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LUCAS HEIGHTS RESEARCH LABORATORIES

RESULTS OF PIPE BEND ANALYSIS
PART V: FLEXIBILITY AND PRESSURE DEFLECTION FACTORS OF OVAL PIPE BENDS

by

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ABSTRACT

A bending flexibility factor and a pressure deflection factor are defined for pipe bends with oval cross-sections, and numerical values of these factors are presented for pipe bends with major-to-minor axis ratios from 1 (circular) to 2 over a wide range of bend radii and wall thicknesses, ignoring end effects.

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ELLIPTICAL CONFIGURATION; FLEXIBILITY; PIPES; PRESSURE DEPENDENCE

CONTENTS

1. INTRODUCTION

Pipe bends with oval cross-sections have been analysed by thin shell theory neglecting end effects [Whatham 1981]. In this report, numerical values of flexibility and pressure deflection factors from that analysis are presented for design application.

The oval cross-sections considered have major-to-minor axis ratios ranging from 1 (circular) to 2. Adopting the nominal radius r of the cross-section as that of a circular pipe with the same circumference, the bends considered have wall thicknesses t ranging from 0.01 r to 0.1 r and bend radii R ranging from 1.25 r to that giving a bend characteristic $h(=Rt/r^2)$ of 2. Poisson's ratio ν is assumed to be 0.3.

2. PIPE CROSS-SECTIONS

Oval and circular cross-sections with the same middle surface circumference are shown in Figure 1, the middle surface being imagined mid-way between the inner and outer surfaces of a pipe bend. Points A are the same distance from the extrados and the oval contour is defined by

$$\mu = \theta + \mu_2 \sin 2\theta \, . \qquad (1)$$

Cartesian coordinates are given by

$$x = -r \int_0^\theta \cos \mu \, d\theta \, , \text{ and}$$
$$y = a/2 - r \int_0^\theta \sin \mu \, d\theta \, , \qquad (2)$$

from which polar coordinates can be derived. Such coordinates for cross-sections with various degrees of ovality are tabulated in Appendix A, and quadrants of two of these cross-sections are plotted in Figure 2 together with those of elliptical shapes with the same major-to-minor axis ratio b/a; the departure from the elliptical shapes is not considered significant unless b/a >1.4.

3. FLEXIBILITY FACTOR

The flexibility factor f of a curved pipe subjected to pure in-plane bending is defined by

$$f = \frac{\alpha}{\alpha'} \quad , \tag{3}$$

where α is the rotation at the pipe end due to a bending moment M, calculated by thin shell theory, and α' is the rotation at the end of a circular pipe bend of the same circumference acted upon by the same bending moment, calculated by elastic line theory; thus

$$f = \frac{\pi r^3 E t \alpha}{M R \phi} \quad , \tag{4}$$

where ϕ = bend angle radians, and
E = Young's modulus.

Flexibility factors are plotted in Figure 3 against h for pipe bends with three cross-sections, namely circular and with two degrees of ovality, and numerical values for these and other oval cross-sections are given in Appendix B. There is an inverse proportionality to h exhibited by all curves in Figure 3, provided that $f > 4$ and, as expected, the factors increase with cross-section flattening.

4. PRESSURE DEFLECTION FACTOR

The pressurising solution for a non-circular pipe bend consists of virtually a membrane solution plus a bending moment solution which becomes more dominant as cross-section flattening is increased. The bending moment solution is to counteract a moment M_p generated by the membrane solution and its effect is to reduce the curvature of the bend; its magnitude is

$$M_p = \pi C_1 R r^2 p \quad , \tag{5}$$

where p is the pressure and C_1 is a cross-section coefficient which is independent of t/r, R/r and ν, and depends solely on the geometry of the cross-section; for pipe bends with circular cross-sections it is zero. Values of C_1 are given in Table 1 for particular oval sections.

The pressure deflection factor f_p is defined by

$$f_p = \frac{\alpha}{\alpha_p'} \quad , \tag{6}$$

where α is the rotation at the pipe end due to pressure p, calculated by thin shell theory, and α_p' is the rotation at the end of a circular pipe bend of the same circumference acted upon by moment M_p/C_1, calculated by elastic line theory; thus

$$f_p = \frac{rEt\alpha}{pR^2\phi} \quad . \tag{7}$$

Pressure deflection factors are plotted in Figure 4 against h for pipe bends with cross-sections having three degrees of ovality; numerical values for these and other oval cross-sections are given in Appendix C. The dominating effect of M_p causes the pressure deflection factors to show the same inverse proportionality to h as flexibility factors, provided that $f_p > 4$ and $b/a > 1.1$. Then

$$f_p \doteqdot C_1 f \quad . \tag{8}$$

The validity of this approximation is demonstrated in Table 2 for pipe bends with $h = 0.2$. The deflection from pressurising a circular pipe bend is

$$\alpha = \frac{r\phi p}{2Et} \quad \text{[Whatham and Thompson 1979]} \tag{9}$$

whence, by equation (7),

$$f_p = (t/r)^2/(2h^2) \quad . \tag{10}$$

5. CONCLUSIONS

Oval section pipe bends have been considered with cross-section shapes generated by

$$\mu = \theta + \mu_2 \sin 2\theta$$

where μ and θ are defined in Figure 1, $(\mu - \theta)$ being the change in slope from circular around the circumference.

Numerical values of a flexibility factor defined by

$$f = \frac{\pi r^3 E t \alpha}{M R \phi}$$

and a pressure deflection factor

$$f_p = \frac{r E t \alpha}{p R^2 \phi}$$

have been tabulated for a wide range of pipe bends with different degrees of ovality and are plotted versus the pipe bend characteristic h (= Rt/r^2).

Log f is related linearly to log h when $b/a \leq 2$ and $f > 4$, and log f_p related linearly to log h when $1.1 \leq b/a \leq 2$ and $f_p > 4$. As cross-sections approach circular, then f_p approaches

$$f_p = \tfrac{1}{2}(t/r)^2/h^2 \quad ,$$

and provided that $b/a > 1.1$,

$$f_p \doteqdot C_1 f \quad ,$$

where C_1 is a cross-section coefficient which is independent of R/r, t/r and ν ; values of C_1 are tabulated for cross-sections with various degrees of ovality.

6. REFERENCES

Novozhilov, V.V. [1970] - Thin Shell Theory. 2nd Augmented and Revised Edition, Wolters-Noordhoff, Gröningen, The Netherlands.

Whatham, J.F. [1981] - Thin Shell Analysis of Non-circular Pipe Bends. Nucl. Eng. Des., 67(2)287.

Whatham, J.F. and Thompson, J.J. [1979] - The Bending and Pressurising of Pipe Bends with Flanged Tangents. Nucl. Eng. Des., 54(1)17.

TABLE 1
OVAL PIPE SECTION CONSTANTS

b/a	μ_2	C_1
1	0	0
1.1	-0.0714	0.1679
1.2	-0.1360	0.2862
1.3	-0.1945	0.3705
1.4	-0.2476	0.4308
1.5	-0.2960	0.4737
1.6	-0.3400	0.5038
1.8	-0.4171	0.5381
2.0	-0.4821	0.5508

TABLE 2
PRESSURE DEFLECTION FACTOR VERSUS OVALITY

$h = 0.2, \quad \nu = 0.3$

b/a	R/r = 20		10		4		2	
	t/r = 0.01		0.02		0.05		0.1	
	f_p	C_1f	f_p	C_1f	f_p	C_1f	f_p	C_1f
1	0.001	0	0.005	0	0.031	0	0.125	0
1.001	0.016	0.017	0.020	0.017	0.046	0.017	0.139	0.016
1.01	0.146	0.165	0.150	0.164	0.175	0.164	0.265	0.163
1.1	1.43	1.59	1.44	1.59	1.45	1.59	1.51	1.58
1.2	2.78	3.05	2.78	3.05	2.79	3.04	2.82	3.01
1.4	5.11	5.49	5.11	5.49	5.11	5.47	5.09	5.41
2	9.17	9.62	9.17	9.61	9.15	9.57	9.10	9.43

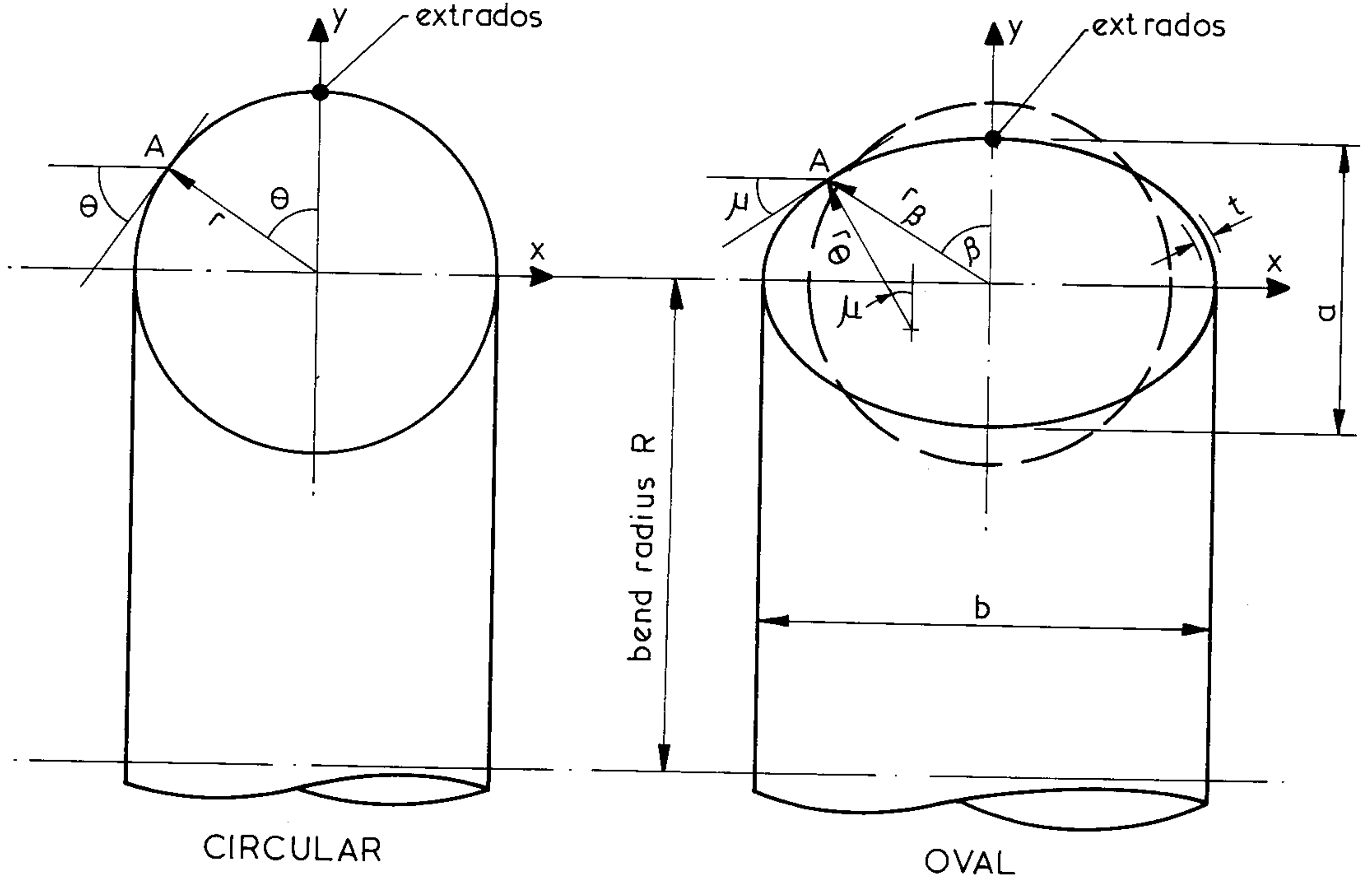

FIGURE 1. DEVELOPMENT OF OVAL SHAPE

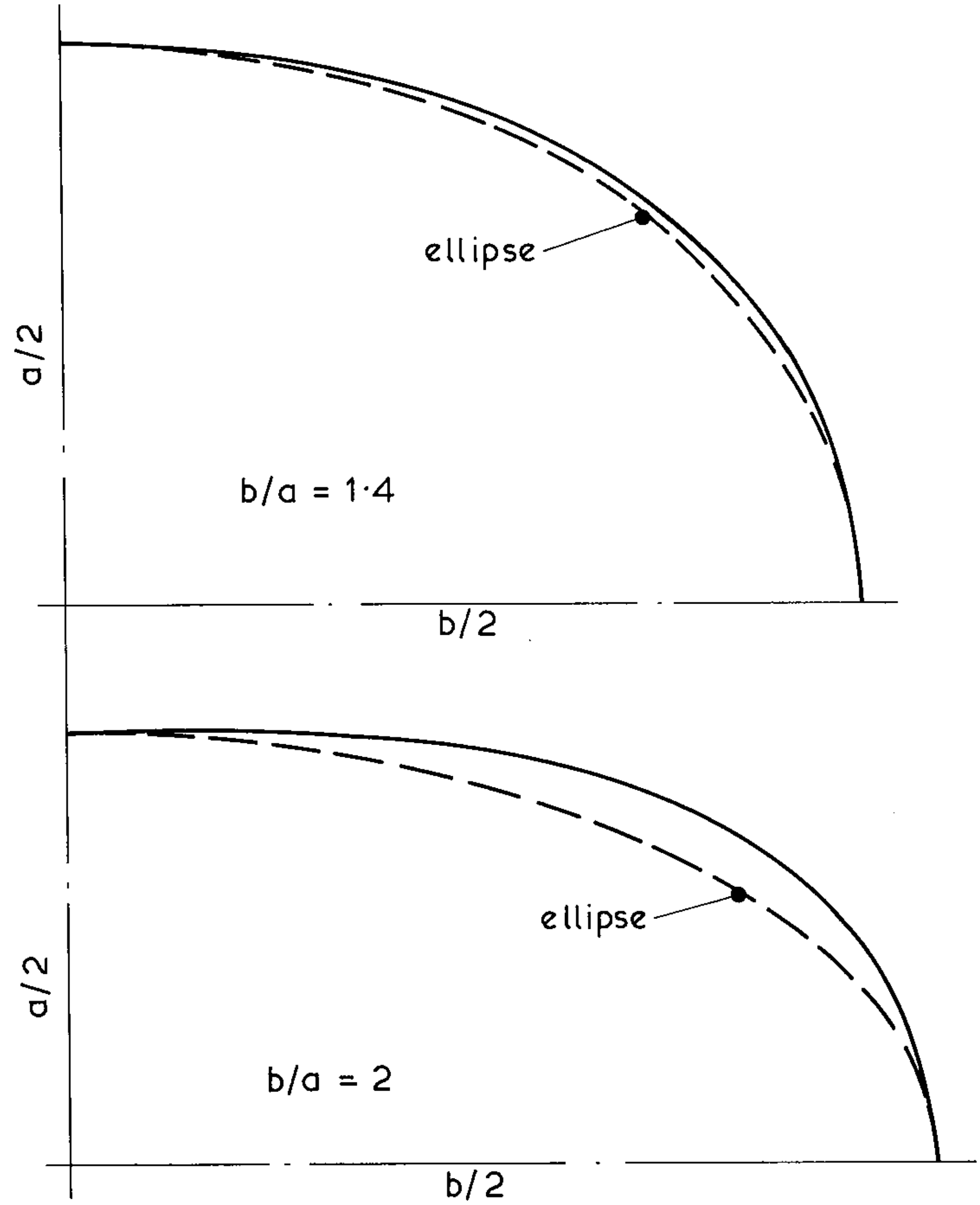

FIGURE 2. QUADRANTS OF OVAL SECTIONS

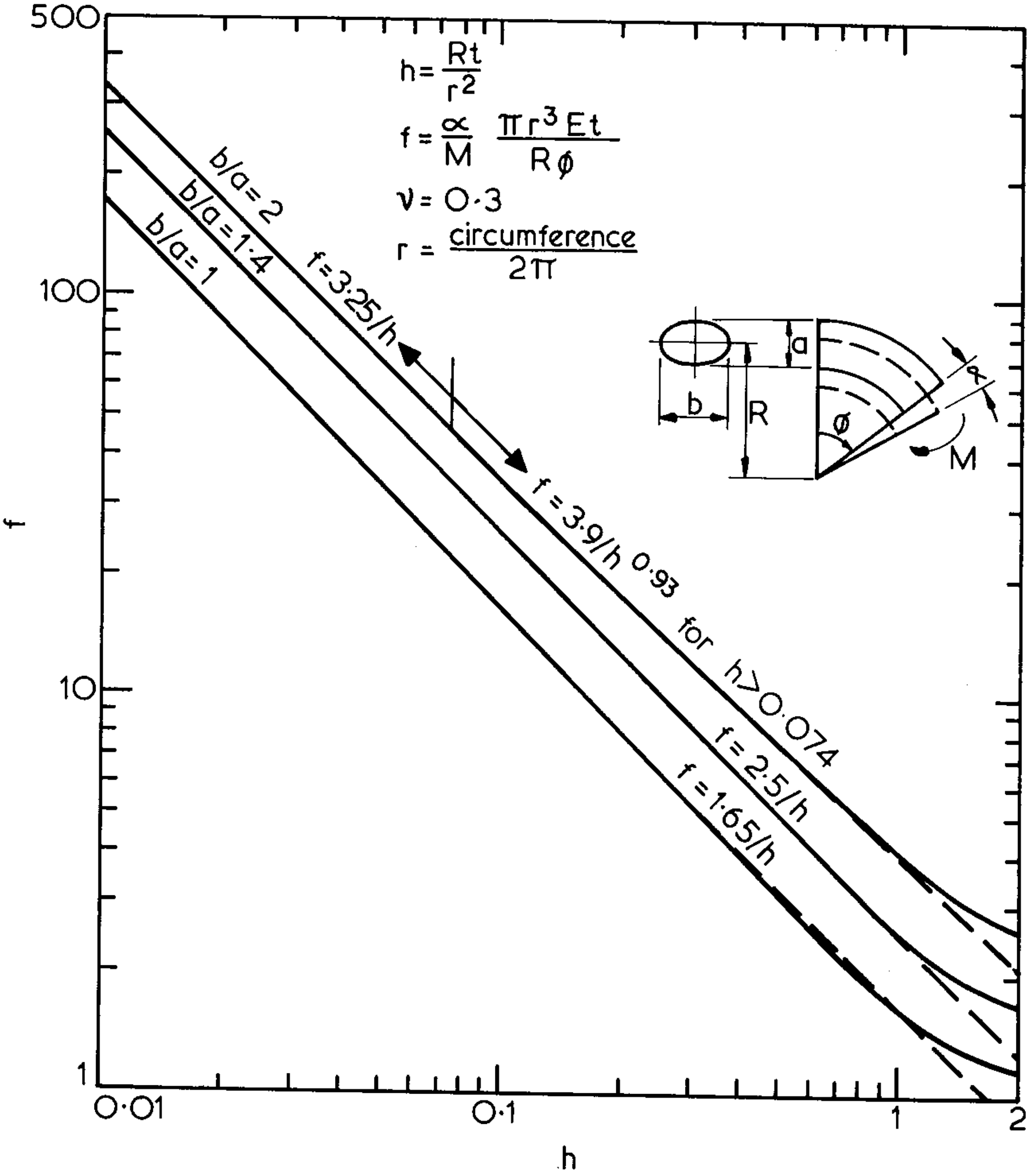

FIGURE 3. FLEXIBILITY FACTORS

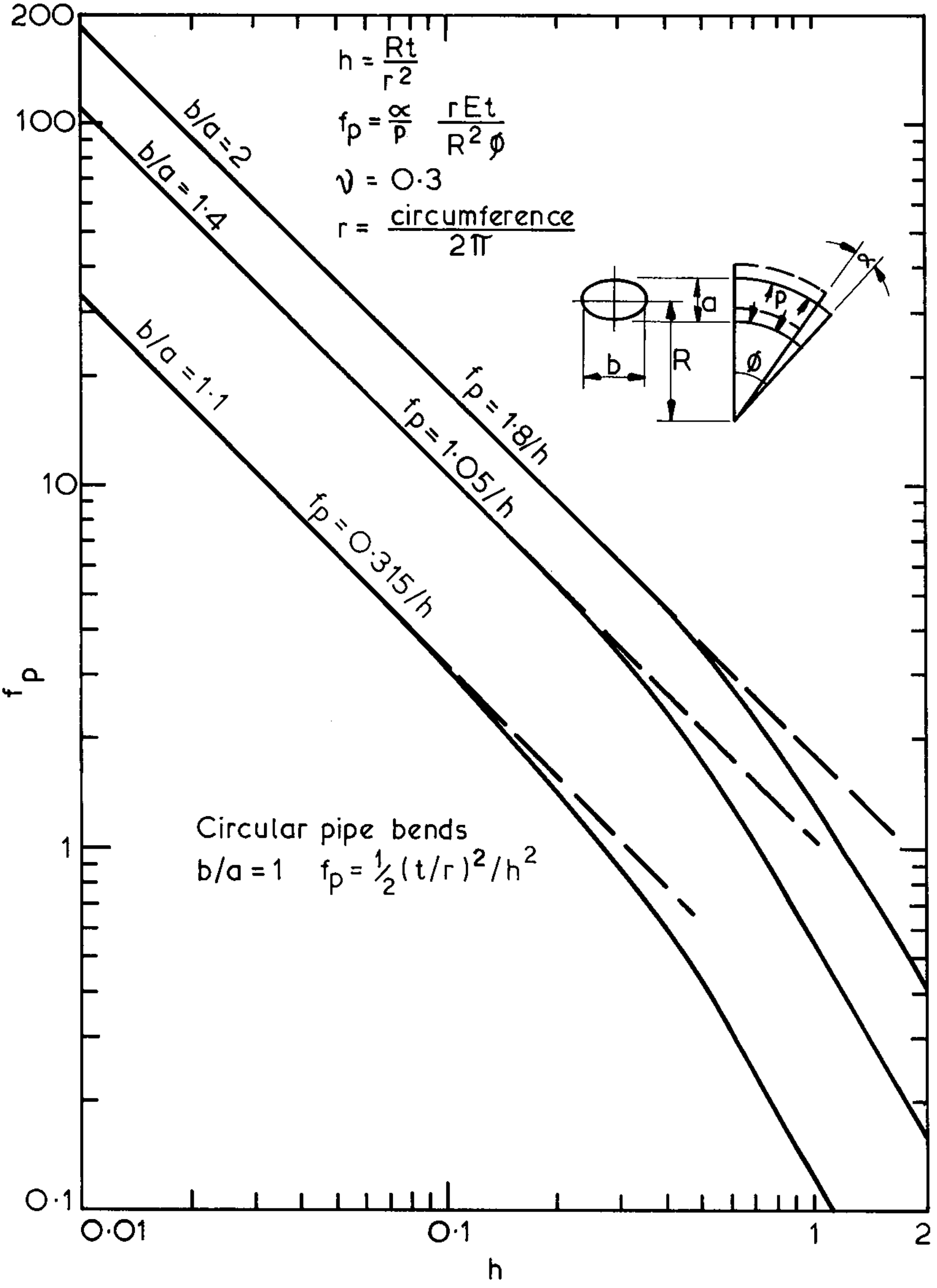

FIGURE 4. PRESSURE DEFLECTION FACTORS

APPENDIX A
OVAL CROSS-SECTIONS IN POLAR COORDINATES

Nomenclature:

b/a = major-to-minor axis ratio of mid-wall surface

β, r_β = polar coordinates,

$r = \dfrac{\text{mid-wall circumference}}{2\pi}$

Only one quadrant is given; cross-sections are symmetrical about both axes.

TABLE A1

b/a	β°	r_β/r	r_β/r (Ellipse)	b/a	β°	r_β/r	r_β/r (Ellipse)
1.1	0.0	0.9511	0.9511	1.2	0.0	0.9046	0.9046
	10.50	0.9540	0.9538		11.02	0.9103	0.9097
	20.93	0.9624	0.9618		21.87	0.9265	0.9244
	31.23	0.9752	0.9741		32.42	0.9512	0.9472
	41.86	0.9908	0.9893		42.63	0.9810	0.9755
	51.33	1.0073	1.0058		52.50	1.0123	1.0066
	61.14	1.0228	1.0215		62.10	1.0414	1.0367
	70.83	1.0353	1.0346		71.50	1.0650	1.0623
	80.43	1.0434	1.0432		80.78	1.0803	1.0795
	90.00	1.0462	1.0462		90.00	1.0855	1.0855
1.3	0.0	0.8608	0.8608	1.4	0.0	0.8198	0.8198
	11.56	0.8691	0.8680		12.11	0.8305	0.8288
	22.32	0.8927	0.8885		23.77	0.8610	0.8545
	33.59	0.9282	0.9202		34.71	0.9065	0.8938
	43.80	0.9709	0.9598		44.90	0.9608	0.9429
	53.54	1.0155	1.0035		54.47	1.0172	0.9973
	62.91	1.0567	1.0467		63.61	1.0692	1.0522
	72.05	1.0900	1.0841		72.51	1.1110	1.1009
	81.05	1.1116	1.1098		81.28	1.1382	1.1352
	90.00	1.1190	1.1190		90.00	1.1476	1.1476
1.5	0.0	0.7814	0.7814	1.6	0.0	0.7457	0.7457
	12.63	0.7945	0.7921		13.25	0.7610	0.7579
	24.72	0.8314	0.8224		25.67	0.8039	0.7924
	35.80	0.8861	0.8683		36.85	0.8670	0.8439
	45.92	0.9508	0.9252		46.87	0.9412	0.9073
	55.31	1.0177	0.9889		56.06	1.0174	0.9786
	64.22	1.0793	1.0541		64.75	1.0874	1.0530
	72.90	1.1288	1.1135		73.22	1.1438	1.1224
	81.46	1.1610	1.1563		81.62	1.1804	1.1738
	90.00	1.1721	1.1721		90.00	1.1931	1.1931
1.8	0.0	0.6815	0.6815	2.0	0.0	0.6260	0.6260
	14.42	0.7010	0.6966		15.60	0.6492	0.6437
	27.52	0.7548	0.7382		29.32	0.7128	0.6912
	38.82	0.8327	0.7986		40.64	0.8032	0.7581
	48.59	0.9230	0.8718		50.09	0.9065	0.8374
	57.36	1.0150	0.9545		58.44	1.0111	0.9275
	65.63	1.0992	1.0438		66.33	1.1067	1.0279
	73.75	1.1671	1.1314		74.14	1.1839	1.1315
	81.86	1.2113	1.2000		82.03	1.2343	1.2173
	90.00	1.2267	1.2267		90.00	1.2519	1.2519

APPENDIX B
FLEXIBILITY FACTORS

Parameters of pipe bends considered:

b/a = 1, 1.2, 1.4, 1.6, 2.0

R/r = 1.25 to $2r/t$

t/r = 0.01, 0.02, 0.05, 0.1

Poisson's ratio = 0.3

$r = \dfrac{\text{mid-wall circumference}}{2\pi}$

ϕ = bend angle radians

TABLE B1

$$\alpha = f\,\frac{MR\phi}{\pi r^3 Et}$$

b/a = 1 (circular pipe)

	f			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	132	65.8	26.1	12.9
1.5	110	54.9	21.9	10.9
2	82.6	41.3	16.5	8.21
2.5	66.1	33.0	13.2	6.55
3	55.1	27.5	11.0	5.43
4	41.3	20.7	8.26	3.99
5	33.0	16.5	6.59	3.14
6.5	25.4	12.7	5.00	2.39
8	20.7	10.3	3.99	1.96
10	16.5	8.27	3.13	1.64
13	12.7	6.33	2.38	1.39
16	10.3	5.09	1.96	1.26
20	8.27	3.99	1.64	1.17
26	6.33	3.00	1.39	
32	5.09	2.42	1.26	
40	3.99	1.96	1.17	
50	3.13	1.64		
65	2.38	1.39		
80	1.96	1.26		
100	1.64	1.17		
130	1.39			
160	1.26			
200	1.17			

TABLE B2

$$\alpha = f \frac{MR\phi}{\pi r^3 Et}$$

b/a = 1.2

	f			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	168	83.7	33.2	16.4
1.5	140	69.9	27.9	13.9
2	105	52.5	21.0	10.5
2.5	84.1	42.1	16.9	8.44
3	70.1	35.1	14.1	7.02
4	52.6	26.4	10.6	5.20
5	42.1	21.1	8.51	4.09
6.5	32.4	16.3	6.51	3.10
8	26.4	13.3	5.22	2.52
10	21.1	10.6	4.10	2.07
13	16.3	8.19	3.10	1.72
16	13.3	6.62	2.52	1.54
20	10.6	5.22	2.07	1.40
26	8.20	3.93	1.72	
32	6.62	3.15	1.54	
40	5.22	2.52	1.40	
50	4.10	2.07		
65	3.10	1.72		
80	2.52	1.54		
100	2.07	1.40		
130	1.72			
160	1.54			
200	1.40			

TABLE B3

$$\alpha = f\,\frac{MR\phi}{\pi r^3 Et}$$

b/a = 1.4

		f		
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	197	98.5	39.2	19.5
1.5	165	82.3	27.9	16.5
2	124	61.9	21.0	12.6
2.5	99.0	49.6	20.0	10.1
3	82.6	41.4	16.8	8.49
4	62.1	31.2	12.7	6.36
5	49.7	25.0	10.2	5.04
6.5	38.3	19.3	7.91	3.82
8	31.2	15.8	6.40	3.10
10	25.0	12.7	5.06	2.53
13	19.4	9.88	3.83	2.08
16	15.8	8.05	3.11	1.84
20	12.7	6.41	2.53	1.66
26	9.89	4.85	2.08	
32	8.05	3.89	1.84	
40	6.41	3.11	1.67	
50	5.06	2.53		
65	3.83	2.08		
80	3.11	1.84		
100	2.53	1.67		
130	2.08			
160	1.84			
200	1.67			

TABLE B4

$$\alpha = f \frac{MR\phi}{\pi r^3 Et}$$

b/a = 1.6

	f			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	222	111	44.2	22.0
1.5	185	92.6	37.1	18.7
2	139	69.7	28.1	14.3
2.5	111	55.9	22.7	11.6
3	93.0	46.7	19.1	9.81
4	69.9	35.2	14.5	7.44
5	56.0	28.3	11.8	5.94
6.5	43.3	22.0	9.18	4.54
8	35.3	18.0	7.49	3.69
10	28.3	14.6	5.97	3.01
13	22.0	11.4	4.55	2.45
16	18.0	9.34	3.70	2.16
20	14.6	7.50	3.01	1.94
26	11.4	5.74	2.46	
32	9.34	4.63	2.16	
40	7.50	3.70	1.94	
50	5.98	3.01		
65	4.56	2.46		
80	3.70	2.16		
100	3.01	1.94		
130	2.46			
160	2.16			
200	1.94			

TABLE B5

$$\alpha = f \frac{MR\phi}{\pi r^3 Et}$$

$b/a = 2.0$

	f			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	259	130	51.8	26.0
1.5	217	108	43.6	22.1
2	163	81.7	33.1	17.1
2.5	130	65.6	26.8	14.1
3	109	54.9	22.6	12.0
4	81.9	41.4	17.4	9.29
5	65.8	33.4	14.2	7.56
6.5	50.8	26.0	11.3	5.88
8	41.5	21.4	9.37	4.83
10	33.4	17.5	7.61	3.95
13	26.0	13.8	5.91	3.22
16	21.4	11.5	4.85	2.82
20	17.5	9.40	3.96	2.52
26	13.8	7.35	3.22	
32	11.5	6.01	2.82	
40	9.40	4.85	2.53	
50	7.63	3.97		
65	5.92	3.23		
80	4.85	2.83		
100	3.97	2.53		
130	3.23			
160	2.83			
200	2.53			

APPENDIX C
PRESSURE DEFLECTION FACTORS

Parameters of pipe bends considered:

b/a = 1.1, 1.2, 1.4, 1.6, 2.0

R/r = 1.25 to $2r/t$

t/r = 0.01, 0.02, 0.05, 0.1

Poisson's ratio = 0.3

$r = \frac{\text{mid-wall circumference}}{2\pi}$

ϕ = bend angle radians

TABLE C1

$$\alpha = f_p \frac{pR^2\phi}{rEt}$$

b/a = 1.1

		f_p		
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	25.4	12.7	5.12	2.57
1.5	21.1	10.6	4.23	2.11
2	15.8	7.87	3.11	1.51
2.5	12.6	6.25	2.45	1.16
3	10.5	5.18	2.00	0.922
4	7.80	3.84	1.45	0.626
5	6.21	3.04	1.12	0.450
6.5	4.74	2.30	0.809	0.294
8	3.82	1.84	0.610	0.206
10	3.02	1.44	0.438	0.137
13	2.29	1.06	0.286	0.084
16	1.83	0.819	0.200	0.057
20	1.43	0.605	0.134	0.037
26	1.06	0.409	0.082	
32	0.818	0.292	0.055	
40	0.605	0.198	0.036	
50	0.434	0.133		
65	0.284	0.081		
80	0.198	0.055		
100	0.132	0.036		
130	0.081			
160	0.055			
200	0.036			

TABLE C2

$$\alpha = f_p \frac{pR^2\phi}{rEt}$$

b/a = 1.2

		f_p		
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	48.0	23.9	9.50	4.67
1.5	40.0	19.9	7.89	3.86
2	29.9	14.9	5.86	2.82
2.5	23.9	11.9	4.63	2.19
3	19.9	9.85	3.82	1.76
4	14.9	7.32	2.79	1.22
5	11.8	5.81	2.17	0.883
6.5	9.05	4.41	1.58	0.585
8	7.31	3.54	1.21	0.412
10	5.80	2.78	0.874	0.277
13	4.41	2.07	0.578	0.171
16	3.54	1.61	0.407	0.116
20	2.78	1.20	0.274	0.077
26	2.07	0.822	0.169	
32	1.61	0.591	0.115	
40	1.20	0.405	0.075	
50	0.872	0.273		
65	0.576	0.168		
80	0.405	0.114		
100	0.272	0.075		
130	0.168			
160	0.114			
200	0.075			

TABLE C3

$$\alpha = f_p \frac{pR^2\phi}{rEt}$$

$b/a = 1.4$

	f_p			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	84.8	42.3	16.8	8.22
1.5	70.7	35.3	14.0	6.86
2	53.0	26.4	10.5	5.09
2.5	42.4	21.1	8.32	4.00
3	35.3	17.6	6.90	3.26
4	26.4	13.1	5.11	2.30
5	21.1	10.5	4.02	1.70
6.5	16.2	7.99	2.98	1.15
8	13.1	6.45	2.31	0.824
10	10.5	5.11	1.70	0.563
13	7.99	3.86	1.15	0.354
16	6.45	3.04	0.820	0.244
20	5.11	2.31	0.560	0.164
26	3.86	1.61	0.352	
32	3.04	1.18	0.242	
40	2.31	0.819	0.163	
50	1.70	0.559		
65	1.15	0.351		
80	0.818	0.241		
100	0.558	0.162		
130	0.350			
160	0.241			
200	0.162			

TABLE C4

$$\alpha = f_p \frac{pR^2 \phi}{rEt}$$

b/a = 1.6

R/r	f_p t/r = 0.01	0.02	0.05	0.1
1.25	112	55.6	22.1	10.9
1.5	93.0	46.4	18.5	9.13
2	69.8	34.8	13.9	6.84
2.5	55.8	27.9	11.1	5.44
3	46.5	23.2	9.23	4.48
4	34.9	17.4	6.90	3.22
5	27.9	13.9	5.48	2.42
6.5	21.4	10.7	4.13	1.67
8	17.4	8.66	3.24	1.21
10	13.9	6.91	2.43	0.838
13	10.7	5.27	1.67	0.535
16	8.66	4.21	1.21	0.374
20	6.91	3.24	0.834	0.255
26	5.27	2.30	0.532	
32	4.21	1.71	0.371	
40	3.24	1.21	0.253	
50	2.43	0.833		
65	1.67	0.531		
80	1.21	0.370		
100	0.832	0.252		
130	0.530			
160	0.370			
200	0.252			

TABLE C5

$$\alpha = f_p \frac{pR^2 \phi}{rEt}$$

$b/a = 2.0$

		f_p		
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	143	71.2	28.4	14.3
1.5	119	59.5	23.8	12.0
2	89.4	44.7	18.0	9.10
2.5	71.5	35.8	14.4	7.34
3	59.6	29.9	12.1	6.14
4	44.8	22.5	9.16	4.56
5	35.9	18.0	7.38	3.53
6.5	27.6	13.9	5.69	2.52
8	22.5	11.4	4.57	1.88
10	18.0	9.17	3.52	1.34
13	13.9	7.11	2.50	0.884
16	11.4	5.78	1.86	0.634
20	9.17	4.56	1.32	0.447
26	7.11	3.35	0.864	
32	5.78	2.55	0.617	
40	4.56	1.85	0.432	
50	3.51	1.31		
65	2.49	0.854		
80	1.84	0.608		
100	1.30	0.425		
130	0.851			
160	0.605			
200	0.423			