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RESEARCH ESTABLISHMENT

LUCAS HEIGHTS RESEARCH LABORATORIES

RESULTS OF PIPE BEND ANALYSIS
PART IV: FLEXIBILITY FACTORS OF PIPE BENDS WITH FLANGED TANGENTS

by

J.F. WHATHAM

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ABSTRACT

An equivalent flexibility factor is defined for pipe bends with flanged tangent pipes under pure in-plane bending, and numerical values of this factor are presented for a range of 90° and 180° bends with various tangent lengths.

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FLANGES; FLEXIBILITY; PIPES

CONTENTS

1. INTRODUCTION

Pipe bends with equal length flange-ended tangent pipes have been analysed by Whatham and Thompson [1979] using the thin shell theory of Novozhilov [1970]. In this report, numerical values of flexibility factors from that analysis are presented for pipework design purposes.

The tangent lengths range from zero (flange on bend) to $2\pi r$, then greater than $2\pi r$, in which case the flanges are ignored. Loading is by a pure in-plane bending moment and the flanges are assumed to be rigid.

2. EQUIVALENT FLEXIBILITY FACTOR

For the pipe bend configuration shown in Figure 1, the equivalent flexibility factor f' is defined by

$$f' = \frac{\gamma - 2\gamma_L}{\gamma'}$$

and the flexibility factor f is obtained when L = 0,

where

γ = relative flange rotation due to bending moment M, calculated by thin shell theory,

γ_L = relative end rotation of each tangent pipe under bending M, calculated by elastic line theory

$$= \frac{ML}{\pi r^3 Et},$$

γ' = relative end rotation of the curved pipe alone under bending moment M, calculated by elastic line theory

$$= \frac{MR\phi'}{180r^3Et}, \quad \text{and}$$

E = Young's modulus.

Flexibility factors are plotted in Figure 2 versus pipe bend characteristic $h(=Rt/r^2)$ and equivalent flexibility factors are plotted in Figures 3 and 4 for various tangent lengths, with bend angles of 90° and 180°; numerical values of these factors are given in Appendix A. The pipe bends considered range from h = 1.25 t/r to h = 2 when all equivalent flexibility factors approximate the flexibility factor for a pipe bend without end effects (Table A17); this value is

$$f^e = 1.17 \quad .$$

For h > 2, the following relationship, derived from the work of von Kármán [1911] applies:

$$f^e = 1 + 3(1 - \nu^2)/(4h^2)$$

$$= 1 + 0.68/h^2 \text{ for } \nu = 0.3 \quad ,$$

where ν = Poisson's ratio.

3. REFERENCES

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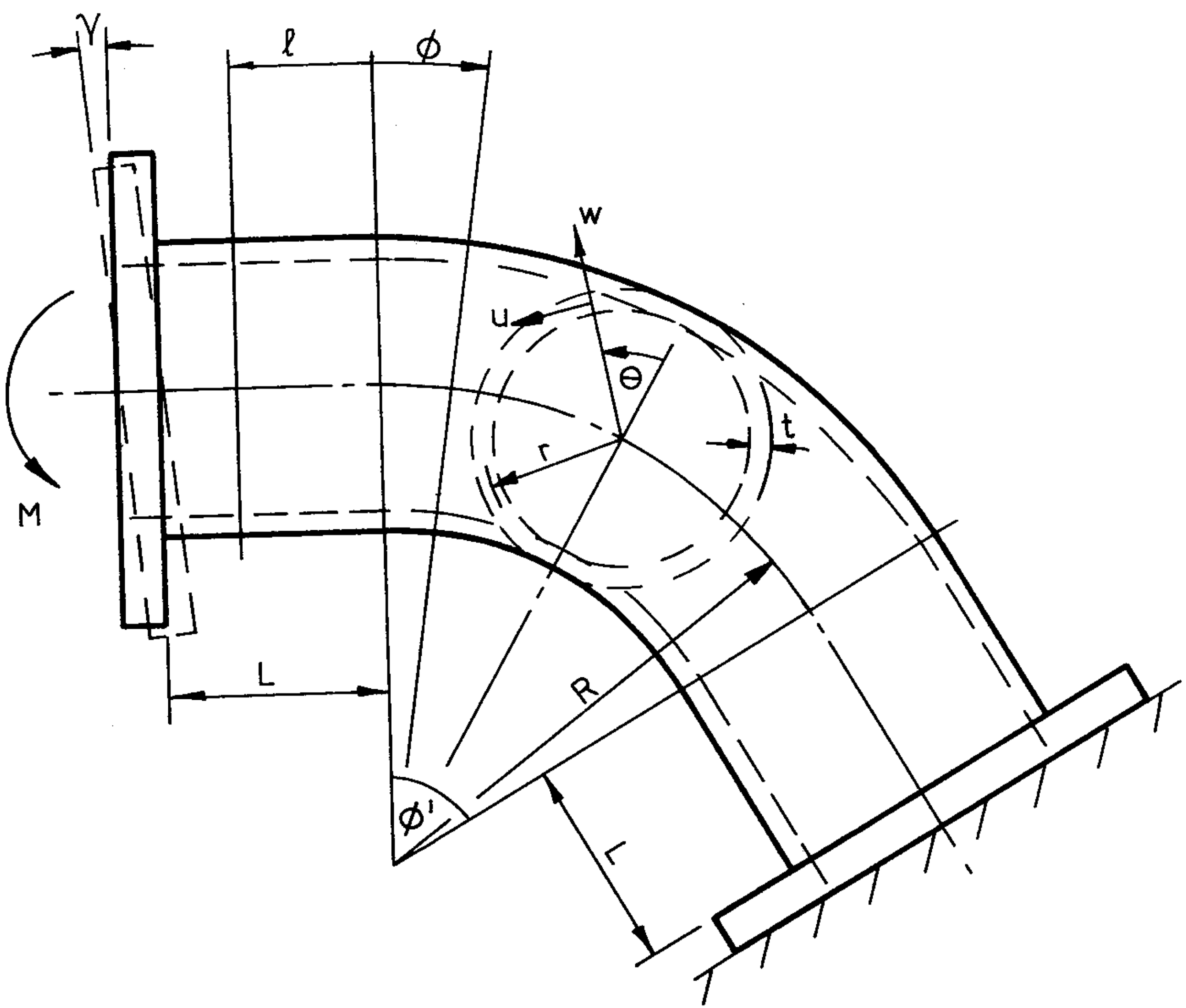


FIGURE 1. PIPE BEND CONFIGURATION

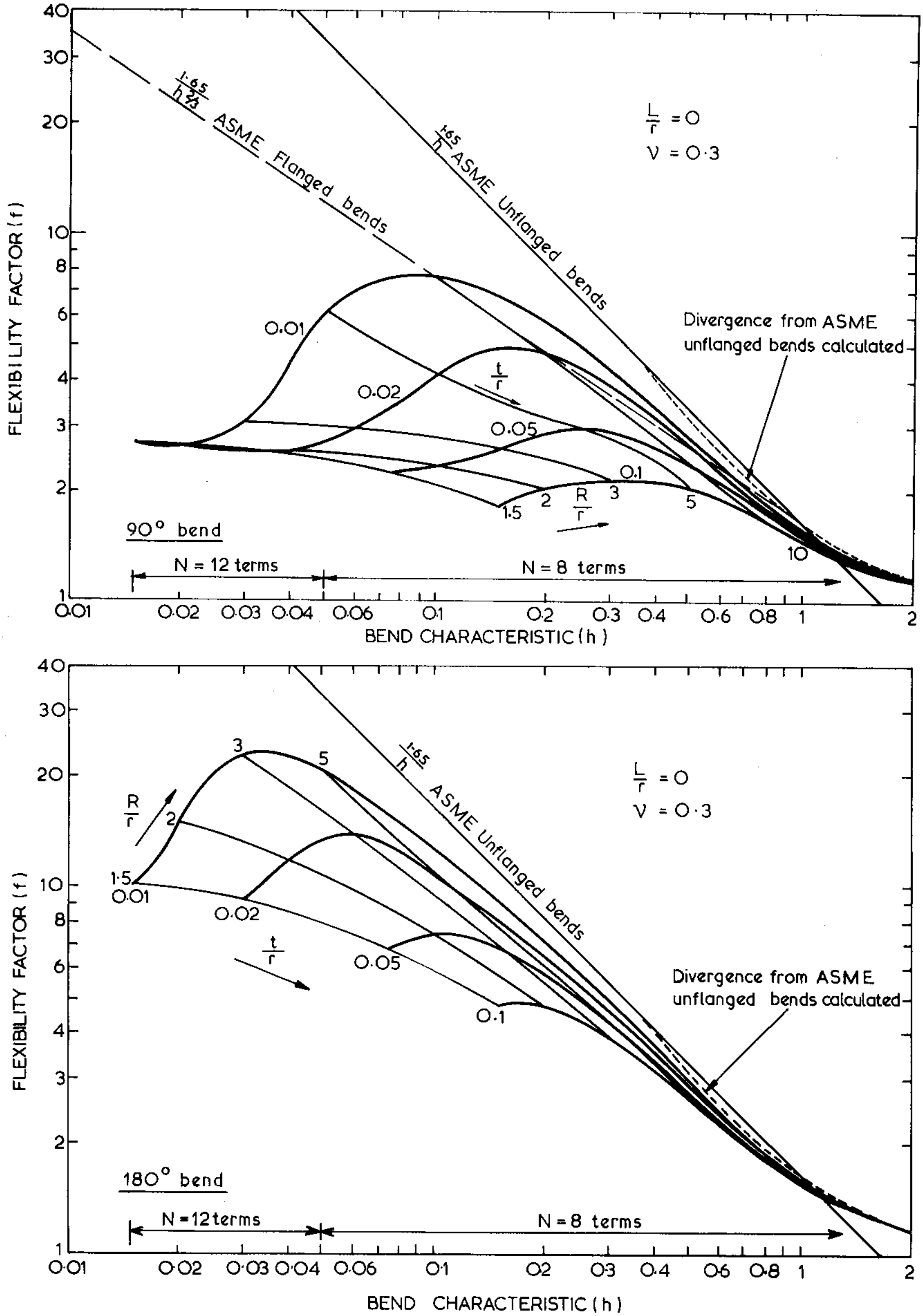


FIGURE 2. FLEXIBILITY OF FLANGED BENDS

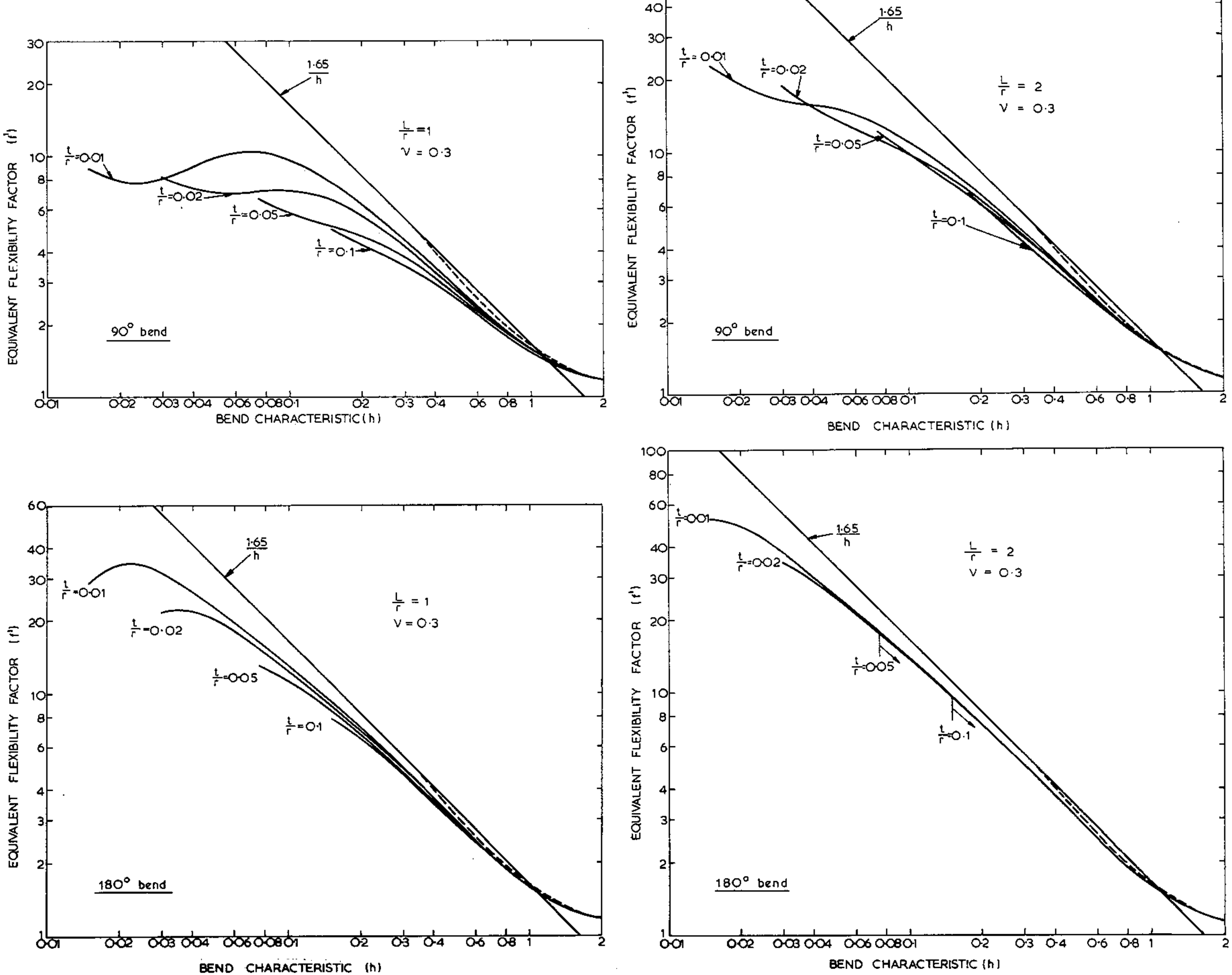
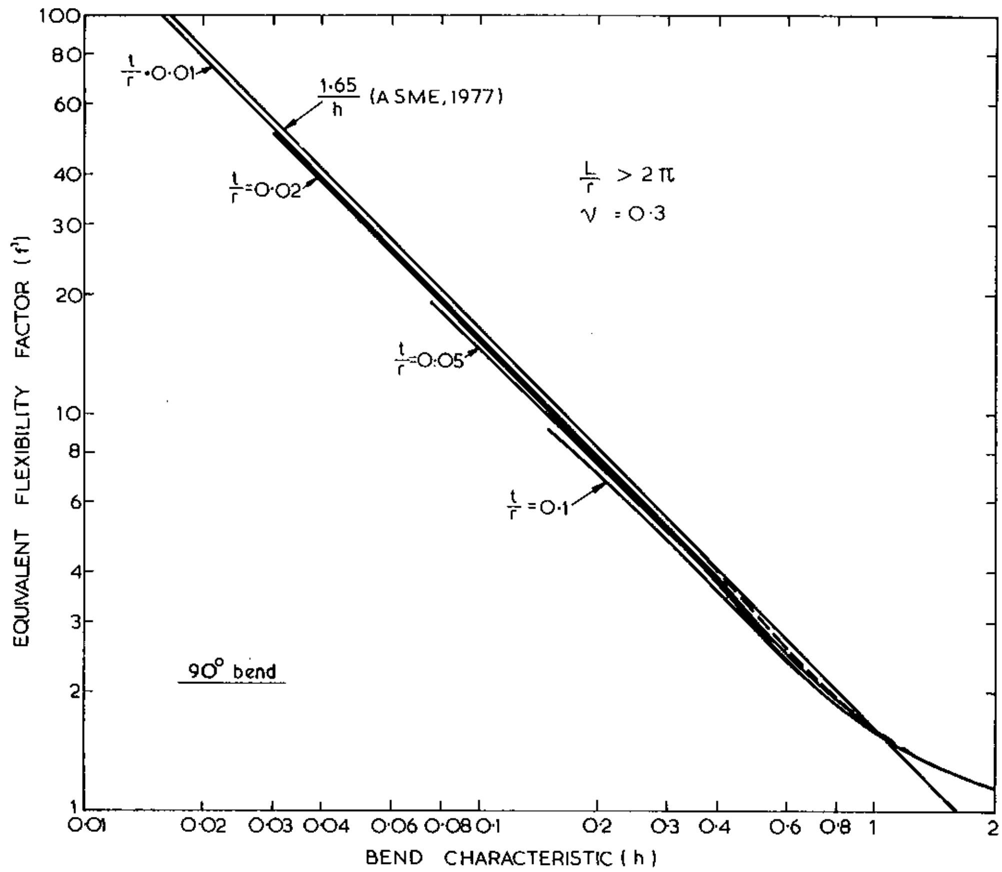


FIGURE 3. EQUIVALENT FLEXIBILITY WITH FLANGED TANGENTS



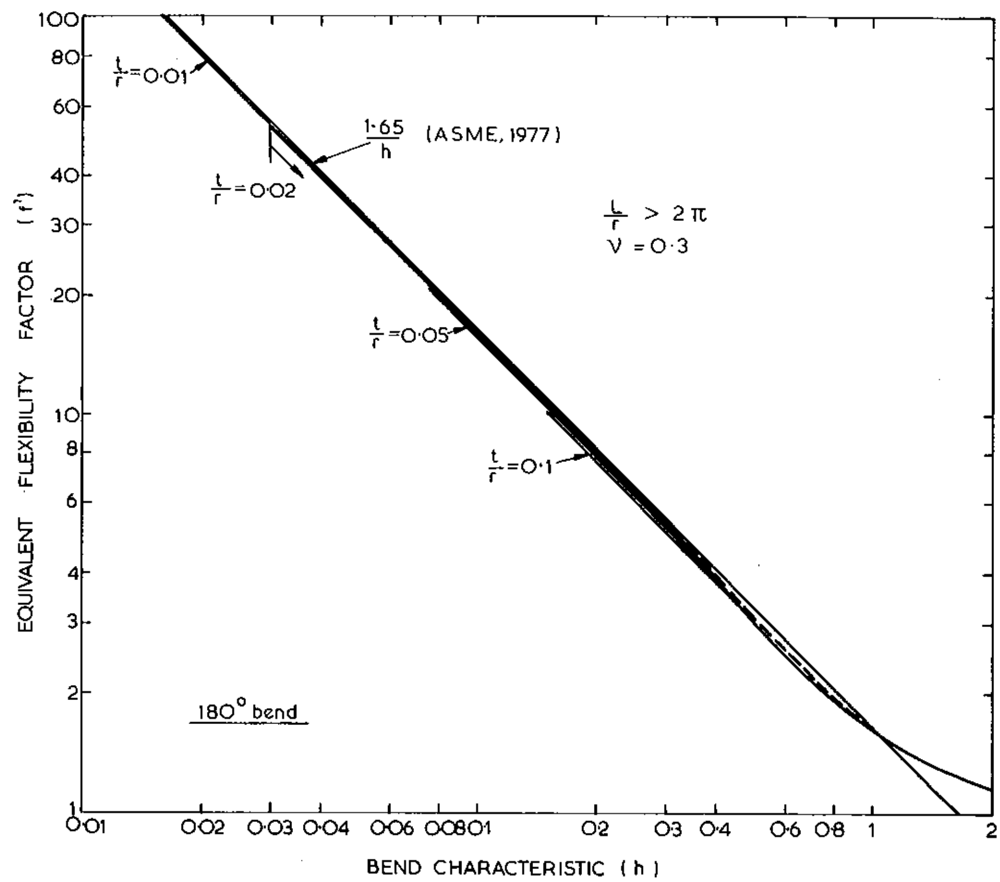


FIGURE 4. EQUIVALENT FLEXIBILITY WITH INFINITE TANGENTS

APPENDIX A
EQUIVALENT FLEXIBILITY FACTORS FOR PIPE BENDS WITH FLANGED TANGENTS

Parameters of pipe bends considered:

R/r = 1.25 to $2r/t$

t/r = 0.01, 0.02, 0.05, 0.1

L/r = 0, 0.5, 1, 2, 3, 4, 2π, $>2\pi$

ϕ' = 90° and 180°

ν = 0.3

Flexibility factors for pipe bends without end effects are also given.

TABLE A1

$$\gamma = f \frac{MR}{2r^3Et}$$

L/r = 0, 'ϕ' = 90°

	f			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	2.95	2.64	1.95	1.54
1.5	2.68	2.57	2.25	1.83
2	2.63	2.56	2.34	2.02
2.5	2.80	2.71	2.46	2.10
3	3.08	2.95	2.60	2.15
4	3.88	3.56	2.86	2.16
5	4.89	4.18	2.98	2.05
6.5	6.36	4.79	2.91	1.84
8	7.32	4.96	2.69	1.65
10	7.69	4.78	2.37	1.47
13	7.28	4.24	1.99	1.31
16	6.61	3.72	1.73	1.21
20	5.78	3.15	1.51	1.14
26	4.81	2.54	1.33	
32	4.09	2.15	1.23	
40	3.38	1.81	1.15	
50	2.77	1.55		
65	2.19	1.35		
80	1.85	1.24		
100	1.58	1.16		
130	1.36			
160	1.24			
200	1.16			

TABLE A2

$$\gamma = \frac{MR}{2r^3Et}\left(f' + \frac{2r}{\pi R}\right)$$

$L/r = 0.5, \quad \phi' = 90°$

	f'			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	5.72	5.42	4.62	3.62
1.5	5.01	4.78	4.18	3.39
2	4.62	4.40	3.84	3.11
2.5	4.72	4.45	3.76	2.96
3	5.06	4.67	3.77	2.83
4	6.08	5.23	3.75	2.57
5	7.19	5.67	3.62	2.30
6.5	8.41	5.88	3.28	1.95
8	8.88	5.70	2.91	1.71
10	8.69	5.22	2.49	1.50
13	7.83	4.49	2.05	1.32
16	6.96	3.87	1.76	1.22
20	6.00	3.23	1.53	1.15
26	4.94	2.59	1.34	
32	4.17	2.17	1.23	
40	3.43	1.82	1.15	
50	2.79	1.56		
65	2.21	1.35		
80	1.86	1.24		
100	1.58	1.16		
130	1.36			
160	1.25			
200	1.16			

TABLE A3

$$\gamma = \frac{MR}{2r^3Et}\left(f' + \frac{4r}{\pi R}\right)$$

$L/r = 1, \quad \phi' = 90°$

	f'			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	9.98	9.19	7.34	5.41
1.5	8.86	8.17	6.61	4.92
2	7.94	7.28	5.79	4.25
2.5	7.77	7.00	5.36	3.80
3	8.03	7.00	5.07	3.46
4	8.98	7.17	4.62	2.91
5	9.85	7.19	4.20	2.49
6.5	10.4	6.87	3.60	2.05
8	10.3	6.35	3.10	1.76
10	9.58	5.62	2.60	1.53
13	8.33	4.71	2.10	1.33
16	7.28	4.01	1.79	1.23
20	6.20	3.32	1.55	1.15
26	5.06	2.63	1.34	
32	4.24	2.20	1.24	
40	3.47	1.84	1.15	
50	2.82	1.57		
65	2.22	1.36		
80	1.86	1.24		
100	1.59	1.16		
130	1.36			
160	1.25			
200	1.16			

TABLE A4

$$\gamma = \frac{MR}{2r^3Et}\left(f' + \frac{8r}{\pi R}\right)$$

L/r = 2, ϕ' = 90°

R/r	f' t/r = 0.01	0.02	0.05	0.1
1.25	26.1	21.5	13.9	8.53
1.5	22.9	18.8	12.2	7.46
2	19.3	15.6	9.86	5.96
2.5	17.5	13.8	8.42	4.98
3	16.6	12.6	7.41	4.28
4	15.8	11.1	6.03	3.33
5	15.2	9.95	5.09	2.72
6.5	14.0	8.55	4.09	2.16
8	12.7	7.44	3.39	1.83
10	11.1	6.30	2.76	1.56
13	9.20	5.09	2.18	1.35
16	7.85	4.25	1.84	1.24
20	6.57	3.47	1.57	1.16
26	5.27	2.71	1.36	
32	4.38	2.24	1.24	
40	3.55	1.86	1.16	
50	2.87	1.58		
65	2.24	1.36		
80	1.88	1.25		
100	1.59	1.16		
130	1.37			
160	1.25			
200	1.16			

TABLE A5

$$\gamma = \frac{MR}{2r^3Et}\left(f' + \frac{12r}{\pi R}\right)$$

$L/r = 3, \quad \phi' = 90°$

	f'			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	49.4	35.4	18.8	10.07
1.5	42.7	30.4	16.1	8.64
2	34.3	24.1	12.5	6.70
2.5	29.4	20.2	10.3	5.47
3	26.3	17.7	8.81	4.62
4	22.4	14.3	6.84	3.50
5	19.8	12.1	5.59	2.82
6.5	16.8	9.80	4.37	2.21
8	14.6	8.26	3.57	1.85
10	12.3	6.82	2.86	1.58
13	9.92	5.40	2.23	1.36
16	8.34	4.45	1.87	1.24
20	6.89	3.58	1.59	1.16
26	5.46	2.77	1.36	
32	4.50	2.28	1.25	
40	3.62	1.88	1.16	
50	2.91	1.59		
65	2.26	1.37		
80	1.89	1.25		
100	1.60	1.16		
130	1.37			
160	1.25			
200	1.16			

TABLE A6

$$\gamma = \frac{MR}{2r^3Et}\left(f' + \frac{16r}{\pi R}\right)$$

$L/r = 4, \quad \phi' = 90°$

	f'			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	72.3	46.0	21.3	10.57
1.5	61.6	39.1	18.1	9.02
2	48.1	30.2	13.8	6.93
2.5	40.0	24.8	11.2	5.62
3	34.6	21.1	9.47	4.72
4	27.8	16.4	7.23	3.57
5	23.5	13.5	5.84	2.85
6.5	19.1	10.7	4.51	2.22
8	16.0	8.83	3.65	1.86
10	13.2	7.20	2.91	1.58
13	10.5	5.62	2.26	1.36
16	8.75	4.59	1.88	1.24
20	7.16	3.67	1.60	1.16
26	5.62	2.82	1.37	
32	4.60	2.31	1.25	
40	3.69	1.89	1.16	
50	2.94	1.60		
65	2.28	1.37		
80	1.90	1.25		
100	1.60	1.16		
130	1.37			
160	1.25			
200	1.16			

TABLE A7

$$\gamma = \frac{MR}{2r^3Et}\left(f' + \frac{8r}{R}\right)$$

$L/r = 2\pi$, $\phi' = 90°$

	f'			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	105	57.2	22.7	10.71
1.5	88.5	48.0	19.2	9.12
2	67.3	36.3	14.5	7.00
2.5	54.4	29.3	11.7	5.66
3	45.8	24.5	9.83	4.75
4	35.0	18.6	7.44	3.57
5	28.4	14.9	5.98	2.86
6.5	22.2	11.6	4.59	2.23
8	18.2	9.47	3.71	1.86
10	14.7	7.63	2.94	1.58
13	11.4	5.88	2.28	1.36
16	9.39	4.76	1.89	1.24
20	7.58	3.78	1.60	1.16
26	5.87	2.87	1.37	
32	4.77	2.34	1.25	
40	3.78	1.91	1.16	
50	3.00	1.61		
65	2.31	1.38		
80	1.92	1.25		
100	1.61	1.16		
130	1.38			
160	1.25			
200	1.16			

TABLE A8

$$\gamma = f \frac{MR}{2r^3Et} (f' + \frac{4L}{\pi R})$$

$L/r > 2\pi$, $\phi' = 90°$, without flanges

	f'			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	124	60.4	22.9	10.75
1.5	104	50.6	19.3	9.15
2	77.9	38.1	14.6	7.02
2.5	62.4	30.6	11.8	5.68
3	52.1	25.5	9.87	4.76
4	39.1	19.2	7.47	3.58
5	31.3	15.4	6.00	2.87
6.5	24.1	11.9	4.60	2.23
8	19.6	9.69	3.71	1.87
10	15.7	7.77	2.95	1.59
13	12.1	5.98	2.28	1.36
16	9.87	4.83	1.90	1.25
20	7.92	3.82	1.60	1.16
26	6.08	2.90	1.37	
32	4.90	2.35	1.25	
40	3.87	1.92	1.16	
50	3.05	1.61		
65	2.34	1.38		
80	1.93	1.25		
100	1.62	1.16		
130	1.38			
160	1.26			
200	1.17			

TABLE A9

$$\gamma = f \frac{MR}{r^3 Et}$$

L/r = 0, $\phi' = 180°$

	f			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	9.35	8.34	6.21	4.27
1.5	10.3	9.08	6.75	4.73
2	15.1	11.9	7.41	4.71
2.5	20.2	13.6	7.31	4.33
3	22.5	13.7	6.83	3.89
4	21.8	12.3	5.77	3.15
5	19.6	10.8	4.93	2.62
6.5	16.6	9.06	4.01	2.12
8	14.4	7.79	3.36	1.80
10	12.3	6.56	2.75	1.55
13	10.0	5.29	2.18	1.35
16	8.47	4.40	1.84	1.24
20	7.02	3.57	1.57	1.16
26	5.57	2.77	1.36	
32	4.59	2.28	1.24	
40	3.69	1.88	1.16	
50	2.95	1.59		
65	2.29	1.37		
80	1.90	1.25		
100	1.61	1.16		
130	1.37			
160	1.25			
200	1.16			

TABLE A10

$$\gamma = \frac{MR}{r^3Et}\left(f' + \frac{r}{\pi R}\right)$$

$L/r = 0.5$, $\phi' = 180°$

	f'			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	16.7	14.3	10.1	6.87
1.5	18.3	15.1	9.98	6.51
2	24.6	17.3	9.55	5.69
2.5	27.8	17.3	8.62	4.91
3	27.7	16.2	7.69	4.26
4	24.4	13.6	6.23	3.34
5	21.2	11.6	5.22	2.73
6.5	17.6	9.54	4.18	2.17
8	15.1	8.11	3.46	1.84
10	12.7	6.77	2.81	1.57
13	10.3	5.41	2.21	1.35
16	8.64	4.47	1.86	1.24
20	7.13	3.61	1.58	1.16
26	5.63	2.79	1.36	
32	4.63	2.29	1.25	
40	3.71	1.89	1.16	
50	2.96	1.60		
65	2.29	1.37		
80	1.91	1.25		
100	1.61	1.16		
130	1.37			
160	1.25			
200	1.16			

TABLE A11

$$\gamma = \frac{MR}{r^3Et}\left(f' + \frac{2r}{\pi R}\right)$$

$L/r = 1, \quad \phi' = 180°$

	f'			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	26.7	21.7	14.0	8.79
1.5	28.8	21.8	13.0	7.89
2	34.1	21.9	11.2	6.42
2.5	34.1	20.1	9.64	5.35
3	31.7	18.0	8.38	4.56
4	26.6	14.6	6.63	3.49
5	22.6	12.3	5.48	2.82
6.5	18.5	9.97	4.32	2.22
8	15.7	8.40	3.55	1.86
10	13.1	6.96	2.86	1.58
13	10.5	5.52	2.24	1.36
16	8.80	4.54	1.88	1.24
20	7.24	3.66	1.59	1.16
26	5.69	2.82	1.37	
32	4.66	2.31	1.25	
40	3.73	1.90	1.16	
50	2.97	1.60		
65	2.30	1.37		
80	1.91	1.25		
100	1.61	1.16		
130	1.37			
160	1.25			
200	1.16			

TABLE A12

$$\gamma = \frac{MR}{r^3Et}\left(f' + \frac{4r}{\pi R}\right)$$

$L/r = 2, \quad \phi' = 180°$

	f'			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	53.9	37.9	19.9	10.97
1.5	52.1	34.3	17.3	9.42
2	48.7	28.8	13.6	7.26
2.5	43.0	24.4	11.1	5.87
3	37.8	21.0	9.42	4.91
4	30.1	16.4	7.23	3.68
5	25.0	13.5	5.87	2.93
6.5	20.0	10.7	4.55	2.27
8	16.8	8.91	3.69	1.89
10	13.8	7.29	2.95	1.60
13	10.9	5.71	2.28	1.37
16	9.09	4.67	1.90	1.25
20	7.42	3.73	1.60	1.16
26	5.80	2.86	1.37	
32	4.73	2.33	1.25	
40	3.77	1.91	1.16	
50	3.00	1.61		
65	2.31	1.37		
80	1.92	1.25		
100	1.61	1.16		
130	1.38			
160	1.25			
200	1.16			

TABLE A13

$$\gamma = \frac{MR}{r^3Et}\left(f' + \frac{6r}{\pi R}\right)$$

$L/r = 3, \quad \phi' = 180°$

	f'			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	80.4	49.7	22.8	11.71
1.5	71.6	42.8	19.4	9.95
2	59.1	33.5	14.8	7.56
2.5	49.5	27.4	11.9	6.07
3	42.4	23.1	10.0	5.06
4	32.9	17.7	7.59	3.76
5	26.9	14.4	6.10	2.98
6.5	21.3	11.3	4.68	2.30
8	17.6	9.30	3.78	1.91
10	14.4	7.54	3.00	1.61
13	11.3	5.86	2.31	1.37
16	9.34	4.77	1.91	1.25
20	7.58	3.79	1.61	1.16
26	5.90	2.89	1.38	
32	4.79	2.35	1.25	
40	3.81	1.92	1.16	
50	3.02	1.61		
65	2.32	1.38		
80	1.92	1.25		
100	1.62	1.16		
130	1.38			
160	1.25			
200	1.17			

TABLE A14

$$\gamma = \frac{MR}{r^3Et}\left(f' + \frac{8r}{\pi R}\right)$$

$L/r = 4, \quad \phi' = 180°$

	f'			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	99.1	56.3	24.0	11.92
1.5	85.1	47.6	20.2	10.10
2	66.6	36.3	15.3	7.66
2.5	54.4	29.3	12.3	6.14
3	46.0	24.6	10.3	5.10
4	35.1	18.6	7.76	3.78
5	28.5	15.0	6.22	3.00
6.5	22.3	11.7	4.75	2.30
8	18.4	9.58	3.82	1.91
10	14.9	7.73	3.02	1.61
13	11.6	5.98	2.32	1.37
16	9.54	4.84	1.92	1.25
20	7.72	3.83	1.62	1.16
26	5.98	2.91	1.38	
32	4.85	2.36	1.25	
40	3.84	1.93	1.16	
50	3.04	1.62		
65	2.33	1.38		
80	1.93	1.25		
100	1.62	1.17		
130	1.38			
160	1.25			
200	1.17			

TABLE A15

$$\gamma = \frac{MR}{r^3 Et}\left(f' + \frac{4r}{R}\right)$$

$L/r = 2\pi, \quad \phi' = 180°$

	f'			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	119	61.9	24.6	11.97
1.5	100	51.8	20.7	10.14
2	75.6	39.0	15.6	7.68
2.5	60.7	31.3	12.5	6.15
3	50.8	26.1	10.5	5.11
4	38.3	19.6	7.86	3.79
5	30.8	15.7	6.29	3.00
6.5	23.8	12.1	4.79	2.31
8	19.4	9.90	3.84	1.91
10	15.6	7.95	3.04	1.61
13	12.1	6.11	2.33	1.37
16	9.86	4.92	1.93	1.25
20	7.93	3.88	1.62	1.16
26	6.10	2.94	1.38	
32	4.93	2.38	1.25	
40	3.89	1.93	1.16	
50	3.06	1.62		
65	2.34	1.38		
80	1.94	1.26		
100	1.62	1.17		
130	1.38			
160	1.26			
200	1.17			

TABLE A16

$$\gamma = \frac{MR}{r^3 Et}\left(f' + \frac{2L}{\pi R}\right)$$

$L/r = > 2\pi$, $\phi' = 180°$, without flanges

R/r	f' t/r = 0.01	0.02	0.05	0.1
1.25	128	63.3	24.6	11.99
1.5	107	52.9	20.7	10.15
2	80.3	39.8	15.6	7.69
2.5	64.3	31.8	12.5	6.16
3	53.6	26.5	10.5	5.12
4	40.2	19.9	7.87	3.79
5	32.2	16.0	6.29	3.00
6.5	24.8	12.3	4.80	2.31
8	20.1	10.0	3.85	1.91
10	16.1	8.02	3.04	1.61
13	12.1	6.15	2.33	1.37
16	10.1	4.96	1.93	1.25
20	8.10	3.90	1.62	1.16
26	6.21	2.95	1.38	
32	4.99	2.38	1.25	
40	3.93	1.94	1.17	
50	3.09	1.63		
65	2.36	1.38		
80	1.94	1.26		
100	1.63	1.17		
130	1.38			
160	1.26			
200	1.17			

TABLE A17

$$\gamma = f^e \frac{MR\phi'}{180r^3Et}$$

For ϕ' bend without end effects

	f			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	132	65.8	26.0	12.91
1.5	110	54.9	21.9	10.89
2	82.6	41.3	16.5	8.21
2.5	66.1	33.0	13.2	6.55
3	55.1	27.5	11.0	5.43
4	41.3	20.7	8.26	4.00
5	33.0	16.5	6.59	3.14
6.5	25.4	12.7	5.00	2.39
8	20.7	10.3	3.99	1.96
10	16.5	8.27	3.13	1.64
13	12.7	6.33	2.38	1.39
16	10.3	5.09	1.96	1.26
20	8.27	3.99	1.64	1.17
26	6.33	3.00	1.39	
32	5.09	2.42	1.26	
40	3.99	1.96	1.17	
50	3.13	1.64		
65	2.38	1.39		
80	1.96	1.26		
100	1.64	1.17		
130	1.39			
160	1.26			
200	1.17			

$f^e = 1 + 0.68/h^2$ for $h > 2$