



AUSTRALIAN ATOMIC ENERGY COMMISSION
RESEARCH ESTABLISHMENT
LUCAS HEIGHTS

**CALCULATIONS AND REVIEW OF CRITICALITY DATA FOR HOMOGENEOUS,
WATER-MODERATED, 5 WEIGHT PER CENT ENRICHED URANIUM
DIOXIDE SYSTEMS**

by

P. B. CRIPPS

April 1974

ISBN 0 642 99611 3

AUSTRALIAN ATOMIC ENERGY COMMISSION

RESEARCH ESTABLISHMENT

LUCAS HEIGHTS

CALCULATIONS AND REVIEW OF CRITICALITY DATA FOR HOMOGENEOUS,
WATER-MODERATED, 5 WEIGHT PER CENT ENRICHED URANIUM
DIOXIDE SYSTEMS

by

P.B. CRIPPS

ABSTRACT

The published data on 5 weight per cent enriched uranium dioxide systems are reviewed and compared with AAEC computed values.

The accuracy of the computation is dependent on both the calculational methods and nuclear data used. Both are treated in some detail.

National Library of Australia card number and ISBN 0 642 99611 3

The following descriptors have been selected from the INIS Thesaurus to describe the subject content of this report for information retrieval purposes. For further details please refer to IAEA-INIS-12 (INIS: Manual for Indexing) and IAEA-INIS-13 (INIS: Thesaurus) published in Vienna by the International Atomic Energy Agency.

CRITICALITY; CROSS SECTIONS; CYLINDRICAL CONFIGURATION; DATA;
MIXTURES; SPHERICAL CONFIGURATION; URANIUM DIOXIDE; URANIUM 235;
URANIUM 238; WATER

CONTENTS

	Page
1. INTRODUCTION	1
2. EXPERIMENTAL CRITICAL DATA ON LOW ENRICHED URANIUM SYSTEMS	1
3. SURVEY OF PUBLISHED CALCULATED LOW ENRICHED OXIDE DATA	2
3.1 UKAEA Handbook Data	2
3.2 Calculated Data by Wallis	2
3.3 Calculated Data by Newlon	3
3.4 Calculated Data by Habiger & Breton	4
3.5 Calculated Data by Dubovskiy	4
4. AAEC CALCULATIONS	4
5. CONCLUSIONS	6
6. ACKNOWLEDGEMENT	7
7. REFERENCES	7

Table 1 Uranium Dioxide - Water Mixtures, Full Water Reflected Spheres

Table 2 Uranium Dioxide - Water Mixtures, Unreflected Spheres

Table 3 Critical Diameters of Infinite Cylinders of Uranium Dioxide -
Water Mixtures, Full Water Reflection

Table 4 Critical Diameters of Infinite Cylinders of Uranium Dioxide -
Water Mixtures, Full Water Reflection

Table 5 Uranium Dioxide - Water Mixtures, Full Water Reflected Spheres

Table 6 Uranium Dioxide - Water Mixtures, Full Water Reflected Spheres

Table 7 Uranium Dioxide - Water Mixtures, Full Water Reflected and
Bare Spheres

Table 8 Uranium Dioxide - Water Mixtures, Infinite Cylinders

Table 9 Uranium Dioxide - Water Mixtures, Infinite Slabs

Figure 1 Minimum critical mass against concentration for 5 wt.%
uranium-235 UO_2 -water mixture, full water reflection

Figure 2 Minimum critical volume against concentration for 5 wt.%
uranium-235 UO_2 -water mixture, full water reflection

Figure 3 Minimum critical cylinder diameter against concentration for
5 wt.% uranium-235 UO_2 -water mixture, full water reflection

- Figure 4 Minimum critical slab thickness against concentration for 5 wt.% uranium-235 UO_2 -water mixture, full water reflection
- Figure 5 Minimum critical mass against concentration for 5 wt.% uranium-235 UO_2 -water mixture, unreflected
- Figure 6 Minimum critical volume against concentration for 5 wt.% uranium-235 UO_2 -water mixture, unreflected
- Figure 7 Minimum critical cylinder diameter against concentration for 5 wt.% uranium-235 UO_2 -water mixture, unreflected
- Figure 8 Minimum critical slab thickness against concentration for 5 wt.% uranium-235 UO_2 -water mixture, unreflected
- Appendix A Cross Section Identification Numbers used in the AAEC Calculations
- Appendix B Uranium-238 Absorption Cross Sections in Groups 8 to 12 for Sig P = 100 barns

1. INTRODUCTION

The paucity of experimental data for the nuclear criticality control of low enriched uranium operations necessitates the use of computed parameters, even for clean homogeneous spherical or cylindrical systems. It has not been practicable to determine experimentally the critical parameters of all geometric configurations and/or fuel concentrations that can occur while processing or fabricating low enriched uranium compounds. Therefore, computed critical parameters are widely used for guidance in establishing nuclear criticality safety specifications. Under these conditions the accuracy and reliability of the nuclear data and computational methods must be assessed.

Because of the recent report by Thomas (1969) on the many, and sometimes severe, discrepancies that exist between the calculated critical data reported for plutonium systems, it was decided to survey the literature on 5 wt. per cent uranium dioxide systems to determine if similar discrepancies exist in this data (Section 3). Preceding this survey is a discussion on the experimental critical data (Section 2) used to check the nuclear cross section data and computational methods used. Section 4 contains new calculated data using the transport code ANISN (Engle, Jr. 1967) and revised Hansen & Roach 16-group cross section data (ORNL ANISN XS Library X7210).

2. EXPERIMENTAL CRITICAL DATA ON LOW ENRICHED URANIUM SYSTEMS

Critical experiments with small concentrations of uranium-235 in uranium-238 (4.89 wt.% uranium-235) have been made at Oak Ridge National Laboratory (Johnson & Cronin 1964). The first set of experiments was on bare and water reflected cylinders of UO_2F_2 - water solutions. The uranium concentrations in these experiments were in the range 0.5 g(U) cm^{-3} to $0.95 \text{ g(U) cm}^{-3}$. The range was extended to higher uranium concentrations with experiments on bare and water reflected U_3O_8 -Sterotex mixtures (Stratton 1967). Sterotex is a glycerol tristearate, $(\text{C}_{17}\text{H}_{35}\text{COO})_3\text{C}_3\text{H}_5$, of density 0.862 g cm^{-3} and molecular weight 891.5. The density of the hydrogen in Sterotex is 0.1072 g cm^{-3} . The two powders were well blended and packed in thin-walled (0.159 cm) aluminium boxes whose outside dimensions, in cm, were $20.32 \times 20.32 \times 20.32$; $20.32 \times 20.32 \times 10.16$; $10.16 \times 10.16 \times 10.16$. Thus, all experiments were parallelepipedal in shape and required a calculated conversion to spherical geometry. The range of uranium concentrations now covered by experiment extends to about 4.0 g(U) cm^{-3} .

Experiments have been carried out also on uranium enriched to 2 wt. per cent uranium-235. The critical assemblies were constructed of blocks

of UF_4 dispersed in paraffin to produce essentially homogeneous mixtures with H/U atomic ratios ranging from 4 to 20. The assemblies were self-supporting rectangular parallelepipeds and contained no internal controls. The experiments were performed both bare and reflected with hydrogenous material (Michalczo & Lynn 1960). Also, two careful k_{∞} experiments (Michalczo & Neeley 1962) were carried out with U(2 wt.% uranium-235) F_4 -paraffin compacts with an $H/^{235}U$ moderation ratio of 195.

Experiments at 3 wt. per cent uranium-235 consist of a single critical assembly of UF_4 -paraffin (Thomas & Lynn 1961) and a series of measurements of k_{∞} , the infinite multiplication factor, at several moderations for a uranium trioxide (UO_3)-polyethylene (CH_2) mixture (Neeley, Handler & Masterson 1961).

3. SURVEY OF PUBLISHED CALCULATED LOW ENRICHED OXIDE DATA

For this report, 5 wt. per cent enriched uranium dioxide systems have been considered and, for easy comparison, the published critical data have been reproduced in one format and presented in a number of tables and figures. The calculated data are for uranium dioxide-water mixtures on an atom to atom ratio and no consideration has been given to the resonance self shielding effect of finely dispersed uranium dioxide particles in water (homogeneous slurries).

3.1 UKAEA Handbook Data

The UKAEA Handbook of Criticality Data (Chalmers, Walker & Pugh 1965) contains provisional data for U(5 wt.% uranium-235) O_2 -water mixtures. The calculations were based on four group diffusion theory with the cross section data being derived from the MUFT code for the three epithermal group and from the SOFOCATE code for the thermal group (Chalmers & Walker 1964).

Very little information is available on the experimental data used by Chalmers, Walker & Pugh to check the calculational method and cross section data. In the preamble to the UKAEA Handbook of Criticality Data, the authors say that data checks against known critical systems gave excellent agreement at 5 wt. per cent uranium-235 but overestimated the reactivity by 3 per cent at 2 wt. per cent uranium-235. It seems likely, therefore, that the above experimental data describing uranium enriched to 4.89 wt. per cent uranium-235 and 2.0 wt. per cent uranium-235 were used as experimental data checks. The method and cross section data were therefore checked over a range of systems and enrichments to demonstrate their adequacy for calculating critical data for U(5 wt.% uranium-235) O_2 -water mixtures.

3.2 Calculated Data by Wallis

Critical data for U(5 wt.% uranium-235)-water systems (Wallis 1964) were

derived from transport calculations (DSN or DTK code) using the older set of Hansen & Roach cross sections (1961). These cross sections were designed for fast and intermediate critical assemblies and are considered inadequate for most systems containing low or low-to-intermediate enriched uranium. Tables 1 and 2 present the calculated data in the same format as in the UKAEA Handbook of Criticality Data.

The calculational method and cross section data were checked (Stratton 1967) against the two sets of experiments on 4.89 wt. per cent uranium-235 over the range 0.5 g(U) cm^{-3} to 3.0 g(U) cm^{-3} . The experimental material contains fluorine, carbon and aluminium and, as near as possible, the correct amount of each of these was included in the homogeneous calculations. As all the calculated data presented are in the one-dimensional form, the need to convert the experimental configuration of right cylinders or parallelepipeds to spheres is apparent; the means for doing this shape conversion is based on leakage calculations using theoretical buckling expressions. The important quantity in the buckling expression is the effective extrapolation distance, δ' , which is the sum of an extrapolation distance, δ , and a reflector saving, ΔR . For low enriched systems, the extrapolation distances are poorly known but the errors from this source are expected to be small compared to the average error of ± 2 per cent on the experimental critical radius (Stratton 1967).

For the bare UO_2F_2 systems, the ratio of calculated to 'experimental' radius is 2.2 per cent high for low $\text{H}/^{235}\text{U}$ ratios but 1 per cent low for high $\text{H}/^{235}\text{U}$ ratios. For water-reflected UO_2F_2 systems the calculation seems excellent where the $\text{H}/^{235}\text{U}$ ratio = 524 but 2.6 per cent low where the $\text{H}/^{235}\text{U}$ ratio = 1,025. For the U_3O_8 -Sterotex mixtures also the calculation seems relatively high for the lower $\text{H}/^{235}\text{U}$ ratios and relatively low for the higher values of $\text{H}/^{235}\text{U}$.

3.3 Calculated Data by Newlon

Newlon (1965) calculated the minimum critical cylinder diameters of $\text{U}(4.9 \text{ wt.}\% \text{ uranium-235})\text{O}_2$ -water systems, with water reflection, using the one-dimensional transport code, DTK, and the Hansen & Roach cross section data (1961). The calculations included the effect of a 0.16 cm aluminium wall cylinder (Table 3).

The method and cross section data were checked for $\text{H}/\text{U} = 24.6$ against the UO_2F_2 solution experiment discussed above. A buckling method was used to relate the experimental dimensions to the diameter of an infinite length cylinder; the ratio of calculated to 'experimental' cylinder diameter was

1.0168. The calculation included the effect of the experimental aluminium cylinder and the fluorine in UO_2F_2 .

3.4 Calculated Data by Habiger & Breton

Critical diameters of infinite cylinders of uranium dioxide (5 wt.% uranium-235)-water mixtures were calculated by Habiger & Breton (1968), assuming a 30 cm water reflector. The calculations were performed using the transport code ANISN in the S4 approximation with 16-group modified Hansen & Roach cross section data (see below). The results are presented in Table 4.

The modifications to the 1961 Hansen & Roach data were adopted by Habiger & Breton because, prior to calculating the critical diameters of the infinite cylinders, the uranium cross sections were checked by comparing the results of computer calculations for uranium systems with the results of critical experiments. The 16-group Hansen & Roach cross sections (1961) tended to predict low values of k_{∞} for ORNL and PCTR experiments on slightly enriched, undermoderated UF_4 -paraffin mixtures and uranium trioxide (UO_3)-polyethylene (CH_2) mixtures. The theoretical value of k_{∞} for a U(2 wt.% uranium-235) F_4 -paraffin assembly at an $\text{H}/^{235}\text{U}$ moderation ratio of 195 was ~ 4 per cent lower than the experimental measurements. To improve the agreement, the basic uranium-238 absorption cross section in groups 8 to 12 was adjusted by applying reduction factors ranging from 0.49 to 1.0 (Newlon & Mallet 1966), such that the computed and experimentally determined k_{∞} values were in agreement. Groups 8 to 12 include the uranium-238 resonance region. These reduction factors, which vary with the scattering cross section per uranium-238 atom, Σ/N_{28} , are given in Appendix 1 of Newlon & Mallet (1966).

3.5 Calculated Data by Dubovskiy

Using a multigroup Pn transport code, Dubovskiy *et al.* (1967) calculated the critical parameters for mixtures of U(5 wt.% uranium-235) O_2 and water, with full water reflection. Two oxide densities were chosen to give an indication of the influence of UO_2 density on the critical parameters. The calculated data are summarised in Table 5.

The calculational method and cross section data were apparently checked against the ORNL UO_2F_2 experiments for 4.89 wt. per cent uranium-235.

4. AAEC CALCULATIONS

Criticality calculations (k_{eff}) have been carried out using the transport code ANISN in the S4 approximation with 16-group revised Hansen & Roach cross section data (ORNL ANISN XS Library X7210).

ANISN solves the one-dimensional Boltzmann neutron transport equation in slab, sphere, or cylindrical geometry. The source of neutrons, defined

by the uranium-235 fission spectrum, and the neutron velocities used in the code are given in Hansen & Roach (1961).

Accuracy can be improved by increasing the number of spatial points in a problem, by increasing the number of angular segmentations, by imposing rigid convergence criteria on the solution and by the inclusion of anisotropic scattering for hydrogen. Computational surveys are usually done with the S4 approximation, moderately tight convergence criteria and sufficient space regions so that fractional changes in neutron flux per space interval are about 10 per cent.

The revised Hansen & Roach cross sections were obtained from a cross section tape (ORNL ANISN XS Library X7210) which contains two sets of cross sections for uranium-235 and uranium-238; the 'R' (revised) is the preferred set. To take into account resonance shielding, the tape contains labelled sets of 16-group cross sections of fissionable isotopes corresponding to a range of Sig P values (Sig P specifies the total scattering cross section per absorber atom). The cross section data used for isotropic and anisotropic hydrogen were averaged over a 1/E spectrum.

To obtain an indication of the effect of cross section data changes on calculations for low enriched uranium dioxide systems, k_{eff} calculations (S4 approximation, convergence criteria of 10^{-3} , 150 mesh points) on a spherical U(5 wt.% uranium-235) O_2 -water mixture were carried out with a number of data assumptions; the results are presented in Table 6. The critical radius taken for the reflected uranium dioxide sphere was that given by Wallis (1964). The potential scattering cross section of the absorbing mixture per atom of uranium-235 and uranium-238 (Sig P) was calculated to be 4,050 barns and 213 barns respectively.

Runs 1 and 2 indicate the different effect of using Connolly's (1964) tabulated form of Hansen & Roach data, and the revised Hansen & Roach data (ORNL ANISN XS Library X7210) for uranium-238. For the Sig P values applicable to this case, the only difference is a decrease in the absorption cross section in group 10 from 8.8 barns to 6.9 barns. As would be expected, the lower the uranium-238 absorption cross section the higher the value of k_{eff} . Therefore the preferred data set (B. McGregor, AAEC private communication) for a critical size search would give a lower critical mass.

The recent cross section libraries based on Hansen & Roach data are in tabular form and contain uranium-238 cross sections for Sig P = 20, 40, 60, 100, 200, 400, 600 etc. barns. If the potential scattering cross section of the absorbing mixture per atom of uranium-238 lies between 100 barns and

200 barns, then there will be an associated error should either 100- or 200-barn uranium-238 cross sections be used. An indication of the difference is shown in runs 2 and 4. The same effect is apparent in the use of uranium-235 cross sections (runs 2 and 3) but to a lesser degree for low enriched uranium dioxide-water systems. Runs 2 and 5 indicate the effect of using P_0 and P_1 cross section data for hydrogen for this water reflected UO_2 -water mixture.

In the calculational survey, the total uranium concentration was chosen to give exact values of Sig P, available in the ANISN cross section library, for uranium-238. The uranium-235 cross section data used were those nearest the calculated Sig P value for the absorbing mixture. The cross section identification numbers used in the calculations are contained in Appendix A. Tables 7, 8 and 9 contain the AAEC critical values for bare and water reflected U(5 wt.% uranium-235) O_2 -water mixtures.

5. CONCLUSION

The results obtained from the literature search and the AAEC calculations are presented in Figures 1 to 8.

Comparison of the critical volumes for reflected and bare systems (Figures 2 and 6) shows the difference between the calculated critical values. The difference is significant for critical values obtained by using anisotropic hydrogen and isotropic hydrogen for total uranium concentrations less than 2 g cm^{-3} .

The critical cylinder diameters, for bare and water reflected systems are shown in Figures 3 and 7. At 4 g(U) cm^{-3} , a discrepancy of 4.1 cm exists between the values of Habiger & Breton (1968) and the AAEC anisotropic values. At $4.21 \text{ g(U) cm}^{-3}$, the uranium-238 cross section used had a Sig P value of 100 barns. Habiger & Breton used Hansen & Roach cross section data (1961) with the uranium-238 absorption cross sections in groups 8 to 12 multiplied by a reduction factor of 0.71. The total uranium concentrations used by Habiger & Breton to produce their critical diameter curve do not have Sig P values available in the Hansen & Roach cross section library, and it is most likely that interpolation was used to obtain the absorption cross section for the calculated Sig P value of the mixture. The data used for the AAEC calculations also used Hansen & Roach cross sections with the uranium-238 absorption cross sections revised in groups 8 to 12. For Sig P = 100 barns, the significant difference occurs in groups 8, 11 and 12 as shown in Appendix B. The smaller critical diameter would be expected from Habiger & Breton because of the overall smaller absorption cross sections in groups 8 to 12,

which includes the resonance region.

If Habiger & Breton's uranium-238 modified cross sections (JKR modification) are correct, then the critical cylinder diameters calculated by the other groups are not conservative. Likewise, the critical masses and volumes for uranium concentrations above approximately 1.5 g(U) cm^{-3} are not conservative. Experimental checks above 1.5 g(U) cm^{-3} are based on the U_3O_8 - Sterotex assemblies. According to checks by Stratton (1967), the calculated critical radius using Hansen & Roach cross sections (1961) was about 5 per cent high at a total uranium concentration of about 3 g cm^{-3} . Habiger & Breton's modified Hansen & Roach cross sections would reduce the critical radius at 3 g(U) cm^{-3} , but additional experimental checks are required before the JKR modified data are accepted.

It may be necessary to do additional homogeneous criticality experiments on low enriched uranium systems to check the applicability of the Hansen & Roach data sets outside the ranges of enrichments and concentrations already having experimental data.

This study of 5 wt. per cent uranium-235 calculated criticality data demonstrates a clear need to check and intercompare published calculated data for all types of systems before they are applied to practical situations.

6. ACKNOWLEDGEMENT

The author wishes to acknowledge the assistance given by P.J.F. Newton in preparing the Tables and Figures for this report.

7. REFERENCES

- Chalmers, J.H. & Walker, G. (1964) - A Four Group Diffusion Theory Calculational Method for Reflected Homogeneous Systems. AHSB(S) R60.
- Chalmers, J.H., Walker, G. & Pugh, J. (1965) - Handbook of Criticality Data, UKAEA, Vol.1, AHSB(S) Handbook 1 (1st Revision).
- Connolly, L.D. (1963) - Los Alamos Group Averaged Cross Sections. LAMS-2941.
- Dubovskiy, B.G. et al. (1967) - Critical Parameters of Fissionable Materials Systems and Nuclear Safety. (A Handbook) JPRS-42322.
- Engle, Jr. W.W. (1967) - A Users Manual for ANISN. A One Dimensional Discrete Ordinance Transport Code with Anisotropic Scattering. K-1693.
- Habiger, K.E. & Breton, D.L. (1968) - Nuclear Safety Studies for Low Enriched Fluoride Volatility Fuel Reprocessing Plants. K-1734.
- Hansen, C.E. & Roach, W.H. (1961) - Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies. LAMS-2543.

- Johnson, E.B. & Cronin, D.F. (1964) - Critical Dimensions of Aqueous UO_2F_2 Solutions Containing 4.9 Per Cent Uranium-235 Enriched Uranium. ORNL-3714, Vol.1.
- Michalczo, J.T. & Lynn, J.J. (1960) - Homogeneous Critical Assemblies of 2% ^{235}U - Enriched UF_4 in Paraffin. ORNL-3016.
- Michalczo, J.T. & Neeley, V.I. (1962) - The Infinite Multiplication Constant of Homogeneous Hydrogen - Moderated 2.0 wt.% - ^{235}U - Enriched Uranium. Nucl. Sci. Eng. 13: 6-11.
- Neeley, V.I., Berberet, J.A. & Masterson, R.H. (1961) - k_{∞} of Three Weight Per Cent U235 Enriched Uranium Trioxide and $\text{UO}_2(\text{NO}_3)_2$ Hydrogeneous Systems. HW-66882.
- Newlon, C.E. (1965) - Minimum Critical Cylinder Diameters of Hydrogen Moderated U (4.9) Systems. K-1629.
- Newlon, C.E. & Mallet, A.J. (1966) - Hydrogen Moderation. A Primary Nuclear Safety Control for Handling and Transporting Low-Enrichment UF_6 . K-1663.
- Stratton, W.R. (1967) - Criticality Data and Factors Affecting Criticality of Single Homogeneous Units. LA-3612.
- Thomas, J.T. & Lynn, J.J. (1961) - Homogeneous Critical Assemblies of 3% ^{235}U - Enriched UF_4 in Paraffin. ORNL-3193.
- Thomas, W. (1969) - Critical and Safe Parameters for Plutonium and Plutonium Compounds. MRR-56.
- Wallis, P.M. (1964) - Computational Survey of Homogeneous Water-Moderated Systems. LA-3166-MS.

TABLE 1
URANIUM DIOXIDE - WATER MIXTURES
FULL WATER REFLECTED SPHERES

Atomic Ratio H/U	Total U Conc. (g cm ⁻³)	²³⁵ U Density (g cm ⁻³)	Critical Radius (cm)	Critical Volume (litre)	Critical Mass (kg ²³⁵ U)	Critical Mass (kg U)
0.55	7.95	0.3973	94.73	3,560.80	1,414.7	28,294.0
0.91	7.16	0.3580	51.28	564.85	202.22	4,044.4
2.74	4.78	0.2389	25.98	73.45	17.55	351.0
8.37	2.38	0.1190	19.70	32.02	3.810	76.2
19.53	1.19	0.0597	19.91	33.06	1.974	39.48
39.20	0.60	0.0299	24.95	65.06	1.945	38.9

Uranium contains 5.0 wt.% uranium-235; 20.3 cm water reflector.

TABLE 2
URANIUM DIOXIDE - WATER MIXTURES
UNREFLECTED SPHERES

Atomic Ratio H/U	Total Conc. (g cm ⁻³)	²³⁵ U Density (g cm ⁻³)	Critical Radius (cm)	Critical Volume (litre)	Critical Mass (kg ²³⁵ U)	Critical Mass (kg U)
0.75	7.49	0.3745	70.08	1,442.00	539.92	10,798.4
1.25	6.55	0.3275	48.04	464.43	152.10	3,042.0
2.5	4.98	0.2493	32.92	149.45	37.26	745.2
5.0	3.37	0.1687	26.17	75.08	12.67	253.4
10.0	2.05	0.1025	23.63	55.27	5.67	113.4
20.0	1.15	0.0574	24.00	57.91	3.324	66.48
30.0	0.80	0.0399	26.20	75.34	3.006	60.01
45.0	0.55	0.0273	31.03	125.16	3.417	68.34

Uranium contains 5.0 wt.% uranium-235.

TABLE 3
CRITICAL DIAMETERS OF INFINITE CYLINDERS OF URANIUM
DIOXIDE - WATER MIXTURES, FULL WATER REFLECTION

Atomic Ratio H/U	Total U Conc. (g cm ⁻³)	Cylinder Diameter (cm)
6	3.05	29.72
8	2.50	27.94
10	2.15	27.43
12	1.85	26.92
15	1.55	27.18
18	1.30	27.69
24	1.02	29.46

Uranium Dioxide Density = 9.7 g cm⁻³

Water Density = 0.998 g cm⁻³

Uranium contains 4.9 wt.% uranium-235

Aluminium cylinder wall thickness = 0.16 cm.

TABLE 4
CRITICAL DIAMETERS OF INFINITE CYLINDERS OF URANIUM
DIOXIDE - WATER MIXTURES, FULL WATER REFLECTION

Uranium Dioxide Volume %	Atomic Ratio H/U	Total U Conc. (g cm ⁻³)	Cylinder Diameter (cm)
5	52.25	0.4804	50.60
10	24.75	0.9608	29.21
20	11.00	1.9216	25.60
30	6.40	2.8824	26.19
40	4.10	3.8432	28.40
60	1.85	5.7648	38.61

Uranium Dioxide Density = 10.9 g cm⁻³

Water Density = 1.0 g cm⁻³

Uranium contains 5.0 wt.% uranium-235

30 cm water reflector.

TABLE 5
URANIUM DIOXIDE - WATER MIXTURES
FULL WATER REFLECTED SPHERES

UO ₂ Density = 3.0 g cm ⁻³			UO ₂ Density = 8.45 g cm ⁻³		
U Conc. (g cm ⁻³)	Critical Mass (kg U)	Critical Volume (litre)	U Conc. (g cm ⁻³)	Critical Mass (kg U)	Critical Volume (litre)
2.40	-	-	5.40	1,400	260
2.00	-	500	4.00	320	86
1.88	-	300	3.60	180	58
1.80	370	200	3.20	130	48
1.36	120	90	2.00	58	32
1.08	74	70	1.44	42	32
0.90	58	67	1.16	38	35
0.76	54	72	0.86	36	41
0.68	52	81	0.80	38	50
0.58	54	95	0.72	40	59
0.54	58	120	0.64	42	70
0.50	62	140	0.58	48	85
0.46	74	180	0.52	56	160

TABLE 6
URANIUM DIOXIDE - WATER MIXTURES
FULL WATER REFLECTED SPHERES

Run	Revised Hansen & Roach Cross Section Data		P ₀ or P ₁	k _{eff}	R Core (cm)
1	U235-10R	U238-5	P ₀	0.9956	19.7
2	U235-10R	U238-5R	P ₀	1.0045	19.7
3	U235-9R	U238-5R	P ₀	1.0048	19.7
4	U235-10R	U238-4R	P ₀	1.0467	19.7
5	U235-10R	U238-5R	P ₁	1.0059	19.7

Uranium contains 5 wt.% uranium-235

20.3 cm water reflector

Water cross sections consist of Hansen & Roach H & O data

H/U atomic ratio 8.4

2.38 g(U) cm⁻³.

TABLE 7
URANIUM DIOXIDE - WATER MIXTURES
FULL WATER REFLECTED AND BARE SPHERES

Hansen & Roach Data		Total U Conc. (g cm ⁻³)	Atomic Ratio H/U	Reflected Critical Radius (cm)		Reflected Critical Volume (litre)		Bare Critical Radius (cm)		Bare Critical Volume (litre)	
				P0	P1	P0	P1	P0	P1	P0	P1
U235-0P	U238-0P										
U235-YR Inf	U238-8	0.59	41	32.30	26.04	141.1	74.1	37.00	30.01	212.1	113.1
U235-12R	U238-7R	0.95	26	23.46	20.83	54.1	37.8	28.40	24.98	93.0	65.3
U235-11R	U238-6R	1.38	17	20.52	19.20	36.1	30.3	25.70	23.52	71.1	54.5
U235-12R	U238-6R	1.38	17	20.46	-	35.9	-	-	-	-	-
U235-10R	U238-5R	2.50	8	19.88	19.84	32.9	32.7	25.80	24.67	71.9	61.3
U235-9R	U238-4R	4.21	3.5	23.66	24.40	55.5	60.8	30.91	30.30	123.6	116.5
U235-8R	U238-2R	6.41	1.4	43.06	40.25	334.3	273.0	48.50	48.11	477.6	466.2

Water cross sections consist of Hansen & Roach H & O data.

Uranium contains 5 wt.% uranium-235.

30.0 cm water reflector

Uranium Dioxide Density = 10.7 g cm⁻³.

TABLE 8
URANIUM DIOXIDE - WATER MIXTURES
INFINITE CYLINDERS

Total U Conc. (g cm ⁻³)	Atomic Ratio H/U	Reflected Critical Diameter (cm)		Bare Critical Diameter (cm)	
		P ₁	P ₀	P ₁	P ₀
0.59	41	36.7	45.8	45.0	55.4
0.95	26	29.3	32.4	37.4	42.3
1.38	17	26.6	27.8	35.0	38.1
2.50	8	27.3	26.4	36.8	38.2
4.21	3.5	33.9	31.2	45.5	46.1
6.41	1.4	56.1	50.3	72.7	73.1

Uranium contains 5 wt.% uranium-235
30 cm water reflector
Uranium Dioxide Density = 10.7 g cm⁻³.

TABLE 9
URANIUM DIOXIDE - WATER MIXTURES
INFINITE SLABS

Total U Conc. (g cm ⁻³)	Atomic Ratio H/U	Reflected Critical Thickness (cm)		Bare Critical Thickness (cm)	
		P ₁	P ₀	P ₁	P ₀
0.59	41	20.6	24.9	28.2	34.8
0.95	26	15.2	16.1	23.2	26.3
1.38	17	13.3	13.0	21.7	23.6
2.50	8	13.2	11.4	22.8	23.6
4.21	3.5	16.6	13.6	28.4	28.8
6.41	1.4	29.4	24.8	46.2	46.4

Uranium contains 5 wt.% uranium-235
30 cm water reflector
Uranium Dioxide Density = 10.7 g cm⁻³.

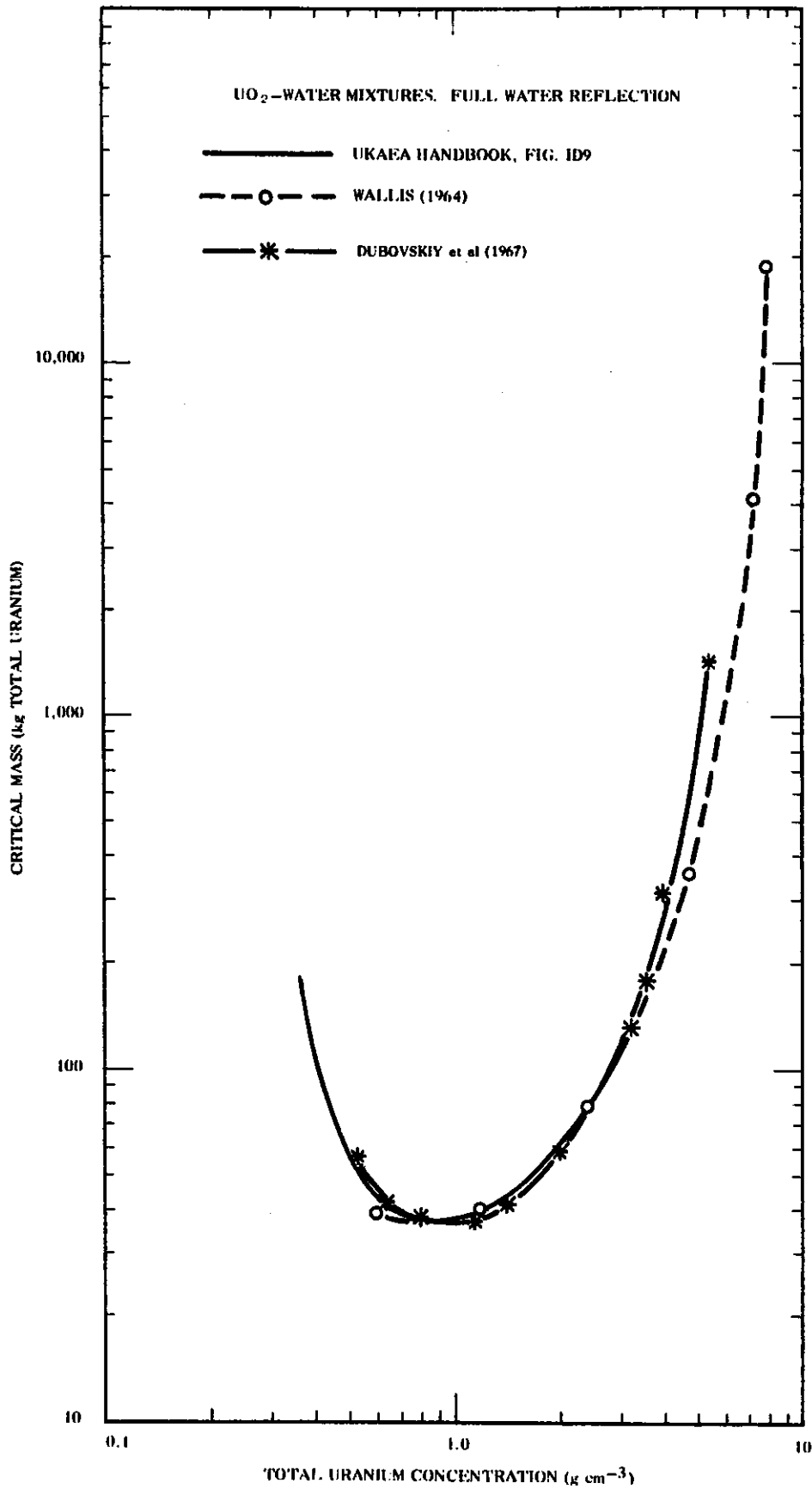
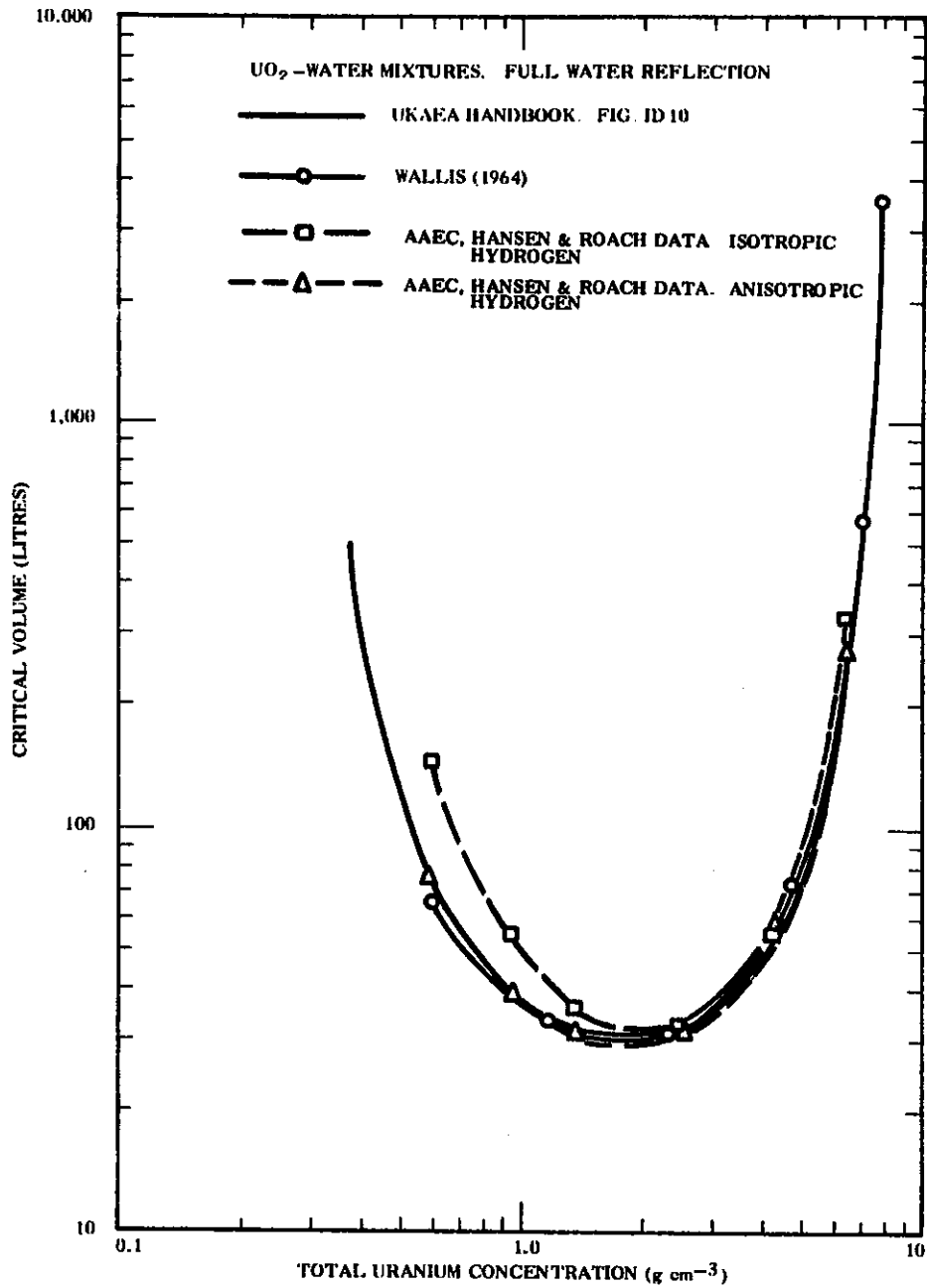


FIGURE 1. MINIMUM CRITICAL MASS AGAINST CONCENTRATION
FOR 5 wt % URANIUM-235



**FIGURE 2. MINIMUM CRITICAL VOLUME AGAINST CONCENTRATION
 FOR 5 wt % URANIUM-235**

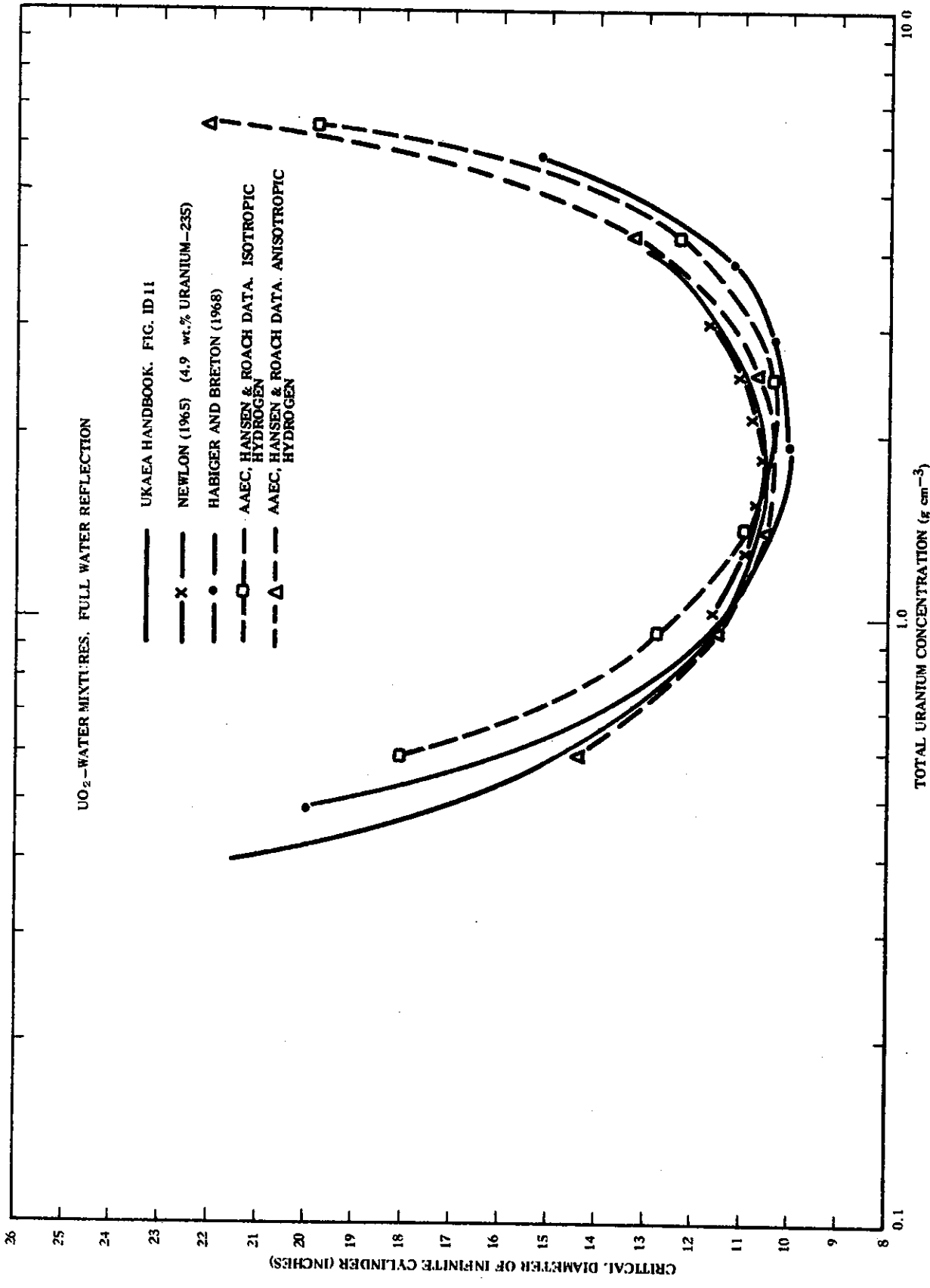


FIGURE 3. MINIMUM CRITICAL CYLINDER DIAMETER AGAINST CONCENTRATION
FOR 5 wt % URANIUM-235

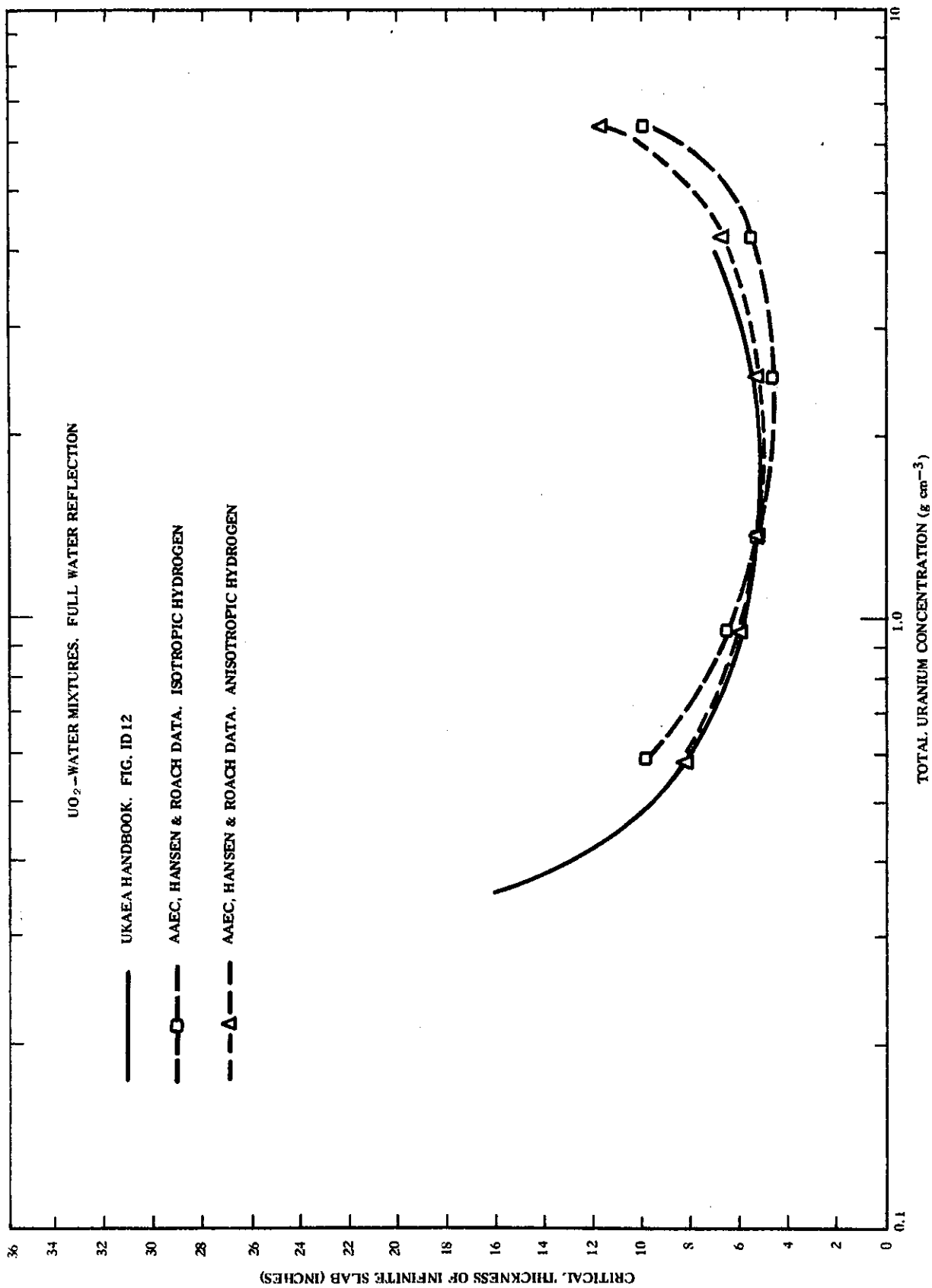


FIGURE 4. MINIMUM CRITICAL SLAB THICKNESS AGAINST CONCENTRATION
FOR 5 wt % URANIUM-235.

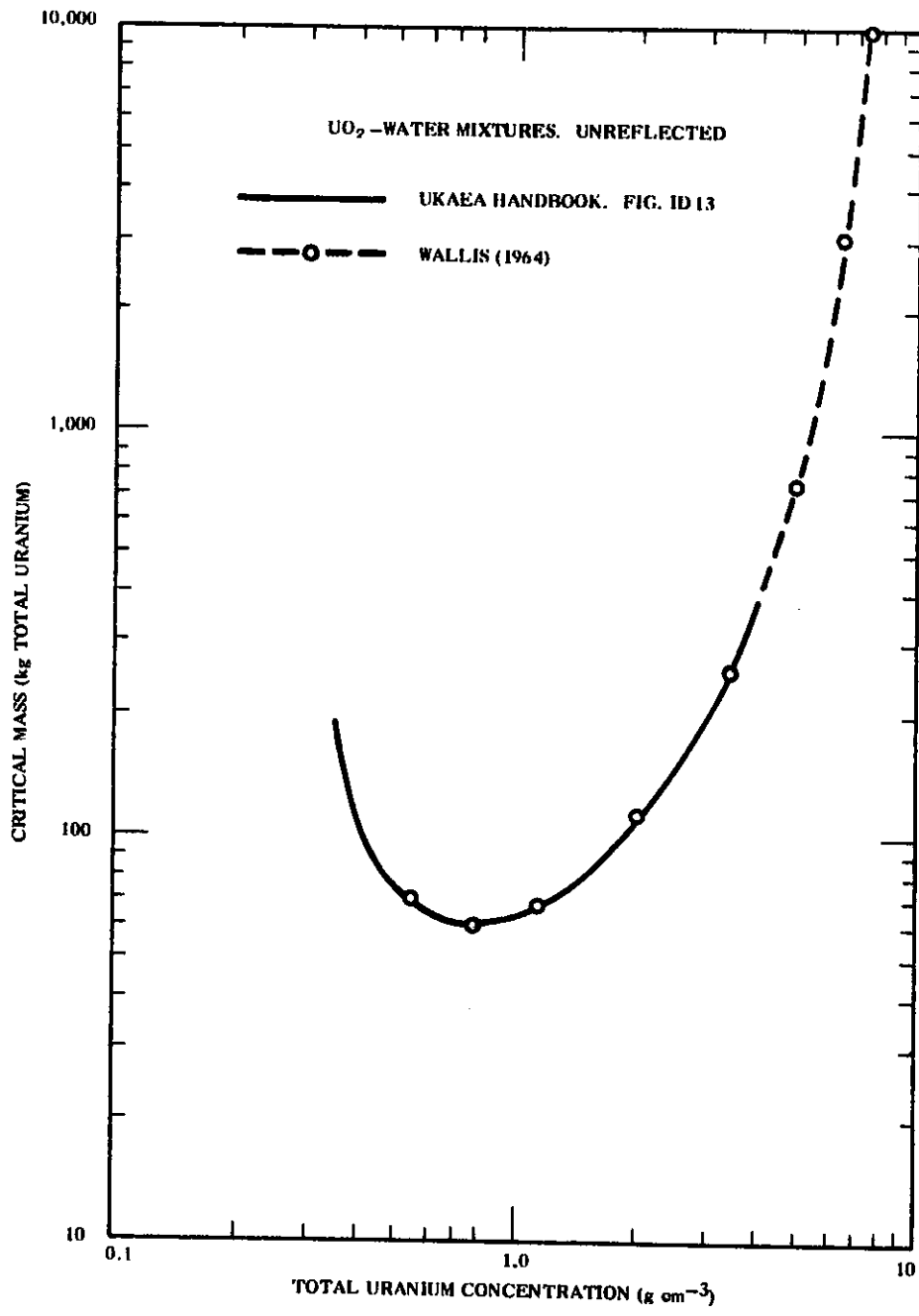


FIGURE 5. MINIMUM CRITICAL MASS AGAINST CONCENTRATION FOR 5 wt % URANIUM-235

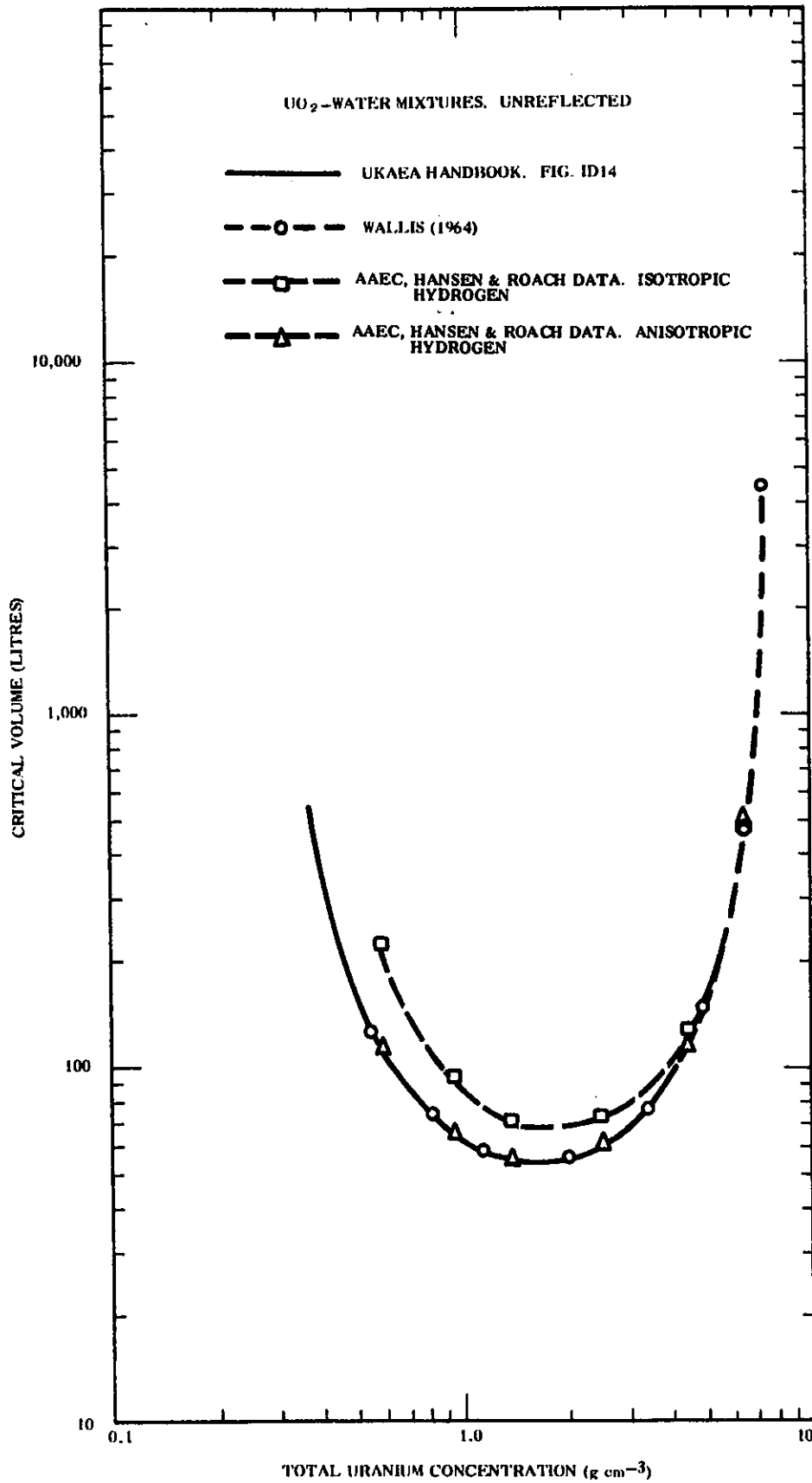


FIGURE 6. MINIMUM CRITICAL VOLUME AGAINST CONCENTRATION FOR 5 wt % URANIUM-235

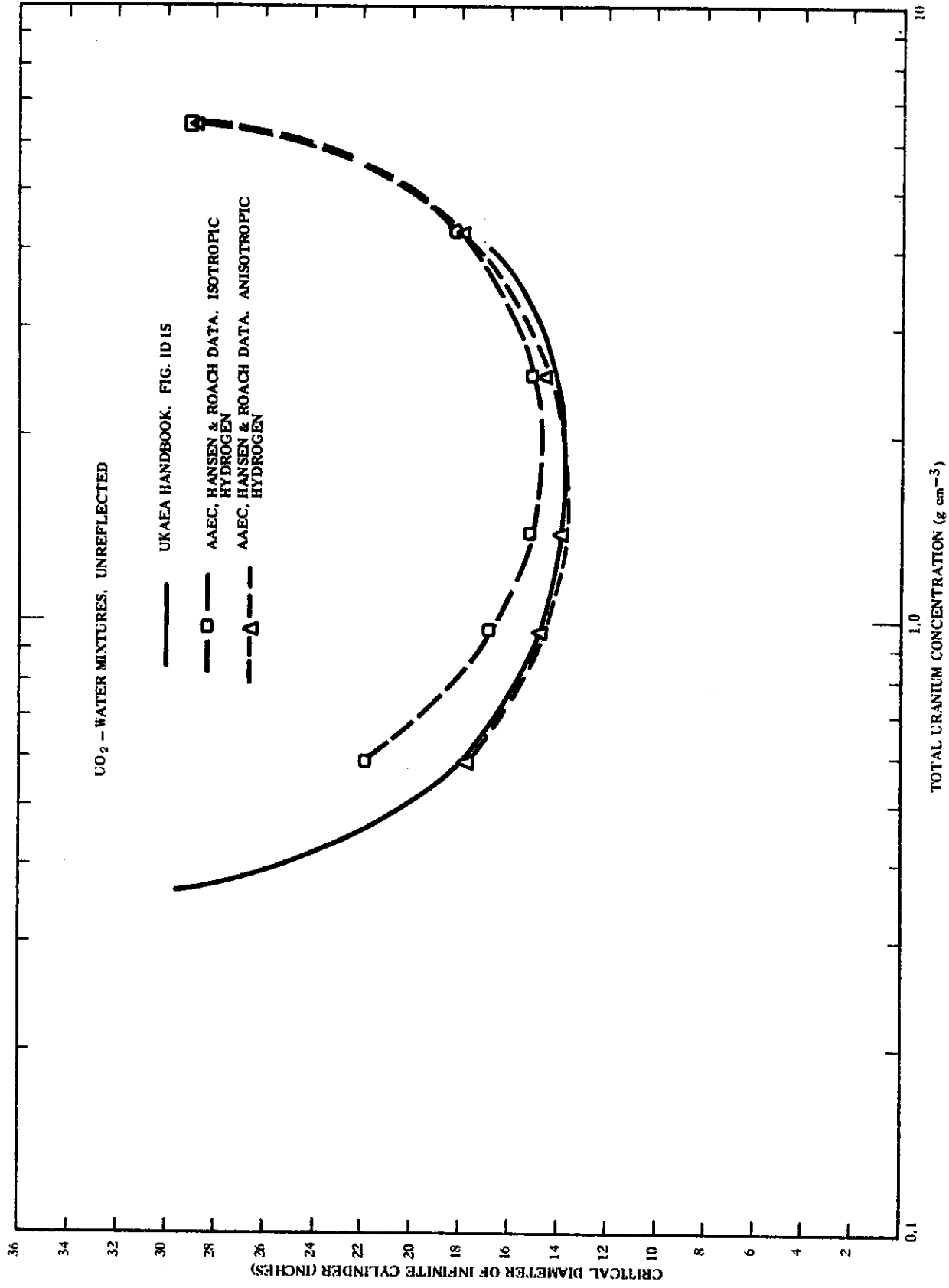


FIGURE 7. MINIMUM CRITICAL CYLINDER DIAMETER AGAINST CONCENTRATION
FOR 5 wt % URANIUM-235

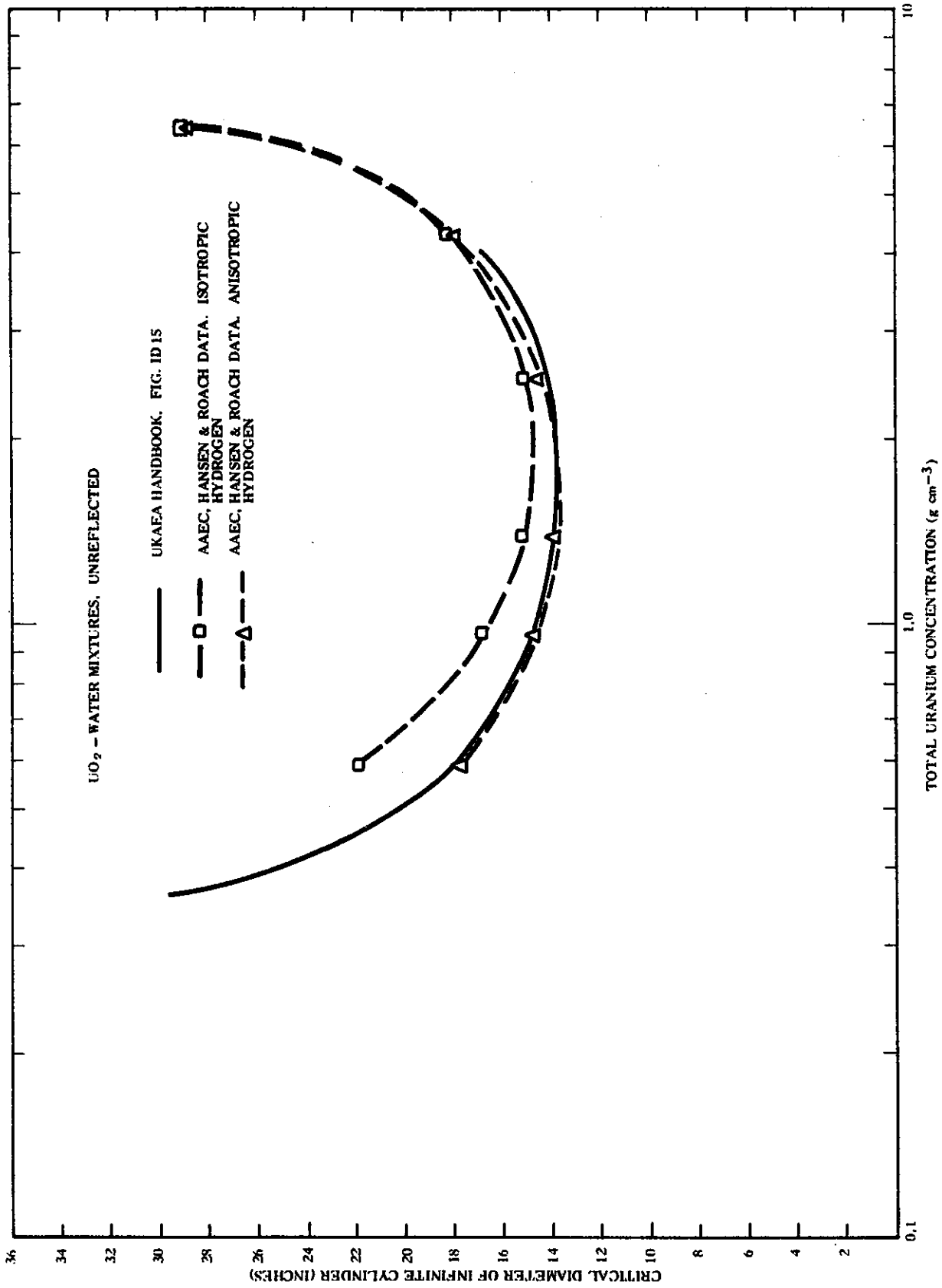


FIGURE 8. MINIMUM CRITICAL SLAB THICKNESS AGAINST CONCENTRATION
FOR 5 wt % URANIUM-235

APPENDIX A
CROSS SECTION IDENTIFICATION NUMBERS USED
IN THE AAEC CALCULATIONS

Identification Number	Material	Source of Data
11	Hydrogen	Hansen & Roach
12	Hydrogen ANISO 1	Hansen & Roach
13	Hydrogen ANISO 2	Hansen & Roach
20	Oxygen	Hansen & Roach
92	U-238-5 Sig P = 200	Hansen & Roach
95	U-238-8 Sig P = 1,000	Hansen & Roach
107	U-235-YR Inf	Hansen & Roach
115	U-235-8R Sig P = 1,000	Hansen & Roach
116	U-235-9R Sig P = 2K	Hansen & Roach
117	U-235-10R Sig P = 4K	Hansen & Roach
128	U-235-11R Sig P = 6K	Hansen & Roach
129	U-235-12R Sig P = 10K	Hansen & Roach
119	U-238-2R Sig P = 40	Hansen & Roach
121	U-238-4R Sig P = 100	Hansen & Roach
122	U-238-5R Sig P = 200	Hansen & Roach
123	U-238-6R Sig P = 400	Hansen & Roach
124	U-238-7R Sig P = 600	Hansen & Roach

APPENDIX B

URANIUM-238 ABSORPTION CROSS SECTIONS IN GROUPS 8 TO 12

FOR SIG P = 100 BARNS

Group	Hansen & Roach	Hansen & Roach JRK Mod.	Hansen & Roach (R)*
8	1.3	0.923	1.0
9	2.9	2.059	2.9
10	7.2	5.112	4.9
11	4.5	3.195	4.5
12	8.2	5.822	8.0

* ORNL ANISN cross section library,
preferred data.

