreme source gradients, the fit between $\phi_d(r)$ and $\phi(r)$ achieved by minimising F , (Equation 2.4), must necessarily be bad. In such cases the assumption of equality in the currents can be regarded as a type of restraint on the form of $\phi_d(r)$ causing the fit to be worse than it need be.

In any situation where diffusion theory is capable of yielding the correct scalar flux, our diffusion coefficient will give the correct result. In the case of a unidirectional neutron beam incident perpendicular to a purely absorbing slab where the conventional diffusion coefficient is $1/3\,\Sigma_a$, our method would produce $D = 1/\Sigma_a$ and a diffusion scalar flux identical to the transport scalar flux.

To discuss the assumption of Equation 2.1 we note from the analytical solutions of Case et al. (1953) that in a uniform isotropically scattering medium the exact solution for the scalar flux can be represented in the form:

$$\phi(r) = \phi_d(r) + \phi_{tnt}(r) \ , \qquad 2.6$$

where the transient portion $\phi_{tnt}(r)$ is of order $e^{-\Sigma d}$, Σd being the optical distance of the position r from the nearest boundary or from the nearest position where the gradient of the source is not zero.

At positions of sufficiently large Σd, Equation 2.1 is valid. In a 1D region of smaller Σd, Equation 2.1 is still valid provided the region is source free and non-capturing. To prove this we designate:

$$A(r) = r^n \ , \qquad 2.7$$

where n = 0, 1 or 2 for slab, cylinder or sphere. If r_1 is the value of the coordinate where Σd is large we have, from neutron conservation:

$$A(r)\,J(r) = A(r_1)J(r_1) \ , \qquad 2.8$$

and

$$A(r)\,J_d(r) = A(r_1)\,J_d(r_1) \ . \qquad 2.9$$

But:

$$J_d(r_1) = J(r_1), \qquad 2.10$$

whence

$$J_d(r) = J(r) \ , \qquad 2.11$$

for r even in the small Σd region.

For a capturing region with sources, $A(r)\,J(r)$ will change from $A(r_1)\,J(r_1)$ by just the excess of captures over sources in the intermediate volume. A similar change in $A(r)\,J_d(r)$ from $A(r_1)\,J_d(r_1)$ will occur, but in this case the captures must be evaluated using the diffusion flux, and the sources, if due to energy transfer from other groups, must be evaluated using the diffusion flux in the other groups. Thus the maximum departure of $A(r)\,J_d(r)$ from $A(r)\,J(r)$ is just the excess of transient captures over transient sources in the volume where Σd is not large. This difference is usually extremely small since the volume of the transient zone is normally small in comparison with the total volume of the uniform region.

3. BOUNDARY CONDITIONS FROM 1D TRANSPORT SOLUTIONS

We assume that the multigroup diffusion coefficients are known throughout the reactor volume.

The homogeneous boundary conditions are obtained by solving the diffusion equations for exactly the same problem as the transport solution was obtained, applying the inhomogeneous condition that, at the boundary, the diffusion net current equals the transport net current. It is stipulated, in addition, that the effective multiplication constant k , in diffusion theory, equals the k value from transport theory. The boundary conditions are obtained from the solution of the inhomogeneous diffusion theory problem as the ratio between diffusion flux and net current at the boundary.

Obviously a diffusion calculation of the homogeneous problem, with homogeneous boundary conditions from the solution of the inhomogeneous problem, will reproduce exactly the net currents in each group at the boundary and will reproduce exactly the effective multiplication constant. Including a self scatter term, the neutron balance equation, in standard notation, is:

$$-\Sigma_{t_g}(r)\,\phi_g(r) + \sum_{g'}\Sigma_{g'\to g}(r)\,\phi_{g'}(r) + \frac{\chi_g}{k}\sum_{g'}\nu\,\Sigma_{f_{g'}}(r)\,\phi_{g'}(r) = \nabla\cdot J_g(r)\ . \qquad 3.1$$

Integrating over the volume of the region enclosed by the selected boundary surface we obtain:

$$-\sum_{n=1}^{N}\Sigma_{t_g}^{(n)}\,\phi_g^{(n)} + \sum_{n=1}^{N}\sum_{g'}\left(\Sigma_{g'\to g}^{(n)} + \frac{\chi_g}{k}\,\nu\,\Sigma_{f_{g'}}^{(n)}\right)\phi_{g'}^{(n)} = L_g\,,\quad g = 1 \text{ to } g_m\ , \qquad 3.2$$

where $\phi_g^{(n)}$ is the volume integrated flux in group g , uniform region n ; L_g is the surface integrated current, group g , directed out of the boundary surface, and N is the number of uniform regions enclosed by the boundary surface. Since the boundary currents are equal in transport and diffusion theory it follows that, in each group, the quantity on the left of Equation 3.2 is the same in transport and diffusion theory. Summing over all groups we find that:

$$\sum_{g}\sum_{n=1}^{N}\left(\frac{1}{k}\,\nu\,\Sigma_{f_g}^{(n)} - \Sigma_{a_g}^{(n)}\right)\phi_g^{(n)}\ , \qquad 3.3$$

is preserved by the diffusion theory.

The above statements are valid irrespective of the values of the diffusion coefficients $D_g^{(n)}$. If the N diffusion coefficients, in each group, are calculated by the method of Section 2, applying the restraints of Equation 2.5, then the only difference between the volume integrated diffusion and transport fluxes arises from the Section 2 assumption of equality in the net currents. The boundary conditions, obtained by solving the inhomogeneous problem over the same N uniform regions, will force such differences to compensate so far as boundary current and multiplication constant are concerned. Note that continuity of diffusion flux, throughout the N uniform regions, was assumed in Section 2. The diffusion flux must be taken as continuous during solution of the inhomogeneous problem over the same N uniform regions.

Much more can be said about the diffusion theory reaction rates if the surface, in the inhomogeneous diffusion theory calculation, is selected to enclose a uniform region of the reactor. With $N=1$ in Equation 3.2 we obtain g_m equations in the g_m fluxes $\phi_g^{(1)}$. Since the equations are the same in diffusion and transport theory the solutions are the same in diffusion and transport theory. It follows that the volume integrated reaction rates, for any reaction, are maintained in each group by the diffusion theory. With $N = 1$ in the method of Section 2, we remove any restraint on the diffusion theory flux at an interface. The diffusion coefficient so obtained will give a better fit between diffusion and transport flux than for $N > 1$.

The diffusion theory is much improved by forcing the diffusion theory net current to equal the transport theory net current at each interface between uniform regions. The inhomogeneous diffusion theory solution must be evaluated separately for each uniform region so that the diffusion theory fluxes will, in general, be discontinuous at interfaces between uniform regions. Of course the net currents will be continuous.

In short, to maintain volume integrated fluxes in the diffusion theory it is necessary to dispense with the flux continuity condition at material interfaces. That the flux continuity condition limits the accuracy of diffusion theory has been recognised by several workers including Davison (1957) and Pomraning and Clark (1963). In our diffusion theory the advantages of the more general interface condition are specific.

The step in the flux $\delta\phi_d$ is related to the interface net current by the interface condition parameter λ in:

$$\delta\phi_d = \lambda J \quad . \tag{3.4}$$

Let J be positive from left to right. If λ^- is the boundary condition parameter in:

$$\phi_d^- = \lambda^- J \quad , \tag{3.5}$$

determined from the inhomogeneous diffusion calculation for the region to the left of the interface and if λ^+ is the boundary condition parameter in:

$$\phi_d^+ = \lambda^+ J \quad , \tag{3.6}$$

determined from the inhomogeneous diffusion calculation for the region to the right of the interface, then:

$$\lambda \equiv \frac{\phi_d^- - \phi_d^+}{J} = \lambda^- - \lambda^+ \quad , \tag{3.7}$$

is the interface parameter, Such a condition can be easily incorporated into existing diffusion codes by separating uniform regions by a thin single mesh region. Finite difference diffusion codes determine the current across the mesh interval, width δ, from:

$$J = D \frac{\phi_d^- - \phi_d^+}{\delta} \quad . \tag{3.8}$$

Comparing Equations 3.7 and 3.8 we see that the interface condition is applied by using the dummy diffusion coefficient:

$$D = \frac{\delta}{\lambda} \quad , \tag{3.9}$$

in the thin interface region.

If we seek solution of the inhomogeneous diffusion theory problem using the conventional source iteration technique we find that convergence cannot always be guaranteed. It is shown in Appendix 1 that a modified technique will yield the solution.

4. ENERGY CONDENSATION FROM TRANSPORT THEORY TO DIFFUSION THEORY

With the 1D transport theory solution available in a large number of neutron energy groups, we desire a condensation to a smaller number of groups for the following 2D or 3D diffusion theory calculation of the more realistic reactor configuration.

A feature of the method of Section 2 is its applicability to the evaluation of a condensed group diffusion coefficient, which is determined simply by condensing the transport flux according to:

$$\bar{\phi}_G(r) = \sum_{g \text{ in } G} \phi_g(r) \quad , \tag{4.1}$$

and the transport net current according to:

$$\bar{J}_G(r) = \sum_{g \text{ in } G} J_g(r) \quad , \tag{4.2}$$

before applying the method of Section 2. The condensed group diffusion coefficient is obtained so that the condensed diffusion flux is an optimum fit to the condensed transport flux.

A feature of the method of Section 3 is that the conclusions about the preservation of volume integrated reaction rates are unchanged, in the condensed group structure, if flux and current are condensed according to Equations 4.1 and 4.2, if the fission spectrum is condensed according to:

$$\bar{\chi}_G = \sum_{g \text{ in } G} \chi_g \quad , \tag{4.3}$$

and if cross sections are condensed according to:

$$\bar{\bar{\Sigma}}^{(n)}_{t_G} = \frac{\sum_{g \text{ in } G} \Sigma^{(n)}_{t_g} \phi^{(n)}_g}{\bar{\phi}^{(n)}_G} \quad , \tag{4.4}$$

$$\overline{\overline{\nu\Sigma}}^{(n)}_{f_G} = \frac{\sum_{g \text{ in } G} \nu\Sigma^{(n)}_{f_g} \phi^{(n)}_g}{\bar{\phi}^{(n)}_G} \quad , \tag{4.5}$$

and

$$\bar{\bar{\Sigma}}^{(n)}_{G \to G'} = \frac{\sum_{g \text{ in } G} \phi^{(n)}_g \sum_{g' \text{ in } G'} \Sigma_{g \to g'}}{\bar{\phi}^{(n)}_G} \quad , \tag{4.6}$$

all before finding the condensed group boundary or interface conditions by solution of the condensed inhomogeneous diffusion equations. Summing Equation 3.2 within condensed group G , and applying Equations 4.1 to 4.6 we arrive at an equation in the condensed volume integrated fluxes which is identical in form to Equation 3.2 . With the leakage term on the right hand side of this condensed group equation forced to have the same value in diffusion and transport theory, the statements made in Section 3 regarding the preservation of volume integrated reaction rates apply equally well in the condensed group structure.

5. SPATIAL CONDENSATION FROM TRANSPORT THEORY TO DIFFUSION THEORY

With the 1D transport solution available for a problem with considerable detail in geometry we consider a reduction in such detail as to reduce the complexity of the following 2D or 3D problem of the more realistic reactor configuration. If the 1D problem is the Wigner–Seitz cell we could desire the complete elimination of the cell spatial fine scale.

In Section 3 we noted that, if the diffusion net current were forced equal to the transport net current at a surface enclosing N adjacent uniform regions, then, unless N = 1 , we could not guarantee the preservation of volume integrated reaction rate, for any reaction, by the diffusion theory.

However if the N regions are smeared according to:

$$\bar{\bar{\Sigma}}_{t_g} = \frac{\sum_n \Sigma^{(n)}_{t_g} \phi^{(n)}_g}{\bar{\phi}_g} \tag{5.1}$$

$$\overline{\overline{\nu\Sigma}}_{f_g} = \frac{\sum_n \nu\Sigma^{(n)}_{f_g} \phi^{(n)}_g}{\bar{\phi}_g} \quad , \tag{5.2}$$

and

$$\bar{\bar{\Sigma}}_{g \to g'} = \frac{\sum_n \Sigma^{(n)}_{g \to g'} \phi^{(n)}_g}{\bar{\phi}_g} \quad , \tag{5.3}$$

$\bar{\phi}_g$ being the integral of the flux over the total volume, then substitution in Equation 3.2 yields:

$$-\overline{\Sigma}_{tg}\,\overline{\phi}_g + \sum_{g'}\left(\overline{\Sigma}_{g'\to g} + \frac{\chi_g}{k}\,\overline{\nu\Sigma}_{fg'}\right)\overline{\phi}_{g'} = L_g\,, \qquad g = 1 \text{ to } g_m\,. \tag{5.4}$$

With L_g and k forced to be the same in diffusion theory and transport theory, the g_m equations (5.4) are the same in both theories. Therefore the volume integrated fluxes are the same in both theories. It follows that, provided a separate inhomogeneous diffusion theory calculation is made for each smeared zone of the reactor, the volume integrated reaction rate for any reaction is preserved in each group by the diffusion theory. The same is true in a condensed group structure if the cross sections on the left of Equations 5.1 to 5.3 are condensed in similar fashion to Equations 4.4 to 4.6.

The above is valid irrespective of the diffusion coefficient, but a diffusion coefficient is needed for the inhomogeneous calculation. A possibility is to weight the individual diffusion coefficients $D_g^{(n)}$, obtained by fitting a diffusion flux to the transport flux within each uniform region of the zone, according to:

$$\overline{D}_g = \frac{\sum_n D_g^{(n)}\,\phi_g^{(n)}}{\overline{\phi}_g}\,. \tag{5.5}$$

Such a procedure is valid if the actual microscopic flux can be expressed as a product of a macroscopic flux with small buckling and a periodic fine scale flux. Applied to the complete Wigner–Seitz cell, the procedure assumes in addition that the ratio of net current to flux gradient determined in the cell calculation is maintained when the flux gradient is changed by the influence of the cell surroundings. In the limit of a small cell, the diffusion coefficient should equal the reciprocal of three times the volume averaged transport cross section. There is no guarantee that Equation 5.5 has this limit.

Though a simple prescription such as Equation 5.5 is probably adequate when the smeared region is a small part of the 1D problem it should be used with caution in the smearing of an entire Wigner–Seitz cell. Better still the methods of Benoist (1959) or Leslie (1961) should be used for the cell problem.

6. FINITE DIFFERENCE DIFFUSION THEORY

In practice, the inhomogeneous diffusion theory calculation, discussed in Section 3, for determining the boundary and interface conditions is made in finite difference diffusion theory. In practice it is the discrete representation of the net current at the boundary which is equated to the transport net current at the boundary. The right hand side of Equation 3.2 is the discrete representation of the current in diffusion theory. $\phi_g^{(n)}$ in Equation 3.2 is the discrete representation of the volume integrated flux in diffusion theory.

It follows that all statements made in Sections 3, 4, and 5 about the preservation of effective multiplication constant, leakages, and volume integrated reaction rates are true, not in analytical diffusion theory, but in the particular finite difference diffusion theory used for solution of the inhomogeneous diffusion theory problem. Of course, if the finite difference diffusion theory mesh is sufficiently fine, analytical and finite difference diffusion theory are equivalent, but if a coarse mesh is desired for the diffusion calculation of the realistic reactor configuration, the calculation will be accurate if the same mesh is used in the inhomogeneous diffusion theory calculation.

For some problems the space coordinate of the 1D model does not correspond to any coordinate of the more realistic reactor model. For example, the radial coordinate, in the cylindrical cell model of a reactor with an array of partially inserted control rods, does not necessarily correspond to a natural coordinate of the reactor.

For such problems the mesh in the inhomogeneous diffusion theory calculation cannot be the same as that for the diffusion calculation of the more realistic reactor. For safety a fine mesh should be used in the inhomogeneous diffusion theory calculation.

For such problems the direct representation of regions which conform naturally to the 1D model is not necessarily possible in the natural mesh geometry for realistic representation of the complete reactor. A boundary perturbation theory had to be developed (Spinks 1965) to account for the difference in the shape of a control rod and the shape of the mesh surfaces. Instead, in representing, say, a cylindrical control rod in an XY mesh, it is suggested that the boundary conditions on the diffusion flux be determined and then applied at a new surface suited to representation by the XY mesh, and located away from the control rod surface. The boundary conditions at the new surface will depend on azimuth even though the conditions at the actual cylindrical control rod surface are independent of azimuth. The boundary conditions at the new surface can be readily determined from the solution of the inhomogeneous problem since this solution is available away from the control rod surface.

7. THE DIFF CODE

The above methods of determining diffusion theory parameters from the results of transport theory were incorporated in the code DIFF written in Fortran 4 for the IBM 7040. This code processes the output tape resulting from a run of the transport code WDSN, Francescon (1963). The diffusion theory parameters are produced in format suited to the diffusion code CRAM, Hassitt (1962).

In its present form DIFF treats each uniform region separately in calculating the diffusion coefficients by the method of Section 2 and in solving the inhomogeneous problem as in Section 3. Thin interface regions are therefore required between each uniform region of the following CRAM calculation. DIFF produces the dummy diffusion coefficients, Equation 3.9, taking δ, the thickness of the interface region, equal to 0.0009999. The dummy diffusion coefficients can be negative. Normally a negative diffusion coefficient is an indication to CRAM that the diffusion coefficient is actually an internal boundary condition to be applied at the surface of the region. A modified version of CRAM tests whether the thickness is not 0.0009999 before using the negative diffusion coefficient in this fashion.

The thin interface regions can effect the convergence of the CRAM calculation when they lie in the channel direction. Since no trouble has been experienced with the thin regions lying perpendicular to the channel direction, the trouble is thought to be due to the CRAM method of solving for all groups in each channel before proceeding to the next channel. It seems likely that the thin regions would have little effect on the convergence of a conventional code, such as CURE (Wachspress 1957), which solves for all space points in the particular energy group before proceeding to the next group.

The procedure of calculation in DIFF is as follows.

1. The WDSN tape is read.

2. The net currents are determined, in the cylindrical geometry, from neutron balance by starting at the axis and calculating outwards.

3. The fluxes and currents are normalised so that the total neutron loss, absorption plus external leakage, equals unity.

4. A table summarising the WDSN calculation is printed. For each uniform region and in each group the code prints the volume integrated flux, the surface integrated current, and the buckling:

$$B_g^2 = 3\,\Sigma_{t_g} \cdot \frac{\text{surface integrated current in group } g}{\text{volume integrated flux in group } g} \quad .$$

The buckling could be used in a group dependent buckling iteration between WDSN and a spectrum code, such as GYMEA, (Pollard and Robinson 1966), which calculates a leakage from $B_g^2 \phi_g / 3 \Sigma_{t_g}$.

5. DIFF prints the net currents in each group crossing each mesh surface.

6. From a condensed group structure specified by the user, the code condenses the fission spectrum according to Equation 4.3, the transport flux and net current according to Equations 4.1 and 4.2, and the multigroup cross sections according to Equations 4.4 to 4.6 .

7. The diffusion coefficient, in each condensed group and in each uniform region, is determined by fitting a diffusion flux to the condensed transport flux using the method of Section 2. The fit is made separately for each uniform region; that is, $N = 1$ in the method of Section 2.

8. The user may specify certain zones of the WDSN problem in which the uniform regions within each zone are to be smeared for the following CRAM calculation. The smeared diffusion coefficients and cross sections (including the total cross section) are calculated by weighting with the volume integrated transport flux in each uniform region of the zone to be smeared; that is, the condensed group versions of Equations 5.1, 5.2, 5.3, and 5.5 are used.

   The above selected zones will be called smeared zones even if the zone contains a single uniform region.

9. As an option to the calculation of the diffusion coefficient by steps 7 and 8, the diffusion coefficient can be set to $1/3\,\Sigma_t$ where Σ_t is the total cross section calculated in step 8. The option is zone dependent.

10. A discrete form of the condensed multigroup diffusion equation is solved separately in each smeared zone with the exact transport currents as fixed quantities at the inner and outer boundaries of the zone and with the exact transport multiplication constant as a fixed multiplying factor. The boundary conditions, obtained from the solution, will force the finite difference diffusion theory to reproduce exactly the transport values of boundary currents, multiplication constant and zone integrated, condensed group dependent, reaction rates.

    The discrete form of the diffusion equation is obtained by integrating over the mesh volume as in CRAM. The mesh itself can be specified or taken equal to the WDSN mesh.

    Convergence of the source iteration technique is achieved not quite by the method of Appendix 1. In the case of a uniform region the error mode corresponding to the largest eigenvalue of the iterating matrix is the only mode which can diverge (See Appendix 1). Furthermore, for a uniform region, this error mode is spatially flat. Divergence is prevented by adding a constant to the group flux, after solution in the group, so as to equate the volume integrated diffusion flux to the volume integrated transport flux.

    The inhomogeneous problem is solved in a subroutine which is also called DIFF. The user can specify a print, after each group solution, of the diffusion flux, but this is optional.

11. After solution of the inhomogeneous problem in a particular zone the code prints out:

    a. The error in the diffusion flux. For each condensed group this is:

$$\epsilon = \frac{\sqrt{\frac{1}{v}\int(\phi_d - \phi)^2\,dv}}{\frac{1}{v}\int\phi\,dv} \quad ,$$

       the integrations being over the volume of the zone.

    b. All transport and diffusion fluxes for the zone.

c. The CRAM boundary conditions, by condensed group, at the inside and outside boundaries of the zone. The boundary condition is the net current crossing the boundary and directed out of the zone divided by the flux at the particular boundary.

d. The condensed multigroup data for the particular zone.

12. The condensed multigroup data, for each zone, and the dummy interface region data are punched in format suited to CRAM. The code also prints the dummy diffusion coefficients for each interface region.

13. Finally DIFF prints and punches the condensed fission spectrum.

The user can choose the position of a CRAM internal boundary surface after running DIFF. CRAM permits internal boundary conditions to be specified in some groups while calculating the region as a diffusing region in other groups. This feature is used by patching the multigroup data from DIFF, for the particular region, to accord with the boundary condition printed by DIFF.

Appendix 2 is a listing of the source deck of DIFF and its subroutine. The input requirements are detailed in this appendix.

Projected changes for DIFF are:

a. The provision of slab and spherical as well as cylindrical geometry.

b. N not necessarily unity in applying the methods of Sections 2 and 3. With this feature included, the application of the flux discontinuity condition, by the user, will be optional.

c. The calculation of the position dependent boundary conditions at a new surface, see Section 6, a surface which does not correspond in shape to the mesh surfaces of the problem but which does correspond to the mesh of the following CRAM problem. It is envisaged that the new surface will be specified by reading the complete CRAM geometry into DIFF.

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APPENDIX 1

SOLUTION OF THE INHOMOGENEOUS DIFFUSION THEORY PROBLEM

The conventional source iteration technique for solving the discrete representation of the multigroup diffusion equations can diverge for our inhomogeneous problem. The technique can be modified to yield the solution.

Consider the matrix equation:

$$M_g \phi_g = \sum_{g'=1}^{g_m} T_{g' \to g} \phi_{g'} + \frac{1}{k} \sum_{g'=1}^{g_m} F_{g' \to g} \phi_{g'} + S_g \quad , \qquad g = 1 \text{ to } g_m \ , \tag{A.1}$$

for the vector of fluxes ϕ_g in group g. The equation results from a discrete representation, such as is used by Hassitt (1962), of the diffusion equation. The matrices are of order p, the number of discrete space points. The matrix M_g contains the diffusion coefficient, removal cross section and mesh geometry. In our 1D geometry M_g is tridiagonal. The matrices $T_{g' \to g}$ and $F_{g' \to g}$ are due to scattering and fission respectively. S_g, the external source vector, has non-zero entries at boundary mesh points only. k is fixed at the transport theory value.

The g_m equations can be cast into $L \equiv g_m \cdot p$ order matrix form:

$$M \phi = T \phi + \frac{1}{k} F \phi + S \ , \tag{A.2}$$

where M is block diagonal, the blocks being the individual M_g's.

The solution is, formally:

$$\phi = \left(M - T - \frac{1}{k} F \right)^{-1} S \ , \tag{A.3}$$

but the matrix is generally far too large to be inverted directly. The inversion of M-T is not so difficult but if T contains upscatter components an iterative process is needed. It can be shown (Spinks 1967b), that this inner iteration will converge.

The conventional source iteration technique is:

$$\phi^{(t+1)} = (M-T)^{-1} \left(\frac{1}{k} F \phi^{(t)} + S \right). \tag{A.4}$$

The process converges if the eigenvalues of $(M-T)^{-1} F$ are less than k in absolute value. In other words, for convergence:

$$M \phi = T \phi + \frac{1}{\lambda} F \phi \ , \tag{A.5}$$

must solve to give $|\lambda_i| < k$ for all eigenvalues λ_i.

Recognising that Equation A.5 describes the reactor with zero net current at the boundary and that the largest λ is the effective multiplication constant of the reactor with zero net current at the boundary we see that the iteration diverges if the multiplication constant of the reactor with zero net current at the boundary is greater than the multiplication constant of the actual reactor.

If the actual net current at the boundary is directed outwards, as at a vacuum boundary, the multiplication constant of the actual reactor will be less than the multiplication constant of the reactor with zero net current at the boundary and the process will diverge.

If the inhomogeneous problem involves a single non-fissile region, which could be the case if the inhomogeneous solution is evaluated separately for each uniform region, then $F = 0$ and the solution is obtained directly.

If the inhomogeneous problem involves a single uniform fissile region then the multiplication constant of the fissile region with zero net current equals k_∞ for that region. There must be at least

APPENDIX 1 (continued)

one such uniform fissile region where $k_\infty > k$. Hence if the solution to the inhomogeneous problem is sought separately in each uniform region of the reactor then, in at least one such region, the iterative technique will diverge.

For the supercritical reactor we expand the error of the initial guess in the eigenvectors x_i of $X \equiv (M-T)^{-1}F$. Thus:

$$\phi^{(0)} = \phi + \sum_{i=1}^{L} \alpha_i x_i \quad . \tag{A.6}$$

After t iterations:

$$\phi^{(t)} = \phi + \sum_{i=1}^{L} \left(\frac{\lambda_i}{k}\right)^t \alpha_i x_i \quad . \tag{A.7}$$

If the eigenvalues are ordered such that $|\lambda_1| > |\lambda_2| > |\lambda_3|$, etc., then for sufficiently large t:

$$\alpha_1 x_1 = \left\{\frac{[\phi^{(t-1)}, \phi^{(t)}]}{[\phi^{(t)}, \phi^{(t)}]}\right\}^t \phi^{(t)} \quad . \tag{A.8}$$

$\alpha_1 x_1$ could now be deducted from $\phi^{(0)}$. The initial guess for a further iteration cycle would be:

$$\phi + \sum_{i=2}^{L} \alpha_i x_i \quad . \tag{A.9}$$

The process could be repeated until all error components corresponding to eigenvalues $|\lambda_i| > k$ are eliminated.

The above method has the disadvantage that round–off errors can accumulate. Though, at the stage of Equation A.8 $\phi^{(t)}$ is parallel to x_1, round–off would cause the two sides of the equation to differ. The round–off error defect is removed by using the shape of x_1 to modify the matrix X for the next iteration cycle.

Let $x_{1,i}$ be the first element of eigenvector x_i. Let r_1 be the row vector identical to the first row of X. Forming the matrix:

$$X^{(1)} = X - \frac{x_1 r_1}{x_{1,1}} \quad , \tag{A.10}$$

we find that all the first row elements are zero and that:

$$X^{(1)} x_i^{(1)} = \lambda_i x_i^{(1)} \quad , \qquad i \geqslant 2 \quad , \tag{A.11}$$

where:

$$x_i^{(1)} = \frac{x_1}{x_{1,1}} - \frac{x_i}{x_{1,i}} \quad . \tag{A.12}$$

Note also that $x_{1,i}^{(1)} = 0$.

The original equation:

$$\phi = \frac{1}{k} X \phi + (M-T)^{-1} S \quad , \tag{A.13}$$

can be written, using Equation A.10,

APPENDIX 1 (continued)

$$\phi = \frac{1}{k} X^{(1)} \phi + \frac{x_1 r_1 \phi}{k\, x_{1,1}} + (M-T)^{-1} S \,, \qquad \text{A.14}$$

or

$$\phi = \phi_1 - \frac{(r_1 \phi_1)}{\lambda_1 - k} \cdot \frac{x_1}{x_{1,1}} \,, \qquad \text{A.15}$$

where:

$$\phi_1 = \left[I - \frac{1}{k} X^{(1)} \right]^{-1} (M-T)^{-1} S \,. \qquad \text{A.16}$$

To determine ϕ_1 we use the process:

$$\phi_1^{(t+1)} = \frac{1}{k} X^{(1)} \phi_1^{(t)} + (M-T)^{-1} S. \qquad \text{A.17}$$

We seek ϕ_1 in the same manner as we sought ϕ except that we iterate with the matrix $X^{(1)}$ instead of with X. Since $X^{(1)}$ has all zeros in the first row we are effectively iterating with the reduced matrix obtained by deleting the first row and first column from $X^{(1)}$. This reduced matrix has eigenvalues λ_i, $i = 2$ to L. The process will converge if $|\lambda_2| < k$. Otherwise the process will diverge to the vector:

$$x_2^{(1)} = \frac{x_1}{x_{1,1}} - \frac{x_2}{x_{1,2}} \,. \qquad \text{A.18}$$

In the case of divergence we use $x_2^{(1)}$ to modify $X^{(1)}$ to $X^{(2)}$ in the same fashion as X was modified to $X^{(1)}$. Thus:

$$X^{(2)} = X^{(1)} - \frac{x_2^{(1)} r_2^{(1)}}{x_{2,2}^{(1)}} \,, \qquad \text{A.19}$$

$r_2^{(1)}$ being the row vector identical to the second row, that is, the first non-zero row, of $X^{(1)}$.

Note that, since $x_2^{(1)}$ has zero in the first position, $X^{(2)}$ will have all zeros in the first two rows. In addition:

$$X^{(2)} x_i^{(2)} = \lambda_i x_i^{(2)} \,, \quad i \geq 3 \,, \qquad \text{A.20}$$

where:

$$x_i^{(2)} = \frac{x_2^{(1)}}{x_{2,2}^{(1)}} - \frac{x_i^{(1)}}{x_{2,i}^{(1)}} \,. \qquad \text{A.21}$$

ϕ_1 can be expressed in terms of a vector ϕ_2 obtained by iterating with the matrix $X^{(2)}$. In similar fashion to the derivation of Equations A.15 and A.17, we find:

$$\phi_1 = \phi_2 - \frac{(r_2^{(1)} \phi_2)}{\lambda_2 - k} \cdot \frac{x_2^{(1)}}{x_{2,2}^{(1)}} \,, \qquad \text{A.22}$$

with ϕ_2 determined from:

$$\phi_2^{(t+1)} = \frac{1}{k} X^{(2)} \phi_2^{(t)} + (M-T)^{-1} S \,. \qquad \text{A.23}$$

APPENDIX 1 (continued)

Since $X^{(2)}$ has all zeros in the first two rows we are effectively iterating with the reduced matrix formed by deleting the first two rows and first two columns from $X^{(2)}$. This reduced matrix has eigenvalues λ_i , $i = 3$ to L. The process will converge if $|\lambda_3| < k$.

In practice it is awkward to form explicitly the matrices $X^{(1)}$, $X^{(2)}$, etc. We prefer to continue iterating with X. This can be done if $(M-T)^{-1} S$ is found separately in an initial iteration. (Set F to zero for this initial iteration). If, at the stage of Equation A.23 we form $X\phi_2^{(t)}$ instead of $X^{(2)}\phi_2^{(t)}$ then the following steps will recover the latter vector.

(a) Obtain $X^{(1)}\phi_2^{(t)}$ by deducting, from $X\phi_2^{(t)}$, a vector proportional to x_1, so as to make the first element zero. For proof, postmultiply Equation A.10 through by $\varphi_2^{(t)}$.

(b) Obtain $X^{(2)}\varphi_2^{(t)}$ by deducting, from $X^{(1)}\varphi_2^{(t)}$, a vector proportional to $x_2^{(1)}$ so as to make the second element zero. For proof, postmultiply Equation A.19 through by $\varphi_2^{(t)}$.

The quantity $(r_2^{(1)}\varphi_2)$ in Equation A.22 is the second element of $X^{(1)}\varphi_2$ obtained from step (a) above after convergence.

Of importance is the number of eigenvalues of the matrix X of modulus greater than k . As mentioned above the eigenvalues are those of the homogeneous equation with the boundary condition of zero net current.

λ_1 corresponds to the eigenvector x_1 , which has the same sign through all space and energy. λ_2 corresponds to the eigenvector which changes sign in space or to the eigenvector which changes sign in energy. Generally the magnitude of the eigenvalue corresponding to the eigenvector with a change of sign in energy is a good deal less than λ_1 . It is possible, for regions of large dimension, for the magnitude of the eigenvalue corresponding to the eigenvector with a change of sign in space, to be quite close to λ_1 . We therefore consider λ_2 as being the eigenvalue corresponding to the eigenvector with a change of sign in space.

If the region under consideration is a uniform slab, the zero point of the eigenvector x_2 occurs at the centre plane of the slab. Remembering that x_2 has zero slope at the boundaries we see, by reflecting the eigenvector at the boundary, that $\lambda_2 = k_{bare}$, the multiplication constant of the slab with zero flux as the boundary condition. It follows that $\lambda_2 < k$, since the multiplication constant of the reactor must exceed that of any portion with zero flux as the boundary condition.

If the region under consideration is a slab reactor symmetrical about the centre plane with a fissile region thickness T_1, reflected at both sides by a thickness T_2, we see, by reflecting the eigenvector x_2 at the outer boundaries, that $\lambda_2 = k_{bare}$, the multiplication constant of the "inside–out" reactor with an inner reflector of thickness $2T_2$, outer fissile regions of thickness $T_1/2$ at both sides, and zero flux as the condition at the outer boundary of the fissile regions. Since the fissile regions have been shifted to positions of lower importance we see again that $\lambda_2 < k$. Conversely if the problem has an inner reflector region we could obtain the result $\lambda_2 > k$, but this would not be a likely situation.

If the region under consideration is a uniform cylindrical or spherical annulus it can be shown that $\lambda_2 < k_{bare}$, the multiplication constant of the annulus with zero flux at the boundaries. Of course $\lambda_2 \to k_{bare}$ as the curvature of the region decreases. Again we see that $\lambda_2 < k$.

To summarise, in most practical situations only λ_1 can be greater than k . In the important case where only uniform regions are considered in the inhomogeneous problem the second eigenvalue is never greater than k . In some rather odd situations involving both fissile and non–fissile regions the second eigenvalue can be greater than k .

APPENDIX 2

FORTRAN SOURCE LIST OF PROGRAMME DIFF AND ITS SUBROUTINE

```
C     DIFF - FINITE DIFFERENCE DIFFUSION THEORY FROM TRANSPORT THEORY.
C
C
C  1. READS WDSN OUTPUT TAPE.
C  2. SUMMARISES WDSN CALCULATION WITH LOSSES(ABSORPTION + EXTERNAL LEAKAGE)
C     NORMALISED TO UNITY.
C  3. CONDENSES THE MULTIGROUP DATA TO FEWER GROUPS. FOR EACH UNIFORM
C     REGION, BY WEIGHTING WITH THE VOLUME INTEGRATED FLUX IN THE REGION
C  4. FINDS A DIFFUSION COEFFICIENT, IN EACH CONDENSED GROUP AND IN EACH REGION,
C     WHICH ACHIEVES THE BEST FIT BETWEEN A DIFFUSION FLUX AND THE TRANSPORT FLUX
C     ASSUMING THAT THE DIFFUSION CURRENT EQUALS THE TRANSPORT CURRENT.
C  5. AVERAGES THE DIFFUSION COEFFICIENTS AND MACROSCOPIC CROSS SECTIONS
C     (INCLUDING THE TOTAL CROSS SECTION) IN SELECTED ZONES BY WEIGHTING
C     WITH THE VOLUME INTEGRATED FLUX IN EACH REGION OF THE SELECTED ZONE.
C
C     (THE ABOVE SELECTED ZONES ARE CALLED SMEARED ZONES. A SELECTED ZONE
C     COULD BE A SINGLE UNIFORM REGION BUT IT IS STILL CALLED A SMEARED ZONE.)
C
C  6. AS AN OPTION THE DIFFUSION COEFFICIENT IN A SMEARED ZONE CAN BE CALCULATED
C     AS THE RECIPROCAL OF 3 TIMES THE SMEARED TOTAL CROSS SECTION.
C  7. SOLVES THE CONDENSED MULTIGROUP DIFFUSION EQUATION (USING SUBROUTINE DIFF).
C     SEPARATELY IN EACH SMEARED ZONE, WITH THE EXACT TRANSPORT CURRENTS AS
C     FIXED SOURCES AT THE INNER AND OUTER BOUNDARIES OF THE ZONE AND WITH THE
C     EXACT TRANSPORT MULTIPLICATION CONSTANT AS A FIXED MULTIPLYING FACTOR.
C     THE DIFFUSION THEORY BOUNDARY CONDITIONS, OBTAINED FROM THE SOLUTION, WILL
C     FORCE THE FINITE DIFFERENCE DIFFUSION THEORY TO REPRODUCE EXACTLY THE
C     TRANSPORT VALUES OF BOUNDARY CURRENTS, MULTIPLICATION CONSTANT, AND
C     ZONE INTEGRATED REACTION RATES. THE QUANTITIES ARE EXACT FOR THE SPECIFIED
C     DIFFUSION THEORY MESH.
C  8. PUNCHES THE CONDENSED ZONE DEPENDENT MULTIGROUP DATA IN FORMAT SUITED
C     TO THE CRAM CODE.
C
C
C MINIMUM DIMENSIONS
C     EACH UNIFORM REGION MUST CONTAIN AT LEASE 2 MESH INTERVALS IN WDSN UNLESS
C     OPTION 6 ABOVE IS USED FOR A ZONE CONTAINING THE PARTICULAR REGION.
C     ZONES MUST CONTAIN AT LEAST 3 INTERVALS IN THE SPECIFIED DIFFUSION THEORY
C     MESH
C
C
C MAXIMUM DIMENSIONS
C     GROUPS            16
C     MATERIAL NUMBER   10
```

```
C     UNIFORM REGIONS 20
C     MESH POINTS     80
C
C
C INPUT INFORMATION
C
C TAPES
C
C 1. WDSN OUTPUT TAPE ON UNIT 4.
C
C CARDS
C
C CARD 1. FORMAT(20A4)
C     TITLE CARD.
C CARD 2. FORMAT(24I3)
C     K(L),L=1 TO THE NUMBER OF CONDENSED GROUPS, - GROUP PARTITION VECTOR
C          EXAMPLE
C                  I I I I I I I I I I ORIGINAL 9 GROUP SET WITH 10 BOUNDARIES.
C                        3     6   8 9 K(L),L=1 TO 4.
C                  I     I     I   I I CONDENSED 4 GROUP SET WITH 5 BOUNDARIES.
C CARD 3. FORMAT(24I3)
C   MIX(L),L=1 TO THE NUMBER OF SMEARED ZONES, - REGION PARTITION VECTOR
C        EXAMPLE
C                  I I I I I I I I     7 ORIGINAL REGIONS WITH 8 BOUNDARIES
C                    1     4     7     MIX(L),L=1 TO 3.
C                  I I     I     I     3 SMEARED ZONES WITH 4 BOUNDARIES
C CARD 4. FORMAT(24I3)
C   MODE(L),L=1 TO THE NUMBER OF SMEARED ZONES, - DIFFUSION COEFFICIENT OPTION
C   MODE = 1 FOR D BY FLUX FITTING                SEE STEP 5 FOR SMEARING RULE
C   MODE = 2 FOR D FROM TOTAL CROSS SECTION       SEE STEP 6 FOR SMEARING RULE
C CARD 5. FORMAT(3I3)
C   IPUNCH - NOT ZERO PUNCHES CONDENSED DATA FOR EACH ZONE IN CRAM FORMAT.
C   IPRINT - SEE DIFF SUBROUTINE. USUALLY 0
C   MAX    - SEE DIFF SUBROUTINE. USUALLY 20
C CARD 6. FORMAT(E12.0)
C   ACC    - SEE DIFF SUBROUTINE. USUALLY 1.E-6
C CARD 7. FORMAT(7(E8.0,I2)) IN AS MANY CARDS AS ARE NEEDED
C     FINITE DIFFERENCE DIFFUSION THEORY MESH
C     EXAMPLE    5.0 20   10.0 30    5.0 40    SPECIFIES 40 MESH INTERVALS IN A
C     REACTOR DIMENSION 100+100+50 CM.
C     NOTE. EACH INTERFACE BETWEEN SMEARED ZONES MUST LIE ON A MESH SURFACE.
C     IF A BLANK CARD IS INSERTED THE WDSN MESH WILL BE USED
C
```

```
C THE SET OF CARDS 2 TO 7 CAN BE REPEATED ANY NUMBER OF TIMES
C
C      CAUTION IN CONFIGURATIONS SUCH AS THOSE EMPLOYING HEAVY
C      ABSORBERS ETC.,A VERSION OF THE SYSTEM FPT ROUTINE WHICH ALLOWS
C      SUFFICIENT UNDERFLOWS MUST BE LOADED WITH THE PROGRAM
C
       DIMENSION IPAR(36),FITR(16,80)
       DIMENSION M(80),DR(80),RDIM(81),R(80),MDIM(82)
       DIMENSION SCAT(10,16,16),NSS(10,16),LBLOCK(10,16),SIGACT(10,16)
       DIMENSION NUSIGF(10,16),SIGTOT(10,16),SIGA(10,16),SIGTR(10,16)
       DIMENSION NFW(16),FIRATE(80),FY(16)
       DIMENSION DVO(80),MATMZ(20),IREF(21),VO(20),NI(20)
       DIMENSION S(16),J(16,80),JDIM(16,81),STERR(16)
       DIMENSION DDUM(16),ZERO(16),BIG(16)
       DIMENSION K(24),MIX(24),DCC(16),STC(16),SRC(16),USFC(16),FYC(16)
       DIMENSION MODE(24)
       DIMENSION SCATC(16,16),FZC(16)
       DIMENSION JC(16,80),JCDIM(16,81),FITRC(16,80),FIDC(16,80),FMZC(16)
       DIMENSION CNTIN(16),CNTOUT(16)
       DIMENSION TITLE(20)
       DIMENSION DELTA(7),MESH(7),DRD(80),RD(80),RDDIM(81),FINT(16,80)
       DIMENSION DVOD(80)
C
       EQUIVALENCE(RDIM(2),R),(MDIM(2),M)
       EQUIVALENCE(JDIM(1,2),J)
       EQUIVALENCE(IJDR,IJDRDM(2))
       EQUIVALENCE(JCDIM(1,2),JC)
       EQUIVALENCE (RDDIM(2),RD)
C
       REAL NUSIGF
       REAL IFIDVO(16,20)
       REAL J,JDIM
       REAL JC,JCDIM
       REAL LOSSES,LOSS
       REAL JTOT,JOUT
       REAL IJDR(80),IJDRDM(81)
       REAL LAMBDA(16,20,2)
C
       INTEGER BRANCH
       INTEGER G,GSIN,GC,GSINC,G1,G2,GSIN1,GSIN2
C
       COMMON SRC,USFC,SCATC,DCC,FYC,CALCK,CNTIN,CNTOUT,FIDC,DRD,RD,DVOD
       COMMON FZC,TVOL
```

```
      COMMON ID1,ID2,NGC
C
C READ TITLE
      READ(5,600) TITLE
  600 FORMAT(20A4)
      WRITE(6,601) TITLE
  601 FORMAT(1H1,20A4)
C
C READ WDSN EDIT TAPE (SEE AEEW-R273)
      REWIND 4
      CALL SETXEM(IERROR,IFTC)
C     CONTROL RETURNS TO HERE WHEN FORTRAN READ LIST EXCEEDS RECORD
C     LENGTH
      IF(IERROR.NE.0) GO TO (101,102,103,104),BRANCH
      BRANCH=1
C RECORD 1
      READ(4) EIG,NG,NR,((FITR(G,I),I=1,NR),G=1,NG),FS,((DUM),IDUM=1,
     12000 )
  101 BRANCH=2
C RECORD 2
      READ(4) IPAR,NS,NM,NGFS,NFG,OB,((DUM),IDUM=1,2000 )
  102 BRANCH=3
C RECORD 3
      READ(4)(M(I),I=1,NR),(DR(I),I=1,NR),((DUM),IDUM=1,2000 )
  103 BRANCH=4
      NGM1=NG-1
      DO 1 NMAT=1,NM
C RECORD 4
      READ(4)NWORDS,(NFW(G),G=1,NG)
C RECORD 5
      READ(4)(NSS(NMAT,G),L,SIGACT(NMAT,G),NUSIGF(NMAT,G),SIGTOT(NMAT,G)
     1,SIGA(NMAT,G),(SCAT(NMAT,G,IG),IG=7,L),G=1,NG)
      DO 2 G=1,NGM1
    2 LBLOCK(NMAT,G)=NFW(G+1)-NFW(G)
      LBLOCK(NMAT,NG)=NWORDS-NFW(NG)+1
    1 CONTINUE
C RECORD N
      IF(ABS(OB+1.).GE..00001) READ(4) ((DUM),IDUM=1,2000 )
C RECORD N+1
  104 IF(NFG .NE.0) READ(4) (FIRATE(I),I=1,NR),(FY(G),G=1,NFG)
      NFGP = NFG+1
      DO 430 G = NFGP,NG
  430 FY(G) = .0
```

```
      CALL REWUN(4)
C     THE REWUN SUBROUTINE REWINDS AND UNLOADS
C
C GEOMETRY
      R(0)=0.0
      DO 17 I=1,NR
   17 R(I)=R(I-1)+DR(I)
      M(0)=M(1)
      VO(1)=0.0
      MZ = 1
      MATMZ(1) = M(1)
      IREF(1) = 1
      NI(MZ) = 0
      DO 45 I = 1,NR
      DVO(I)=(R(I)+R(I-1))*DR(I)
      NI(MZ) = NI(MZ)+1
      IF(M(I).EQ.M(I-1)) GO TO 45
      NI(MZ) = NI(MZ)-1
      MZ = MZ+1
      NI(MZ) = 1
      IREF(MZ) = I
      VO(MZ)=0.0
      MATMZ(MZ) = M(I)
   45 VO(MZ)=VO(MZ)+DVO(I)
      NMZ = MZ
      IREF(NMZ+1)=NR+1
C
C DETERMINE SURFACE INTEGRATED CURRENTS FROM NEUTRON BALANCE
      DO 9 G=1,NG
    9 J(G,0) = .0
      DO 18 I=1,NR
      DO 11 G=1,NG
   11 S(G)=0.0
      MAT=M(I)
      DO 12 G=1,NFG
   12 S(G)=FIRATE(I)*FY(G)
      DO 13 G=1,NG
      NSELF=NSS(MAT,G)
      L=LBLOCK(MAT,G)
      S(G) = S(G)-FITR(G,I)*SIGTOT(MAT,G)
      DO 13 IND = 7,L
      GSIN = G+IND-NSELF
   13 S(GSIN) = S(GSIN)+FITR(G,I)*SCAT(MAT,G,IND)
```

```
      DO18 G=1,NG
   18 J(G,I) = J(G,I-1)+S(G)*DVO(I)
C
C SUMMARY BY ZONE OF WDSN CALCULATION
      LOSSES=0.0
      DO 87 I=1,NR
      MAT=M(I)
      LOSS=0.0
      DO 211  G=1,NG
  211 LOSS = LOSS+FITR(G,I)*SIGA(MAT,G)
   87 LOSSES=LOSSES+LOSS*DVO(I)
      DO 214 G=1,NG
  214 LOSSES=LOSSES+J(G,NR)
      DO 88 I=1,NR
      DO 88 G=1,NG
      FITR(G,I) = FITR(G,I)/LOSSES
   88 J(G,I)= J(G,I)/LOSSES
      WRITE (6,93)
   93 FORMAT(50H0SUMMARY OF WDSN CALCULATION
     1/50H0TOTAL NEUTRON LOSSES MADE=1
     4/115H0BUCKLING IS BASED ON D=1/(3*SIGMA TRANSPORT) AND CAN BE USED
     5 IN A SPECTRUM CODE WHICH CALCULATES D IN THE SAME WAY
     2/62H0ZONE   GROUP   INTL.FLUX WRT VOL.     LEAKAGE           BUCKLING
     3/1H0)
      CALCK=0.0
      DO 89 MZ=1,NMZ
      JTOT=0.0
      FITOT=0.0
      MAT=MATMZ(MZ)
      ISTART=IREF(MZ)
      ISTOP=ISTART+NI(MZ)-1
      DO 90 G=1,NG
   90 IFIDVO(G,MZ)=0.0
      DO 91 G=1,NG
      DO 92 I=ISTART,ISTOP
   92 IFIDVO(G,MZ) = IFIDVO(G,MZ)+FITR(G,I)*DVO(I)
      CALCK = CALCK+NUSIGF(MAT,G)*IFIDVO(G,MZ)
      JOUT = J(G,ISTOP)-J(G,ISTART-1)
      JTOT=JTOT+JOUT
      FITOT = FITOT+IFIDVO(G,MZ)
      BSQR = 3.*SIGTOT(MAT,G)*JOUT/IFIDVO(G,MZ)
   91 WRITE(6,94) MZ,G,IFIDVO(G,MZ),JOUT,BSQR
   94 FORMAT(I4,I8,1P3E18.7)
```

```
   89 WRITE(6,95)MZ,FITOT,JTOT
   95 FORMAT(I4,5X,3HALL 1P2E18.7/1H0)
      OOEIG=1./EIG
      WRITE(6,98)OOEIG,CALCK
   98 FORMAT(1H0/1H01PE16.7,25H= K DIRECT FROM WDSN
     1/1PE17.7,30H= K CALCULATED IN SUMMARY                    )
C
C CHANGE CURRENTS FROM SURFACE INTEGRATED TO PER UNIT AREA
      DO 96 I = 1,NR
      DO 96 G = 1,NG
   96 J(G,I) = J(G,I)/(2.*R(I))
      WRITE(6,115)
  115 FORMAT(18H1CURRENT DENSITIES )
      DO 116 G=1,NG
  116 WRITE(6,117) G,(J(G,I),I=1,NR)
  117 FORMAT(6H0GROUP I3/(1P8E16.7))
C
C READ GROUP PARTITION VECTOR
  500 READ(5,400)(K(GC),GC=1,24)
  400 FORMAT (24I3)
      DO 700 GC=1,NG
      IF(K(GC))701,701,700
  700 CONTINUE
  701 NGC=GC-1
      WRITE(6,602) (K(GC),GC=1,NGC)
  602 FORMAT(25H1GROUP PARTITION VECTOR =,1613)
C
C READ REGION PARTITION VECTOR
      READ(5,400)(MIX(NZ),NZ=1,24)
      DO 702 NZ=1,NMZ
      IF(MIX(NZ))703,703,702
  702 CONTINUE
  703 NZC=NZ-1
      WRITE(6,704)(MIX(NZ),NZ=1,NZC)
  704 FORMAT(26H0REGION PARTITION VECTOR =,24I3)
      READ(5,400)(MODE(NZ),NZ=1,NZC)
C
C READ CALCULATION CONTROL PARAMETERS
      READ(5,400) IPUNCH,IPRINT,MAX
      READ(5,125)ACC
  125 FORMAT(E12.0,68X)
      WRITE(6,603) IPUNCH,IPRINT,MAX,ACC
  603 FORMAT(9H0IPUNCH =,I3,
```

```
     1       /9H0IPRINT =,I3,
     2       /9H0MAX    =,I3,
     3       /9H0ACC    =1PE10.3)
C
C      READ FINITE DIFFERENCE DIFFUSION THEORY MESH
       WRITE(6,930)
  930  FORMAT(40H0FINITE DIFFERENCE DIFFUSION THEORY MESH)
       RD(0)=0.
       ID1=1
  906  READ(5,900)(DELTA(I),MESH(I),I=1,7)
  900  FORMAT(7(E8.0,I2))
       DO 902 I=1,7
       IDUM=I
       IF(DELTA(I).EQ.0.)GO TO 905
       ID2=MESH(I)
       DO 903 ID=ID1,ID2
       DRD(ID)=DELTA(I)
       RD(ID)=RD(ID-1)+DELTA(I)
  903  DVOD(ID)=(RD(ID)+RD(ID-1))*DRD(ID)
  902  ID1=ID2+1
       WRITE(6,901)(DELTA(I),MESH(I),I=1,7)
  901  FORMAT(7(1PE13.6,I3))
       DUM=ABS(RD(ID2)-R(NR))
       IF(DUM.GT.1.E-6)GO TO 906
  905  IF(ID1.EQ.1)GO TO 904
       IDUM=IDUM-1
       WRITE(6,901)(DELTA(I),MESH(I),I=1,IDUM)
       IDC=ID2
       GO TO 907
  904  IDC=NR
       WRITE(6,931)
  931  FORMAT(16H SAME AS IN WDSN)
       DO 908 ID=1,IDC
       DRD(ID)=DR(ID)
       RD(ID)=R(ID)
  908  DVOD(ID)=DVO(ID)
  907  IF(IPUNCH.EQ.0)GO TO 604
       WRITE(7,600) TITLE
       WRITE(7,605) (K(GC),GC=1,NGC)
  605  FORMAT(25H GROUP PARTITION VECTOR =,16I3)
       WRITE(7,699)(MIX(NZ),NZ=1,NZC)
  699  FORMAT(25HREGION PARTITION VECTOR =,24I3)
       WRITE(7,810)(MODE(NZ),NZ=1,NZC)
```

```
  810  FORMAT(25H D OPTION BY ZONE IS      ,2413)
       WRITE(7,470)
  470 FORMAT(19HCRAM CROSS SECTIONS)
C
C FISSION SPECTRUM CONDENSATION
  604 G1 = 1
       DO 458 GC = 1,NGC
       FYC(GC) = .0
       G2 = K(GC)
       DO 458 G = G1,G2
       FYC(GC) = FYC(GC)+FY(G)
  458 G1 = G2+1
C
C FLUX AND CURRENT CONDENSATION
       DO 420 GC = 1,NGC
  420 JC(GC,0) = .0
       DO 421 I = 1,NR
       G1 = 1
       DO 421 GC = 1,NGC
       G2 = K(GC)
       JC(GC,I) = .0
       FITRC(GC,I) = .0
       DO 422 G = G1,G2
       JC(GC,I) = JC(GC,I)+J(G,I)
  422 FITRC(GC,I) = FITRC(GC,I)+FITR(G,I)
  421 G1 = G2+1
C
C CALCULATE BY ZONE
       MZ1=1
       DO 401 NZ=1,NZC
       MZ2=MIX(NZ)
       WRITE(6,457)NZ
  457 FORMAT(5H1ZONE,I3)
       MOOD=MODE(NZ)
       GO TO (811,812),MOOD
  811 WRITE(6,814)
  814 FORMAT(61H D OBTAINED BY FLUX FIT IN EACH UNIFORM REGION THEN AVER
     1AGING)
       GO TO 813
  812 WRITE(6,815)
  815 FORMAT(44H D OBTAINED FROM AVERAGE TOTAL CROSS SECTION)
  813 TVOL=.0
       DO 705 GC=1,NGC
```

```
      FZC(GC)=.0
      DO 415 GSINC=1,NGC
  415 SCATC(GC,GSINC)=.0
      USFC(GC)=.0
      STC(GC)=.0
  705 DCC(GC)=.0
C
C DO LOOP 706 FOR UNIFORM REGIONS WITHIN THE ZONE
      DO 706 MZ=MZ1,MZ2
      TVOL=TVOL+VO(MZ)
      MAT = MATMZ(MZ)
      ISTART = IREF(MZ)
      ISTOP=IREF(MZ+1)-1
      DO 708 GC=1,NGC
  708 FMZC(GC)=.0
C
C SCATTERING REACTIONS
      G1 = 1
      DO 416 GC = 1,NGC
      G2 = K(GC)
      DO 413 G = G1,G2
      NSELF = NSS(MAT,G)
      L = LBLOCK(MAT,G)
      FMZC(GC) = FMZC(GC)+IFIDVO(G,MZ)
      GSIN1 = 1
      DO 413 GSINC = 1,NGC
      GSIN2 = K(GSINC)
      SUM = .0
      DO 412 GSIN = GSIN1,GSIN2
      IND = GSIN-G+NSELF
      IF(IND-7) 412,418,418
  418 IF(IND-L) 419,419,412
  419 SUM = SUM+SCAT(MAT,G,IND)
  412 CONTINUE
      SCATC(GC,GSINC) = SCATC(GC,GSINC)+IFIDVO(G,MZ)*SUM
  413 GSIN1 = GSIN2+1
      FZC(GC)=FZC(GC)+FMZC(GC)
  416 G1 = G2+1
C
C OTHER REACTIONS
      G1 = 1
      DO 402 GC = 1,NGC
      G2 = K(GC)
```

```
      DO 403 G = G1,G2
      STC(GC)=STC(GC)+IFIDVO(G,MZ)*SIGTOT(MAT,G)
  403 USFC(GC) = USFC(GC)+IFIDVO(G,MZ)*NUSIGF(MAT,G)
  402 G1 = G2+1
C
C FIND DIFFUSION COEFFICIENT, BY REGION, WHICH BEST FITS A DIFFUSION FLUX
C TO THE CONDENSED TRANSPORT FLUX. ESTIMATE THE DIFFUSION FLUX.
      ISTP1=ISTART+1
      DO 46 GC = 1,NGC
      BETA = 0.0
      GAMMA = 0.0
      EPSI = 0.0
      IJDR(ISTART)=0.0
      DO 78 I=ISTP1,ISTOP
   78 IJDR(I) = IJDR(I-1)+JC(GC,I-1)*.5*(DR(I)+DR(I-1))
      DO 47 I = ISTART,ISTOP
      DVOIJ = IJDR(I)*DVO(I)
      BETA = BETA+DVOIJ
      EPSI = EPSI+DVOIJ*IJDR(I)
   47 GAMMA=GAMMA + DVOIJ*FITRC(GC,I)
      ALPHA=FMZC(GC)/VO(MZ)
      DMZC=(EPSI-BETA*BETA/VO(MZ))/(ALPHA*BETA-GAMMA)
      FIDC(GC,ISTART)=ALPHA+BETA/(DMZC*VO(MZ))
      DO 707 I=ISTP1,ISTOP
  707 FIDC(GC,I)=FIDC(GC,ISTART)-IJDR(I)/DMZC
   46 DCC(GC)=DCC(GC)+DMZC*FMZC(GC)
  706 CONTINUE
C
C AVERAGE CROSS SECTIONS
      DO 709 GC=1,NGC
      DO 710 GSINC=1,NGC
  710 SCATC(GC,GSINC)=SCATC(GC,GSINC)/FZC(GC)
      STC(GC)=STC(GC)/FZC(GC)
      USFC(GC)=USFC(GC)/FZC(GC)
      SRC(GC)=STC(GC)-SCATC(GC,GC)
      SCATC(GC,GC)=.0
      GO TO(800,801),MOOD
  800 DCC(GC)=DCC(GC)/FZC(GC)
      GO TO 709
  801 DCC(GC)=1./(3.*STC(GC))
  709 CONTINUE
      ISTART=IREF(MZ1)
      ISTOP=IREF(MZ2+1)-1
```

```
      ID1=0
      ID2=0
      DO 909 ID=0,IDC
      DUM1=ABS(RD(ID)-R(ISTART-1))
      DUM2=ABS(RD(ID)-R(ISTOP))
      IF(DUM1.LT.1.E-6)ID1=ID+1
      IF(DUM2.LT.1.E-6)ID2=ID
  909 CONTINUE
      IF(ID1.NE.0.AND.ID2.NE.0.AND.ID2-ID1.GT.1)GO TO 920
      WRITE(6,932)
  932 FORMAT(11H MESH ERROR)
      GO TO 500
C
C INTERPOLATE TRANSPORT FLUX TO DIFFUSION MESH POINTS
  920 IR=ISTART+1
      DO 910 ID=ID1,ID2
      RAD=RD(ID)-.5*DRD(ID)
      R1=R(ISTART)-.5*DR(ISTART)
      IF(RAD.GT.R1)GO TO 911
      I1=ISTART
      I2=ISTART+1
      R2=R(I2)-.5*DR(I2)
      GO TO 913
  911 R2=R(ISTOP)-.5*DR(ISTOP)
      IF(RAD.LT.R2)GO TO 912
      I2=ISTOP
      I1=ISTOP-1
      R1=R(I1)-.5*DR(I1)
      GO TO 913
  912 DO 914 I=IR,ISTOP
      I2=I
      RAT=R(I)-.5*DR(I)
      IF(RAT.GT.RAD)GO TO 915
  914 CONTINUE
  915 R2=RAT
      I1=I2-1
      R1=R(I1)-.5*DR(I1)
      IR=I2
  913 DO 910 GC=1,NGC
      FINT(GC,ID)=FITRC(GC,I1)+(RAD-R1)*(FITRC(GC,I2)-FITRC(GC,I1))/(R2-
     1R1)
  910 FIDC(GC,ID)=FINT(GC,ID)
C
```

```
C SOLVE THE DIFFUSION EQUATION IN THE SMEARED ZONE USING THE TRANSPORT
C CURRENTS AS FIXED BOUNDARY SOURCES. OBTAIN THE DIFFUSION THEORY
C BOUNDARY AND INTERFACE CONDITIONS FROM THE SOLUTION.
      DO 431 GC=1,NGC
      CNTIN(GC) = 2.*R(ISTART-1)*JC(GC,ISTART-1)
  431 CNTOUT(GC) = -2.*R(ISTOP)*JC(GC,ISTOP)
      CALL DIFF(IPRINT,MAX,ACC)
  718 DO 200 GC=1,NGC
      FN=0.0
      DO 201 ID=ID1,ID2
      DEV=FIDC(GC,ID)-FINT(GC,ID)
  201 FN=FN+DEV*DEV*DVOD(ID)
  200 STERR(GC)=SQRT(FN*TVOL)/FZC(GC)
      WRITE(6,202)(STERR(GC),GC = 1,NGC)
  202 FORMAT(20HOFLUX ERROR BY GROUP/16F8.4)
C
C WRITE FLUXES IN THE ZONE
      DO 118 GC = 1,NGC
      WRITE(6,51)GC,(FINT(GC,ID),ID=ID1,ID2)
   51 FORMAT(21HOTRANSPORT FLUX GROUP,I3/(1P12E10.3))
  118 WRITE(6,121)GC,(FIDC(GC,ID),ID=ID1,ID2)
  121 FORMAT(21HODIFFUSION FLUX GROUP,I3/(1P12E10.3))
C
C BOUNDARY CONDITIONS
      DO 64 GC = 1,NGC
      FSTART=FIDC(GC,ID1)+JC(GC,ISTART-1)*.5*DRD(ID1)/DCC(GC)
   64 LAMBDA(GC,NZ,1)=-JC(GC,ISTART-1)/FSTART
      DO 66 GC = 1,NGC
      FSTOP=FIDC(GC,ID2)-JC(GC,ISTOP)*.5*DRD(ID2)/DCC(GC)
   66 LAMBDA(GC,NZ,2)= JC(GC,ISTOP)/FSTOP
      WRITE(6,43)
   43 FORMAT(87HOCRAM BOUNDARY CONDITION I.E. NET CURRENT ACROSS BOUNDAR
     1Y DIRECTED OUT OF REGION / FLUX)
      WRITE(6,42)(LAMBDA(GC,NZ,1),GC=1,NGC)
   42 FORMAT(17HOINSIDE  BOUNDARY/(1P8E16.7))
      WRITE(6,67)(LAMBDA(GC,NZ,2),GC=1,NGC)
   67 FORMAT(17HOOUTSIDE BOUNDARY/(1P8E16.7))
C
C PUNCHED OUTPUT
      IF(IPUNCH) 454,401,454
  454 WRITE(6,32)
   32 FORMAT(20HOCRAM CROSS SECTIONS)
      WRITE(6,450)NZ
```

```
      IF(IPUNCH.NE.0)WRITE(7,450)NZ
  450 FORMAT(2H I,I2,1HD)
      WRITE(6,451)(DCC(GC),GC = 1,NGC)
      IF(IPUNCH.NE.0)WRITE(7,451)(DCC(GC),GC=1,NGC)
  451 FORMAT(1P5E13.5)
      WRITE(6,451)(SRC(GC),GC = 1,NGC)
      IF(IPUNCH.NE.0)WRITE(7,451)(SRC(GC),GC=1,NGC)
      WRITE(6,451)(USFC(GC),GC = 1,NGC)
      IF(IPUNCH.NE.0)WRITE(7,451)(USFC(GC),GC=1,NGC)
      DO 452 GC=1,NGC
      IF(IPUNCH.NE.0)WRITE(7,451)(SCATC(GC,GSINC),GSINC=1,NGC)
  452 WRITE(6,451)(SCATC(GC,GSINC),GSINC=1,NGC)
  401 MZ1=MZ2+1
C
C INTERFACE CONDITIONS
      IF(NZC.EQ.1)GO TO 460
      DO 453 GC = 1,NGC
      ZERO(GC) = .0
  453 BIG(GC) = 1.E9
      WRITE(6,207)
  207 FORMAT(47H1INTERFACE DATA ASSUMES A MESH WIDTH = .0009999)
      NZCM1=NZC-1
      DO 204 NZ=1,NZCM1
      DO 205 GC = 1,NGC
  205 DDUM(GC)=0.0009999/(1./LAMBDA(GC,NZ,2)+1./LAMBDA(GC,NZ+1,1))
      WRITE(6,206)NZ,(DDUM(GC),GC=1,NGC)
  206 FORMAT(42H DUMMY DIFFUSION COEFFICIENTS AT INTERFACE,I3/(1P8E16.7)
     1)
      IF(IPUNCH.EQ.0) GO TO 204
      INTF=NZC+NZ
      WRITE(7,450) INTF
      WRITE(7,451)(DDUM(GC),GC = 1,NGC)
      NGCP2 = NGC+2
      DO 455 G  = 1,NGCP2
  455 WRITE(7,451)(ZERO(GC),GC = 1,NGC)
  204 CONTINUE
      IF(IPUNCH.EQ.0) GO TO 460
      WRITE(6,456) NZC
  456 FORMAT(36H0N.B. CRAM INTERFACE ZONE NUMBER IS ,I2,28H PLUS ABOVE I
     1INTERFACE NUMBER)
C
C CONDENSED FISSION SPECTRUM
  460 WRITE(6,462)
```

```
462 FORMAT(27H0CONDENSED FISSION SPECTRUM)
    WRITE(6,451)(FYC(GC),GC=1,NGC)
    IF(IPUNCH.EQ.0) GO TO 500
     WRITE(7,463)
463 FORMAT(3H SP)
    WRITE(7,451)(FYC(GC),GC=1,NGC)
    WRITE(7,606)
606 FORMAT(80H**********************************************************
   1**********************)
    GO TO 500
    END
```

```
C   SUBROUTINE DIFF
C
C   FINITE DIFFERENCE MULTI-GROUP DIFFUSION THEORY SOLUTION IN UNIFORM ANNULAR
C   REGION WITH FIXED MULTIPLICATION AND GROUP DEPENDENT BOUNDARY SOURCE(CURRENT)
C
C
C   DATA
C
C   SUBROUTINE ARGUMENTS
C
C   IPRINT = 0 NO PRINT
C          = 1 PRINT OF BOUNDARY FLUXES ONLY       ) PRINT OCCURS AFTER EACH
C          = 2 PRINT OF ALL FLUXES                 )   GROUP FLUX SOLUTION
C          = 3 DEBUG PRINT
C   MAX - MAXIMUM NUMBER OF SOURCE ITERATIONS
C   ACC - SOURCE ITERATION TERMINATES WHEN FRACTIONAL CHANGE IN BOUNDARY FLUXES
C         IS LESS THAN ACC FOR ALL GROUPS AND FOR THREE CONSECUTIVE ITERATIONS
C
C      COMMON
C
C   SR - REMOVAL CROSS SECTION
C   USF - NU-FISSION CROSS SECTION
C   SCAT - SCATTERING MATRIX
C   DC - DIFFUSION COEFFICIENT
C   FY - FISSION SPECTRUM
C   AK - MULTIPLICATION CONSTANT
C   CNTIN - SOURCE AT INNER BOUNDARY
C   CNTOUT - SOURCE AT OUTER BOUNDARY
C   FLUX - ON ENTRY IS INITIAL FLUX GUESS
C        - ON EXIT IS THE SOLUTION
C   DR - MESH WIDTH
C   R(I) - OUTER RADIUS OF MESH INTERVAL I
C   VOL - MESH VOLUME
C   ISTART TO ISTOP - PORTION OF MESH CONSIDERED
C   NG - NUMBER OF GROUPS
C
      SUBROUTINE DIFF(IPRINT,MAX,ACC)
      DIMENSION SR(16),USF(16),SCAT(16,16),DC(16),FY(16)
      DIMENSION CNTIN(16),CNTOUT(16),FLUX(16,80),DR(80),R(80),VOL(80)
      DIMENSION T(16,16),C(16,80),B(16,80),S(80),W(80),U(16,80)
```

```
      DIMENSION FBL(16),FBR(16),IT(16)
      DIMENSION FTOT(16)
      COMMON SR,USF,SCAT,DC,FY,AK,CNTIN,CNTOUT,FLUX,DR,R,VOL
      COMMON FTOT,TVOL
      COMMON ISTART,ISTOP,NG
C
C  MACROSCOPIC CROSS SECTIONS
      DO 5 L = 1,NG
      DO 5 LD = 1,NG
    5 T(L,LD) = SCAT(L,LD)+FY(LD)*USF(L)/AK
      IF(IPRINT-3) 100,101,101
  101 WRITE(6,250)
  250 FORMAT(15H0CROSS SECTIONS)
      WRITE(6,102)(DC(L),L = 1,NG)
      WRITE(6,102)(SR(L),L = 1,NG)
      WRITE(6,102)(USF(L),L = 1,NG)
      DO 103 L = 1,NG
  103 WRITE(6,102)(SCAT(L,LD),LD = 1,NG)
  102 FORMAT(1X,1P12E10.3)
C
C  FINITE DIFFERENCE COEFFICIENTS
  100 ISTOPM=ISTOP-1
      ISTRTP=ISTART+1
      DO 13 L=1,NG
      DO 12 I=ISTART,ISTOPM
   12 C(L,I) = DC(L)*4.*R(I)/(DR(I)+DR(I+1))
      C(L,ISTOP)=1.
      B(L,ISTART) = SR(L)*VOL(ISTART)+C(L,ISTART)
      DO 3 I=ISTRTP,ISTOPM
    3 B(L,I) = SR(L)*VOL(I)+C(L,I-1)+C(L,I)
   13 B(L,ISTOP) = SR(L)*VOL(ISTOP)+C(L,ISTOPM)
      IF(IPRINT-3)105,106,106
  106 WRITE(6,251)
  251 FORMAT(29H0MESH WIDTH,RADIUS AND VOLUME)
      WRITE(6,102) (DR(I),I = ISTART,ISTOP)
      WRITE(6,102)(R(I),I=ISTART,ISTOP)
      WRITE(6,102)(VOL(I),I=ISTART,ISTOP)
C
C  FIXED RECURRENCE COEFFICIENT
      EQUIVALENCE (U,B)
  105 DO 14 L=1,NG
      U(L,ISTART)=C(L,ISTART)/B(L,ISTART)
      DO 14 I=ISTRTP,ISTOP
```

```
   14 U(L,I)=C(L,I)/(B(L,I)-C(L,I-1)*U(L,I-1))
      IF(IPRINT-3)107,108,108
  108 WRITE(6,252)
  252 FORMAT(26H0BOUNDARY SOURCE (CURRENT))
      WRITE(6,102) (CNTIN(L),L = 1,NG)
      WRITE(6,102) (CNTOUT(L),L = 1,NG)
      WRITE(6,253)
  253 FORMAT(11H0FLUX GUESS)
      DO 123 L=1,NG
  123 WRITE(6,102)(FLUX(L,I),I=ISTART,ISTOP)
C
C  SELECT GROUPS REQUIRING SOURCE ITERATION (IT=2)
  107 DO 304 L = 1,NG
      IT(L) = 1
      DO 303 LD = 1,NG
      IF(T(LD,L).NE.0.) GO TO 305
  303 CONTINUE
      GO TO 304
  305 IT(L) = 2
  304 CONTINUE
      DO 300 L = 1,NG
      IF(IT(L).EQ.1) GO TO 300
      DO 302 LD = 1,NG
      IF(T(LD,L).EQ.0.) GO TO 302
      IF(IT(LD).EQ.2) GO TO 300
  302 CONTINUE
      IT(L) = 1
  300 CONTINUE
      ITALL = 2
      DO 307 L = 1,NG
      IF(IT(L).EQ.2) GO TO 309
  307 CONTINUE
      ITALL = 1
C
C  INITIALISE
  309 KOUNT = 0
      L = 1
      KGOOD = 0
C
C  STORE BOUNDARY FLUXES
   41 DO 306 LD= 1,NG
      FBL(LD) = FLUX(LD,ISTART)
  306 FBR(LD) = FLUX(LD,ISTOP)
```

```
C
C  GROUP SOURCE
      DO 15 I = ISTART,ISTOP
      S(I) = .0
      DO 4 LD = 1,NG
    4 S(I) = S(I)+T(LD,L)*FLUX(LD,I)
   15 S(I) = S(I)*VOL(I)
      S(ISTART) = S(ISTART)+CNTIN(L)
      S(ISTOP) = S(ISTOP)+CNTOUT(L)
C
C  SOURCE DEPENDENT RECURRENCE COEFFICIENT
      W(ISTART) = S(ISTART)*U(L,ISTART)/C(L,ISTART)
      DO 16 I = ISTRTP,ISTOP
   16 W(I) = U(L,I)*(S(I)+C(L,I-1)*W(I-1))/C(L,I)
C
C  NEW GROUP FLUX
      FLUX(L,ISTOP) = W(ISTOP)
      DO 17 I = ISTART,ISTOPM
      II = ISTOPM-I+ISTART
   17 FLUX(L,II) = U(L,II)*FLUX(L,II+1)+W(II)
C
C  NORMALISE FLUX BY ADDING A CONSTANT
      SUM=.0
      DO 400 I=ISTART,ISTOP
  400 SUM=SUM+FLUX(L,I)*VOL(I)
      CONST=(FTOT(L)-SUM)/TVOL
      DO 401 I=ISTART,ISTOP
  401 FLUX(L,I)=FLUX(L,I)+CONST
C
C  PRINTED FLUXES
      IF(IPRINT-1)50,51,52
   51 WRITE(6,31) L,FLUX(L,ISTART),FLUX(L,ISTOP)
   31 FORMAT(6H0GROUP,I3,7H FLUXES/(1X,1P8E16.7))
      GO TO 50
   52 WRITE(6,31) L,(FLUX(L,II),II = ISTART,ISTOP)
   50 L = L+1
      IF(L.GT.NG.AND.ITALL.EQ.1) GO TO 90
      IF(L.GT.NG) GO TO 310
      IF(KOUNT.EQ.0) GO TO 41
  311 IF(IT(L).EQ.2) GO TO 41
      L = L+1
      GO TO 311
  310 KOUNT = KOUNT+1
```

```
      IF(KOUNT-MAX) 22,91,91
   91 WRITE(6,92)
   92 FORMAT(38HOMAXIMUM NUMBER OF ITERATIONS EXCEEDED)
      GO TO 90
   22 EMAX = .0
      DO 19 L = 1,NG
      ERRORL = ABS(1.-FBL(L)/FLUX(L,ISTART))
      ERRORR = ABS(1.-FBR(L)/FLUX(L,ISTOP))
      IF(ERRORL.GT.EMAX) EMAX = ERRORL
      IF(ERRORR.GT.EMAX) EMAX = ERRORR
   19 CONTINUE
      L = 1
      IF(EMAX-ACC) 312,312,313
  313 KGOOD = 0
      GO TO 311
  312 KGOOD = KGOOD+1
      IF(KGOOD.LT.3) GO TO 311
   90 IF(IPRINT-3)109,110,110
  110 WRITE(6,254)
  254 FORMAT(14HOFLUX SOLUTION)
      DO 112 L = 1,NG
  112 WRITE(6,102)(FLUX(L,I),I=ISTART,ISTOP)
  109 RETURN
      END
```