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A TWO–GROUP, THREE–REGION, FULLY REFLECTED CYLINDRICAL REACTOR PROGRAM FOR THE IBM 1620

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J. J. THOMPSON *
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ABSTRACT

This program was prepared as a pilot program for a larger computer and handles symmetrical reactors with core, side reflector and end reflectors, using 10 radial and 10 axial mesh regions. The output consists of the effective multiplication constant, the two flux distributions, and the fission source distribution. The extrapolated Liebmann process is used for the inner iterations.

CONTENTS

1. INTRODUCTION

Although a 1620 computer with 60,000 digits is not large enough or fast enough to cope with realistic two-dimensional reactor calculations, it is extremely useful for testing small scale programs which can later be expanded in size for use on a larger machine. At the same time, such programs may fill an urgent need for preliminary engineering design estimates of power distribution and the effects of variations in geometry.

It was decided to prepare a program for a fully reflected cylinder, axially symmetrical about the centre plane, which would check out the computation of parameters in the finite difference form of the reactor equations for variable mesh length, and test also the speed of the inner and outer iterations. It was also decided that although the initial application should be to a reactor with only three main regions of uniform nuclear properties, the program should be capable of use as a sub-routine for a more ambitious calculation involving space dependent properties over each region.

This report describes the formulation of the reactor equations, the method used for solution, and the program as it exists at the present time. Some information is also given of the results of applications to bare and reflected systems.

2. FORMULATION OF REACTOR EQUATIONS

The upper portion of the cylindrical reactor is as shown in Figure 1. In every cell (i,j) formed by mesh lines i, i+1, j, j+1, parallel to the z and r axes, the coefficients are constant and the diffusion equations for the fluxes are:

$$D_1 \nabla^2 \phi_1 - \Sigma_1 \phi_1 + S_1 = 0 \tag{1a}$$

$$D_2 \nabla^2 \phi_2 - \Sigma_2 \phi_2 + S_2 = 0 \tag{1b}$$

$$S_1 = S/k \tag{2}$$

$$S = \eta_1 \Sigma_{f1} \phi_1 + \eta_2 \Sigma_{f2} \phi_2 \tag{3}$$

$$S_2 = \Sigma_s \phi_1 \ , \tag{4}$$

where k is the effective multiplication factor, that is, the eigenvalue, to be determined. The boundary conditions are, for each group:

$$\left.\begin{array}{l}
D\,\dfrac{\partial\phi}{\partial r} \text{ and } \phi \text{ are continuous across a line of constant } r \\[2ex]
D\,\dfrac{\partial\phi}{\partial z} \text{ and } \phi \text{ are continuous across a line of constant } z \\[2ex]
\dfrac{\partial\phi}{\partial r} = 0 \text{ for } r = 0 \\[2ex]
\dfrac{\partial\phi}{\partial z} = 0 \text{ for } z = 0 \\[2ex]
\phi = 0 \text{ at } r = R_c + R_R \\[2ex]
\phi = 0 \text{ at } z = Z_c + Z_R \ .
\end{array}\right\} \tag{5}$$

To discuss the formulation of the finite difference equations it is convenient to use the notation shown in Figure 2 for a particular cell (i,j).

In Figure 2 the parameters of the cell (i,j) are:

$$\delta P_i = \pi(r_{i+1}^2 - r_i^2) = \pi(r_i + r_{i+1})\,\Delta_i \tag{6a}$$

$$\delta P_{i,j} = 2\pi r_i \delta_j \tag{6b}$$

$$\delta P_{i+1,j} = 2\pi r_{i+1}\delta_j \quad . \tag{6c}$$

The volume element V_{ij} is then:

$$V_{ij} = \delta P_i \;.\; \delta_j \quad , \tag{7}$$

and the means over the cell are:

$$\phi_{ij} = \frac{1}{V_{ij}} \int_{r_i}^{r_{i+1}} \int_{z_j}^{z_{j+1}} \phi(r,z)\, 2\pi r\, dr\, dz \tag{8a}$$

$$S_{ij} = \frac{1}{V_{ij}} \int_{r_i}^{r_{i+1}} \int_{z_j}^{z_{j+1}} S(r,z)\, 2\pi r\, dr\, dz \quad . \tag{8b}$$

The basic equation of diffusion in a group is, from (1):

$$D\nabla^2\phi - \Sigma\phi + S = 0 \quad .$$

Integrating over cell (i,j) gives:

$$\delta P_{i+1,j}\, \overline{(D\nabla_r\phi)}_{i+1,j} - \delta P_{i,j}\, \overline{(D\nabla_r\phi)}_{i,j} + \delta P_i \overline{(D\nabla_z\phi)}_{i,j+1} - \delta P_i \overline{(D\nabla_z\phi)}_{i,j} -$$

$$\Sigma_{ij}\,\phi_{ij}\,V_{ij} + S_{ij}\,V_{ij} = 0 \quad , \tag{9}$$

where the mean currents are defined by:

$$\overline{(D\nabla_r\phi)}_{i+1,j} = \frac{1}{\delta P_{i+1,j}} \int_{z_j}^{z_{j+1}} D\, \frac{\partial}{\partial r}\phi(r_{i+1}, z)\, 2\pi r_{i+1}\, dz$$

$$= \frac{D_{i,j}}{\delta_j} \int_{z_j}^{z_{j+1}} \frac{\partial\,\phi(r_{i+1}, z)}{\partial r}\, dz \quad , \tag{10}$$

with the flux ϕ defined inside the cell (i,j).

Now consider the radial variation of flux between cells (i,j) and (i + 1, j) as shown in Figure 3. The approximation is made that, in terms of a boundary flux ϕ^*,

$$\overline{(D\nabla_r\phi)}_{i+1,j} = \frac{2}{\Delta_i}(\phi^* - \phi_{i,j})\, D_{i,j} \quad \text{for cell (i,j)} \tag{11a}$$

$$\overline{(D\nabla_r\phi)}_{i+1,j} = \frac{2}{\Delta_{i+1}}(\phi_{i+1,j} - \phi^*)\, D_{i+1,j} \quad \text{for cell (i+1,j)} \quad . \tag{11b}$$

From continuity of neutron current,

$$\frac{D_{i,j}}{\Delta_i}(\phi^* - \phi_{i,j}) = \frac{D_{i+1,j}}{\Delta_{i+1}}(\phi_{i+1,j} - \phi^*) \quad .$$

$$\therefore\ \phi^{*} = \left(\frac{D_{i+1,j}}{\Delta_{i+1}}\ \phi_{i+1,j} + \frac{D_{i,j}}{\Delta_i}\ \phi_{i,j}\right) \Big/ \left(\frac{D_{i+1,j}}{\Delta_{i+1}} + \frac{D_{i,j}}{\Delta_i}\right) .$$

$$\therefore \overline{(D\nabla_r \phi)}_{i+1,j} = 2\ \frac{D_i}{\Delta_i} \cdot \frac{D_{i+1}}{\Delta_{i+1}} (\phi_{i+1,j} - \phi_{i,j}) \Big/ \left(\frac{D_{i+1,j}}{\Delta_{i+1}} + \frac{D_{i,j}}{\Delta_i}\right) \qquad (12)$$

A similar argument applies for the z direction and the results can be written:

$$\overline{(D\nabla_r \phi)}_{i+1,j} = D_j^{i,i+1}(\phi_{i+1,j} - \phi_{i,j}) \qquad (13a)$$

$$\overline{(D\nabla_r \phi)}_{i,j+1} = D_i^{j,j+1}(\phi_{i,j+1} - \phi_{i,j}) , \qquad (13b)$$

where
$$D_j^{i,i+1} = 2\left(\frac{D_{i+1,j}}{\Delta_{i+1}}\right)\left(\frac{D_{i,j}}{\Delta_i}\right) \Big/ \left(\frac{D_{i+1,j}}{\Delta_{i+1}} + \frac{D_{i,j}}{\Delta_i}\right) \qquad (14a)$$

$$D_i^{j,j+1} = 2\left(\frac{D_{i,j+1}}{\delta_{j+1}}\right)\left(\frac{D_{i,j}}{\delta_j}\right) \Big/ \left(\frac{D_{i,j+1}}{\delta_{j+1}} + \frac{D_{i,j}}{\delta_j}\right) . \qquad (14b)$$

Equation 9 now becomes, for the general interior point:

$$\begin{aligned} &\delta P_{i+1,j}\ D_j^{i,i+1}(\phi_{i+1,j} - \phi_{i,j}) - \delta P_{i,j}\ D_j^{i-1,i}(\phi_{i,j} - \phi_{i-1,j}) + \\ &\quad \delta P_i\ D_i^{j,j+1}(\phi_{i,j+1} - \phi_{i,j}) - \delta P_i\ D_i^{j-1,j}(\phi_{i,j} - \phi_{i,j-1}) - \\ &\quad \Sigma_{ij}\ \phi_{i,j} V_{ij} + S_{ij} V_{ij} = 0 , \end{aligned} \qquad (15)$$

that is,

$$\begin{aligned} &\tilde{b}_{ij}(\phi_{i+1,j} - \phi_{i,j}) - \tilde{c}_{ij}(\phi_{i,j} - \phi_{i-1,j}) + \tilde{d}_{ij}(\phi_{i,j+1} - \phi_{i,j}) - \\ &\quad \tilde{e}_{ij}(\phi_{i,j} - \phi_{i,j-1}) - \Sigma_{ij} V_{ij}\phi_{i,j} + \tilde{a}_{ij} S_{ij} = 0 , \end{aligned} \qquad (16)$$

that is,

$$\phi_{i,j} = a_{ij} S_{ij} + b_{ij}\phi_{i+1,j} + c_{ij}\phi_{i-1,j} + d_{ij}\phi_{i,j+1} + e_{ij}\phi_{i,j-1} , \qquad (17a)$$

where
$$a_{ij} = \tilde{a}_{ij}/Z_{ij} \qquad (17b)$$

and
$$Z_{ij} = \tilde{b}_{ij} + \tilde{c}_{ij} + \tilde{d}_{ij} + \tilde{e}_{ij} + \Sigma_{ij}V_{ij} . \qquad (17c)$$

The boundary conditions are satisfied as follows:

Along the radial axis $z = 0$,

$$e_{ij} = 0$$

$$Z_{ij} = \tilde{b}_{ij} + \tilde{c}_{ij} + \tilde{d}_{ij} + \Sigma_{ij} V_{ij} .$$

Along the z axis $r = 0$,

$$c_{ij} = 0$$

$$Z_{ij} = b_{ij} + d_{ij} + e_{ij} + \Sigma_{ij} V_{ij} .$$

At the boundary $r = R_c + R_R$,

$$b_{ij} = 0$$

and $$Z_{ij} = 2\tilde{b}_{ij} + \tilde{c}_{ij} + \tilde{d}_{ij} + \tilde{e}_{ij} + \Sigma_{ij} V_{ij} \quad .$$

At the boundary $z = Z_c + Z_R$,

$$d_{ij} = 0$$

and $$Z_{ij} = \tilde{b}_{ij} + \tilde{c}_{ij} + 2\tilde{d}_{ij} + \tilde{e}_{ij} + \Sigma_{ij} V_{ij} \ .$$

However, it is more convenient to compute the values of Z_{ij} correctly, but to use the general expression (17a) for every cell by setting the fluxes always equal to zero in the cells outside the true boundaries, that is, in the cells:

$(1,j)$, $(11,j)$, $j = 1$ to 10

$(i,1)$, $(i,11)$, $i = 1$ to 10 .

3. INNER AND OUTER ITERATIONS

In terms of the vector ϕ whose elements are the cell fluxes, the equations for a group can be written, from (17a):

$$\phi = M\phi + AS , \tag{18}$$

where M has zero diagonal elements. For a given S, (18) is solved by the extrapolated Liebmann process:

$$\phi^{n}_{i,j} = \phi^{n-1}_{i,j} + \epsilon(\phi^{*}_{i,j} - \phi^{n-1}_{i,j}) \tag{19a}$$

$$\phi^{*}_{i,j} = a_{ij} S_{ij} + b_{ij}\,\phi^{n-1}_{i+1,j} + c_{ij}\,\phi^{n}_{i-1,j} + d_{ij}\,\phi^{n-1}_{i,j+1} + e_{ij}\,\phi^{n}_{i,j-1} \tag{19b}$$

The optimum extrapolation parameter ϵ is given by

$$\epsilon = 2/\{1 + \sqrt{1-\lambda^2_M}\} , \tag{20}$$

where λ_M is the dominant eigenvalue of the matrix M.

In practice λ_M is estimated from

$$\lambda_M = \psi' M \psi / \psi' \psi \tag{21}$$

where ψ is an easily calculated function that satisfies the boundary conditions on the flux. Note that it is convenient to use ψ as the initial guess to start the iterations as well as to estimate λ_M and hence ϵ . In this program ψ is taken as parabolic in both radial and axial directions initially and provision is made to enable ϵ to be estimated from the current flux solutions.

Inner iterations in each group continue until the convergence criterion is satisfied, that is, if ϵ^n is the error vector, $\phi^n - \phi^{n-1}$, then

$$|\epsilon^{n'}\epsilon^n| < \eta\,|\phi^{n'}\phi^n| \ . \tag{22}$$

On completion of the solution of the group equations, the effective multiplication constant k is estimated by:

$$(S_1)^n_{ij} = \eta_1\,\Sigma_{f1}\,(\phi_1)^n_{ij} + \eta_2\,\Sigma_{f2}\,(\phi_2)^n_{ij} \tag{23a}$$

$$k = \frac{\sum\limits_i \sum\limits_j (S_1)^n_{ij}\ V_{ij}\ (S_1)^n_{ij}}{\sum\limits_i \sum\limits_j (S_1)^n_{ij}\ V_{ij}\ S^n_{ij}} \quad . \tag{23b}$$

Replacement of S_{ij} by $(S_1)_{ij}/k$ provides the new source for a new outer iteration.

In this program no convergence criterion for k is written in, but several measures of accuracy are provided to indicate the progression of the solution. These consist of the upper and lower limits on k :

$$k_{max} = \left| \frac{(S_1)_{ij}}{S_{ij}} \right|_{max}$$

$$k_{min} = \left| \frac{(S_1)_{ij}}{S_{ij}} \right|_{min} , \tag{24}$$

and measures of the relative error, that is,

$$\sum_i \sum_j \left| S_{ij}^{n} - S_{ij}^{n+1} \right|^2 \quad \text{and} \quad \sum_i \sum_j \left| S_{ij}^{n+1} \right|^2 . \tag{25}$$

4. PROBLEM SPECIFICATION

The location relative to the mesh lines of the three regions (1) side reflector, (2) top reflector, and (3) core, are specified by the indices $i = M$ and $j = N$. The dimensions are given by the length R_c, R_R, Z_c, and Z_R .

The locations of the mesh lines are defined by two non-dimensional parameters p and q defined by:

$$p = r/R_c \quad (0 < r \le R_c)$$

$$p = (r - R_c)/R_R \; (R_c < r \le R_c + R_R 0)$$

$$q = z/Z_c \quad (0 < z \le Z_c)$$

$$q = (z - Z_c)/Z_R (Z_c < z \le Z_c + Z_R) .$$

Note that both sets of the 10 values p and q must contain two values 1.0 corresponding to the core reflector interface and the outer boundary. For example, for a system of overall radius 70 cm, half height 90 cm,

$M = 6$, $N = 5$

$p = .2 \quad .4 \quad .6 \quad .8 \quad 1.0 \quad .2 \quad .4 \quad .6 \quad .8 \quad 1.0$

$q = .3 \quad .6 \quad .8 \quad 1.0 \quad .3 \quad .5 \quad .7 \quad .8 \quad .9 \quad 1.0$

$R_c = 40$, $Z_c = 40$ cm

$R_R = 30$, $Z_R = 50$ cm ,

corresponds to mesh lines located at:

$r = 8, \; 16, \; 24, \; 32, \; 40, \; 46, \; 52, \; 58, \; 64, \; 70$ cm

$z = 12, \; 24, \; 32, \; 40, \; 55, \; 65, \; 75, \; 80, \; 85, \; 90$ cm.

On completion of a calculation of k and the associated flux distributions it is possible to change the core radius and height via the console typewriter and repeat the calculation. Note that in this new calculation the thicknesses of the top and side reflectors are not changed, and that the previously computed fluxes are used as the starting values for the iterative procedure.

5. PROGRAM DESCRIPTION

The problem flow chart is shown in Figure 4, and the FORTRAN program is listed in Table 1. It will be seen that the basic nuclear data are stored for each cell, although in this program the data are common for all cells of a particular region (1), (2), or (3). This feature will enable the program to be readily modified to cope with fully space dependent properties.

Note that use is made of sense switches, pauses, and the console typewriter to enable the operator to control the flow, to alter accuracy figures, and extrapolation parameters, and to obtain information as to speed of convergence. The numbers adjacent to blocks in the flow chart refer to the associated statement numbers in the FORTRAN program.

6. ILLUSTRATIVE CALCULATIONS

In the process of developing and testing the program, an extensive series of calculations has been performed for a cylindrical core of fixed composition with $R_c = Z_c$, with and without side and end reflectors of the same thickness (that is, $R_R = Z_R$) also of fixed composition. Some of these results are given here.

(a) For a bare cylinder of radius 49.5 cm, two group theory predicts $k = 1.042$. The computed value was $1.044 \pm .001$ where the limits refer to the difference between k, k_{min}, and k_{max}.

(b) $R_c = Z_c = 45.0, \quad R_R = Z_R = 20$ cm

$$k = .9869 \pm .0006 \qquad (10),$$

that is, 10 outer iterations were required.

(c) $R_c = Z_c = 47.5, \quad R_R = Z_R = 20$ cm

$$k = 1.00197 \pm .00001 \qquad (16).$$

This calculation followed directly after (b) by using the accept instruction to change the core size.

(d) $R_c = Z_c = 35, \quad R_R = Z_R = 40$ cm

$$k = .9812 \pm .003 \qquad (10).$$

(e) $R_c = Z_c = 37.5, \quad R_R = Z_R = 40$ cm

$$k = 1.0006 \pm .0002 \qquad (11).$$

Again (e) followed directly from (d).

The above calculations were performed with both core and reflector subdivided in both directions into equal parts, that is,

$$p_i = q_j = .2,\ .4,\ .6,\ .8,\ 1.0,\ .2,\ .4,\ .6,\ .8,\ 1.0$$

Figure 5 shows the computed cell fluxes in the axial direction for different radii, for case (e). As these computed values refer to mean fluxes over a cell, the lines joining the points in Figure 5 are intended only to delineate the families of points. With only 10 points it is not possible to obtain a continuous flux pattern, but it is apparent that the results shown in Figure 5 do give a good picture of the flux and hence the power variation over the core.

To check on the effect of varying the mesh spacing, problems (c) and (e) were recomputed with

$$p_i = q_j = .3,\ .6,\ .8,\ .9,\ 1.0,\ .1,\ .2,\ .4,\ .7,\ 1.0$$

and gave the following results:

(c) (i) $R_c = Z_c = 47.5$, $R_R = Z_R = 20$ cm

$$k = 1.0029 \pm .0001 \qquad (28).$$

(e) (i) $R_c = Z_c = 37.5$, $R_R = Z_R = 40$ cm

$$k = 1.0065 \pm .0001 \qquad (21).$$

From (b) and (c), the system with the 20 cm reflector has

$$\frac{\partial k}{\partial R_c} = .006/\text{cm} .$$

while from (d) and (e) the system with the 40 cm reflector has

$$\frac{\partial k}{\partial R_c} = .008/\text{cm} .$$

It follows that the differences in critical radius and height obtained by using the two different mesh spacings investigated will be less than 1 cm for both systems. As might be expected, the more heavily reflected reactor is more sensitive to mesh spacing because of the greater thermal flux rise and fast flux dip near the core reflector boundary.

These results indicate that the existing program can be a valuable tool for initial critical size determinations and power distributions for the symmetrical fully reflected reactors for which it was designed. However, it would appear to be worth improving the convergence rate of the outer iterations. The non-uniform mesh spacing obviously produced significant decreases in the convergence rates obtained with the uniform spacings.

The speed of the computations may be estimated from the following performance figures.

Time to compute all co-efficients = 4 minutes

Time for each inner iteration = 30 seconds

Time for each outer iteration comprising one inner iteration in each group = 1 minute 10 seconds .

7. CONCLUSION

The program as described is a successful two-dimensional, two-group, diffusion program but it has obvious limitations in its existing form. While it could undoubtedly be simply enlarged in scope with more points and more groups for use on a larger computer, it is probably more useful to continue with developmental work on the 1620, aimed at increasing the rate of convergence of the outer iterations.

TABLE 1

TWO-GROUP REFLECTED CYLINDER PROGRAM

```
C     REFLECTED CYLINDER, TWO GROUP, 1620
      DIMENSION DD1(10,10),DC1(10,10),DD2(10,10),DC2(10,10),FM2(10,10)
      DIMENSION FM1(10,10),SD(10,10)
      DIMENSION S1(10,10),S(10,10),PH1(12,12),PH2(12,12)
      DIMENSION AS1(10,10),AS2(10,10),A1(10,10),A2(10,10),B1(10,10)
      DIMENSION B2(10,10),C1(10,10),C2(10,10),D1(10,10),D2(10,10)
      DIMENSION E1(10,10),E2(10,10),R(11),DR(10),Z(11)
      DIMENSION DZ(10),P(10),Q(10),V(10,10)
      READ101,M,N
      K=1
      MA=M
      MB=10
      NA=1
      NB=10
      DO111J=2,11
      PH1(1,J)=0.
      PH2(1,J)=0.
      PH1(12,J)=0.
  111 PH2(12,J)=0.
      DO112I=2,11
      PH1(I,1)=0.
      PH2(I,1)=0.
      PH1(I,12)=0.
  112 PH2(I,12)=0.
    1 READ102,AD1,AC1,AM1
      READ103,AD2,AC2,AM2,ASD
      DO204 I=MA,MB
      DO204 J=NA,NB
      DD1(I,J)=AD1
      DC1(I,J)=AC1
      FM1(I,J)=AM1
      DD2(I,J)=AD2
      DC2(I,J)=AC2
      FM2(I,J)=AM2
  204 SD(I,J)=ASD
      GO TO(2,3,4),K
    2 MA=1
      MB=M-1
      NA=N
      K=2
      GO TO 1
    3 NA=1
      NB=N-1
      K=3
      GO TO 1
```

TABLE 1 (continued)

```
  4 READ104,RC,RR,ZC,ZR
    READ105,P(1),P(2),P(3),P(4),P(5),P(6),P(7),P(8),P(9),P(10)
    READ106,Q(1),Q(2),Q(3),Q(4),Q(5),Q(6),Q(7),Q(8),Q(9),Q(10)
    KC=1
    R(1)=0.
    Z(1)=0.
  5 DO113I=2,M
113 R(I)=RC*P(I-1)
    MC=M+1
    DO114I=MC,11
114 R(I)=RC+RR*P(I-1)
    DO115J=2,N
115 Z(J)=ZC*Q(J-1)
    NC=N+1
    DO116J=NC,11
116 Z(J)=ZC+ZR*Q(J-1)
    DO117I=1,10
117 DR(I)=R(I+1)-R(I)
    DO118J=1,10
    DZ(J)=Z(J+1)-Z(J)
    B1(10,J)=2.*3.1415926*R(11)*DZ(J)/DR(10)
    B2(10,J)=B1(10,J)*DD2(10,J)
    B1(10,J)=B1(10,J)*DD1(10,J)
    C1(1,J)=0.
    C2(1,J)=0.
    DO118I=1,9
    B1(I,J)=4.*3.1415926*DZ(J)*R(I+1)
    WA=DD1(I,J)/DR(I)
    B=DD2(I,J)/DR(I)
    WC=DD1(I+1,J)/DR(I+1)
    WD=DD2(I+1,J)/DR(I+1)
    B2(I,J)=B1(I,J)*WB*WD/(WB+WD)
    B1(I,J)=B1(I,J)*WA*WC/(WA+WC)
    C1(I+1,J)=B1(I,J)
118 C2(I+1,J)=B2(I,J)
    DO119I=1,10
    D1(I,10)=3.1415926*DR(I)*(R(I)+R(I+1))
    V(I,10)=D1(I,10)
    D1(I,10)=D1(I,10)/DZ(10)
    D2(I,10)=D1(I,10)*DD2(I,10)
    D1(I,10)=D1(I,10)*DD1(I,10)
    E1(I,1)=0.
    E2(I,1)=0.
    DO120J=1,9
    V(I,J)=V(I,10)*DZ(J)
    WA=DD1(I,J)/DZ(J)
    WB=DD2(I,J)/DZ(J)
    WC=DD1(I,J+1)/DZ(J+1)
    WD=DD2(I,J+1)/DZ(J+1)
    D2(I,J)=2.*V(I,10)*WB*WD/(WB+WD)
    D1(I,J)=2.*V(I,10)*WA*WC/(WA+WC)
```

TABLE 1 (continued)

```
      E1(I,J+1)=D1(I,J)
  120 E2(I,J+1)=D2(I,J)
  119 V(I,10)=V(I,10)*DZ(10)
      DO121I=1,10
      DO122J=1,10
      A1(I,J)=DC1(I,J)*V(I,J)+B1(I,J)+C1(I,J)+D1(I,J)+E1(I,J)
  122 A2(I,J)=DC2(I,J)*V(I,J)+B2(I,J)+C2(I,J)+D2(I,J)+E2(I,J)
      A1(I,10)=D1(I,10)+A1(I,10)
      A2(I,10)=D2(I,10)+A2(I,10)
      D1(I,10)=0.0
  121 D2(I,10)=0.
      DO123J=1,10
      A1(10,J)=A1(10,J)+B1(10,J)
      A2(10,J)=A2(10,J)+B2(10,J)
      B1(10,J)=0.
  123 B2(10,J)=0.
      DO124I=1,10
      DO124J=1,10
      A1(I,J)=1./A1(I,J)
      A2(I,J)=1./A2(I,J)
      B1(I,J)=B1(I,J)*A1(I,J)
      B2(I,J)=B2(I,J)*A2(I,J)
      C1(I,J)=C1(I,J)*A1(I,J)
      C2(I,J)=C2(I,J)*A2(I,J)
      D1(I,J)=D1(I,J)*A1(I,J)
      D2(I,J)=D2(I,J)*A2(I,J)
      E1(I,J)=E1(I,J)*A1(I,J)
      E2(I,J)=E2(I,J)*A2(I,J)
      A1(I,J)=V(I,J)*A1(I,J)
  124 A2(I,J)=V(I,J)*A2(I,J)
      GO TO(6,7)KC
    6 RLA=1./(RC+RR)
      RZA=1./(ZC+ZR)
      AMA=DC2(1,1)/SD(1,1)
      DO125I=1,10
      PH2(I+1,11)=1.-(R(I)*RLA)**2
      DO125J=1,10
      PH2(I+1,J+1)=PH2(I+1,11)*(1.-(Z(J)*RZA)**2)
      PH1(I+1,J+1)=AMA*PH2(I+1,J+1)
      S1(I,J)=FM1(I,J)*PH1(I+1,J+1)+FM2(I,J)*PH2(I+1,J+1)
  125 AS1(I,J)=A1(I,J)*S1(I,J)
    7 SUM1=0.
      SUM2=0.
      DO126I=1,10
      DO126J=1,10
      SUM1=SUM1+PH1(I+1,J+1)*(B1(I,J)*PH1(I+2,J+1)+C1(I,J)*PH1(I,J+1))
      SUM1=SUM1+PH1(I+1,J+1)*(D1(I,J)*PH1(I+1,J+2)+E1(I,J)*PH1(I+1,J))
  126 SUM2=SUM2+PH1(I+1,J+1)*PH1(I+1,J+1)
      EP1=SUM1/SUM2
      EP1=2./(1.+SQR(1.-EP1*EP1))
      SUM1=0.
```

TABLE 1 (continued)

```
      SUM2=0.
      DO127I=1,10
      DO127J=1,10
      SUM1=SUM1+PH2(I+1,J+1)*(B2(I,J)*PH2(I+2,J+1)+C2(I,J)*PH2(I,J+1))
      SUM1=SUM1+PH2(I+1,J+1)*(D2(I,J)*PH2(I+1,J+2)+E2(I,J)*PH2(I+1,J))
127   SUM2=SUM2+PH2(I+1,J+1)*PH2(I+1,J+1)
      EP2=SUM1/SUM2
      EP2=2./(1.+SQR(1.-EP2*EP2))
      TYPE 128,EP1,EP2
      ACCEPT129,EP1,EP2,ACP
  8   L=1
  9   ERR=0.
      SUM=0.
      DO130I=1,10
      DO130J=1,10
      COR=AS1(I,J)+B1(I,J)*PH1(I+2,J+1)+C1(I,J)*PH1(I,J+1)
      COR=COR+D1(I,J)*PH1(I+1,J+2)+E1(I,J)*PH1(I+1,J)
      COR=EP1*(COR-PH1(I+1,J+1))
      ERR=ERR+COR*COR
      PH1(I+1,J+1)=PH1(I+1,J+1)+COR
130   SUM=SUM+PH1(I+1,J+1)*PH1(I+1,J+1)
 10   IF(ACP*SUM-ERR)11,21,21
 21   IF(SENSE SWITCH3)15,22
 22   TYPE 139,L,ERR,SUM
      GO TO 15
 11   IF(SENSE SWITCH1)12,14
 12   TYPE 131,L,ERR,SUM
      PAUSE
      IF(SENSE SWITCH2)13,14
 13   ACCEPT132,ACP
 14   GO TO(9,18),L
 15   GO TO(16,17),L
 16   DO132I=1,10
      DO132J=1,10
132   AS2(I,J)=SD(I,J)*PH1(I+1,J+1)*A2(I,J)
      L=2
 18   ERR=0.
      SUM=0.
      DO133I=1,10
      DO133J=1,10
      COR=AS2(I,J)+B2(I,J)*PH2(I+2,J+1)+C2(I,J)*PH2(I,J+1)
      COR=COR+D2(I,J)*PH2(I+1,J+2)+E2(I,J)*PH2(I+1,J)
      COR=EP2*(COR-PH2(I+1,J+1))
      ERR=ERR+COR*COR
      PH2(I+1,J+1)=PH2(I+1,J+1)+COR
133   SUM=SUM+PH2(I+1,J+1)*PH2(I+1,J+1)
      GO TO 10
 17   SUM1=0.0
      SUM2=0.0
      S(1,1)=FM1(1,1)*PH1(2,2)+FM2(1,1)*PH2(2,2)
      SMIN=S(1,1)/S1(1,1)
```

TABLE 1 (continued)

```
      SMAX=SMIN
      DO134I=1,MB
      DO134J=1,NB
      S(I,J)=FM1(I,J)*PH1(I+1,J+1)+FM2(I,J)*PH2(I+1,J+1)
      SRO=S(I,J)/S1(I,J)
      IF(SRO-SMIN)77,77,78
   77 SMIN=SRO
      GO TO 80
   78 IF(SMAX-SRO)79,79,80
   79 SMAX=SRO
   80 SUM1=SUM1+S(I,J)*V(I,J)*S(I,J)
  134 SUM2=SUM2+V(I,J)*S(I,J)*S1(I,J)
      AK=SUM1/SUM2
      RAK=SUM2/SUM1
      ERS=0.
      SUMS=0.
      DO135I=1,MB
      DO135J=1,NB
      S(I,J)=RAK*S(I,J)
      ERS=ERS+(S1(I,J)-S(I,J))**2
      S1(I,J)=S(I,J)
      AS1(I,J)=A1(I,J)*S1(I,J)
  135 SUMS=SUMS+S1(I,J)*S1(I,J)
      TYPE136,AK,ERS,SUMS
      TYPE199,SMIN,SMAX
      PAUSE
      IF(SENSE SWITCH1)20,19
   19 IF(SENSE SWITCH2)7,8
   20 DO137I=1,10
      DO137J=1,10
      PUNCH138,R(I),Z(J),S1(I,J)
      PUNCH1381,R(I),Z(J),PH1(I+1,J+1),PH2(I+1,J+1)
  137 CONTINUE
      ACCEPT 141,RC,ZC
      KC=2
      GO TO 5
      END
```

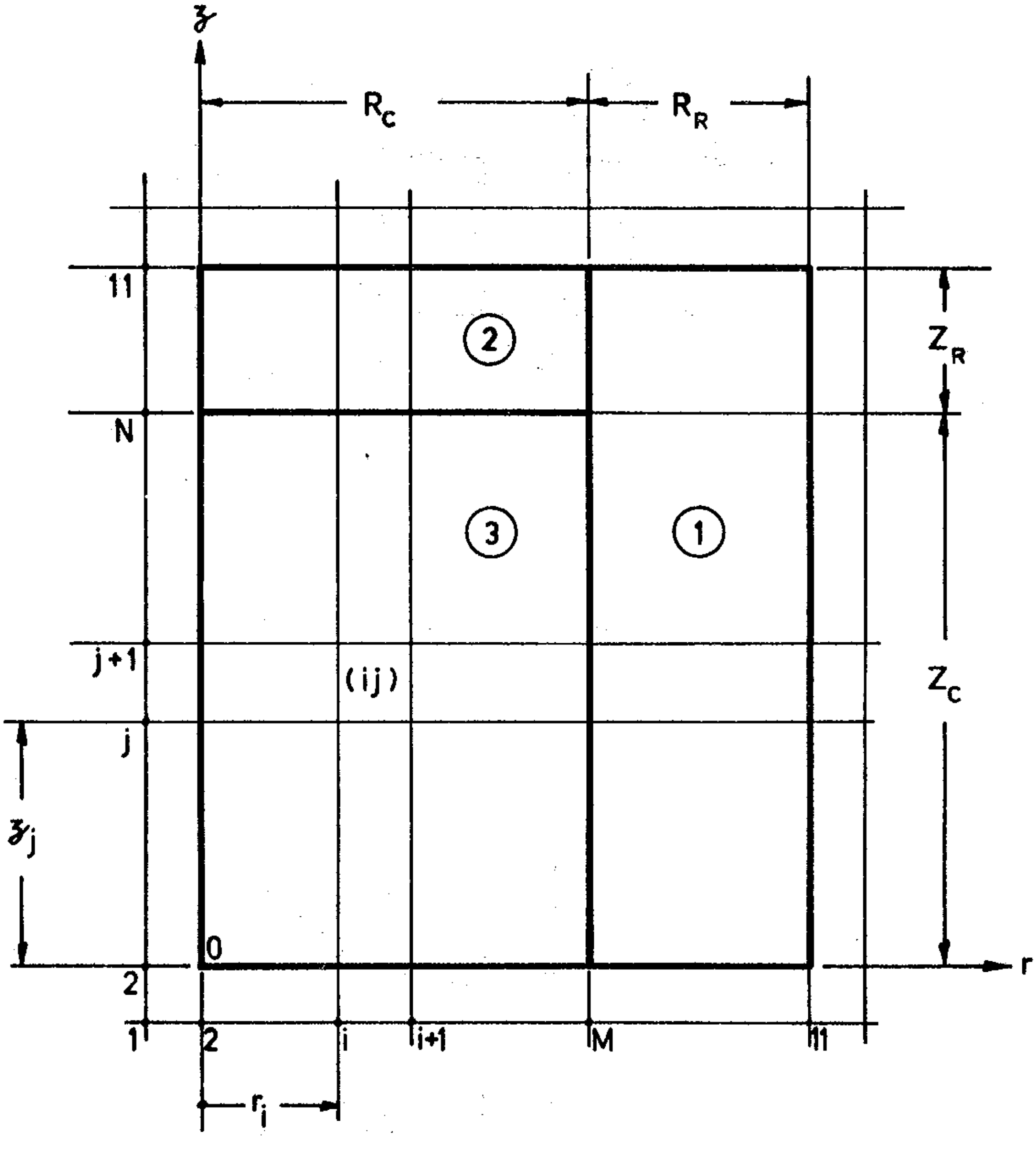

FIGURE 1 REACTOR AND MESH

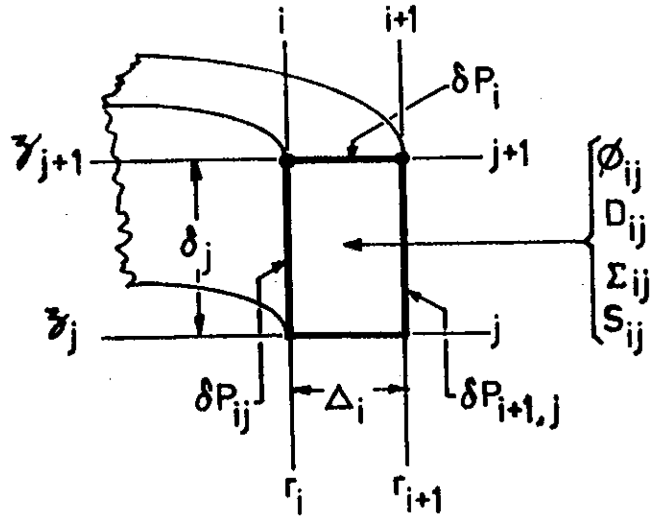

FIGURE 2 CELL NOTATION

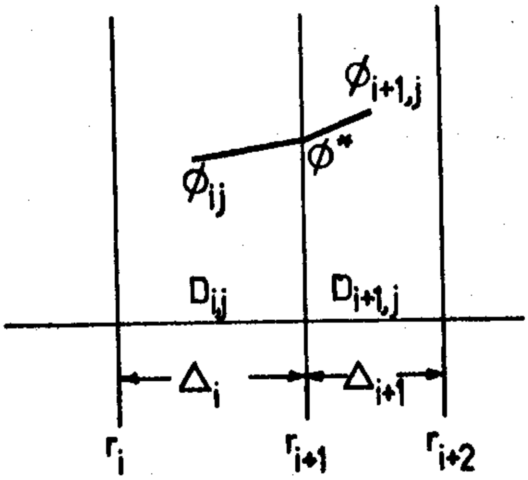

FIGURE 3 RADIAL BOUNDARIES

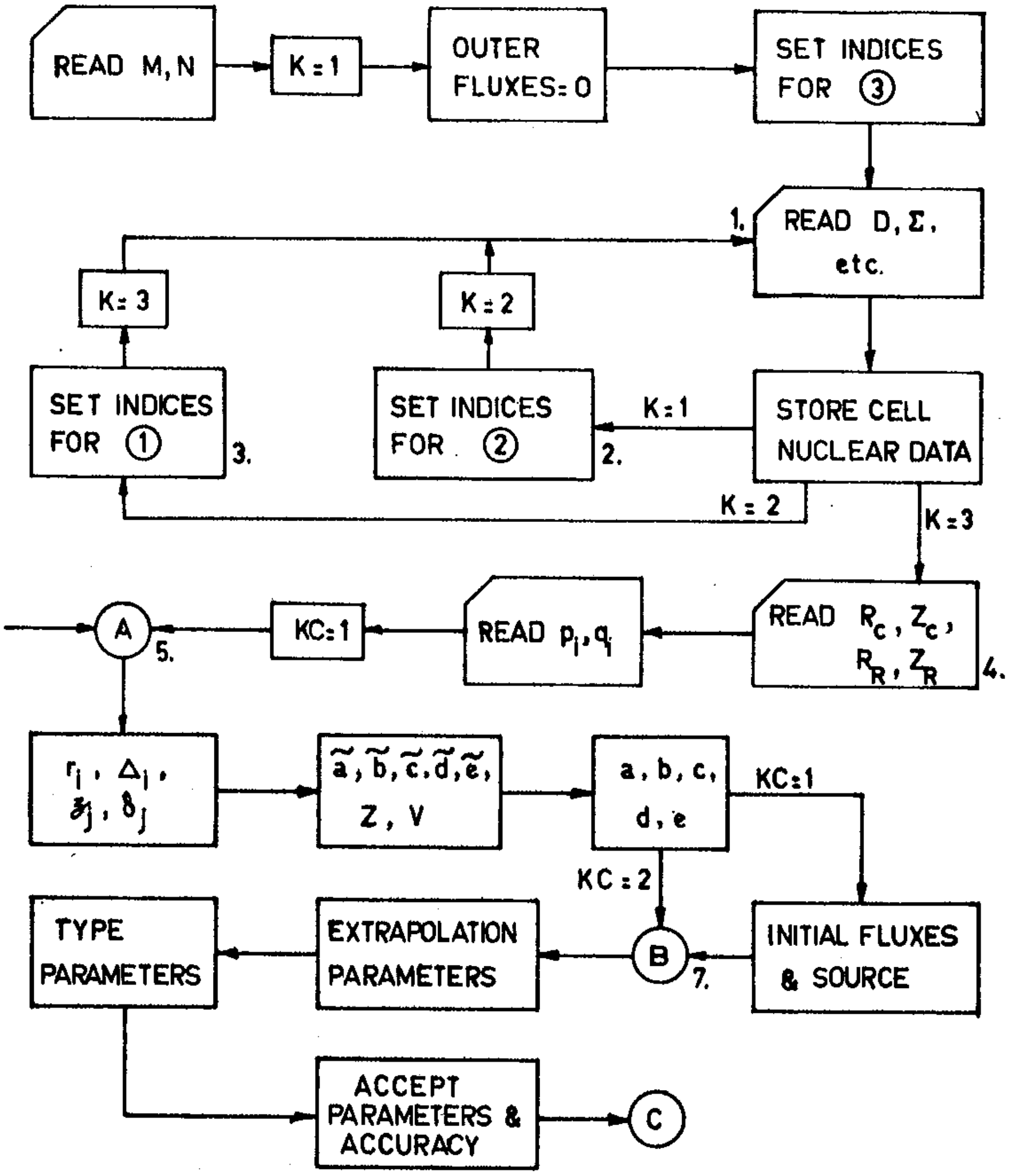

FIGURE 4 (a) FLOW CHART (a) – CALCULATION OF COEFFICIENTS AND PARAMETERS

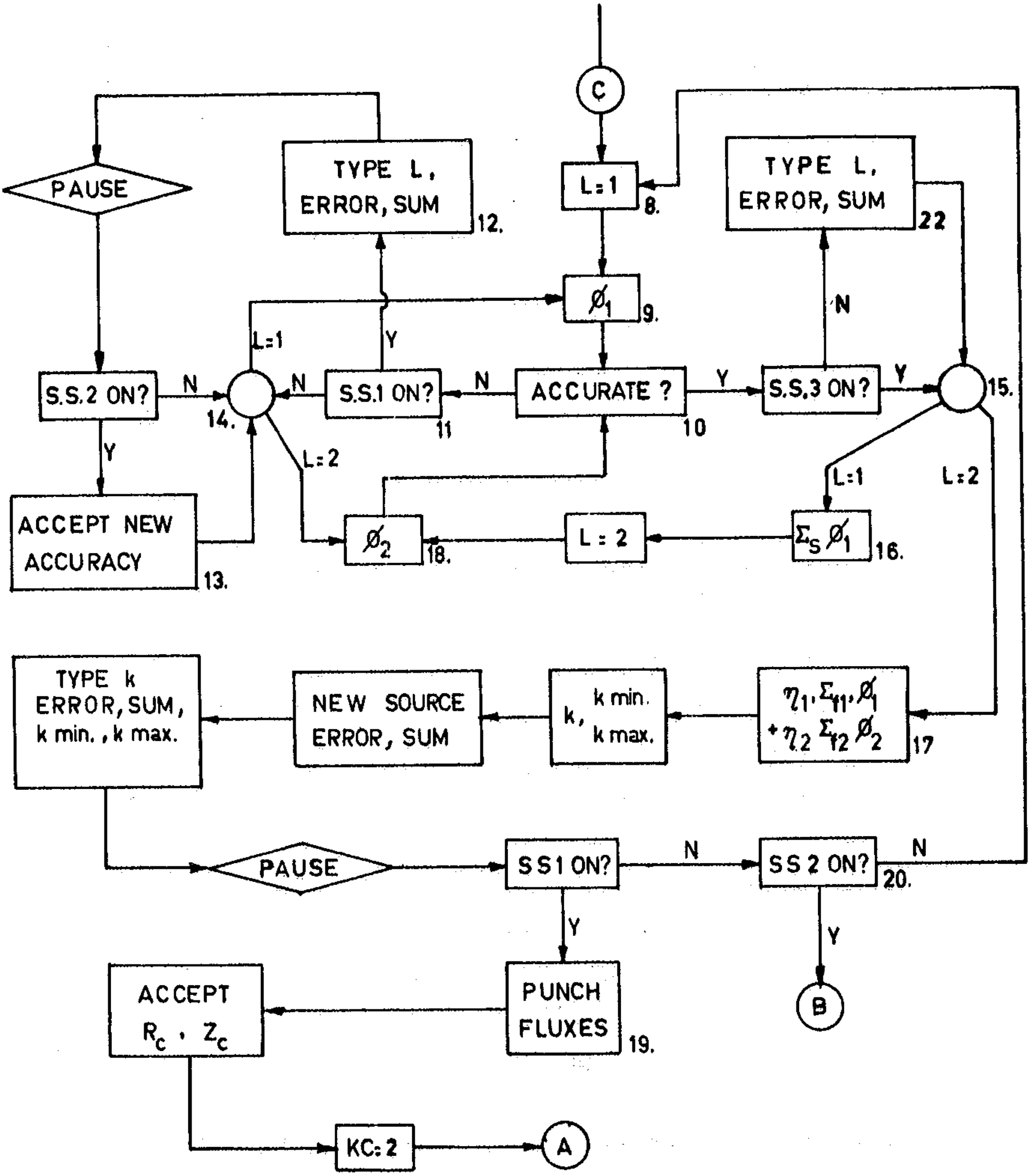

FIGURE 4 (b) FLOW CHART (b) – SOURCE ITERATION

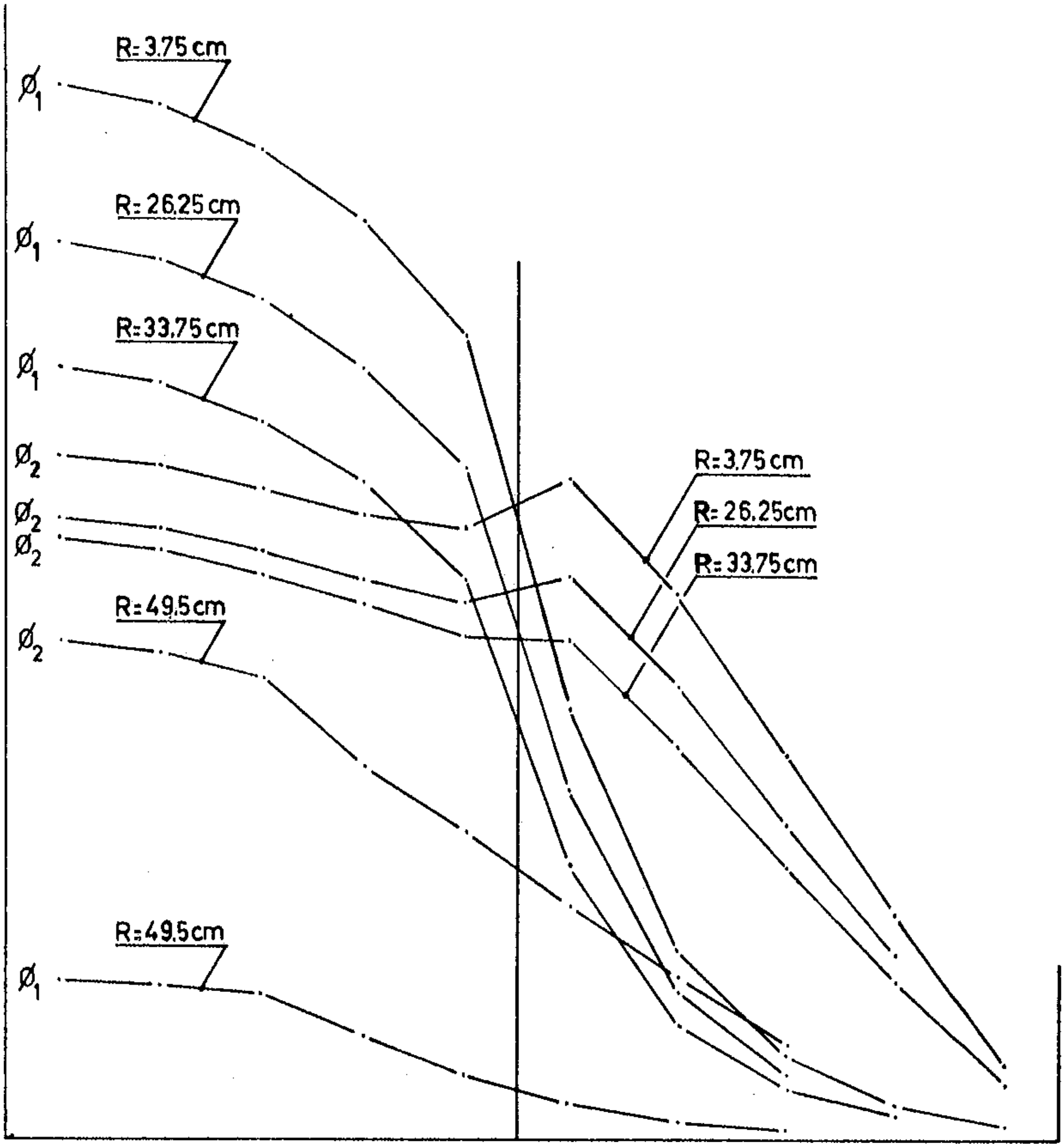

FIGURE 5 AXIAL FLUX PATTERNS

• Computed Values

$R_c = Z_c = 37.5$ cm

$R_R = Z_R = 40$ cm