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**AUSTRALIAN ATOMIC ENERGY COMMISSION
RESEARCH ESTABLISHMENT
LUCAS HEIGHTS**

EQUIVALENCE RELATIONS FOR HETEROGENEOUS REACTOR SYSTEMS

by

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ABSTRACT

A derivation is given of an equivalence relation between heterogeneous and homogeneous systems, which allows for close-packing without the need for a Dancoff correction and which includes fuels dispersed in granular form. An alternative derivation of the equivalence relation is also presented, giving insight into the physical significance of the rational approximation.

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1. INTRODUCTION

The first theoretical determination of resonance absorption in a heterogeneous assembly was due to Wigner et al. (1955), who considered a fuel lump in an isotropic neutron bath and obtained a volume term corresponding to neutrons which had their last collision in the fuel lump and a surface term from neutrons crossing the lump surface from the moderator into the fuel. Wigner applied a penetrability factor to the surface term to allow for neutrons entering the lump and crossing it without a collision. The penetrability factor was evaluated for a sphere and then approximated by a rational expression which was later generalised by Chernick and Vernon (1958) for arbitrary shaped lumps and became widely known as the "rational approximation". Wigner's work was actually carried out in 1941, but it was not published until 1955.

As early as 1944 Dancoff and Ginsburg (1944) derived a correction for the screening of one fuel rod by another, thus allowing for flux depressions in the moderator. However, they assumed in their calculations that the fuel rods were black at resonance energies and the implications of greyness as introduced by Wigner were largely overlooked until the 1955 Geneva Conference when the work of Gurevich and Pomeranchouk (1955) was reported. This work, which had also been carried out several years before publication, produced a different surface dependence for the effective resonance integral by making the penetrability of the fuel lump the dominant factor in the theory.

Spinrad, Chernick and Corngold (1958) at the second Geneva Conference used a spatially averaged flux in the fuel region and established a neutron balance for a lump in an isotropic neutron bath from which they established an equivalence with a homogeneous system. In their paper they attributed the suggestion of the equivalence of homogeneous and heterogeneous systems to P. Bakshi (1957).

The way was opened for a consistent treatment of the flux depressions in both fuel and moderator regions by the introduction of generalised escape probabilities by Bell (1959). These were used by Keane, McKay and Cox (1959) in a generalisation of the neutron balance equation of Spinrad et al. (1958) to derive an equivalence relation for a close packed lattice. This expression incorporated an allowance both for greyness of the rods and the flux depression in the moderator and thus made the use of the Dancoff correction redundant. The full implications of the equivalence were not seen at the time owing to a cumbersome notation but the method is re-presented in this report in a manner more consistent with current usage.

For some years attention was restricted to a study of the validity of the approximations made in deriving the equivalence relation. The error involved in the rational approximation was investigated analytically by Dresner (1959), Bakshi (1959) and McKay and Keane (1960), while the effect of spatially averaging the flux has been studied with the aid of large computer codes which have been developed at most research establishments. Transport calculations (Carlson, Lee, and Worlton 1960) and Monte Carlo studies (Morton 1958, Richtmeyer 1956) have obtained improved numerical results at the expense of considerable complication and effort.

Interest in the analytic treatment of heterogeneous systems has been latterly revived in the study of dispersion fuels. Lane, Nordhein, and Sampson (1962) have used the formulation of Spinrad et al. (1958) and applied the Dancoff correction to derive an equivalence relation for small fuel particles. They have, however, used the rational approximation which, while being adequate for large rods, involves considerable error for the small fuel particles under consideration. Assuming spherical fuel particles and using the exact expression for the distribution of neutron paths in a sphere, Keane (1964) has derived a correction to the equivalence relation which compensates for the error introduced by the rational approximation. To derive an equivalence relation for dispersed particles which gives particular attention to the 1 eV resonance of Pu240, Dyos and Pomraning (1966) have effectively used the formulation of Spinrad et al. (1958) in combination with an infinite mass approximation. However, they have not taken into account the flux depression in the moderator which is a major consideration for dispersion fuels in a practical situation.

In this report the method of Keane, McKay, and Cox (1959) is employed to derive an equivalence relation which is valid for closely packed lattices as well as for isolated lumps and which may also be adapted to cover the case of fuels dispersed in grains. An alternative derivation is also presented, which gives insight into the physical significance of the rational approximation.

2. FORMULATION OF THE PROBLEM

For a homogeneous system, the narrow-resonance approximation leads to an expression for the resonance escape probability in the form:

$$p = \exp \left\{ - \frac{1}{\xi} \int \frac{\Sigma_a}{\Sigma} \frac{dE}{E} \right\}, \quad (1)$$

where Σ_a and Σ are respectively the macroscopic absorption and total cross sections of the mixture at energy E and ξ is the average lethargy gain of a neutron on collision with an atom of the mixture.

To apply this expression to a heterogeneous assembly, we may weight the cross sections of the various constituents with the appropriate fluxes and volumes. Thus, if lumps of fuel, each of volume V_f , are dispersed in moderator with relative volume V_m , the resonance escape probability is given by:

$$p = \exp \left\{ - \frac{1}{\xi} \int \frac{\Sigma_{af} \phi_f}{\Sigma_f \phi_f + \Sigma_m \phi_m V_m / V_f} \frac{dE}{E} \right\}, \quad (2)$$

where ϕ_f and ϕ_m are the spatially averaged fluxes in the fuel and moderator respectively and the subscripts f and m are also used to indicate the appropriate cross sections.

A consideration of the neutron balance shows that the total number of collisions in the fuel involving neutrons of energy E is equal to the number of collisions involving neutrons which reached this energy as a result of their last collision in the fuel plus the number involving neutrons "born" at this energy in the moderator. Hence, defining P_{ff} as the probability that a neutron born in the fuel will have its next collision in the fuel and P_{mm} as a similar probability for the moderator, we have:

$$\Sigma_f \phi_f = P_{ff} \sum_i \int_E^{E/\alpha_i} (\Sigma_{sfi} + \Sigma_{pfi}) \phi_f \frac{dE'}{(1-\alpha_i)E'} + (1-P_{mm}) \sum_j \int_E^{E/\alpha_j} \Sigma_{mj} \phi_m \frac{V_m}{V_f} \frac{dE'}{(1-\alpha_j)E'}, \quad (3)$$

where the first summation is taken over the constituents of the fuel lump and the second summation over the constituents of the moderator, with Σ_{sfi} and Σ_{pfi} respectively the macroscopic resonance scattering and potential scattering cross sections of the i th constituent of the fuel and α_k the minimum fraction of a neutron's energy which may be retained on collision with the k th nuclear species. Similarly:

$$\frac{V_m}{V_f} \Sigma_m \phi_m = (1-P_{ff}) \sum_i \int_E^{E/\alpha_i} (\Sigma_{sfi} + \Sigma_{pfi}) \phi_f \frac{dE'}{(1-\alpha_i)E'} + P_{mm} \sum_j \int_E^{E/\alpha_j} \Sigma_{mj} \phi_m \frac{V_m}{V_f} \frac{dE'}{(1-\alpha_j)E'}. \quad (4)$$

The narrow-resonance approximation assumes that neutrons entering resonance energies come from off-resonance energies where $\Sigma_{sf} = 0$ and $\phi_f = \phi_m = \phi_o$, so that Equations 3 and 4 can be approximated by:

$$\Sigma_f \phi_f = P_{ff} \Sigma_{pf} \phi_o + (1-P_{mm}) \frac{V_m}{V_f} \Sigma_m \phi_o, \quad (5)$$

and

$$\frac{V_m}{V_f} \Sigma_m \phi_m = (1-P_{ff}) \Sigma_{pf} \phi_o + P_{mm} \frac{V_m}{V_f} \Sigma_m \phi_o \quad (6)$$

Adding (5) and (6) gives:

$$\Sigma_f \phi_f + \frac{V_m}{V_f} \Sigma_m \phi_m = \left(\Sigma_{pf} + \frac{V_m}{V_f} \Sigma_m \right) \phi_o \quad (7)$$

Substituting from (7) into Equation 2 now gives:

$$p = \exp \left\{ - \frac{1}{\xi [\Sigma_{pf} + \Sigma_m V_m / V_f]} \int \Sigma_{af} \frac{\phi_f}{\phi_o} \frac{dE}{E} \right\}, \quad (8)$$

so that it only remains to evaluate $\frac{\phi_f}{\phi_o}$ from Equation 5.

3. ESCAPE PROBABILITIES

Let P_o be the probability that a neutron born in an isolated fuel lump has its next collision in that fuel lump and P_1 the probability that a neutron born in moderator will collide with a moderator atom before entering a fuel lump. Let G_o be the probability that a neutron incident on a fuel lump collides with a fuel atom on that transit and G_1 the probability that a neutron incident on moderator collides with a moderator atom before entering another fuel lump.

Case, de Hoffmann, and Placzek (1953) have shown that for an isotropic distribution of neutrons and for convex fuel lumps:

$$P_o = 1 - \frac{S}{4V_f \Sigma_f} G_o = 1 - \frac{G_o}{\Sigma_f \bar{l}}, \quad (9)$$

where S is the surface area of a fuel lump and \bar{l} is the average length of a chord through the lump. The argument is easily adapted to the moderator (See Appendix) and for a sufficiently large system we obtain:

$$P_1 \approx 1 - \frac{G_1}{\Sigma_m L}, \quad (10)$$

where L is the average length of a straight path through the moderator. Now:

$$G_o = \int f(\ell) (1 - e^{-\Sigma_f \ell}) d\ell, \quad (11)$$

$$\text{and } G_1 = \int g(\ell) (1 - e^{-\Sigma_m \ell}) d\ell, \quad (12)$$

where $f(\ell)$ and $g(\ell)$ are probability distributions of neutron paths in the fuel and moderator respectively.

P_o , P_1 , G_o , and G_1 are average probabilities and the derivations of Equations 9 to 12 are based on the assumptions that the distribution of neutrons born in either fuel lumps or moderator is uniform and isotropic, and that the neutrons penetrating the surface either into fuel or into moderator have an isotropic distribution. Although the flux varies considerably near the boundaries between fuel and moderator at resonance energies, the conditions of the derivations are not grossly violated. Provided the resonance is not too wide, most neutrons "born" at resonance energies either in moderator or fuel have been degraded from off-resonance energies and are thus isotropically distributed. The rapid decrease in the flux in passing from moderator to fuel is due to the much smaller number of neutrons passing from fuel to moderator compared with those crossing the surface in the opposite direction. Since the neutrons crossing the surface in either direction are the remnants of an isotropic distribution it is reasonable to assume that, although there is a great difference between inward and outward currents, there is an isotropic distribution of neutrons over the solid angle of 2π sterads in either direction across the surface.

The rational approximation first proposed by Wigner et al. (1955) when applied to Equations 11 and 12 gives:

$$G_0 \approx 1 - e^{-\sum_f \bar{\ell}} \approx \frac{\sum_f \bar{\ell}}{1 + \sum_f \bar{\ell}} \quad (13)$$

and
$$G_1 \approx 1 - e^{-\sum_m L} \approx \frac{\sum_m L}{1 + \sum_m L} \quad (14)$$

Inserting these approximations into (9) and (10), we get:

$$P_0 \approx G_0 \quad (15)$$

and
$$P_1 \approx G_1 \quad (16)$$

We are now in a position to obtain approximations for P_{ff} and P_{mm} , since:

$$\begin{aligned} P_{ff} &= P_0 + (1-P_0)(1-G_1)G_0 + (1-P_0)(1-G_1)(1-G_0)(1-G_1)G_0 + \dots \\ &\approx \frac{G_0}{1-(1-G_0)(1-G_1)} \end{aligned}$$

using (15). Hence, using (13) and (14), we have:

$$P_{ff} \approx \frac{\sum_f \bar{\ell}(1 + \sum_m L)}{\sum_f \bar{\ell} + \sum_m L + \sum_f \bar{\ell} \sum_m L} \quad (17)$$

and similarly,

$$P_{mm} \approx \frac{\sum_m L(1 + \sum_f \bar{\ell})}{\sum_f \bar{\ell} + \sum_m L + \sum_f \bar{\ell} \sum_m L} \quad (18)$$

4. EQUIVALENCE RELATION

We now restrict our attention to a single absorber contained in the fuel and we write σ_m for the scattering cross section of the moderator per absorber atom, so that:

$$\sigma_m = \frac{\sum_m V_m}{NV_f}$$

where N is the number of absorber nuclei per cm^3 of the fuel. We also denote the potential scattering and total cross sections of the fuel per absorber atom by σ_{pf} and σ respectively. With this notation, if σ_a and σ_s are respectively the microscopic resonance absorption and resonance scattering cross sections of the absorber, we see that the spatially averaged flux in the fuel, given by Equation 5, may be written in the form:

$$\phi_f = \phi_0 \left\{ P_{ff} \frac{\sigma_{pf}}{\sigma_f} + (1-P_{mm}) \frac{\sigma_m}{\sigma_f} \right\} \quad (19)$$

Also, since

$$L/\bar{\ell} \approx V_m/V_f$$

we obtain from Equations 17 and 18:

$$P_{ff} = \frac{\sigma_f (1 + N\bar{\ell}\sigma_m)}{\sigma_f + \sigma_m + \sigma_f N\bar{\ell}\sigma_m} \quad (20)$$

$$\text{and } P_{mm} = \frac{\sigma_m(1+N\bar{\ell}\sigma_f)}{\sigma_f + \sigma_m + \sigma_f N\bar{\ell}\sigma_m} \quad (21)$$

Substituting from (20) and (21) into (19), we get:

$$\begin{aligned} \frac{\phi_f}{\phi_o} &= \frac{\sigma_{pf}(1+N\bar{\ell}\sigma_m) + \sigma_m}{\sigma_f(1+N\bar{\ell}\sigma_m) + \sigma_m} \\ &= \frac{\sigma_{pf}(1+N\bar{\ell}\sigma_m) + \sigma_m}{(\sigma_a + \sigma_s)(1+N\bar{\ell}\sigma_m) + \sigma_{pf}(1+N\bar{\ell}\sigma_m) + \sigma_m} \end{aligned} \quad (22)$$

Equation 22 may be written as:

$$\frac{\phi_f}{\phi_o} = \frac{\sigma_p}{\sigma_a + \sigma_s + \sigma_p} \quad (23)$$

$$\text{where } \sigma_p = \sigma_{pf} + \frac{\sigma_m}{1+N\bar{\ell}\sigma_m} \quad (24)$$

Equation 8 for the resonance escape probability may now be expressed in the form:

$$P = \exp \left\{ - \frac{I}{\xi(\sigma_{pf} + \sigma_m)} \right\} \quad (25)$$

where I is the effective resonance integral, given by:

$$\begin{aligned} I &= \int \sigma_a \frac{\phi_f}{\phi_o} \frac{dE}{E} \\ &= \sigma_p \int \frac{\sigma_a}{\sigma_a + \sigma_s + \sigma_p} \frac{dE}{E} \end{aligned} \quad (26)$$

using Equation 23 for ϕ_f/ϕ_o .

Comparison of Equation 25 with Equation 1 shows that the effective resonance integral defined in Equation 26 has the same form as its counterpart in the homogeneous case, where σ_p denotes the potential scattering cross section of the mixture per atom of the absorber. This is the equivalence relation between heterogeneous and homogeneous systems, arising from the use of the rational approximations (13) and (14) for the collision probabilities G_0 and G_1 .

5. ALTERNATIVE DERIVATION OF THE EQUIVALENCE RELATION

In order to gain more insight into the physical situation, we now consider an alternative derivation of the equivalence relation discussed in Section 4.

If it is assumed that the flux and scattering are spatially uniform and isotropic throughout a fuel lump, so that for every neutron scattered out of a given path at a given point there is one scattered back into that path at the same energy, the mean chord length in a fuel lump, $\bar{\ell}$, is the mean free path for a neutron in the fuel to have a "collision" with the fuel surface. Thus, treating the surface as though it were homogeneously distributed throughout the fuel lump, $1/\bar{\ell}$ is its macroscopic neutron "cross section", that is, the probability per unit volume of the fuel for neutrons within the fuel to "collide" with the surface of the lump.

The probability per unit volume of the fuel for a neutron in the fuel to have a scattering collision with outside moderator is equal to the probability of a "collision" with the surface, multiplied by the probability that a neutron traversing the moderator will have a scattering collision. Thus the effective potential scattering cross section per unit volume of the fuel is given by:

$$\Sigma_p = \Sigma_{pf} + G_1/\bar{l} , \quad (27)$$

where, as in Section 3, G_1 is the probability that a neutron incident on moderator collides with a moderator atom before entering another fuel lump.

The mean free path in the moderator for collision with a fuel surface is L , so that, considering the surfaces of the fuel lumps to be homogeneously distributed throughout the moderator, the probability that a neutron will travel a distance r in the moderator without collision with either a fuel surface or with moderator is:

$$\exp \left\{ -\frac{r}{L} - r\Sigma_m \right\} = \exp \left\{ -\frac{r}{L} (1 + \Sigma_m L) \right\} . \quad (28)$$

Hence the probability of a collision with moderator for a path length dr at r is:

$$\exp \left\{ -\frac{r}{L} (1 + \Sigma_m L) \right\} \Sigma_m dr , \quad (29)$$

and the probability of colliding with moderator on a traverse is:

$$\begin{aligned} G_1 &= \int_0^{\infty} \exp \left\{ -\frac{r}{L} (1 + \Sigma_m L) \right\} \Sigma_m dr , \\ &= \frac{\Sigma_m L}{1 + \Sigma_m L} , \\ &\simeq \frac{N\bar{l}\sigma_m}{1 + N\bar{l}\sigma_m} , \end{aligned} \quad (30)$$

since $L \simeq \frac{V_m}{V_f} \bar{l} ,$

and $\sigma_m = \frac{\Sigma_m V_m}{N V_f} .$

Substitution in Equation 27 gives

$$\Sigma_p = \Sigma_{pf} + \frac{N\sigma_m}{1 + N\bar{l}\sigma_m} ,$$

or $\sigma_p = \sigma_{pf} + \frac{\sigma_m}{1 + N\bar{l}\sigma_m} , \quad (31)$

which is identical with Equation 24.

This derivation, which follows the method used to derive the Dancoff correction, makes it clear that the physical significance of the rational approximation is that the surface of each fuel lump is considered to be distributed homogeneously throughout the lump and that the surfaces of all the fuel lumps are considered to be distributed homogeneously throughout the moderator.

6. DEDUCTIONS

Most of the earlier expressions for the calculation of resonance absorption can be derived as special cases of the foregoing theory.

(i) For a fuel rod in an isotropic neutron bath it may be assumed that $N\bar{\ell}\sigma_m \gg 1$, so that the equivalence relation (24) becomes:

$$\sigma_p = \sigma_{pf} + \frac{1}{N\bar{\ell}} \quad (32)$$

This expression, which was derived by Spinrad, Chernick and Corngold (1958), can be seen to follow from (27) on the assumption that neutrons cannot cross from one lump to another without colliding with moderator.

For a Calder Hall type of reactor $N \simeq 2 \times 10^{22}$, $\bar{\ell} \simeq 2.5$ cm, and σ_m is very large, making $N\bar{\ell}\sigma_m$ considerably greater than unity and giving $\frac{1}{N\bar{\ell}} \simeq 20b$.

(ii) The Dancoff correction, which was introduced to allow for the mutual shielding of fuel lumps, is in effect the probability that a neutron escaping from a lump will enter another lump before colliding with moderator. Denoting this correction by C , as usual, and applying to Equation 32 we have:

$$\sigma_p = \sigma_{pf} + \frac{1}{N\bar{\ell}}(1-C) = \sigma_{pf} + \frac{G_1}{N\bar{\ell}} \quad (33)$$

which is identical with Equation 27. Thus the Dancoff correction is included in the equivalence relation specified by Equation 24 [or (31)].

(iii) For fuels dispersed in small granular particles, $\sigma_m = 0(10^3)$, $N \simeq 2 \times 10^{22}$, and $\bar{\ell} \simeq 0.01$ cm, so that $N\bar{\ell}\sigma_m \simeq 0.2$. Hence $\frac{\sigma_m}{1+N\bar{\ell}\sigma_m}$ is still $0(10^3)$, so that σ_{pf} , which is $0(10)$, can be neglected.

This gives the equivalence relation for dispersed fuel particles as $\sigma_p = \frac{\sigma_m}{1+N\bar{\ell}\sigma_m}$ (34)

However, the fact that the surface-volume ratio for small particles is so large introduces a considerable error into the approximations used to derive Equation 24. Using the exact chord distribution for a sphere it has been shown by Keane (1964) that:

$$\sigma_p = \frac{\sigma_m}{1+aN\bar{\ell}\sigma_m} \quad (35)$$

where $\frac{9}{16} \leq a \leq 1$,

and that the most accurate results for the important large resonances are obtained when a is given an intermediate value of $\frac{3}{4}$.

7. INFINITE-MASS AND INTERMEDIATE-RESONANCE APPROXIMATIONS

For simplicity the above discussion has been confined to the narrow-resonance approximation, but it is not difficult to extend the theory to cover the infinite-mass approximation. Extension to the intermediate-resonance approximation is straightforward provided that this approximation is only applied to the fuel, while the outside moderator is treated in the narrow-resonance approximation (Hill and Schaefer 1962, Goldstein and Brooks 1964). However, application of the intermediate-resonance approximation to the external moderator introduces the complication that the intermediate-resonance parameters become functions of the heterogeneity as well as of the atomic mass and density of the constituents. This means that there is in general no equivalence between heterogeneous and homogeneous systems when the intermediate-resonance approximation is applied to the outside moderator, as pointed out by Sehgal and Goldstein (1966).

8. ACKNOWLEDGEMENT

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APPENDIX 1

COLLISION PROBABILITIES AND MEAN CHORD LENGTH IN MODERATOR

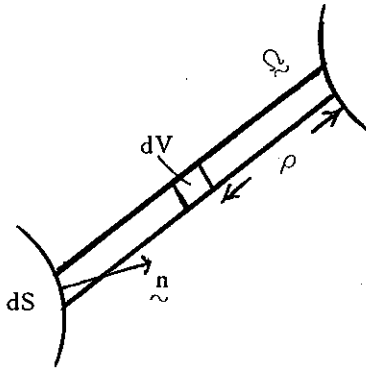
Consider a system of identical fuel lumps dispersed in moderator. Let S be the surface area and V_f the volume of a fuel lump and let V_m/V_f be the ratio of moderator to fuel in the system. Take an imaginary surface S' in the moderator which encloses a large number of fuel lumps. Let the surface area of S' be αa^2 and the volume enclosed be βa^3 where a is a linear dimension through the volume and α and β are constants depending on the shape of the region. Thus the total volume of moderator is $\frac{V_m}{V_m + V_f} \beta a^3 = V$ and the total surface area of the region containing moderator is:

$$\frac{S}{V_m + V_f} \beta a^3 + \alpha a^2 = \mathcal{S}$$

Let ρ be the distance from a volume element dV in the moderator in a direction $\hat{\Omega}$ to the first boundary surface. Then the average probability of hitting a boundary surface before colliding with moderator is:

$$1 - P_1 = \frac{1}{4\pi V} \iint e^{-\Sigma \rho} d\hat{\Omega} dV, \quad (A1)$$

where Σ is the macroscopic cross section of the moderator.



If \hat{n} is the unit normal from a surface element into the moderator, then:

$$dV = (\hat{\Omega} \cdot \hat{n}) dS d\rho,$$

so that on changing the order of integration in (A1):

$$\begin{aligned} 1 - P_1 &= \frac{1}{4\pi V} \iiint e^{-\Sigma \rho} d\rho (\hat{\Omega} \cdot \hat{n}) d\hat{\Omega} dS, \\ &= \frac{1}{4\pi \Sigma V} \iint (1 - e^{-\Sigma \ell}) (\hat{\Omega} \cdot \hat{n}) d\hat{\Omega} dS, \end{aligned} \quad (A2)$$

where ℓ is the chord length in the direction $\hat{\Omega}$, and the integration is restricted to $(\hat{\Omega} \cdot \hat{n}) > 0$.

Now consider neutrons falling isotropically on the surface \mathcal{S} and entering the moderator. The number entering the solid angle $d\hat{\Omega}$ about $\hat{\Omega}$ is proportional to $(\hat{\Omega} \cdot \hat{n}) d\hat{\Omega}$. Since the chord length through the moderator from surface element dS in direction $\hat{\Omega}$ is ℓ then the average value of any function, $h(\ell)$, of the chord length is proportional to $\iint h(\ell) (\hat{\Omega} \cdot \hat{n}) d\hat{\Omega} dS$.

If we now define $g(\ell)d\ell$ as the probability of a chord having length between ℓ and $\ell + d\ell$, where the number of chords through the surface element dS in the solid angle $d\hat{\Omega}$ about $\hat{\Omega}$ is proportional to $(\hat{\Omega} \cdot \hat{n}) d\hat{\Omega}$, then

$$\int h(\ell) g(\ell) d\ell = \frac{\iint h(\ell) (\hat{\Omega} \cdot \hat{n}) d\hat{\Omega} dS}{\iint (\hat{\Omega} \cdot \hat{n}) d\hat{\Omega} dS},$$

where the denominator on the right has been introduced to normalise $g(\ell)$ so that $\int g(\ell) d\ell = 1$, and the integrations are restricted to $(\hat{\Omega} \cdot \hat{n}) > 0$.

$$\begin{aligned} \text{But } \iint (\hat{\Omega} \cdot \hat{n}) d\hat{\Omega} dS &= \mathcal{S} \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos \theta \sin \theta d\theta, \\ &= \pi \mathcal{S}. \end{aligned}$$

(continued)

APPENDIX I (continued)

$$\text{Thus } \pi \delta \int h(\ell) g(\ell) d\ell = \iint h(\ell) (\tilde{\Omega}, \tilde{n}) d\tilde{\Omega} dS, \quad (A3)$$

so that (A2) becomes

$$1 - P_1 = \frac{\delta}{4V\Sigma} \int (1 - e^{-\Sigma\ell}) g(\ell) d\ell. \quad (A4)$$

$$\begin{aligned} \text{Since } \iint d\tilde{\Omega} dV &= \iiint (\tilde{\Omega}, \tilde{n}) d\rho dS d\tilde{\Omega}, \\ &= \iint \ell(\tilde{\Omega}, \tilde{n}) dS d\tilde{\Omega}, \end{aligned}$$

$$\text{and also } \iint d\tilde{\Omega} dV = 4\pi V,$$

$$\text{we see from (A3) that } \int \ell g(\ell) d\ell = \frac{4\pi V}{\pi \delta},$$

$$\text{or mean chord length, } L = \frac{4V}{\delta}.$$

$$\text{Thus } L = \frac{4V_m}{S + (V_m + V_F) \frac{\alpha}{\beta a}},$$

so that if the system is large ($a \rightarrow \infty$) we have:

$$L = \frac{4V_m}{S}. \quad (A5)$$

Let G_1 be the average probability that a neutron which enters the moderator through a boundary surface will escape through another part of the surface without colliding with moderator. Then:

$$G_1 = \int (1 - e^{-\Sigma\ell}) g(\ell) d\ell. \quad (A6)$$

Thus from (A4) and (A5),

$$1 - P_1 = \frac{1}{\Sigma L} G_1. \quad (A7)$$