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RESEARCH ESTABLISHMENT

LUCAS HEIGHTS

REACTIVITY WORTH OF CONTROL RODS IN THE BERYLLIUM OXIDE MODERATED PEBBLE BED REACTOR

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ABSTRACT

Preliminary and final studies are reported for the worth of tubular control rods in a beryllium oxide moderated pebble bed reactor system. In the preliminary study the consideration of ball flow forces the investigation of a number of control rod arrangements. In addition a range of rod sizes is considered. In the final study a particular rod size and pattern is chosen for more detailed investigation.

For the intermediate reactor cores under consideration the control rods must absorb appreciably in the keV region. A feature of the final study is the accurate way the epithermal absorption can be taken into account while using a three group model in the reflected reactor. The preliminary study is not so accurate but is useful for comparative purposes.

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Figure 1 Core cross section for two reactor designs

Figure 2 Survey results

1. INTRODUCTION

Calculations are presented of the reactivity worth of tubular control rods inserted in the beryllium oxide moderated pebble bed reactor described by Ebeling and Hayes (1966).

For the core compositions under consideration about half of the neutron flux is at energies greater than 1 keV, so, for reactivity worth [illegible], the control rods must absorb appreciably in this energy region. Boron is chosen as being the best absorber material in the keV region.

Section 2 describes initial survey calculations where a range of control rod sizes and a number of control rod patterns are considered. A clean core composition is considered at this stage.

From that survey a particular control rod size and arrangement is selected for the final design study (Section 3) in which the core composition is changed from clean to one determined by the equilibrium burn-up model of Bicevskis and Hesse (1966).

2. SURVEY CALCULATIONS

The isotopic composition of the clean homogeneous core, under consideration in the survey calculations, is presented in Table 1.

TABLE 1

CORE COMPOSITION IN THE SURVEY

Isotope	Be^9	O^{16}	Th^{232}	Pu^{239}	Pu^{240}	Pu^{241}
Atoms/barn cm	4.121×10^{-2}	4.166×10^{-2}	2.018×10^{-4}	1.937×10^{-5}	4.204×10^{-6}	1.232×10^{-6}

Two reactor ball flow designs are considered and are shown in Figure 1. In the first design, seven ball inlet and outlet facilities are provided. For the study of ball flow, the core can be considered subdivided into seven hexagonal zones. In the second design, nineteen inlet and outlet facilities are provided.

If ball flow through the core is to be unrestricted, control rods can be positioned only on the boundaries of the hexagonal zones or at the centres of such zones. In the centre positions however, control rods introduce problems at the ball inlet or outlet facility. Therefore three control rod arrangements are considered:

(I) Control rods are positioned at each corner of the seven zone design. The rods are spaced at 80 cm pitch in hexagonal pattern.

(II) In addition to the rods in (I), rods are positioned at the centres of the seven hexagonal zones. The rods are spaced at 80 cm pitch in triangular pattern.

(III) Control rods are positioned at each corner of the nineteen zone design. The rods are spaced at 48.4 cm pitch in hexagonal pattern.

At the stage of the survey calculations, the reactivity worth requirement of the control rods was not very well known. Hence, for each arrangement, a range of control rod sizes was considered. Also, at that stage a complex energy and space description of the problem was not justified. In fact the energy and spatial complexities of the problem were simplified to the extent where control rod worth could be calculated using a simple analytical technique.

2.1 Cylindrical Cell Space Model

By ignoring the finite extent of the control rod patterns of Figure 1, thereby assuming that the rod pattern repeats to infinity in the plane of the Figure, we obtain the boundary condition of zero neutron flux gradient at the triangular or hexagonal cell surrounding each control rod.

The spatial complexity of the problem is then further simplified by considering the control rod fully inserted on the axis of an axially bare cylindrical cell. The volume of the cylindrical cell is equated to the volume of the actual cell. Cohen (1956) showed that this prescription is accurate for square arrays. It should apply even more accurately to triangular arrays which lead to hexagonal cells, but some doubt exists as to the accuracy of application to hexagonal arrays which lead to triangular cells.

2.2 Black-White Two-Group Energy Model

The model used for the energy description of the problem (Spinks 1966) permits epithermal absorption to be included while retaining a two group description. Such a description is necessary for the application of simple analytical techniques in the space dependent calculation.

In the black-white model, a group dividing energy E_b is chosen so the blackness of the rod equals 1/2 at this energy. The model considers the rod transparent above and black below this energy.

As discussed (Spinks 1966), the two-group model can be quite accurate provided

account is taken, in the space-independent spectrum calculation, of absorption by the rod at energies less than E_b. At the stage of the survey calculations the method for improving the spectrum calculation had not been developed. The two-group data were condensed over the spectrum resulting from an energy independent bare core buckling.

2.3 Control Tube Wall Thickness

Choosing the black-white boundary at 1.23 keV we require the dimension of the control tube to achieve this boundary.

With the control tube blackness equal to 1/2 at this energy we find, from the tables of Case et al. (1953) that:

$$\Sigma_a \quad (E_b) \quad \bar{\ell} \;=\; 0.838 \quad \text{for slabs}$$

and

$$\Sigma_a \quad (E_b) \quad \bar{\ell} \;=\; 0.750 \quad \text{for cylinders,}$$

where $\bar{\ell}$ is the mean chord length. Since $\Sigma_a \bar{\ell}$ will approach the slab result for small tube wall thickness, we use the slab value.

We obtain $\bar{\ell} = 5.77$ cm for a tube containing B^{10} at 0.00840 atoms/barn.cm, a concentration which can be achieved using a 30 per cent. dispersion of natural B_4C in steel. Note however that the results are valid for all tubes with the same product of B^{10} atom density and mean chord length. When the tube outside diameter is known the inside diameter can be found from:

$$\bar{\ell} \;=\; \left(d_o^2 \;-\; d_i^2\right) \Big/ d_o \quad .$$

2.4 Analytical Method

Solving the two-group diffusion equations together with the cell and control rod boundary conditions, the requirement for a non-trivial solution produces $D(k') = 0$, where D is a fourth order determinant. With the rod-in multiplication constant k' obtained from this implicit relation, the reactivity worth of the rod is obtained from:

$$\rho \;=\; \frac{k - k'}{k} \quad ,$$

where k is the rod-out multiplication constant.

2.5 Survey Results

Figure 2 shows ρ versus rod outside diameter for each of the three control rod arrangements.

More accurate calculations have been made using the better space-independent spectrum calculation mentioned above and better basic data. For the 6 inch O.D. rod in arrangement (II) we obtain a rod worth of 12.75 per cent. instead of the 7.76 per cent. plotted in Figure 2. The difference, 4.99 per cent., is made up from:

(i)	Black-white model	1.19 per cent.
(ii)	Two group model	2.02 per cent.
(iii)	Basic data	1.78 per cent.

3. FINAL STUDY OF SELECTED ROD SIZE AND ARRANGEMENT

With a detailed investigation of reactivity requirements indicating a shutdown reactivity of about 8 per cent. the 6 inch O.D. rod in triangular arrangement (II) was chosen for more detailed study.

At this stage, Bicevskis and Hesse (1966) had developed an equilibrium spectrum model for the calculation of the average nuclide concentrations in the core at equilibrium burn-up. Using the Gymea code (Pollard and Robinson 1966), the equilibrium model was applied to the feed composition of Table 2 to obtain the average nuclide concentrations corresponding to a value of F.I.F.A. (fissions per initial fissile atom) of 1.75 and a mean power density of 10 watts/cm^3. The control rod worth results of this section are based on such an equilibrium composition.

TABLE 2

FEED COMPOSITION FOR EQUILIBRIUM BURN-UP CALCULATION

Isotope	Be^9	O^{16}	Th^{232}	Pu^{239}	Pu^{240}	Pu^{241}
Atoms/barn.cm	4.06×10^{-2}	4.14×10^{-2}	4.009×10^{-4}	2.315×10^{-5}	5.019×10^{-6}	1.471×10^{-6}

A change in control rod design was made. The axially movable control tube was located on a fixed BeO spine. The spine served two purposes: first a reduction in neutron streaming through the control rod hole in the rod-out situation; second an increase in the capture rate of high energy (> 1 keV) neutrons by the control rod from the slowing down of such neutrons by the spine. For maximum slowing down effect and maximum reduction in streaming, the spine diameter should

be a maximum or the tube wall thickness a minimum. As shown by Spinks (1966) hollow tubes with the same product (mean chord length x absorber concentration) are equivalent. The reactivity worth of the hollow tube will be preserved if the absorber concentration is increased in proportion to the decrease in mean chord length. The B^{10} density in the tube was increased from 0.00840 atoms/barn.cm to 0.0183 atoms/barn.cm to allow the mean chord length to be decreased from 5.77 cm to 2.646 cm. The increased B^{10} density is easily achieved by an enrichment of the natural boron.

The final calculations consider the triangular array of control rods fully inserted in an axially bare core reflected at the side by graphite, atom density 0.0903 per barn cm. The axial reflectors in the actual design (Ebeling and Hayes 1966) are most inefficient so account need not be taken of their effect on control rod worth.

3.1 Improved Few Group Data

We retain the black-white model for the final calculations but improve the spectrum calculation so as to permit the use of a small number of neutron energy groups.

In the spectrum and space-dependent calculations, axial leakage is accounted for by a buckling $B^2 = 0.943 \times 10^{-4}$. For the rod-in spectrum calculation an additional buckling κ^2, due to the rod in the cell, is included for the energy range $E < E_b$. κ^2 is obtained by solving:

$$\frac{J_o(\kappa a) + \lambda \kappa J_1(\kappa a)}{J_1(\kappa b)} = \frac{Y_o(\kappa a) + \lambda \kappa Y_1(\kappa a)}{Y_1(\kappa b)} ,$$

where a and b are the rod and cell radii and λ the black rod extrapolation distance.

A three group energy description is used for the reflected reactor calculation. E_b(1.23 keV) is retained for the high energy boundary, the additional boundary being inserted at 0.3 eV to divide the thermal and epithermal regions and so account for core to reflector spectrum changes. (Rod worth is evaluated at a common core and reflector temperature of 300°K).

3.2 Cell Calculations

Since the black-white and three-group models do incur some error and since the determination of the black-white boundary (Spinks 1966) does not account for moderation by the BeO spine, a series of cell calculations is first made, to determine a correction factor to be applied to the black-white, three-group reflected reactor results.

The accurate cell calculations are made in 8 neutron energy groups. E_b is retained and the remaining group boundaries chosen so as to achieve approximately equal total flux in each group.

The ratio between entries 5 and 2 of Table 3 is the correction factor. Entries 1 and 3 show the good accuracy of the two group model. Entries 3 and 4 show that, in the situation for which the black-white model was developed, it underestimates rod worth by about 1 in 10.

TABLE 3

CONTROL ROD REACTIVITY WORTH IN EQUILIBRIUM CELL

Control Rod	Method of Calculation	Groups	Rod Worth (1) per cent.
Hollow Tube	Black-White Diffusion	2	12.27
Hollow Tube	Black-White Diffusion	3	12.39
Hollow Tube	Black-White Diffusion	8	12.74
Hollow Tube	WDSN (S6)	8	13.79
BeO in Tube	WDSN (S6)	8	15.41

(1) Based on k = 0.9622 for the cell without rod, a result obtained from the rod-out spectrum calculation.

3.3 Side-Reflected-Reactor Calculations

The CRAM finite difference diffusion code is used for the side-reflected reactor calculations. Except at the centre of the reactor, the finite difference mesh cannot depict a circular control rod. We chose a near square rod for the off-centre locations. We obtain equivalence between the square (13 cm x 13 cm) and the circle (7.62 cm radius) using the Hurwitz-Roe (1955) absorption area method.

The results are presented in Table 4. Ring 0 is the central rod, ring 1 is the set of 6 rods at 80 cm radius, ring 2 is the set of 6 rods at 80 $\sqrt{3}$ cm radius, and ring 3 is the set of 6 rods at 160 cm radius. The core radius is 192 cm and the graphite side reflector thickness is 60 cm.

TABLE 4

REACTIVITY WORTH OF CONTROL RODS IN SIDE REFLECTED REACTOR

Ring				Groups	Reactivity Worth (1)	
0	1	2	3		Black-White	Corrected
x	x	x	x	8	11.0%	
x	x	x	x	3	11.2%	13.8%
x	x	x		3	8.3%	10.3%
	x	x		3	7.7%	9.6%
			x	3	2.4%	3.0%

(1) Based on k = 0.9422 for the side reflected reactor without rods, a result obtained using 8 groups, data condensed over the rod-out cell spectrum.

The first and second entries of Table 4 show excellent accuracy in the three group model. That the accuracy is better than in the cell calculation is fortuitous.

Comparing entry 2 of Table 3 to entry 2 of Table 4 we see that the cylindrical cell is a good model for the complete rod array.

Entry 5 of Table 4 indicates the suitability of the outer ring of 6 rods for regulation purposes.

Entry 4 shows that sufficient shutdown worth is available even when the centre rod is lost.

4. CONCLUDING REMARKS

The low multiplication constant of 0.94 for the rod-out reactor of Section 3 is, of course, not satisfactory. Bicevskis (1966) showed that a factor-of-two reduction in feed thorium concentration, with a reduction in F.I.F.A. from 1.75 to 1.4, will produce a sufficiently reactive core.

Noting that the change in rod worth going from the clean core of Section 2 to the more thermal equilibrium core of Section 3 was only from 12.75 to 13.79 per cent. (we compare the cell results), we can be quite confident that the spectrum change, resulting from a factor-of-two change in thorium concentration, will cause only a small change in control rod worth. Also, a decrease in thorium concentration will lead to a more thermal core spectrum and an increase in rod

worth. Therefore the Section 3 results will underestimate control rod worth in the more reactive core.

Finally, the comparison between the results of Sections 2 and 3 shows that control rod worth will not change greatly during the approach to equilibrium.

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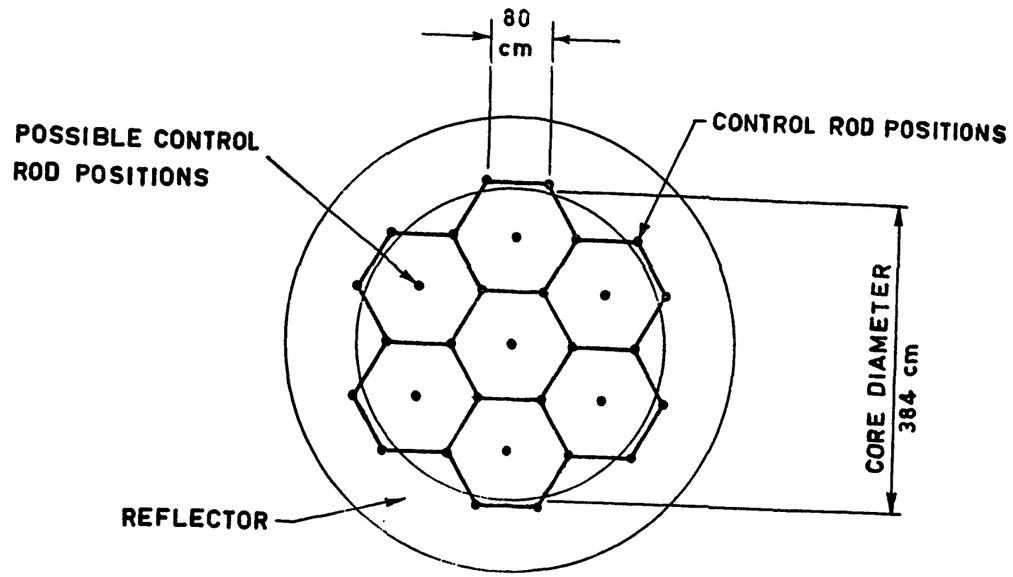

FIGURE 1. CORE CROSS SECTION FOR TWO REACTOR DESIGNS

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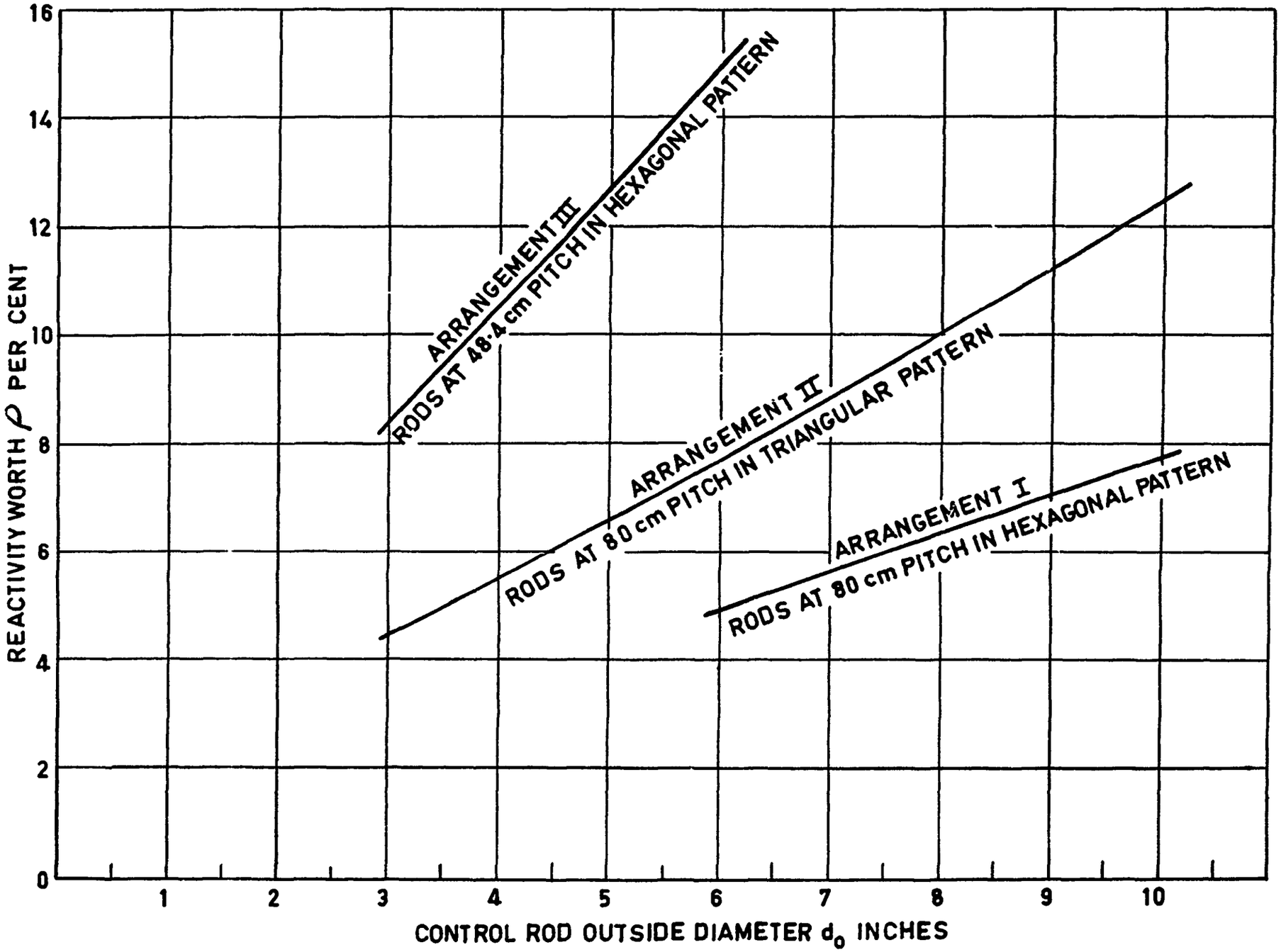

FIGURE 2. SURVEY RESULTS