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TWO GROUP THEORY FOR EXPLORATORY
H.T.G.C. REACTOR CALCULATIONS

by

J. J. THOMPSON

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Summary

A method is described for the manipulation of the solutions of two-group diffusion theory for a reflected sphere. Matrices are used to give a simple treatment useful for practical computation. The critical equation is formulated in terms of a second-order determinant only, which yields a measure of the amount of hypothetical absorber required between core and reflector for criticality and therefore a basis for a comparative study of systems. Sufficient equations are presented to enable both flux and adjoint flux distributions to be calculated. In addition, the basic results for the formulation of the critical equation for cylinders with either radial or end reflectors are given.

1. INTRODUCTION

1.1 To explore the sizes, investments and initial conversion factors of homogeneous gas cooled reactors resulting from the utilization of uranium, thorium, beryllium and graphite in various ways, a simple and attractive formulation of the basic critical equation is very desirable. It would also be convenient if the scheme of computation were easily adaptable for use on high speed digital computers. It is not proposed to introduce errors by the approximate analysis of realistic configurations such as fully reflected cylinders, so only spheres and cylinders for which exact solutions of the diffusion problem are available will be considered.

1.2 The use of matrices, and the extension of the albedo concept leads to a simple presentation for a two-group two-region problem, and a critical equation is obtained as the vanishing of a second-order determinant. This method is described briefly elsewhere, (1), but it has since been modified to reduce the amount of computation and to yield results more suitable for the survey of possible systems. It is proposed therefore to present this version in more detail, and to use it for the initial H.T.G.C. calculations.

2. THEORY

2.1 The two-group equations, including capture and fission in the fast group are:-

$$D_1 \nabla^2 \phi_1 - (\Sigma_s + \Sigma_1 + \Sigma_{f1}) \phi_1 + \epsilon_2 \eta_2 \Sigma_{f2} \phi_2 + \epsilon_1 \eta_1 \Sigma_{f1} \phi_1 = 0$$

$$D_2 \nabla^2 \phi_2 - (\Sigma_2 + \Sigma_{f2}) \phi_2 + \Sigma_s \phi_1 = 0$$

As the interest here is in the form of the equations, they will be written

$$\begin{bmatrix} \nabla^2 - K_1^2 (1+x) & kK_2^2 D_2 / D_1 \\ K_1^2 D_1 / D_2 & \nabla^2 - K_2^2 \end{bmatrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix} = 0$$

with

$$K_1^2 = \Sigma_s / D_1 \quad x = (\Sigma_1 + \Sigma_{f1} (1 - \eta_1 \epsilon_1)) / \Sigma_s$$

$$K_2^2 = (\Sigma_2 + \Sigma_{f2}) / D_2 \quad k = \epsilon_2 \eta_2 \Sigma_{f2} / (\Sigma_2 + \Sigma_{f2})$$

The solution is therefore obtained as a linear combination of the permissible solutions of the equations

$$(\nabla^2 + K^2) \phi = 0 \quad ; \quad (\nabla^2 - \mu^2) \phi = 0$$

$$\text{where } K^2 \mu^2 = (k-1-x) K_1^2 K_2^2$$

$$\mu^2 - K^2 = K_1^2 (1+x) + K_2^2$$

These functions will be denoted by ϕ_K and ϕ_μ . Then

$$\begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix} = \begin{bmatrix} S_K & -S_\mu \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \phi_K & 0 \\ 0 & \phi_\mu \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix}$$

where

$$S_K = (K^2 + K_2^2) / (D_1 K_1^2 / D_2)$$

$$S_\mu = (K^2 + K_1^2 (1 + x)) / (D_1 K_1^2 / D_2)$$

Note that $S_K S_\mu = \frac{k(D_2)^2 (K_2)^2}{(D_1)(K_1)}$

2.2 Let $\bar{\Phi} = \{ \phi_1(R) \phi_2(R) \}$ refer to the neutron flux at the boundary $r = R$ of a spherical core, and let $\bar{J} = \{ J_1(R) J_2(R) \}$ refer to the total neutron current at the boundary directed into the core. Then, using dashes to denote differentiation with respect to r

$$\bar{\Phi} = \begin{bmatrix} S_K & -S_\mu \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \phi_K(R) & 0 \\ 0 & \phi_\mu(R) \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix}$$

$$\bar{J} = \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix} \begin{bmatrix} S_K & -S_\mu \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \phi_K^1(R) & 0 \\ 0 & \phi_\mu^1(R) \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix}$$

Eliminating the constants A and B gives:

$$\bar{J} = \lambda \bar{\Phi}$$

$$\lambda = \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix} \begin{bmatrix} S_K & -S_\mu \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \phi_K^1(R) & 0 \\ 0 & \phi_\mu^1(R) \end{bmatrix} \begin{bmatrix} \phi_K(R) & 0 \\ 0 & \phi_\mu(R) \end{bmatrix}^{-1} \begin{bmatrix} S_K & -S_\mu \\ 1 & 1 \end{bmatrix}^{-1}$$

$$= \frac{1}{S_K + S_\mu} \begin{bmatrix} D_1 S_K / R & -D_1 S_\mu / R \\ D_2 / R & D_2 / R \end{bmatrix} \begin{bmatrix} R \phi_K^1(R) / \phi_K(R) & 0 \\ 0 & R \phi_\mu^1(R) / \phi_\mu(R) \end{bmatrix} \begin{bmatrix} 1 & S_\mu \\ -1 & S_K \end{bmatrix}$$

2.3 For a spherical core $\phi_K = \sin(Kr)/r$, and

$\phi_\mu = (\sinh \mu r)/r$. Hence, writing

$$X_K = KR \cot KR - 1$$

$$X_\mu = \mu R \coth \mu R - 1$$

the required matrix λ is:

$$\lambda = \frac{1}{S_K + S_\mu} \begin{bmatrix} D_1 S_K/R & - D_1 S_\mu/R \\ D_2/R & D_2/R \end{bmatrix} \begin{bmatrix} X_K & 0 \\ 0 & X_\mu \end{bmatrix} \begin{bmatrix} 1 & S_\mu \\ -1 & S_K \end{bmatrix}$$

This is adequate for numerical work, but expansion is simple and leads to

$$\lambda = \frac{1}{S_K + S_\mu} \begin{bmatrix} \frac{D_1}{R}(S_K X_K + S_\mu X_\mu) & \frac{D_1}{R} S_K S_\mu (X_K - X_\mu) \\ \frac{D_2}{R}(X_K - X_\mu) & \frac{D_2}{R} (S_\mu X_K + S_K X_\mu) \end{bmatrix}$$

2.4 In the reflector or blanket, assuming no multiplication, the two-group equations are:

$$D_1 \nabla^2 \bar{\phi}_1 - (\Sigma_s + \Sigma_1) \bar{\phi}_1 = 0$$

$$D_2 \nabla^2 \bar{\phi}_2 - \Sigma_2 \bar{\phi}_2 + \Sigma_s \bar{\phi}_1 = 0$$

These are written as before:

$$\begin{bmatrix} \nabla^2 - K_1^2(1+x) & 0 \\ \frac{D_1}{D_2} K_1^2 & \nabla^2 - K_2^2 \end{bmatrix} \begin{Bmatrix} \bar{\phi}_1 \\ \bar{\phi}_2 \end{Bmatrix} = 0$$

The solution again is a linear combination of the solutions of:

$$(\nabla^2 - k_1^2)\phi = 0 \quad ; \quad (\nabla^2 - k_2^2)\phi = 0$$

with $k_1 = K_1(1+x)^{1/2}$ and $k_2 = K_2$

These functions $\bar{\phi}_1$ and $\bar{\phi}_2$ satisfy the boundary condition of zero flux at $r = R + T$. The required solution is now:

$$\begin{pmatrix} \bar{\phi}_1 \\ \bar{\phi}_2 \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ S_1 & 1 \end{bmatrix} \begin{bmatrix} \bar{\phi}_1 & 0 \\ 0 & \bar{\phi}_2 \end{bmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$$

with $S_1 = (D_1 K_1^2 / D_2) / (K_2^2 - K_1^2(1+x))$

2.5 Again, using $\bar{\Phi}$ and \bar{J} to refer to the fluxes at R, and the total neutron currents directed into the reflector, it may easily be shown that:

$$\begin{aligned} \bar{J} &= \bar{\lambda} \bar{\Phi} \\ \bar{\lambda} &= - \begin{bmatrix} D_1/R & 0 \\ S_1 D_2/R & D_2/R \end{bmatrix} \begin{bmatrix} R \bar{\phi}_1^1(R) / \bar{\phi}_1(R) \\ 0 & R \bar{\phi}_2^1(R) / \bar{\phi}_2(R) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -S_1 & 1 \end{bmatrix} \end{aligned}$$

In a spherical system $\bar{\phi}_1 = (\sinh k_1 (R+T-r))/r$ etc., so writing

$$X_1 = 1 + k_1 R \coth k_1 T \text{ etc.},$$

$$\begin{aligned} \bar{\lambda} &= \begin{bmatrix} D_1/R & 0 \\ S_1 D_2/R & D_2/R \end{bmatrix} \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -S_1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} D_1 X_1 / R & 0 \\ D_2 S_1 (X_1 - X_2) / R & D_2 X_2 / R \end{bmatrix} \end{aligned}$$

2.6 The condition that core and reflector fit together is simply

$$\begin{aligned} \bar{\Phi} &= \bar{\Phi} \text{ and } \bar{J} = -\bar{J} \\ \text{i.e. } (\lambda + \bar{\lambda}) \bar{\Phi} &= 0 \\ \therefore |\lambda + \bar{\lambda}| &= 0 \end{aligned}$$

This is the critical equation. Its formulation involves essentially the evaluation of the functions X_K , X_μ , X_1 and X_2 .

2.7 In Ref. (1), a flux matrix was defined by $\Phi = \gamma J$ and an albedo matrix by $J_0 = \beta J_1$ where J_1 and J_0 are the components of the current in the directions of J and J reversed respectively. Thus:

$$J = J_1 - J_0 \text{ and } \Phi = 2(J_1 + J_0)$$

$$\therefore \Phi = 2(U + \beta)J_1 \text{ and } \Phi = \gamma(U - \beta)J_1 \left(U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$$

$$\therefore 2(U + \beta) = \gamma(U - \beta)$$

$$\text{i.e. } \beta = U - 4(\gamma + 2U)^{-1} = 2(2\lambda + U)^{-1}U$$

Other forms of the critical equations are:

$$|\gamma + \bar{\gamma}| = 0 \text{ and } |U - \beta \bar{\beta}| = 0$$

2.8 A reactor of given composition and reflector thickness will be critical at only one value of the core radius R . For any other value of R , it could be critical if the number of neutrons per capture in fissile material in a thermal group were changed from its actual value η_2 to η_2^1 . For a given composition, $k \propto \eta_2$ and can therefore be regarded as the variable parameter. Fixing composition and size, k^1 can be found by trial and error. A choice of k will determine K and μ and enable all the elements of λ to be calculated. Note that λ does not depend on the core properties. Having found k^1 to satisfy the critical equation, the effective multiplication constant of the actual physical system is simply $\eta_2/\eta_2^1 = k/k^1$ and the reactivity is $1 - (\eta_2^1/\eta_2) = 1 - (k^1/k)$.

2.9 Using this method, several repeated calculations are required to determine one value of reactivity. To reduce this work and to bring the model closer to reality where the object is to maintain $k_{\text{eff}} = 1$, it is useful to consider the possibility of representing the essential features of a control system. It is not possible to obtain simple solutions for partially inserted control rods in a reactor of a realistic shape. To maintain the simplicity of a reflected sphere it must be assumed that a thin shell of material which strongly captures thermal neutrons is inserted between core and reflector. For simplicity the thickness Δt can be assumed very small and the cross section σ_a can be assumed very large, the variable quantity being the number of absorber atoms present per unit area. Then, if the flux in the shell is Φ_c and the total neutron current in the positive direction changes from J_i to J_0 in passing through the shell,

$$J_i - J_0 = \nu \Phi_c$$

$$\text{where } \nu = \begin{bmatrix} 0 & 0 \\ 0 & \Sigma_a \Delta t \end{bmatrix}$$

The critical equation now comes from the conditions,

$$J = -J_i, J_0 = \bar{J}, \Phi = \Phi_c = \bar{\Phi}$$

and this gives

$$(\lambda + \bar{\lambda} + \nu) \Phi = 0$$

$$\text{i.e. } |\lambda + \bar{\lambda} + \nu| = 0$$

It follows that the amount of absorber required to maintain the system critical is:

$$\Sigma_a \Delta t = - | \lambda + \bar{\lambda} | / (\lambda_{11} + \bar{\lambda}_{11})$$

The calculations can now all be directed towards the evaluation of $\Sigma_a \Delta t$, which must of course be non-negative. It provides a convenient fiction for evaluating the various systems as well as the effect of changes in any one system. Although not strictly correct, it does give an indication of the amount of control required, and it is possible that its usefulness can be increased by the application of perturbation theory.

2.10 To complete the analysis, it is necessary to show how the actual flux distribution can be calculated. From the critical equation written in terms of the fast and thermal flux at the control shell, $\phi_1(R)$ and $\phi_2(R)$ the ratio $\phi_1(R) / \phi_2(R) = \zeta$ can be determined. Then,

$$\begin{aligned} \begin{Bmatrix} A \\ B \end{Bmatrix} &= \phi_2(R) \begin{bmatrix} \phi_K(R) & 0 \\ 0 & \phi_\mu(R) \end{bmatrix}^{-1} \begin{bmatrix} S_K & -S_\mu \\ 1 & 1 \end{bmatrix}^{-1} \begin{Bmatrix} \zeta \\ 1 \end{Bmatrix} \\ \therefore \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix} &= \phi_2(R) \begin{bmatrix} S_K & -S_\mu \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \phi_K & 0 \\ 0 & \phi_\mu \end{bmatrix} \begin{bmatrix} \phi_K(R) & 0 \\ 0 & \phi_\mu(R) \end{bmatrix}^{-1} \begin{bmatrix} S_K & -S_\mu \\ 1 & 1 \end{bmatrix}^{-1} \begin{Bmatrix} \zeta \\ 1 \end{Bmatrix} \\ &= \frac{\phi_2(R)}{S_K + S_\mu} \begin{bmatrix} S_K & -S_\mu \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \phi_K / \phi_K(R) & 0 \\ 0 & \phi_\mu / \phi_\mu(R) \end{bmatrix} \begin{bmatrix} 1 & S_\mu \\ -1 & S_K \end{bmatrix} \begin{Bmatrix} \zeta \\ 1 \end{Bmatrix} \end{aligned}$$

Similarly, the flux distribution in the reflector is:

$$\begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix} = \phi_2(R) \begin{bmatrix} 1 & 0 \\ S_1 & 1 \end{bmatrix} \begin{bmatrix} \bar{\phi}_1 / \bar{\phi}_1(R) & 0 \\ 0 & \bar{\phi}_2 / \bar{\phi}_2(R) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -S_1 & 1 \end{bmatrix} \begin{Bmatrix} \zeta \\ 1 \end{Bmatrix}$$

2.11 For perturbation studies, particularly for burn-ups attainable, knowledge of the adjoint fluxes is required. In the core the governing equations are:

$$\begin{bmatrix} \nabla^2 - K_1^2(1+x) & K_1^2 \\ kK_2^2 & \nabla^2 - K_2^2 \end{bmatrix} \begin{Bmatrix} \phi_1^* \\ \phi_2^* \end{Bmatrix}$$

The solution is

$$\begin{Bmatrix} \phi_1^* \\ \phi_2^* \end{Bmatrix} = \begin{bmatrix} 1 & 1 \\ D_1 S_\mu / D_2 & -D_1 S_K / D_2 \end{bmatrix} \begin{bmatrix} \phi_K & 0 \\ 0 & \phi_\mu \end{bmatrix} \begin{Bmatrix} A^* \\ B^* \end{Bmatrix}$$

which leads to $J^* = \lambda^* \Phi^*$ with λ^* given by

$$\lambda^* = \frac{1}{S_K + S_\mu} \begin{bmatrix} D_1/R & D_1/R \\ S_\mu D_1/R & -S_K D_1/R \end{bmatrix} \begin{bmatrix} X_K & 0 \\ 0 & X_\mu \end{bmatrix} \begin{bmatrix} S_K & D_2/D_1 \\ S_\mu & -D_2/D_1 \end{bmatrix}$$

Expansion shows that $\lambda^* = \lambda^1$ i.e., the transpose of λ .

2.12 In the reflector

$$\begin{bmatrix} \nabla^2 & -K_1^2(1+x) & K_1^2 \\ 0 & \nabla^2 - K_2^2 & 0 \end{bmatrix} \begin{Bmatrix} \bar{\phi}_1^* \\ \bar{\phi}_2^* \end{Bmatrix} = 0$$

$$\therefore \begin{Bmatrix} \bar{\phi}_1^* \\ \bar{\phi}_2^* \end{Bmatrix} = \begin{bmatrix} 1 & -S_1 D_2 / D_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{\bar{\phi}}_1 & 0 \\ 0 & \bar{\bar{\phi}}_2 \end{bmatrix} \begin{Bmatrix} \bar{A}^* \\ \bar{B}^* \end{Bmatrix}$$

It also follows that $\bar{\lambda}^* = \bar{\lambda}^1$. For an absorbing shell, $\Phi_c^* = \Phi_c$ and the critical equation therefore leads to:

$$\Sigma_a \Delta t = - |\lambda^* + \bar{\lambda}^*| / (\lambda_{11}^* + \bar{\lambda}_{11}^*)$$

As was to be expected, this is identical with the result obtained from the flux equations, because of the transpose relations indicated above.

2.13 The adjoint flux distributions are derived in a manner similar to that for the flux, and the results are:-

$$\begin{Bmatrix} \phi_1^* \\ \phi_2^* \end{Bmatrix} = \frac{\phi_2^*(R)}{S_K + S_\mu} \begin{bmatrix} 1 & 1 \\ D_1 S_\mu / D_2 & -D_1 S_K / D_2 \end{bmatrix} \begin{bmatrix} \phi_K / \phi_K(R) & 0 \\ 0 & \phi_\mu / \phi_\mu(R) \end{bmatrix} \begin{bmatrix} S_K & D_2 / D_1 \\ S_\mu & -D_2 / D_1 \end{bmatrix} \begin{Bmatrix} \zeta^* \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} \bar{\phi}_1^* \\ \bar{\phi}_2^* \end{Bmatrix} = \phi_2^*(R) \begin{bmatrix} 1 & -S_1 D_2 / D_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{\phi}_1 / \bar{\phi}_1(R) & 0 \\ 0 & \bar{\phi}_2 / \bar{\phi}_2(R) \end{bmatrix} \begin{bmatrix} 1 & S_1 D_2 / D_1 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \zeta^* \\ 1 \end{Bmatrix}$$

2.14 A similar analysis can be carried out for the other two simple systems, a cylindrical core with radial reflector only, and a cylindrical core with end reflector only. The formulae for λ and Λ only will be given here.

(a) Radially reflected cylinder, radius R, length L, reflector thickness T.

$$\lambda = \frac{1}{S_K + S_\mu} \begin{bmatrix} D_1 S_K / R & -D_1 S_\mu / R \\ D_2 / R & D_2 / R \end{bmatrix} \begin{bmatrix} X_K & 0 \\ 0 & X_\mu \end{bmatrix} \begin{bmatrix} 1 & S_\mu \\ -1 & S_K \end{bmatrix}$$

$$\left. \begin{aligned} X_K &= -\tilde{K}R J_1(\tilde{K}R) / J_0(\tilde{K}R) \\ X_\mu &= \tilde{\mu}R I_1(\tilde{\mu}R) / I_0(\tilde{\mu}R) \end{aligned} \right\} \tilde{K}^2 = K^2 - \left(\frac{\pi}{L}\right)^2 ; \tilde{\mu}^2 = \mu^2 + \left(\frac{\pi}{L}\right)^2$$

$$\bar{\lambda} = \begin{bmatrix} D_1 / R & 0 \\ S_1 D_2 / R & D_2 / R \end{bmatrix} \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -S_1 & 1 \end{bmatrix}$$

$$X_1 = \tilde{k}_1 R \left\{ \frac{K_1(\tilde{k}_1 R) I_0[\tilde{k}_1(R+T)] + I_1(\tilde{k}_1 R) K_0[\tilde{k}_1(R+T)]}{K_0(\tilde{k}_1 R) I_0[\tilde{k}_1(R+T)] - I_0(\tilde{k}_1 R) K_0[\tilde{k}_1(R+T)]} \right\}$$

$$\tilde{k}_1^2 = k_1^2 + \left(\frac{\pi}{L}\right)^2 \text{ with similar expressions for } X_2 \text{ and } \tilde{k}_2.$$

(b) End reflected cylinder, core height 2L, reflector thickness T.

$$\lambda = \frac{1}{S_K + S_\mu} \begin{bmatrix} D_1 S_K / L & -D_1 S_\mu / L \\ D_2 / L & D_2 / L \end{bmatrix} \begin{bmatrix} X_K & 0 \\ 0 & X_\mu \end{bmatrix} \begin{bmatrix} 1 & S_\mu \\ -1 & S_K \end{bmatrix}$$

$$\left. \begin{aligned} X_K &= -\tilde{K}L \tan \tilde{K}L \\ X_\mu &= \tilde{\mu}L \tanh \mu L \end{aligned} \right\} \tilde{K}^2 = K^2 - \frac{(2.405)^2}{(R)^2} ; \tilde{\mu}^2 = \mu^2 + \frac{(2.405)^2}{(R)^2}$$

$$\bar{\lambda} = \begin{bmatrix} D_1/L & 0 \\ S_1 D_2/L & D_2/L \end{bmatrix} \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -S_1 & 1 \end{bmatrix}$$

$$X_1 = \tilde{k}_1 L \coth \tilde{k}_1 T$$

$$\tilde{k}_1^2 = k_1^2 + \frac{(2.405)^2}{\left(\frac{R}{L}\right)^2} \text{ with similar expressions for } X_2 \text{ and } \tilde{k}_2$$

2.15 It remains to indicate how Conversion Ratios can be calculated. The initial Conversion Ratio, I.C.R., is the ratio of the number of neutrons captured in fertile material to the number of atoms of fissionable material destroyed per unit time, at the beginning of the irradiation history. Thus

$$\text{I.C.R.} = \frac{\int_{C+R} (\Sigma_1)_s \phi_1 dv + \int_{C+R} (\Sigma_2)_s \phi_2 dv}{\int_C [\Sigma f_1] \phi_1 dv + \int_C [\Sigma f_2] \phi_2 dv}$$

assuming that fertile material can be present in the reflector.

2.16 For a spherical system with uniformly distributed material in core and reflector, integration of the basic differential equations and the application of Gauss' Theorem leads to

$$\begin{Bmatrix} \int_c \phi_1 dv \\ \int_c \phi_2 dv \end{Bmatrix} = \frac{4\pi R^2}{(1+x-k)_c} \begin{bmatrix} \frac{1}{\Sigma_s} & \frac{k}{\Sigma_s} \\ \frac{1}{\Sigma_2 + \Sigma_{f2}} & \frac{1+x}{\Sigma_2 + \Sigma_{f2}} \end{bmatrix} \begin{Bmatrix} J_1 \\ J_2 \end{Bmatrix}_c$$

For the reflector, it will be assumed that leakage from the outer boundary is very small and can be neglected. The same technique yields

$$\begin{Bmatrix} \int_R \bar{\phi}_1 dv \\ \int_R \bar{\phi}_2 dv \end{Bmatrix} = \frac{4\pi R^2}{(1+x)_R} \begin{bmatrix} \frac{1}{\Sigma_s} & 0 \\ \frac{1}{\Sigma_2} & \frac{1+x}{\Sigma_2} \end{bmatrix} \begin{Bmatrix} \bar{J}_1 \\ \bar{J}_2 \end{Bmatrix}_R$$

From the basic definitions of λ , $\bar{\lambda}$ and ζ it follows that the flux integrals can be expressed in terms of $\phi_2(R)$, which then cancels out of the I.C.R. formula. Finally

$$\text{I.C.R.} = \text{S.C./F.D.}$$

$$\text{S.C.} = \left(\frac{1}{(1+x-k)_c} [(\Sigma_1)_s \quad (\Sigma_2)_s] \begin{bmatrix} \frac{1}{\Sigma_s} & \frac{k}{\Sigma_s} \\ \frac{1}{\Sigma_2 + \Sigma_{f2}} & \frac{1+x}{\Sigma_2 + \Sigma_{f2}} \end{bmatrix} [\lambda] \right)_c$$

$$+ \frac{1}{(1+x)} [(\Sigma_1)_s \quad (\Sigma_2)_s]_R \begin{bmatrix} \frac{1}{\Sigma_s} & 0 \\ \frac{1}{\Sigma_2} & \frac{1+x}{\Sigma_2} \end{bmatrix}_R [\bar{\lambda}] \begin{Bmatrix} \zeta \\ 1 \end{Bmatrix}$$

$$\text{F.D.} = \frac{1}{(1+x-k)_c} [(\Sigma_{f1}) \quad (\Sigma_{f2})]_C \begin{bmatrix} \frac{1}{\Sigma_s} & \frac{k}{\Sigma_s} \\ \frac{1}{\Sigma_2 + \Sigma_{f2}} & \frac{1+x}{\Sigma_2 + \Sigma_{f2}} \end{bmatrix} [\lambda] \begin{Bmatrix} \zeta \\ 1 \end{Bmatrix}$$

3. CONCLUSION

3.1 The fact that only second order matrices are involved in this formulation of the boundary value problem arising in the two-group theory of a two region reactor of simple geometry, is a considerable advantage of the method. It is suggested, therefore, as being very suitable for the many calculations required for a preliminary reactor survey, and it is hoped that it may be readily programmed for use with a high speed digital computer.

3.2 Although λ has not the immediate physical interpretation of the albedo matrix β , only extremely simple functions appear in the individual matrices involved in its determination, it results in a straightforward presentation of the flux distribution and it lends itself to the interpretation of the critical matrix $\lambda + \bar{\lambda}$ in terms of the amount of thermal absorber required at the boundary for control.

4. REFERENCES

1. Thompson, J.J., Generalised Albedo Concepts in Two-Group Reactor Theory. A.E.R.E., L.M.F.S. P/9.

$$\bar{\lambda} = \begin{bmatrix} D_1/L & 0 \\ S_1 D_2/L & D_2/L \end{bmatrix} \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -S_1 & 1 \end{bmatrix}$$

$$X_1 = \tilde{k}_1 L \coth \tilde{k}_1 T$$

$$\tilde{k}_1^2 = k_1^2 + \left(\frac{2.405}{R}\right)^2 \text{ with similar expressions for } X_2 \text{ and } \tilde{k}_2$$

2.15 It remains to indicate how Conversion Ratios can be calculated. The initial Conversion Ratio, I.C.R., is the ratio of the number of neutrons captured in fertile material to the number of atoms of fissionable material destroyed per unit time, at the beginning of the irradiation history. Thus

$$\text{I.C.R.} = \left(\int_{C+R} (\Sigma_1)_s \phi_1 dv + \int_{C+R} (\Sigma_2)_s \phi_2 dv \right) / \left(\int_C [\Sigma_{f1}] \phi_1 dv + \int_C [\Sigma_{f2}] \phi_2 dv \right)$$

assuming that fertile material can be present in the reflector.

2.16 For a spherical system with uniformly distributed material in core and reflector, integration of the basic differential equations and the application of Gauss' Theorem leads to

$$\begin{Bmatrix} \int_c \phi_1 dv \\ \int_c \phi_2 dv \end{Bmatrix} = \frac{4\pi R^2}{(1+x-k)_c} \begin{bmatrix} \frac{1}{\Sigma_s} & \frac{k}{\Sigma_s} \\ \frac{1}{\Sigma_2 + \Sigma_{f2}} & \frac{1+x}{\Sigma_2 + \Sigma_{f2}} \end{bmatrix} \begin{Bmatrix} J_1 \\ J_2 \end{Bmatrix}_c$$

For the reflector, it will be assumed that leakage from the outer boundary is very small and can be neglected. The same technique yields

$$\begin{Bmatrix} \int_R \bar{\phi}_1 dv \\ \int_R \bar{\phi}_2 dv \end{Bmatrix} = \frac{4\pi R^2}{(1+x)_R} \begin{bmatrix} \frac{1}{\Sigma_s} & 0 \\ \frac{1}{\Sigma_2} & \frac{1+x}{\Sigma_2} \end{bmatrix} \begin{Bmatrix} \bar{J}_1 \\ \bar{J}_2 \end{Bmatrix}_R$$

From the basic definitions of λ , $\bar{\lambda}$ and ζ it follows that the flux integrals can be expressed in terms of $\phi_2(R)$, which then cancels out of the I.C.R. formula. Finally

$$\text{I.C.R.} = \text{S.C./F.D.}$$

$$\text{S.C.} = \left(\frac{1}{(1+x-k)_c} [(\Sigma_1)_s \ (\Sigma_2)_s] \begin{bmatrix} \frac{1}{\Sigma_s} & \frac{k}{\Sigma_s} \\ \frac{1}{\Sigma_2 + \Sigma_{f2}} & \frac{1+x}{\Sigma_2 + \Sigma_{f2}} \end{bmatrix} [\lambda] \right)_c$$

$$+ \frac{1}{(1+x)} [(\Sigma_1)_s \quad (\Sigma_2)_s]_R \begin{bmatrix} \frac{1}{\Sigma_s} & 0 \\ \frac{1}{\Sigma_2} & \frac{1+x}{\Sigma_2} \end{bmatrix} [\bar{\lambda}] \begin{Bmatrix} \zeta \\ 1 \end{Bmatrix}$$

$$\text{F.D.} = \frac{1}{(1+x-k)_c} [(\Sigma_{f1}) \quad (\Sigma_{f2})]_C \begin{bmatrix} \frac{1}{\Sigma_s} & \frac{k}{\Sigma_s} \\ \frac{1}{\Sigma_2 + \Sigma_{f2}} & \frac{1+x}{\Sigma_2 + \Sigma_{f2}} \end{bmatrix} [\lambda] \begin{Bmatrix} \zeta \\ 1 \end{Bmatrix}$$

3. CONCLUSION

3.1 The fact that only second order matrices are involved in this formulation of the boundary value problem arising in the two-group theory of a two region reactor of simple geometry, is a considerable advantage of the method. It is suggested, therefore, as being very suitable for the many calculations required for a preliminary reactor survey, and it is hoped that it may be readily programmed for use with a high speed digital computer.

3.2 Although λ has not the immediate physical interpretation of the albedo matrix β , only extremely simple functions appear in the individual matrices involved in its determination, it results in a straightforward presentation of the flux distribution and it lends itself to the interpretation of the critical matrix $\lambda + \bar{\lambda}$ in terms of the amount of thermal absorber required at the boundary for control.

4. REFERENCES

1. Thompson, J.J., Generalised Albedo Concepts in Two-Group Reactor Theory. A.E.R.E., L.M.F.S. P/9.