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LUCAS HEIGHTS

THE TEMPERATURE AND POROSITY DEPENDENCE OF THE MODULUS OF RIGIDITY AND POISSON'S RATIO OF BERYLLIA

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ABSTRACT

Using torsional equipment designed for the task, the modulus of rigidity of isostatically pressed and sintered beryllia was determined as a function of temperature and porosity.

The temperature dependence was given by

$$G_t = G_{20}(1-at),$$

where G_t is the modulus at t°C, G_{20} is the modulus at 20°C and a is a constant approx. 3×10^{-5} degC independent of porosity up to 800°C.

Porosity dependence was given by

$$G_t = G_o (1-bP),$$

where G_o is the modulus at zero porosity, P is the fractional volume porosity and b is a constant approx. 1.58 independent of temperature up to 800°C.

Poisson's ratio was found to be 0.30 ± 0.05 and was independent of both temperature and porosity.

CONTENTS

1. INTRODUCTION

Neither the modulus of rigidity nor Poisson's ratio of beryllia is well documented in the literature and values of the latter parameter range from 0.204 (Lillie 1961) for hot pressed beryllia to 0.35 (Ryshkewitch 1951) for cold pressed and sintered material.

The work reported here was done as part of a programme to characterize Brush UOX beryllia fabricated at Lucas Heights by isostatic pressing and sintering. The temperature and porosity dependence of the modulus of rigidity and Poisson's ratio are established and the results compared with published data.

2. SPECIMEN PREPARATION

Blocks of beryllia of the required porosity (ranging from 3 to 23 per cent) were prepared by isostatically pressing Brush UOX powder at 20,000 p.s.i. followed by sintering for 1 to 5 hours in dry nitrogen at 1400°C. Specimens to the design and dimensions of Figure 1 were then prepared from these blocks by surface and centreless grinding. Porosity (P) was the fractional volume porosity calculated from the measured density (ρ_{meas}) of the fully machined specimen according to the relation $P = 1 - \rho_{meas}/3.01$. Grain sizes ranged from 1 to 5μ.

3. APPARATUS AND METHOD

The modulus of rigidity was measured statically in torsion using the apparatus illustrated in Figure 2. The specimen was held between two hollow ceramic (Al_2O_3) grips whose ends protruded from the furnace. These grips were, in turn, held by hollow steel grips attached to the frame of the apparatus. The upper steel grip was rigidly attached, the lower grip was supported on an air bearing within the framework and was free to rotate about an axis concentric with the upper grip (and specimen). Torsional loads were applied to the specimen through the lower grips by rotation of the grip under a load W applied via a pulley wheel (radius R) concentric with and forming part of the lower steel grip.

Silica rods were cemented to the ends of the specimen and passed through the hollow grips (ceramic and steel) to points outside the framework of the apparatus. A small mirror was attached to each of the outer ends of the silica rods and these together with two conventional lamp and scale systems were used to measure the deflection of each end of the specimen under load. The torsional deflection of the specimen (x), equalled the difference in deflection of these two mirrors (measured at the scale).

The modulus of rigidity (G) was then calculated from the relation

$$G = \frac{20.37\,R\ell L}{d^4} \cdot \frac{W}{x} \quad , \qquad .....(1)$$

where L = distance from scale to mirror

ℓ = specimen gauge length,

and d = specimen diameter.

Since the accuracy of moduli measurements was known to be affected adversely by creep in the specimens from about 900°C upwards, a temperature limit of 800°C was placed on the present measurements. The temperatures were obtained using a nichrome-wound furnace constructed so as to reduce axial temperature gradients to 3 degC per in. over the specimen. Temperatures were measured and controlled from chromel-alumel thermocouples tied to the specimen.

4. RESULTS

Eleven sets of results involving 76 data points were obtained for the variation of the modulus of rigidity with porosity and temperature and are given in Table 1.

(a) The effect of porosity: the results at constant temperature were fitted, using a least mean square analysis, to the linear equation

$$G = G_o(1-bP) \quad , \qquad .....(2)$$

where G_o is the modulus of rigidity extrapolated to zero porosity and b is a constant. The results are given in Table 2 and show that G_o and b are approximately constant at 20.4 and 1.58 respectively. *

(b) The effect of temperature: the results at constant porosity were fitted to the linear equation

$$G_t = G_{20}(1-at) \quad , \qquad .....(3)$$

where G_{20} is the modulus of rigidity at 20°C, t is the temperature in degrees Centigrade, and a is a constant. The results given in Table 2 show that the value of a varies from 3×10^{-5} degC^{-1} at $P = 0.05$ to 8×10^{-5} degC^{-1} at $P = 0.25$. With an estimated variation in a of $\pm 3 \times 10^{-5}$ degC^{-1}, this suggests that Equation 3 is essentially independent of porosity.

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* Replots of the data according to the empirical relationship of Spriggs (1962), $G = G_o e^{-bP}$, and the semi-theoretical relation of Hasselman (1962), $G = G_o \left[1 + \frac{AP}{1-(A+1)P} \right]$ showed no significant improvement over the linear form.

5. DISCUSSION

5.1 Errors in the Measurement of G

The value of G is subject to some uncertainty due to variations in material properties of the specimens and errors inherent in the method of measurement.

5.1.1 Effect of material properties

(a) Grain size: wide variations in grain size (2-100μ) have been shown to have only a slight (1%) effect on the modulus of rigidity (General Electric 1963). The small range of grain size (1-5μ) in the present work should not affect the results significantly.

(b) Purity: the presence of impurities could affect the modulus of rigidity since impurity atoms present as a second phase or, to a lesser extent, in solution would be expected to have moduli different from the matrix. The magnitude of the effect would depend on both the concentration and difference in moduli of the impurity. The deliberate addition of a large amount of magnesium (0.5 wt %) has been shown (General Electric 1963) to increase the value of G_o by only 4 per cent (with a 3% scatter) in Brush UOX material. Consequently the relatively small variations in impurity content of the high purity Brush UOX powder would not be expected to affect G_o significantly for samples prepared by a standard fabrication route, as in the present work.

(c) Preferred orientation: isostatically pressed and sintered beryllia does not have a significant preferred orientation (K. G. Watson, private communication), so the present results are not subject to this error. However, this factor may affect a comparison of the present results with those of other workers, see for example General Electric (1963).

5.1.2 Experimental errors

(a) Errors in R, L, ℓ and W: the maximum error in the radius R of the pulley, the length ℓ of the specimen, the radius L of the light beam, and the load W was estimated to be ± 0.3, ± 0.1, ± 0.2 and 0.1 per cent respectively.

(b) Errors in deflection (x): the high modulus of rigidity of beryllia together with the small loads (< 4.5 lb) which could be applied without fear of breakage of the specimens limited x to a maximum of 3.2 cm. With an error in reading of ± 0. 5 mm, the accuracy of the x determination was ± 1.6 per cent.

(c) Errors in diameter (d): Measurement of the diameter was made to 0.002 in. at twelve different positions along the length and circumference of the gauge lengths of the specimens. However, owing to 'bowing' of the specimen during grinding, the average diameter of the specimen could not be defined to better than ± 0.001 in. corresponding to an error of ± 0.53 per cent in d.

From Brownlee (1949):

$$\text{Error in } G = \pm\sqrt{\text{sum of (errors)}^2 \text{ in } R,L,\ell,W \text{ and } x + 4\ (\text{error in } d)^2}$$

Substitution of the estimated individual errors in this equation gives an error in the modulus of rigidity of ± 2.0 per cent. This is in good agreement with the experimental results as shown by the variation in G_o values (Table 1).

5.2 Poisson's Ratio

Elasticity theory shows that Poisson's ratio can be calculated from the relation

$$\mu = \frac{E}{2G} - 1 \quad , \qquad .....(4)$$

where E is the elastic modulus. Using the present data for G and that of Veevers and Rotsey (1965) for E (since it was obtained for beryllia fabricated in the same manner), $\mu = 0.30 \pm 0.05$ at room temperature for material of theoretical density.

(a) Temperature dependence: replotting Rotsey and Veevers' data in linear form gives

$$E = E_{20}(1-3 \times 10^{-5}t) \quad .$$

This equation together with Equations 3 and 4 indicates that Poisson's ratio is independent of temperature up to 800°C.

(b) Porosity dependence: Rotsey and Veevers' data give

$$E = E_o(1-1.47P) \quad .$$

This, together with Equation 2 suggests that G varies more rapidly with porosity than E does. However, when the accuracy of this result is taken into account (estimated error in $b = \pm 0.08$), this difference is insignificant. Thus the present results show that μ is essentially independent of P up to 25 per cent porosity.

5.3 Comparison with Other Published Data

The room temperature values of $G_o = 20.4$ and $\mu = 0.30$ found here are in good agreement with those of Ryshkewitch (1951), $\mu = 0.35$, using a static method similar to the present method; Budnikov and Belyaev (1960), $\mu = 0.34$, method unknown; and the extensive data of General Electric (1963), $G_o = 21.17$ and $\mu = 0.315$ using a dynamic resonant frequency method. The only data differing substantially from these results is that originating at Atomics International (Lillie 1961) where a value of $\mu = 0.204$ was obtained using a dynamic, ultrasonic pulse technique. This result was obtained from three separate runs on a single specimen with a density of 3.008 g cm^{-3} ($P = 0.0007$). This value is surprisingly low compared with the other dynamic method result of General Electric and well beyond the error of the latter measurement (± 0.03). It is reasonable to suppose that it represents an error in measurement but this cannot be checked since no details of this work were reported. On the other hand if it is true, it means that G and/or E is not linearly dependent on porosity for material close to zero porosity ($P < 0.01$) and values of G_o and E_o cannot be obtained by extrapolation from results on lower density material as was done in both the General Electric and present work.

The only data with which the porosity and temperature dependence of the modulus of rigidity can be compared is that of General Electric (1963). Their average value of $b = 1.86 \pm 0.08$ is significantly higher than the present result of $b = 1.58 \pm 0.08$. However, the difference is not unexpected. Spriggs (1961) has noted that, in alumina, b varies according to the fabrication techniques, namely for hot pressing, b ranges from 4.08 to 4.35; for cold pressing and sintering, 3.44 to 3.55; and for slip casting and sintering, 2.73. Consequently, the small difference found here for extruded and sintered beryllia compared with isostatically pressed and sintered beryllia is reasonable and is probably related to a variation in the ratio of closed to open porosity, with open pores showing the greater influence. The small difference in temperature dependence, $a = 3$ to 8×10^{-5} compared with $a = 9 \times 10^{-5}$ for the General Electric work, is probably attributable to the same cause.

6. CONCLUSION

For Brush UOX beryllia fabricated at Lucas Heights by isostatic pressing and sintering:

(1) The temperature dependence of the modulus of rigidity is expressed by the relation

$$G_t = G_{20}\ (1 - at) \quad ,$$

where G_t is the modulus at t°C, G_{20} is the modulus at 20°C and a is a constant $\sim 3\text{x}10^{-5}$ degC^{-1} independent of porosity up to 800°C.

(2) The porosity dependence of the modulus of rigidity is given by

$$G_t = G_o\ (1 - bP) \quad ,$$

where G_o is the modulus for theoretically dense material, P is the fractional volume porosity, and b is a constant ~ 1.58, independent of temperatures up to 800°C.

(3) Poisson's ratio equals 0.30 ± 0.05 and is independent of both temperature and porosity.

7. ACKNOWLEDGEMENTS

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TABLE 1

VARIATION OF MODULUS OF RIGIDITY WITH POROSITY AND TEMPERATURE

Spec. No.	2	3	4	16	17	6	18	8	7	10	15
P (%)	5	3	4	7	12	13	19	22	23	24	24
$G \times 10^{-6}$ → ↓ Temp °C											
20	19.65	19.9	20.15	18.20	16.60	14.9	14.35	13.74	14.20	12.72	12.45
	19.60	-	19.15	18.10	16.45		14.20		13.55		12.30
200	19.45	19.45	20.40	18.00	16.45		14.10	14.25	14.24		12.25
	19.75		19.45	17.90	16.30		14.00		13.50		12.15
400	19.70		19.45	17.85	16.25		13.95		13.84		12.05
	19.60		19.20	17.80	16.15		13.85		13.42		11.95
600	19.60		19.35	17.80	16.15		13.75		13.05		11.95
	19.25		18.95	17.65	15.95		13.50		13.75		11.70
800	19.10		19.20	17.62	15.85		13.40		13.05		11.60
	19.10		19.20	17.62	15.85		13.40		13.05		11.60

TABLE 2

THE EFFECT OF TEMPERATURE AND POROSITY

t (°C)	$G_o \times 10^{-6}$ p.s.i.	b	P	$a \times 10^{5}$ (degC^{-1})
20	20.4	1.58	0.03 to 0.05	3 to 5
200	20.0	1.51		
400	19.9	1.64		
600	20.9	1.50	0.22 to 0.24	5 to 8
800	20.8	1.65		

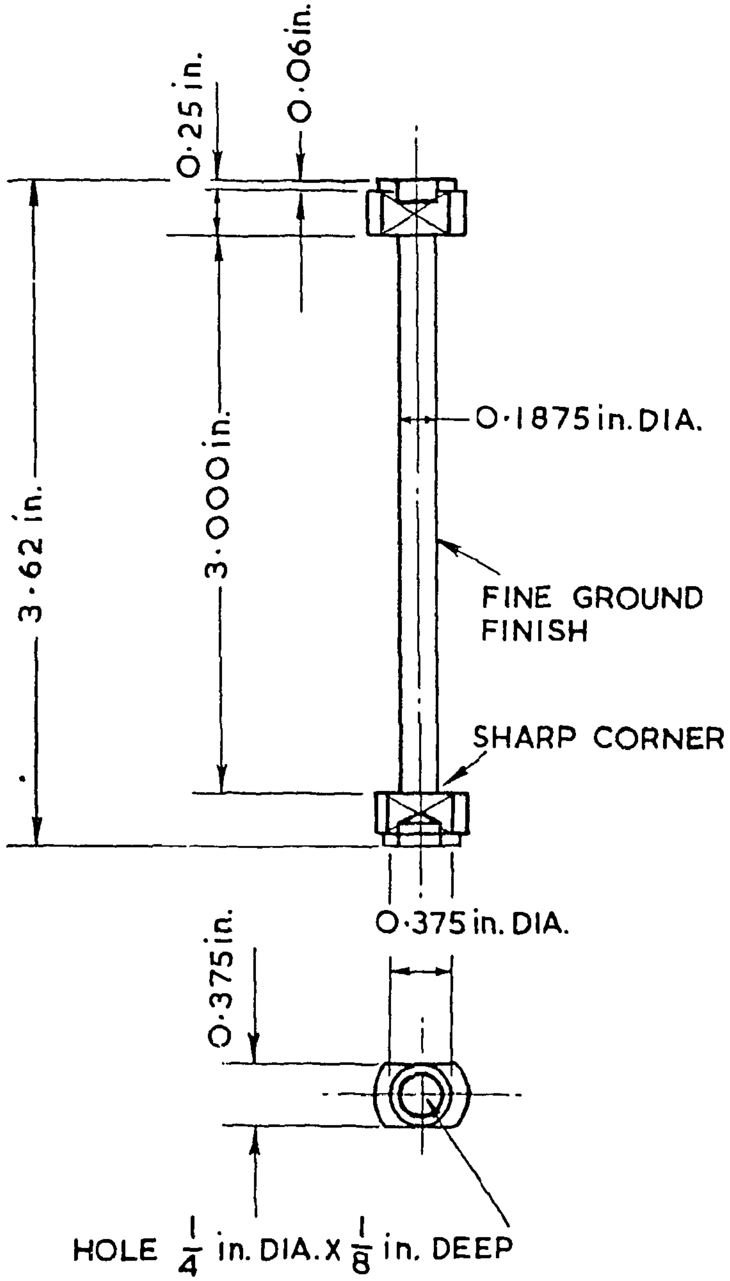

FIGURE 1. BERYLLIA SPECIMEN FOR MEASUREMENT OF MODULUS OF RIGIDITY

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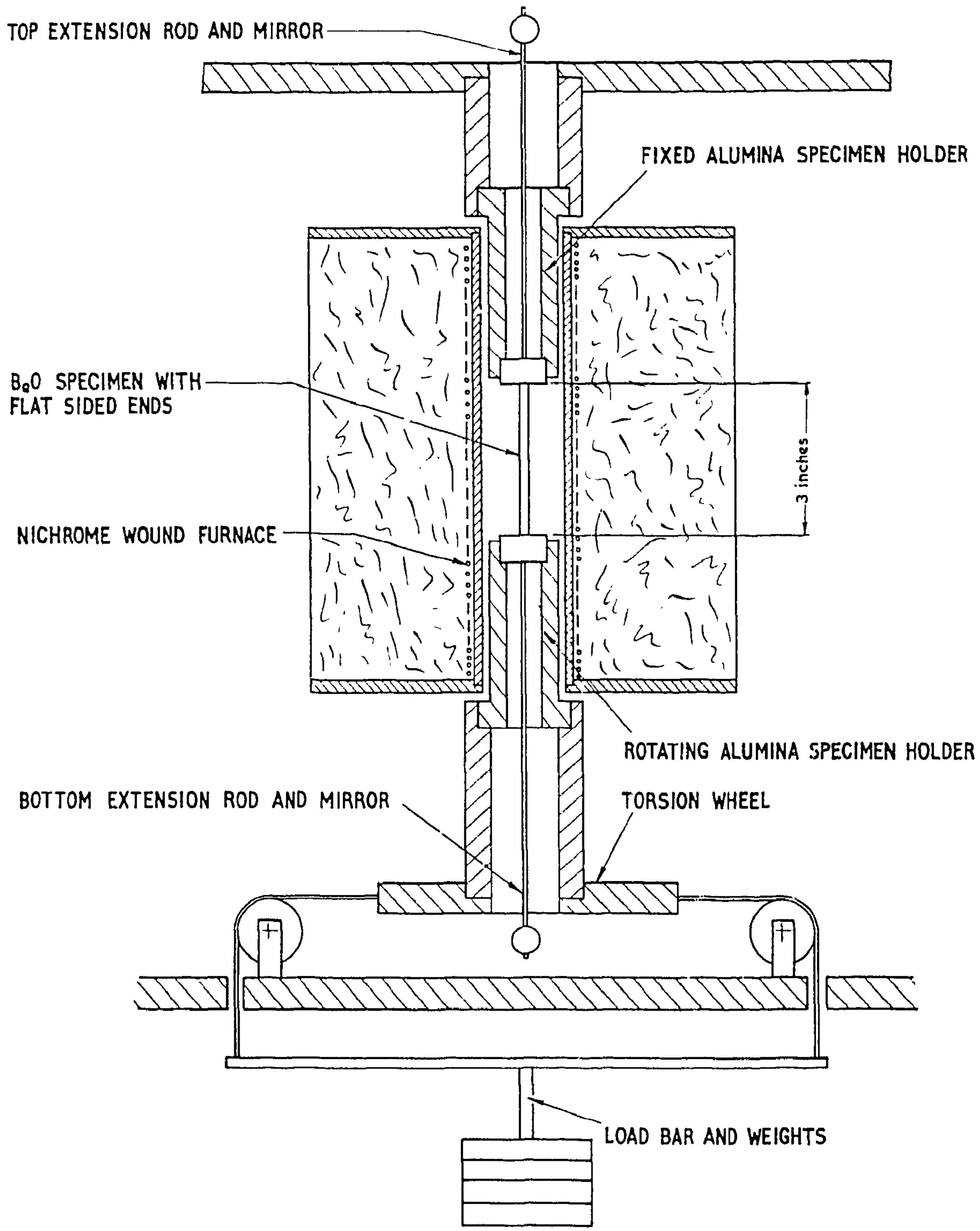

FIGURE 2. RIGIDITY MODULUS RIG

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