

AUSTRALIAN NUCLEAR SCIENCE & TECHNOLOGY ORGANISATION
LUCAS HEIGHTS RESEARCH LABORATORIES

CAMPION'S COINCIDENCE-COUNTING
CORRECTIONS REVISITED

by

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ABSTRACT

Detailed derivations are given of some of Campion's formulae. Improved versions of some formulae are also derived. Efficiency extrapolation is dealt with briefly.

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EDITORIAL NOTE

From 27 April 1987, the Australian Atomic Energy Commission (AAEC) is replaced by Australian Nuclear Science and Technology Organisation (ANSTO). Serial numbers for reports with an issue date after April 1987 have the prefix ANSTO with no change of the symbol (E, M, S or C) or numbering sequence.

CONTENTS

	Page No.
1. INTRODUCTION	1
2. FIRST TABLE OF SYMBOLS	1
3. THE ELEMENTARY COINCIDENCE-COUNTING EQUATIONS	1
4. FURTHER SYMBOLS AND DEFINING EQUATIONS	2
5. DEAD-TIME CORRECTION EQUATIONS FOR ONE CHANNEL	2
6. THE CAMPION EQUATION WHICH CORRECTS FOR DEAD-TIME LOSSES AND ACCIDENTAL COINCIDENCES	3
6.1 Introduction	3
6.2 Campion's First Equation for C_{True}	4
6.3 Campion's Dead-Time Equation for One Channel	7
6.4 Campion's Second Equation for C_{True}	9
6.5 The Campion Equation for N which Corrects for the Effects of Dead Time and Resolving Time	10
6.6 Note on Equations (14) and (15)	12
7. CALCULATION OF E_A AND E_B	12
8. A BETTER VERSION OF EQUATION (25) FOR E_A	13
8.1 The Rate of Dead-Time Overlapping	13
8.2 Total Duration of Overlaps in Unit Time	13
8.3 Dead-Time Correction Equation for C_{True}	15
8.4 Equation for E_A	16
9. A BETTER VERSION OF EQUATION (20) FOR N	16
10. SOURCES CONTAINING A β -EMITTER AND A $\beta\gamma$ -EMITTER	17
10.1 Third Table Symbols	17
10.2 The β Efficiency with Respect to the $\beta\gamma$ Emitter, $E_{A\beta\gamma}$	17
10.3 Efficiency Extrapolation	18
11. BREMSSTRAHLUNG EFFECTS IN COINCIDENCE COUNTING	20
11.1 Fourth Table of Symbols	20
11.2 Limitations of the Analysis	21
11.3 The Bremsstrahlung Correction Equation	21
12. ACKNOWLEDGEMENTS	23
13. REFERENCES	24

"I have no satisfaction in formulas unless I feel their numerical magnitude"

LORD KELVIN, Life by Sylvanus Thompson, p.827.

1. INTRODUCTION

P.J. Campion [1959] in his well-known paper discussed comprehensively the corrections involved in the coincidence counting of radioactive sources of around a few kilobecquerel. His treatment of the subject was very concise and not without difficulties for those making an initial approach to the subject. More extensive versions of Campion's derivations have been formulated by S.L. Sherlock [1985] and S.M. Buckman [1990].

This report considers some of the equations in Campion's paper. Detailed step-by-step derivations are given, and the various approximations made are justified, where possible, by calculations using typical experimental data. Some formulae are derived in an improved form, and the technique of efficiency extrapolation is dealt with briefly.

2. FIRST TABLE OF SYMBOLS

Quantity	Symbol	
	This Report	Campion [1959]
β efficiency	E_A	ϵ_β
γ efficiency	E_B	ϵ_γ
Disintegration Rate	N	N_0
"Ideal" β -count rate, with no background or dead time	A_I	
"Ideal" γ -count rate, with no background or dead time	B_I	
"Ideal" coincidence-count rate, with no background, dead time or accidental coincidences	C_I	

3. THE ELEMENTARY COINCIDENCE-COUNTING EQUATIONS

$$E_A = \frac{A_I}{N}, \text{ by definition.} \quad (1)$$

$$E_B = \frac{B_I}{N}, \text{ by definition.} \quad (2)$$

$$C_I = N E_A E_B. \quad (3)$$

$$E_A = \frac{C_I}{B_I} \quad (4)$$

$$E_B = \frac{C_I}{A_I} \quad (5)$$

4. FURTHER SYMBOLS AND DEFINING EQUATIONS

Quantity	Symbol	
	This Report	Campion [1959]
β-count Rates: Background Source plus background $A'' - A_b =$	A_b A'' A'	N_{β}' N_{β}
γ-count Rates: Background Source plus background $B'' - B_b =$	B_b B'' B'	N_{γ}' N_{γ}
Dead Time	T	τ
Resolving Time	T_R	τ_R
True-coincidence count rate	C_{True}	N_c
Accidental-coincidence count rate	C_{Acc}	N_{Acc}
Observed-coincidence count rate minus background	C'	N_c'
Duration of overlap, i.e. extent of partial coincidence of a β and a γ dead time	x	

$$C' = C_{True} + C_{Acc} \quad ; \quad N_c' = N_c + N_{Acc}$$

5. DEAD-TIME CORRECTION EQUATIONS FOR ONE CHANNEL

$$NE_A + A_b = \frac{A''}{1 - A''T} \quad (6)$$

$$NE_B + B_b = \frac{B''}{1 - B''T} \quad (7)$$

Equation (6) can be "simply" derived [NCRP 1985:62] by assuming that the lost counts per second are equal to the product of the total dead time per second, $A''T$, and $(NE_A + A_b)$; thus,

$$NE_A + A_b = A'' + A''T (NE_A + A_b), \text{ which, when rearranged, gives equation (6).}$$

Equation (6), neglecting background, can be derived more elaborately; e.g. see Parzen [1962].

6. THE CAMPION EQUATION WHICH CORRECTS FOR DEAD-TIME LOSSES AND ACCIDENTAL COINCIDENCES

6.1 Introduction

The Campion equation for the disintegration rate, which corrects for dead-time losses and accidental coincidences, is given in the following form by J. Bryant [1963 : equation (5)]:-

$$N = \frac{A'B'}{C' - 2T_R A'B'} \times \left\{ 1 + \frac{B'T - T_R(A'+B')}{1 - B'T} \right\} \quad (8)$$

The derivation of equation (8), which is given below, is preceded by the derivation of two separate expressions for C_{True} . To justify arithmetically the approximations made, we shall consider a system in which

$$E_A = 0.9,$$

$$E_B = 0.04,$$

$$A' = 4000 \text{ s}^{-1},$$

$$T = 5 \mu\text{s},$$

$$T_R = 1 \mu\text{s},$$

$$A_b = 1,$$

$$B_b = 2.$$

Rearranging equation (6) and inserting numerical values given above,

$$N = \frac{1}{E_A} \left[\frac{A''}{1 - A''T} - A_b \right] = 4535.19.$$

Rearranging equation (7),

$$B'' = \frac{NE_B + B_b}{1 + (NE_B + B_b)T} = 183.2396.$$

6.2 Campion's First Equation for C_{True}

We shall derive an expression for C_{Acc} , which will then be substituted into the equation

$$C_{True} = C' - C_{Acc} \quad (9)$$

We shall thus obtain the first equation for C_{True} (equation 13). A rough derivation of C_{Acc} will suffice if we show that $C_{Acc} \ll C'$.

6.2.1 Equation for C_{Acc}

We assume that a β and a γ pulse which result from the same disintegration will arrive at the mixer exactly simultaneously and produce the "shaped" pulses whose width is the resolving time of the mixer, as shown in figure 1. Such an event in the mixer constitutes a true coincidence. The overlapping "shaped" pulses in figures 2 and 3 result from the detection of two disintegrations, and such an overlap constitutes an accidental coincidence.

If the dead time, T , is set at $> 2T_R$, there will be no double overlapping of resolving times, of which there are two types as shown in figure 4. This is proved elsewhere [Wyllie 1992].

In figure 5, the rate of the shaped pulses in channel A is $A'' \text{ s}^{-1}$. The number of shaped pulses per second in channel A which are not involved in true coincidences = $A'' - C_{True}$.

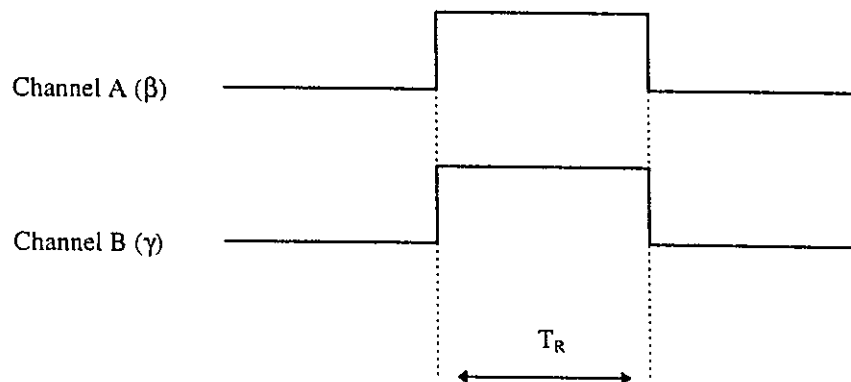


Figure 1 - Shaped pulses produced in the mixer by the same disintegration. This is a true coincidence.

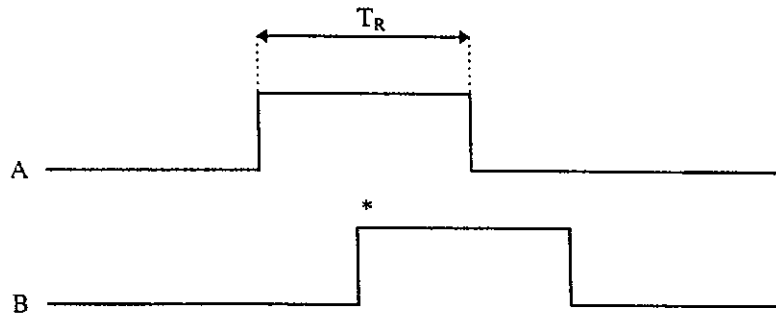


Figure 2 - A disintegration (*) occurring during a channel A resolving time is detected by channel B. This is an accidental coincidence (type A).

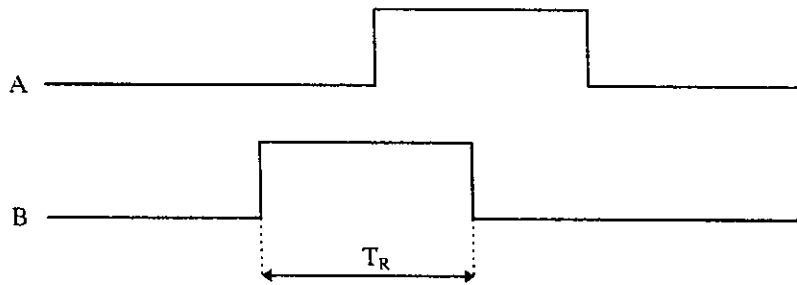


Figure 3 - Accidental coincidence (type B).

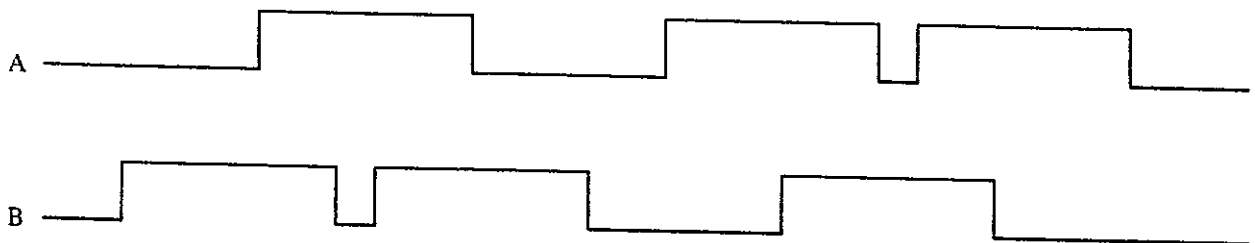


Figure 4 - Double overlapping of resolving times.

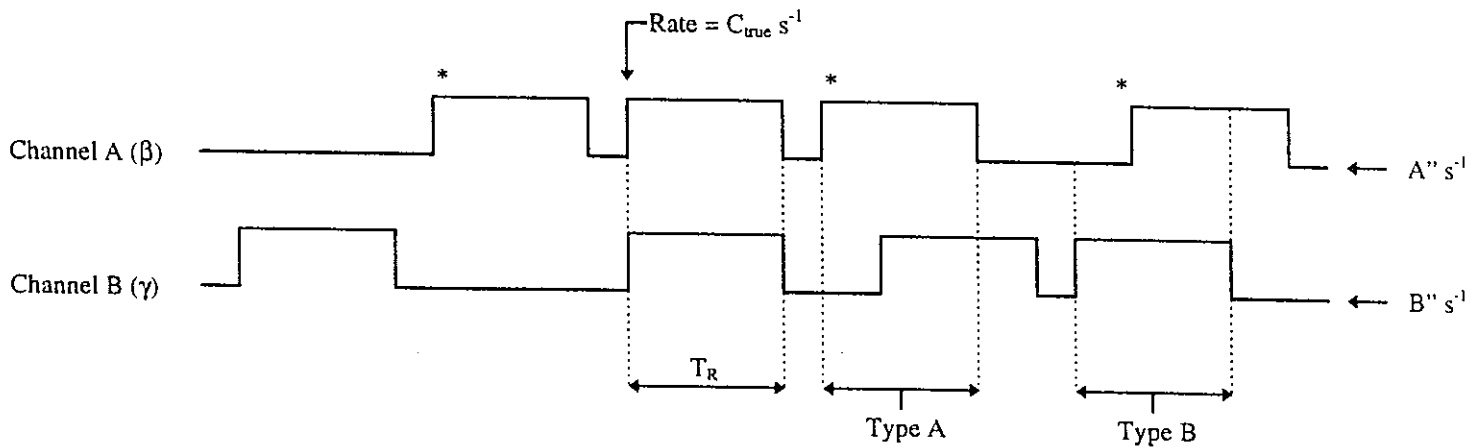


Figure 5 - The rate of pulses in channel A which are not involved in true coincidences is the difference between the total rate, A'' , and the rate of true coincidences, C_{True} ; i.e. rate of pulses marked $*$ = $(A'' - C_{True})s^{-1}$.

The total duration per second of these latter pulses is equal to

$$(A'' - C_{True})T_R.$$

In the manner of the "simple" derivation of equations (6) and (7), we say that the number of counts recorded in channel B during $(A'' - C_{True})T_R$ is $(A'' - C_{True})T_R B''$. Each of these counts is followed by a dead time, T . Hence, during each shaped pulse in channel A, there is a maximum of one count in channel B. Thus,

$$(A'' - C_{True})T_R B'' \tag{10}$$

is also the count rate per second of the accidental coincidences designated type A in figure 5.

Similarly, the rate of accidental coincidences, type B, is equal to

$$(B'' - C_{True})T_R A'' \tag{11}$$

Adding equations (10) and (11),

$$C_{Acc} = [2A''B'' - C_{True}(A'' + B'')]T_R \tag{12}$$

We now show that $C_{Acc} \ll C'$. Referring to equation (5),

$$E_B = \frac{C_I}{A_I} \approx \frac{C_{True}}{A'}$$

$$\begin{aligned}\therefore C_{\text{True}} &\approx E_B A', \text{ which, substituting the values in section 6.1,} \\ &= 0.04 \times 4000 = 160.\end{aligned}$$

Substituting this last value and others from section 6.1 into equation (12),

$$\begin{aligned}C_{\text{Acc}} &\approx \{(2 \times 4001 \times 183.24) - [160 \times (4001 + 183.24)]\} \times 10^{-6} \\ &\approx 0.80.\end{aligned}$$

$$C' = C_{\text{True}} + C_{\text{Acc}} \approx 160 + 0.80 = 160.8.$$

Hence, $C_{\text{Acc}} \approx 0.80 \ll 160.8 \approx C'$, thus justifying the rough derivation of equation (12) for C_{Acc} .

6.2.2 The Equation for C_{True}

Substituting the right-hand side of equation (12) in equation (9),

$$C_{\text{True}} = C' - 2A''B''T_R + C_{\text{True}}(A'' + B'')T_R.$$

Rearranging,

$$C_{\text{True}} = \frac{C' - 2A''B''T_R}{1 - (A'' + B'')T_R}, \quad (13)$$

which is the second version of Champion's [1959] equation (9).

6.3 Champion's Dead-Time Equation for One Channel

The probability of no disintegration occurring during an interval equal to the dead time T is

$$\begin{aligned}e^{-NT} &= 1 - NT + \frac{(NT)^2}{2} - \frac{(NT)^3}{6} + \dots \\ &\approx 1 - NT, \text{ when } NT \text{ is small.}\end{aligned}$$

The probability of at least one disintegration occurring during T is $1 - e^{-NT} \approx NT$. In the following derivation, we neglect the dead time caused by the background counts.

The β count rate, A' , is the number of disintegrations per second which occur when channel A is live, and which are detected. These disintegrations are of two types.

A disintegration of the first type at instant I_2 in figure 6, is not preceded by a disintegration in the interval I_1I_2 ; the probability of this occurring is $1 - NT$. Therefore, immediately before I_2 the channel must be live. The disintegration at I_2 is detected with probability E_A . Thus, the rate of the first type

$$\approx N(1 - NT)E_A.$$

A disintegration of the second type at instant I_3 in figure 7, is preceded by one or more disintegrations in the interval I_1I_3 , with a probability NT . When N is low, we may neglect the possibility of two or more disintegrations occurring in I_1I_3 ; we consider only the case of one disintegration occurring (e.g. at I_2) which is not detected with the probability $1 - E_A$. The disintegration at I_3 is detected with probability E_A . Thus, the rate of the second type

$$\approx N(NT)(1 - E_A)E_A.$$

At low count rates we neglect the possibility, shown in figure 8, that the channel is dead at I_2 , in which case the channel cannot be dead at I_3 as a result of the disintegration at I_2 .

Adding the rates of the two types of disintegration,

$$A' = NE_A [(1 - NT) + (NT)(1 - E_A)];$$

i.e. $A' = NE_A (1 - NT E_A).$ (14)

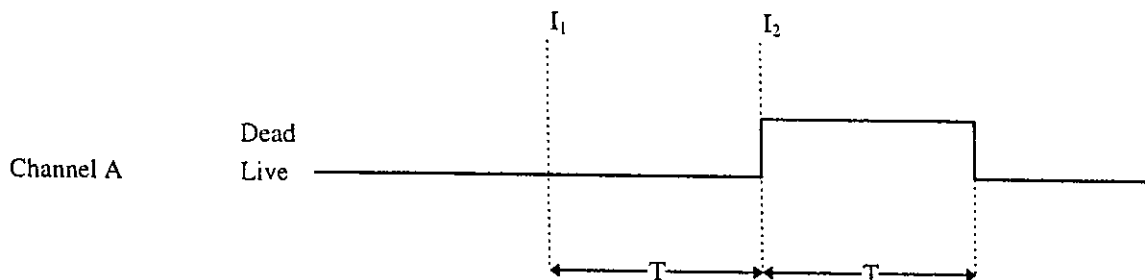


Figure 6 - There is no disintegration in the interval I_1I_2 with a probability $1 - NT$. The disintegration at instant I_2 is detected with probability E_A .

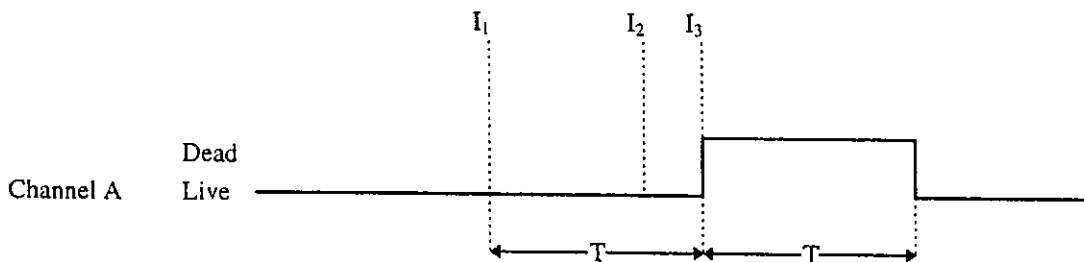


Figure 7 - A disintegration at I_2 occurs in the interval I_1I_3 with a probability NT and is not detected with a probability $1 - E_A$. The disintegration at I_3 is detected with probability E_A .

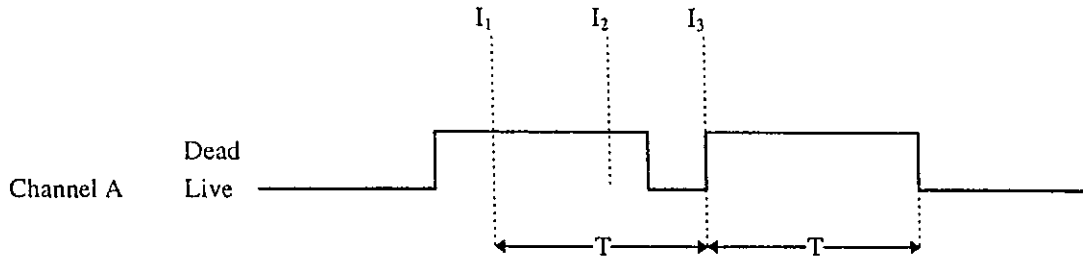


Figure 8 - The disintegration at I_2 in the interval I_1I_3 cannot result in the channel being already dead when the disintegration occurs at I_3 .

Similarly,

$$B' = NE_B (1 - NT E_B). \quad (15)$$

The latter two equations are Campion's [1959] equations (10) and (12), respectively.

6.4 Campion's Second Equation for C_{True}

In the following derivation, which is similar to that of equation (14), we neglect the dead time caused by the β and γ background counts.

The rate of true coincidences, C_{True} , is the number of disintegrations per second which occur when both channels are live, and which are detected in both channels. These disintegrations are of two types.

A disintegration of the first type at instant I_2 in figure 9, is not preceded by a disintegration in the interval I_1I_2 , with a probability $1 - NT$. Therefore, immediately before I_2 both channels are live. The disintegration at I_2 is detected in channels A and B with probabilities E_A and E_B , respectively. Thus, the rate of the first type of disintegration

$$= N(1 - NT) E_A E_B.$$

A disintegration of the second type at instant I_3 in figure 10 is preceded by a disintegration with probability NT in the interval I_1I_3 , at the instant I_2 , and is not detected in channels A and B with probabilities $(1 - E_A)$ and $(1 - E_B)$, respectively. The disintegration at I_3 is detected in channels A and B with probabilities E_A and E_B , respectively. Thus, the rate of the second type of disintegration

$$= N(NT)(1 - E_A)(1 - E_B)E_A E_B.$$

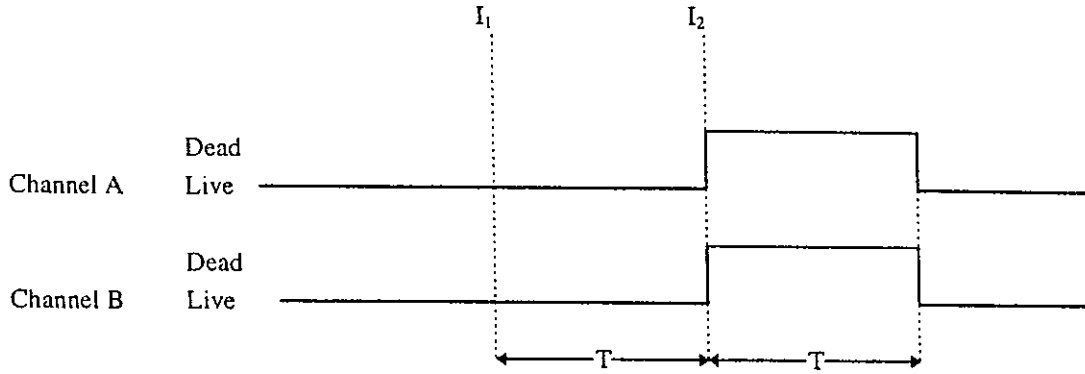


Figure 9 - There is no disintegration in the interval I_1I_2 with a probability $1 - NT$. The disintegration at instant I_2 is detected in both channels with probability $E_A E_B$.

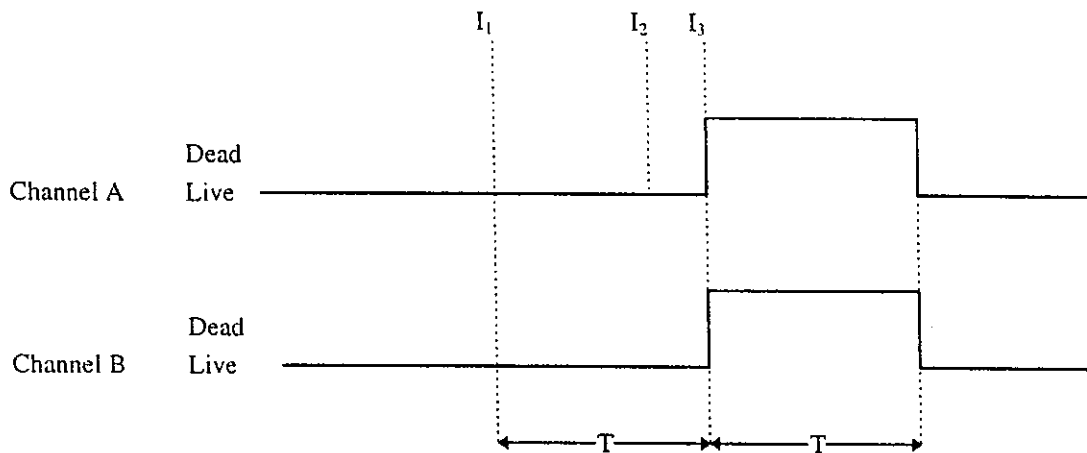


Figure 10 - A disintegration occurs with probability NT in the interval I_1I_3 , at the instant I_2 , and is not detected in either channel with probability $(1 - E_A)(1 - E_B)$. The disintegration at I_3 is detected in both channels with probability $E_A E_B$.

Adding the rates of the two types of disintegration,

$$C_{\text{True}} = NE_A E_B [1 - NT + NT(1 - E_A)(1 - E_B)];$$

i.e. $C_{\text{True}} = NE_A E_B [1 - NT(E_A + E_B - E_A E_B)].$ (16)

6.5 The Campion Equation for N which Corrects for the Effects of Dead Time and Resolving Time

Combining equations (14), (15) and (16),

$$\frac{A'B'}{C_{\text{True}}} = \frac{N(1-NT E_A)(1-NT E_B)}{1-NT(E_A + E_B - E_A E_B)}$$

Rearranging,

$$\begin{aligned} N &= \frac{A'B'}{C_{\text{True}}} + \frac{1-NT E_A - NT E_B + (NT)^2 E_A E_B}{1-NT(E_A + E_B - E_A E_B)} \\ &= \frac{A'B'}{C_{\text{True}}} + \frac{1-NT(E_A + E_B) + (NT E_A E_B - NT E_A E_B) + (NT)^2 E_A E_B}{1-NT(E_A + E_B - E_A E_B)} \\ &= \frac{A'B'}{C_{\text{True}}} + \frac{1-NT(E_A + E_B - E_A E_B) - NT E_A E_B (1-NT)}{1-NT(E_A + E_B - E_A E_B)} \\ &= \frac{A'B'}{C_{\text{True}}} + \left[1 - \frac{NT E_A E_B (1-NT)}{1-NT(E_A + E_B - E_A E_B)} \right], \end{aligned} \quad (17)$$

which, since $NT \ll 1$, reduces to

$$N = \frac{A'B'}{C_{\text{True}}(1-NT E_A E_B)}. \quad (18)$$

Campion shows arithmetically that "no appreciable error is introduced" if equation (18) is replaced by

$$N = \frac{A'B'}{C_{\text{True}}(1-B'T)}, \quad (19)$$

which is Campion's equation (14). Perhaps this replacement was suggested by:

$$NE_A E_B = C_I > C_{\text{True}} < B'.$$

The reduction of equation (17) to equation (19) can be justified by considering the ratio

$$\left[1 - \frac{NT E_A E_B (1-NT)}{1-NT(E_A + E_B - E_A E_B)} \right] + (1-B'T).$$

The calculated value of the above expression, using values from section 6.1, is 1.000092, which is close enough to unity.

Substituting the right-hand side of equation (13) in equation (19) and writing A' for A'' , B' for B'' ,

$$N = \frac{A'B'[1-(A'+B')T_R]}{(1-B'T)(C'-2A'B'T_R)}. \quad (20)$$

By inserting $(-B'T + B'T)$ within the square brackets, we can obtain equation (8).

Bryant derived an equation for N [Bryant 1963: equation(4)] which he showed was more accurate than equation (8) at high count rates.

6.6 Note on Equations (14) and (15)

Equation (14) can be derived from equation (6). Neglecting background, we delete A_b from equation (6), so that $A'' = A'$, and then rearrange:

$$NE_A - NE_A A'T = A' ;$$

$$A'(1 + NE_A T) = NE_A ;$$

$$A' = NE_A(1 + NE_A T)^{-1}.$$

If $NT \ll 1$, then according to the binomial theorem, the above equation becomes

$$A' \approx NE_A(1 - NE_A T),$$

which is equation (14). Similarly, equation (15) can be derived from equation (7).

7. CALCULATION OF E_A AND E_B

The values of E_A and E_B can be obtained by substituting the value of N calculated by equation (20) into rearrangements of equations (6) and (7); viz

$$E_A = \frac{1}{N} \left[\frac{A''}{1 - A''T} - A_b \right], \quad (21)$$

$$E_B = \frac{1}{N} \left[\frac{B''}{1 - B''T} - B_b \right]. \quad (22)$$

Alternatively, we can use Campion's equations for E_A and E_B which are derived as follows. Neglecting background, we delete A_b and B_b from equations (21) and (22), respectively, and let $A'' = A'$, and $B'' = B'$. We then substitute the right-hand side of equation (19) for N . Thus,

$$E_A = \frac{A'}{N(1 - A'T)} = \frac{C_{\text{True}}(1 - B'T)}{A'B'} \times \frac{A'}{1 - A'T},$$

i.e. $E_A = \frac{C_{\text{True}}(1 - B'T)}{B'(1 - A'T)} \quad (23)$

$$E_B = \frac{B'}{N(1 - B'T)} = \frac{C_{\text{True}}(1 - B'T)}{A'B'} \times \frac{B'}{1 - B'T},$$

$$\text{i.e. } E_B = \frac{C_{\text{True}}}{A'} \quad (24)$$

Equations (23) and (24) are both numbered (15) in Campion's [1959] paper.

It may be noted that the derivation of equations (23) and (24) involves:

- (a) The equations (6) and (7), i.e. the conventional dead-time equations, rearranged, with backgrounds neglected; and
- (b) The equations (14) and (15) in which a further approximation is made, and which are used to obtain the formula for N given in equation (19).

Equations (23) and (24) can be written in terms of the experimental data by substituting for C_{True} from equation (13). Thus,

$$E_A = \frac{(1 - B'T)(C' - 2A''B''T_R)}{B'(1 - A'T)[1 - (A'' + B'')T_R]} \quad (25)$$

$$E_B = \frac{C' - 2A''B''T_R}{A'[1 - (A'' + B'')T_R]} \quad (26)$$

8. A BETTER VERSION OF EQUATION (25) FOR E_A

8.1 The Rate of Dead-Time Overlapping

An accidental coincidence occurs when a shaped pulse of width T_R in one channel overlaps a similar pulse in the other channel. The number of overlaps per unit time is the accidental coincidence count rate.

We now consider overlapping of dead times. The situation is identical with that in figure 5, except that T_R is replaced by T . It can be seen from the derivation of formula (10) that the number per unit time of overlaps, type A, is equal to

$$(A'' - C_{\text{True}})TB'' \quad (27)$$

8.2 Total Duration of Overlaps in Unit Time

The duration of an overlap is defined as the interval x in figure 11, which shows a type A overlap. It is assumed that all values of x between 0 and T are equally probable. Referring to the probability density graph with constant ordinate equal to $\frac{1}{T}$, the fraction of overlaps whose duration lies between x and $x + \delta x$ is $\frac{1}{T} \times \delta x$. Thus,

$$\int_0^T \frac{1}{T} dx = 1.$$

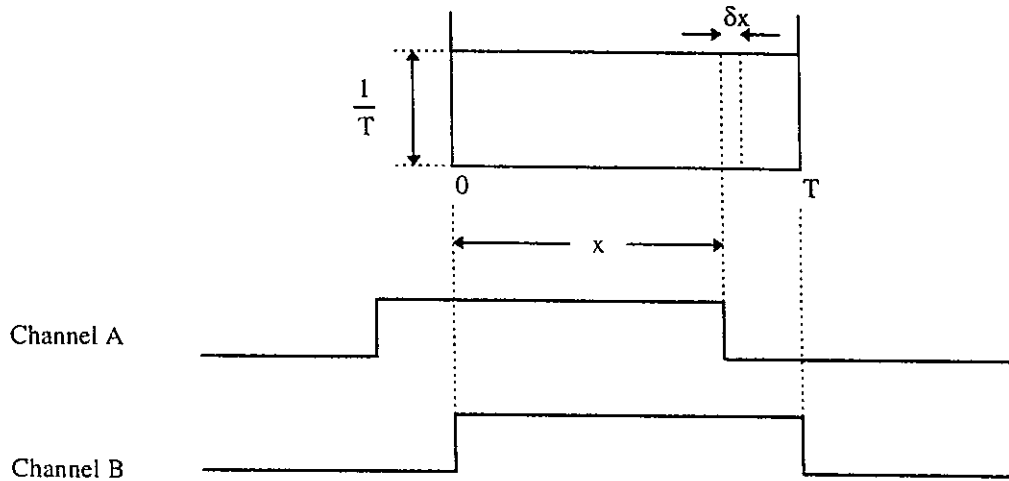


Figure 11 - The interval x is the duration of the overlap of the two dead times. It is assumed that all degrees of overlapping are equally probable; the probability density graph is a horizontal line whose ordinate is $\frac{1}{T}$.

Recalling formula (27), the number per unit time of overlaps, type A, whose duration lies between x and $x + \delta x$ is

$$(A'' - C_{\text{True}})TB'' \times \frac{1}{T} \times \delta x,$$

and the duration of these overlaps is

$$(A'' - C_{\text{True}})B''x \delta x.$$

The total duration of the overlaps, type A, per unit time is

$$\int_0^T (A'' - C_{\text{True}})B''x dx$$

$$= \frac{1}{2}(A'' - C_{\text{True}})B''T^2.$$

The corresponding formula for type B overlaps is

$$\frac{1}{2}(B'' - C_{\text{True}})A''T^2.$$

Thus, considering both types, the total duration of overlaps in unit time is

$$[A''B'' - \frac{1}{2} C_{\text{True}}(A'' + B'')]T^2. \quad (28)$$

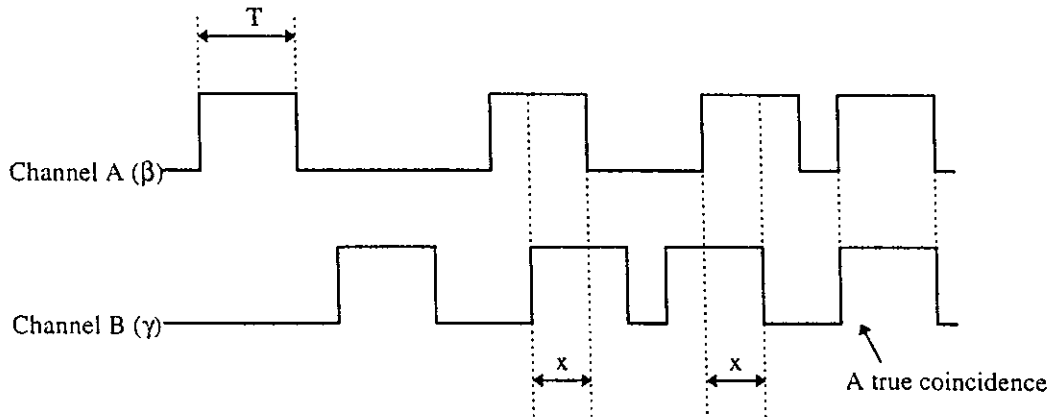


Figure 12 - The duration, x , of an overlap varies between 0 and T . A true coincidence can occur only when both channels are live. In one second, the dead time for true coincidences is equal to the sum of the A dead times, plus the B dead times not started by a true coincidence, minus the overlaps, Σx .

8.3 Dead-Time Correction Equation for C_{True}

The dead-time correction equation for C_{True} is an equation for C_I . Referring to figure 12, the dead time per second during which no true coincidence can be recorded at the mixer is equal to $A''T + (B'' - C_{\text{True}})T - \Sigma x$, where Σx , the sum, during unit time, of the dead-time overlaps is given by formula (28). Thus,

$$C_I = C_{\text{True}} + C_I[A''T + (B'' - C_{\text{True}})T - \Sigma x];$$

$$C_I \{1 - [A''T + (B'' - C_{\text{True}})T - \Sigma x]\} = C_{\text{True}};$$

$$C_I = \frac{C_{\text{True}}}{1 - [A''T + (B'' - C_{\text{True}})T - \Sigma x]} \quad (29)$$

It is of interest to calculate the values of the terms within the square brackets in equation (29).

$$\begin{aligned} A''T &= 4001 \times 5 \times 10^{-6} \\ &= 0.0200. \end{aligned}$$

$$\begin{aligned} (B'' - C_{\text{True}})T &= (183.24 - 160) \times 5 \times 10^{-6} \\ &= 0.000116. \end{aligned}$$

$$\begin{aligned} \Sigma x &= \{(4001 \times 183.24) - [\frac{1}{2} \times 160(4001 + 183.24)]\} \times 25 \times 10^{-12} \\ &= 0.0000100. \end{aligned}$$

8.4 Equation for E_A

The equation for E_A is now obtained by substituting in equation (4) from equations (2), (7) and (29):

$$E_A = \frac{C_I}{B_I} = \frac{C_I}{NE_B}; \quad \text{i.e.,}$$

$$E_A = \frac{C_{\text{True}}}{\{1 - [A''T + (B'' - C_{\text{True}})T - \Sigma x]\} \left\{ \frac{B''}{1 - B''T} - B_b \right\}}, \quad (30)$$

$$\text{where } \Sigma x = [A''B'' - \frac{1}{2}C_{\text{True}}(A'' + B'')]T^2, \quad (28)$$

$$\text{and } C_{\text{True}} = \frac{C' - 2A''B''T_R}{1 - (A'' + B'')T_R}. \quad (13)$$

It can be seen that equation (30) reduces to equation (25) [i.e., Campion's equation for E_A] if we delete the terms $(B'' - C_{\text{True}})T$ and Σx , and partly neglect background.

9. A BETTER VERSION OF EQUATION (20) FOR N

A better version of equation (20) for N is the following rearrangement of equation (6):

$$N = \frac{1}{E_A} \left[\frac{A''}{1 - A''T} - A_b \right], \quad (31)$$

with E_A given by equation (30).

We can reduce equation (31) to equation (20) [i.e., Campion's equation for N] if E_A is given by equation (25), and background is neglected. Thus,

$$N = \frac{B'(1 - A'T)[1 - (A'' + B'')T_R]}{(1 - B'T)(C' - 2A''B''T_R)} \times \frac{A'}{1 - A'T};$$

i.e.

$$N = \frac{A'B'[1 - (A' + B')T_R]}{(1 - B'T)(C' - 2A'B'T_R)}. \quad (20)$$

It may be noted that Campion's equation for E_A is derived from his equation for N; whereas, equation (30) for E_A is a precursor to equation (31) for N.

10. SOURCES CONTAINING A β -EMITTER AND A $\beta\gamma$ -EMITTER

10.1 Third Table of Symbols

Quantity	Symbol
β efficiency with respect to the $\beta\gamma$ emitter	$E_{A\beta\gamma}$
β efficiency with respect to the β emitter	E_{Ax}
β count rate of the $\beta\gamma$ emitter	$A'_{\beta\gamma}$
β count rate of the β emitter	A'_x
β count rate, including background, of a source for which $E_{A\beta\gamma} = 1$, and $E_{Ax} = 1$	A''_1
Mass (in mg.) of solution used to make a source	m
Disintegration rate of the $\beta\gamma$ emitter	$N_{\beta\gamma}$
Unknown disintegration rate of the β emitter	N_x
β intensity of the β emitter	I_x
Radioactivity concentration with respect to the $\beta\gamma$ emitter	$R_{\beta\gamma}$
Radioactivity concentration with respect to the β emitter	R_x

10.2 The β Efficiency with Respect to the $\beta\gamma$ Emitter, $E_{A\beta\gamma}$

10.2.1 Campion's First Equation for C_{True}

In a source containing a β -emitter and a $\beta\gamma$ -emitter,

$$A'' = A'_{\beta\gamma} + A'_x + A_b.$$

This does not make any difference to the derivation of the rate of accidental coincidences which is again given by equation (12). Hence, C_{True} is again given by equation (13).

10.2.2 The Rate of Dead-Time Overlapping

The number per unit time of overlaps, type A, is again equal to

$$(A'' - C_{True})TB'', \tag{27}$$

and the type B formula is likewise unaltered.

10.2.3 Total Duration of Overlaps in Unit Time

As in section 8.2, the total duration of overlaps in unit time (Σx) is given by formula (28).

10.2.4 Dead-Time Correction Equation for C_{True}

As in section 8.3, the true coincidence count rate corrected for dead time, C_I , is given by equation (29).

10.2.5 Equation for $E_{A\beta\gamma}$

Since $C_I = N_{\beta\gamma} E_{A\beta\gamma} E_B$,

$$E_{A\beta\gamma} = \frac{C_I}{N_{\beta\gamma} E_B}, \text{ where } C_I \text{ is given by equation (29) and } N_{\beta\gamma} E_B = \frac{B''}{1 - B''T} - B_b.$$

Thus the formula for $E_{A\beta\gamma}$ is identical with formula (30) for E_A .

10.3 Efficiency Extrapolation

The method of efficiency extrapolation is used for the determination of the radioactivity concentration of a solution of a nuclide which emits β particles, but no γ photons [NCRP 1985:93]. Sources are made from a solution which is prepared by quantitatively mixing the solution of the β -emitting nuclide with a solution of a $\beta\gamma$ -emitting nuclide whose radioactivity concentration has been measured by coincidence counting. It is required that the mean β -particle energy of the $\beta\gamma$ emitter be less than that of the β emitter, and that the sources be completely homogeneous. This ensures that if in a hypothetical source the β efficiency with respect to the $\beta\gamma$ emitter, $E_{A\beta\gamma}$, is unity, then so also will be the β efficiency with respect to the β emitter, E_{Ax} .

The sources are prepared so that one of them has the highest possible β efficiency, and the others have progressively lower efficiencies. They are then subjected to coincidence counting.

Let m = mass (in mg.) of solution deposited on the source mount,
 $N_{\beta\gamma}$ = the known disintegration rate of the $\beta\gamma$ emitter in the source,
 N_x = the unknown disintegration rate of the β emitter in the source,
 I_x = the β intensity of the β emitter.

In the absence of dead time the β count rate would be: $N_{\beta\gamma} E_{A\beta\gamma} + N_x I_x E_{Ax} + A_b$

$$= A'' + A''T(N_{\beta\gamma} E_{A\beta\gamma} + N_x I_x E_{Ax} + A_b).$$

Rearranging,

$$(N_{\beta\gamma}E_{A\beta\gamma} + N_x I_x E_{Ax} + A_b) (1 - A''T) = A'';$$

$$N_{\beta\gamma}E_{A\beta\gamma} + N_x I_x E_{Ax} = \frac{A''}{1 - A''T} - A_b. \quad (32)$$

The radioactivity concentrations of the two nuclides in the source-making solution are denoted respectively by

$$R_{\beta\gamma} = \frac{N_{\beta\gamma}}{m}, \text{ and } R_x = \frac{N_x}{m}. \quad (33)$$

Dividing both sides of equation (32) by m , and substituting from equations (33),

$$R_{\beta\gamma}E_{A\beta\gamma} + R_x I_x E_{Ax} = \frac{A''}{m(1 - A''T)} - \frac{A_b}{m}. \quad (34)$$

Let A_1'' denote the β count rate (including background) of the hypothetical source for which $E_{A\beta\gamma} = 1$, and $E_{Ax} = 1$.

For such a source, equation (34) becomes:

$$R_{\beta\gamma} + R_x I_x = \frac{A_1''}{m(1 - A_1''T)} - \frac{A_b}{m}. \quad (35)$$

The results of the coincidence counting are plotted as shown in figure 13. The abscissae are the values of $E_{A\beta\gamma}$ calculated by formulae (30) or (25), and the ordinates are the corresponding values of $\frac{A''}{m(1 - A''T)} - \frac{A_b}{m}$.

Extrapolation gives the ordinate corresponding to $E_{A\beta\gamma}$ equal to one, i.e.,

$$\frac{A_1''}{m(1 - A_1''T)} - \frac{A_b}{m},$$

which, by equation (35) is equal to $R_{\beta\gamma} + R_x I_x$. Since $R_{\beta\gamma}$ and I_x are known, R_x can now be calculated.

It may be possible to achieve a more nearly linear extrapolation by plotting

$$\frac{A''}{m(1 - A''T)} - \frac{A_b}{m} \text{ against } \frac{1 - E_{A\beta\gamma}}{E_{A\beta\gamma}}, \text{ and extrapolating to } \frac{1 - E_{A\beta\gamma}}{E_{A\beta\gamma}} = 0.$$

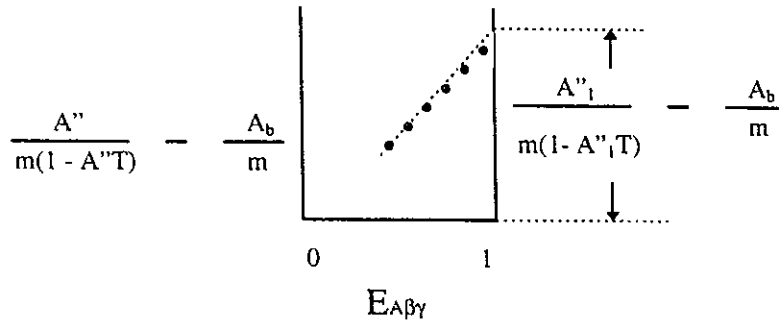


Figure 13 - The value of $\frac{A''}{m(1-A''T)} - \frac{A_b}{m}$ is plotted against $E_{A\beta\gamma}$ for a series of sources and extrapolated to obtain the value corresponding to $E_{A\beta\gamma} = 1$.

Efficiency extrapolation is also used for the determination of the radioactivity concentration of a solution containing only a $\beta\gamma$ -emitting nuclide. The method eliminates the need to make corrections which vanish when $E_{A\beta\gamma} = 1$ [NCRP 1985: 89]. For a source made from such a solution, $R_x = 0$, $R_{\beta\gamma}$ is unknown, and equation (35) becomes

$$R_{\beta\gamma} = \frac{A''_1}{m(1-A''_1T)} - \frac{A_b}{m} \quad (36)$$

Referring to figure 13, $R_{\beta\gamma}$ is equal to the ordinate corresponding to $E_{A\beta\gamma} = 1$, which is obtained by extrapolation.

11. BREMSSTRAHLUNG EFFECTS IN COINCIDENCE COUNTING

11.1 Fourth Table of Symbols

Quantity	Symbol
The minimum energy which a β particle must possess to ensure that it produces a β -amplifier output pulse whose amplitude is greater than the β discriminator level.	E_1
The rate of accepted input pulses to the γ SCA, resulting from the detection of bremsstrahlung (B-photons) by the γ detector.	$N_{\gamma B}$

11.2 Limitations of the Analysis

For simplification the study is restricted as follows:-

11.2.1

Dead-time losses are neglected. Pile-up in the γ amplifier is not considered.

11.2.2

The concept of E_1 (as defined in section 11.1) necessitates the assumption that β -particles of energy greater than E_1 never lose all of their energy in the production of B-photons.

11.2.3

It is supposed that the lower level of the γ SCA is set so that a B-photon input pulse to the γ SCA will be rejected if it is produced by a β -particle whose energy was less than E_1 . This ensures that if a disintegration results in a B-photon output pulse from the γ SCA, the disintegration must be recorded in the β channel; i.e., it is not possible to record a B-photon without also recording the associated β particle.

11.3 The Bremsstrahlung Correction Equation

The detection in the NaI crystal of a γ -photon and a B-photon from the same disintegration gives rise to a sum pulse at the input of the γ SCA. A disintegration may not produce a γ pulse that is acceptable to the γ SCA, but may produce a B-photon pulse which is accepted, thus giving rise to a spurious γ count; and since, according to section 11.2.3, that disintegration will be recorded by the β channel, a spurious coincidence count will be recorded.

Let $N_{\gamma B}$ denote the rate of the following class of detected B-photon: each photon has sufficient energy to provide a γ -SCA input pulse whose amplitude is greater than the γ -SCA lower level. The $N_{\gamma B}$ disintegrations which produce the B-photons also produce $N_{\gamma B} \epsilon_\gamma$ γ -photons, thus giving rise to $N_{\gamma B} \epsilon_\gamma$ sum pulses at the input of the γ SCA. Thus the rate of spurious γ pulses is equal to $N_{\gamma B}(1-\epsilon_\gamma)$. The genuine γ count rate is $N_0 \epsilon_\gamma$. Thus, the observed γ count rate, N_γ is given by:

$$N_\gamma = N_0 \epsilon_\gamma + N_{\gamma B}(1 - \epsilon_\gamma) \quad (37)$$

This derivation is illustrated in figure 14.

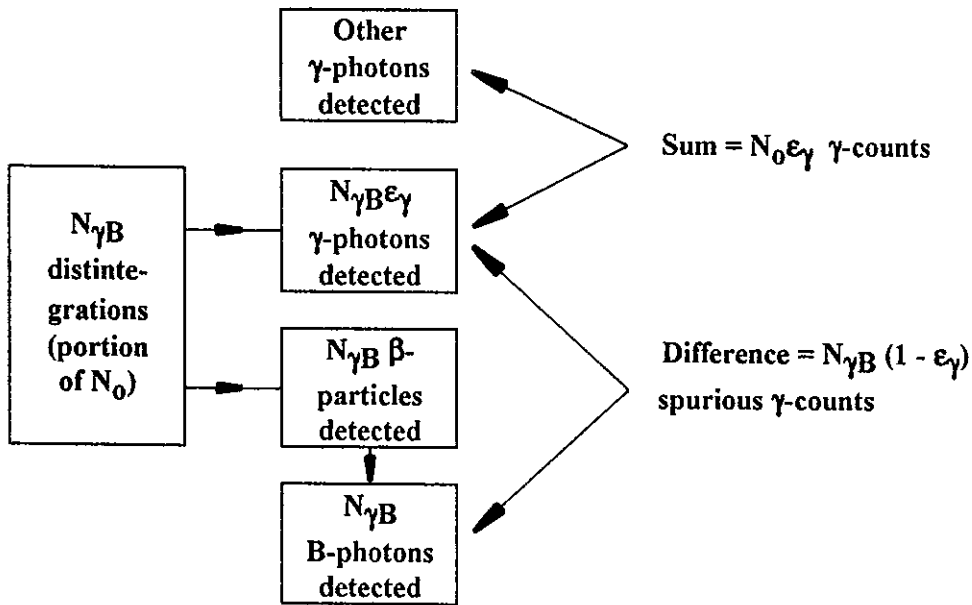


Figure 14 - Diagrammatic representation of the increase in the γ -count rate by bremsstrahlung detected in the NaI crystal.

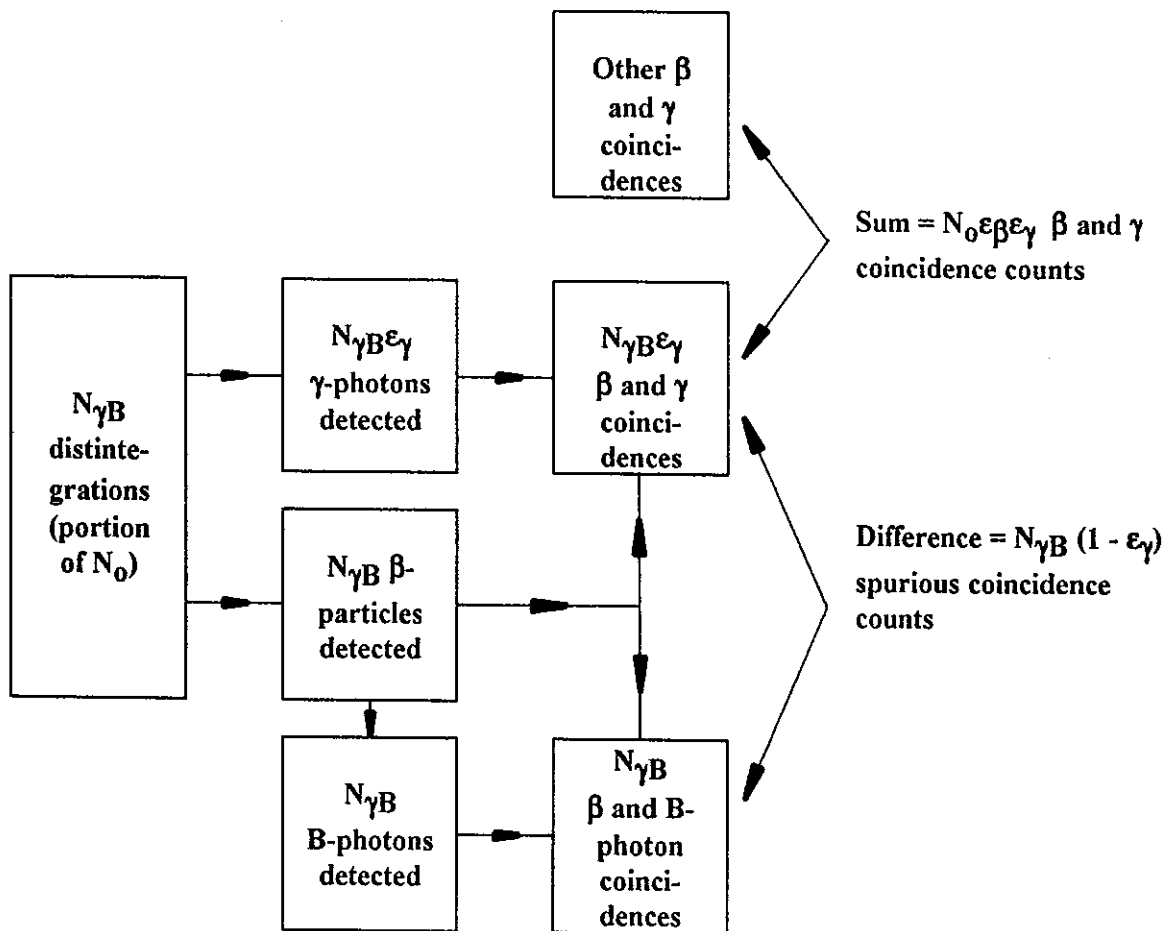


Figure 15 - Diagrammatic representation of the increase in the coincidence-count rate by β and B-photon coincidences.

Each of the $N_{\gamma B}(1 - \epsilon_{\gamma})$ spurious γ pulses entering the coincidence mixer will coincide with one of the $N_{\gamma B}$ β pulses which result from the $N_{\gamma B}$ disintegrations. Thus, the rate of spurious coincidences is $N_{\gamma B}(1 - \epsilon_{\gamma})$. The rate of genuine coincidences is $N_0\epsilon_{\beta}\epsilon_{\gamma}$. Thus, N_c , the "true-coincidence" count rate, is given by:

$$N_c = N_0\epsilon_{\beta}\epsilon_{\gamma} + N_{\gamma B}(1 - \epsilon_{\gamma}). \quad (38)$$

This derivation is illustrated in figure 15.

Substituting from equations (37) and (38), and recalling that $N_{\beta} = N_0\epsilon_{\beta}$,

$$\begin{aligned} \frac{N_{\beta}N_{\gamma}}{N_c} &= \frac{N_0\epsilon_{\beta} [N_0\epsilon_{\gamma} + N_{\gamma B}(1 - \epsilon_{\gamma})]}{N_0\epsilon_{\beta}\epsilon_{\gamma} + N_{\gamma B}(1 - \epsilon_{\gamma})} \\ &= \frac{N_0\epsilon_{\beta}N_0\epsilon_{\gamma} \left[1 + \frac{N_{\gamma B}(1 - \epsilon_{\gamma})}{N_0\epsilon_{\gamma}} \right]}{N_0\epsilon_{\beta}\epsilon_{\gamma} \left[1 + \frac{N_{\gamma B}(1 - \epsilon_{\gamma})}{N_0\epsilon_{\beta}\epsilon_{\gamma}} \right]}, \end{aligned}$$

which, if $\frac{N_{\gamma B}}{N_0}$ is small enough,

$$= N_0 \left[1 + \frac{N_{\gamma B}(1 - \epsilon_{\gamma})}{N_0\epsilon_{\gamma}} \left(1 - \frac{1}{\epsilon_{\beta}} \right) \right].$$

Thus,

$$\frac{N_{\beta}N_{\gamma}}{N_c} = N_0 \left[1 - \frac{N_{\gamma B}(1 - \epsilon_{\gamma})(1 - \epsilon_{\beta})}{N_0\epsilon_{\gamma}\epsilon_{\beta}} \right].$$

The above is equation (6) in Campion's paper, which states that $\frac{N_{\gamma B}}{N_0}$ can be calculated or found experimentally.

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