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ABSTRACT

Equations are derived for the determination of the intrinsic dead time of the components which precede the paralysis unit in a counting system for measuring radioactivity. The determination depends on the extension of the set dead time by the intrinsic dead time.

Improved formulae are given for the dead-time correction of the count rate of a radioactive source in a single-channel system. A variable in the formulae is the intrinsic dead time which is determined concurrently with the counting of the source. The only extra equipment required in a conventional system is a scaler.

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COINCIDENCE METHODS; DEAD TIME; MATHEMATICAL MODELS; PROBABILITY; RADIATION DETECTION; RADIATION DETECTORS; RADIOACTIVITY; SENSITIVITY.

EDITORIAL NOTE

From 27 April 1987, the Australian Atomic Energy Commission (AAEC) is replaced by Australian Nuclear Science and Technology Organisation (Ansto). Serial numbers for reports with an issue date after April 1987 have the prefix ANSTO with no change of the symbol (E, M, S or C) or numbering sequence.

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1. INTRODUCTION

Systems for counting radioactive sources experience an intrinsic dead time immediately after a nuclear event is detected. During this period a subsequent event cannot be recorded. To facilitate the making of accurate corrections for events lost during dead time, a paralysis unit is added (paralysis unit A in figures 1 and 2) with dead time set to be greater than the intrinsic dead time of the preceding components. The paralysis unit imposes, for correction calculations, a set dead time which is exactly the same for each event detected. The set dead time can be measured with considerable accuracy. However, as the count rate increases, the set dead time is increasingly extended by the intrinsic dead time [NCRP 1985:70]. As a result, the non-extendable dead-time correction equation [NCRP 1985:62] becomes less accurate.

This report describes a method for measuring intrinsic dead time while a source is being counted. The method depends on the intrinsic dead-time's extension of the set dead time, and requires only an additional scaler. Equations are derived for a more accurate total dead-time correction in a single-channel system, taking into account the intrinsic dead time of the system. System 1, shown in figure 1, contains a $4\pi\beta$ proportional counter, preamplifier, amplifier, timing Single-Channel Analyser (SCA), and paralysis unit A whose dead time is set to be greater than the intrinsic dead time of the amplifier and timing SCA. The set dead time is measured by the double pulse method, and is used for correcting the count rate of scaler A by the non-extendable dead-time equation [NCRP 1985:62]. The scaler I is required for the measurement of the intrinsic dead time. Paralysis unit W and scaler W are used when the intrinsic dead-time method is checked by measuring the dead time of paralysis unit A.

System 2, shown in figure 2, has a discriminator inserted between the amplifier and timing SCA. This is necessary because the maximum amplitude of the output pulses of the System 2 amplifier is greater than the maximum upper level of the timing SCA, whereas the disciminator output pulses are not too high to be recognised by the timing SCA.

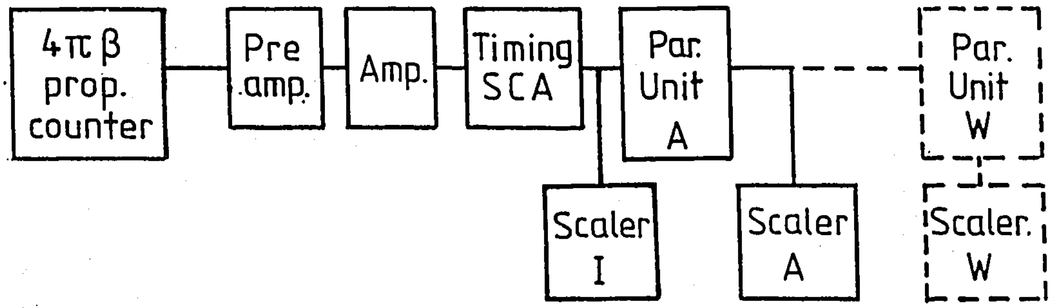


Figure 1. A β-counting system consisting of a $4\pi\beta$ proportional counter, preamplifier, amplifier, timing single-channel analyser, paralysis unit A and scaler A, with an extra scaler, I, which is used in the measurement of the intrinsic dead time of the components preceding paralysis unit A. The extra paralysis unit W and scaler W are required when the method for measuring intrinsic dead time is tested by measuring the dead time of paralysis unit A.

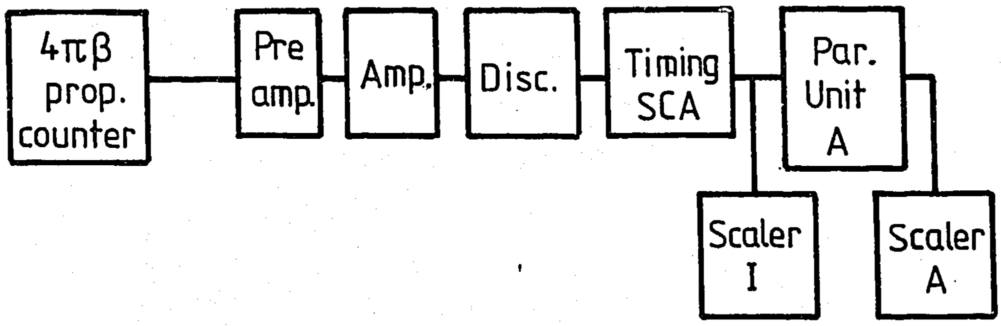


Figure 2. A β-counting system similar to that shown in Figure 1. The discriminator is necessary because the amplitude of the β-amplifier output pulses is greater than the maximum upper level of the timing single-channel analyser.

2. OUTLINE OF THE REPORT

The first part of the report deals with the origins of the intrinsic dead times of systems 1 and 2. The topics dealt with are as follows:

(a) The time of recognition of an input pulse by the timing SCA.

(b) The measurement of the delay in the timing SCA.

(c) How the intrinsic dead time of the combined components preceding the paralysis unit is related to the amplifier output pulse width and to the delay in the timing SCA.

The next part of the report shows (figures 7, 8, 9 and 10) that the set dead time, which is equal to the paralysis-unit dead time, T, starts at the same instant as the intrinsic dead time, T_I.

A derivation is given of the non-extendable dead-time correction equation for the count rate of scaler A. Then follows the derivation of a similar equation for the count rate of scaler I.

Formulae for the determination of intrinsic dead time are derived, one for $T_I > \frac{1}{2}T$, and one for $T_I < \frac{1}{2}T$. The determination requires the measurement of T and the count rates of scalers I and A.

Dead-time correction formulae are derived which take into account the intrinsic dead time. Two formulae, one for $T_I > \frac{1}{2}T$ and one for $T_I < \frac{1}{2}T$, are given as functions of T and the count rates of scalers I and A. Two alternative formulae, derived for $T_I > \frac{1}{2}T$ and $T_I < \frac{1}{2}T$, are given as functions of T_I, T and the count rate of scaler A. The alternative formulae can be used when T_I is measured by some other method.

Most of the abovementioned formulae are roots of quadratic equations. The ways of selecting the correct roots are given.

When the intrinsic dead time is determined by the method described, two values of T_I are calculated from the equations for $T_I > \frac{1}{2}T$ and $T_I < \frac{1}{2}T$. The way of selecting the correct value is given.

Graphs are plotted (figures 16 and 17) which show the error incurred by using the non-extendable dead-time correction equation instead of the formulae which take the intrinsic dead time into account.

An analysis of the coincidence-counting model is given. This shows that when coincidence counting is carried out, there is, in addition to having $T > T_I$, an advantage in having $T > 2T_R$, where T_R is the resolving time of the coincidence mixer.

Experiments are described in which intrinsic dead times are obtained by the method derived in this report and by Baerg's [1965] method. The results from the two methods are compared with one another, with the amplifier output pulse width and with the delay in the timing SCA.

3. THE β AMPLIFIER AND TIMING SINGLE-CHANNEL ANALYSER OF SYSTEM 1

The β amplifier is overloaded and its output pulses are mostly clipped. The β-amplifier ouput pulse, shown in a simplified form in figure 3, is the input pulse to the timing single-channel analyser (SCA) at instant I_1. An internally-generated signal in the SCA rises to 50% of the input peak amplitude and is stretched at that level. The time of recognition of the input pulse by the SCA is instant I_3 when the input pulse decays through the 50% level. This recognition technique, called Constant Fraction Discrimination [ORTEC], eliminates the effect of variation in input amplitude on the timing of the SCA output pulse; it is of no great advantage when the amplifier is overloaded and the leading edge of the input pulse is virtually vertical, as in this case. The SCA output pulse is delayed until instant I_4. The delay, from I_3 to I_4, is adjusted on the front panel to between 0.1 and 11 μs.

An amplifier input pulse at instant I_2, between I_1 and I_3, will find the system dead. This is because the two amplifier input pulses at I_1 and I_2 result in the recognition, at I_3, of the trailing edge of only one SCA input pulse.

4. MEASUREMENT OF THE TIMING SCA DELAY IN SYSTEM 1

The delay of the SCA was measured by means of a digital storage CRO and a pulse generator which provided a 15 mV, 0.5 μs-wide square pulse to the input of the β amplifier. The β-amplifier output pulse appeared on the screen of the CRO as shown in figure 4. One cursor of the CRO was set on point A which was the point on the trailing edge of the SCA input pulse at 50% of the peak amplitude. The other cursor was set at point B which was the start of the SCA output pulse. The average of 256 determinations of the time interval AB was taken as the SCA delay.

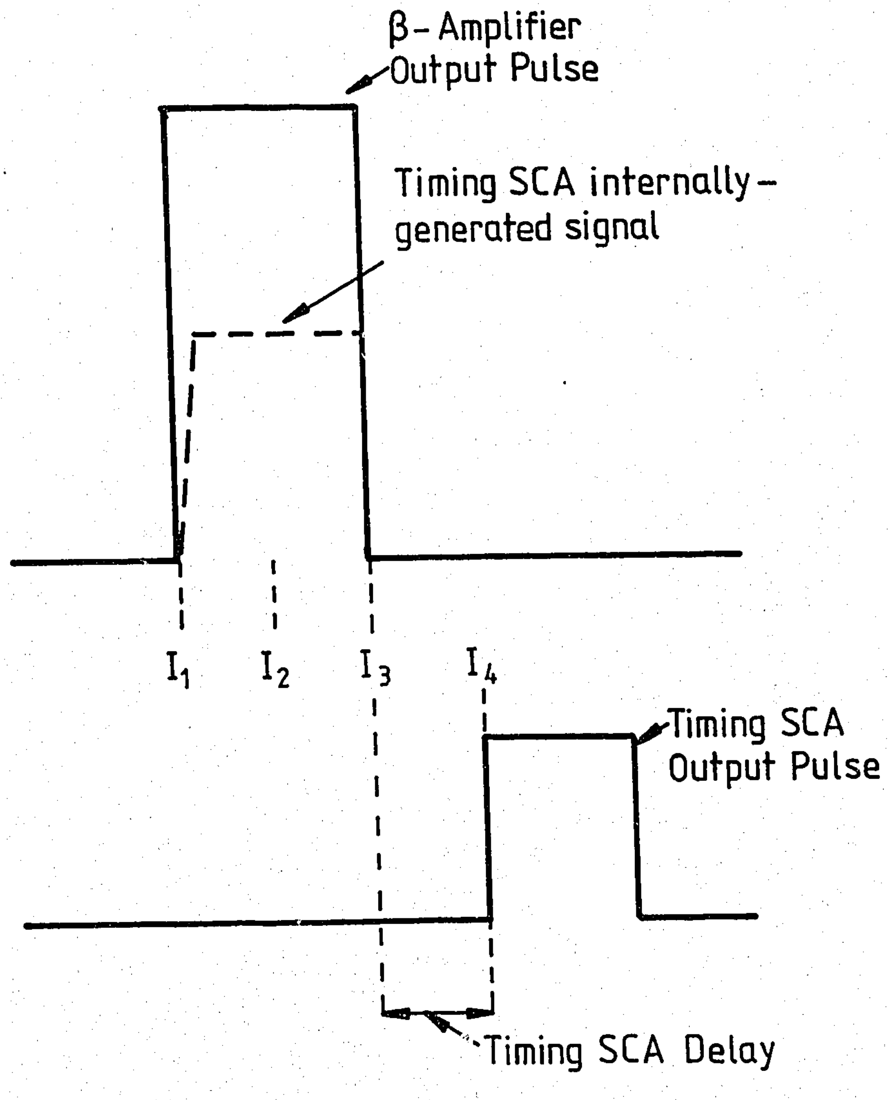


Figure 3. The output pulse of the amplifier of system 1 is recognised by the timing SCA at instant I_3. Generation of the SCA output pulse is delayed until instant I_4.

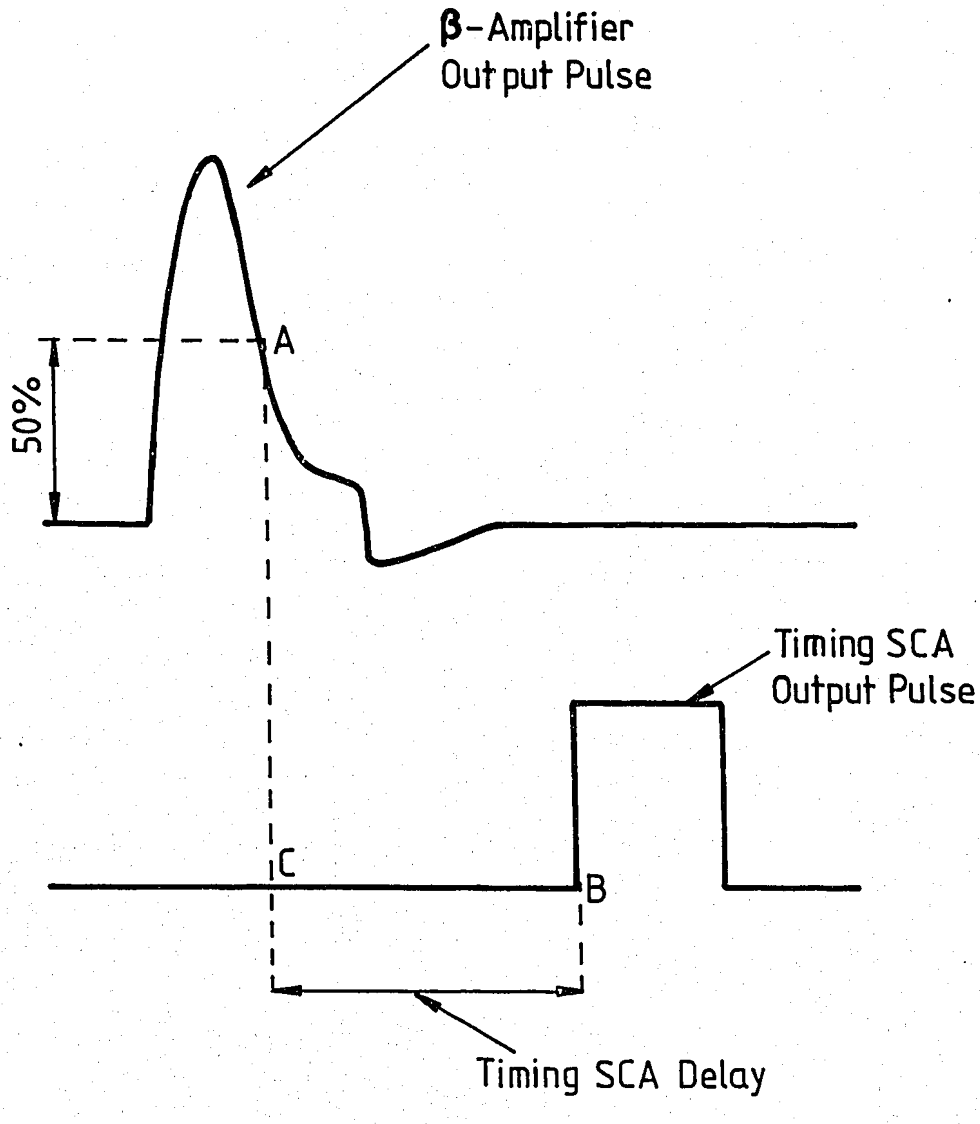


Figure 4. Measurement of the Timing SCA delay in system 1. The β-amplifier output pulse (SCA input pulse) appears on the oscilloscope screen above the SCA output pulse. The SCA delay is the time interval between points A and B.

5. INTRINSIC DEAD TIME OF THE β AMPLIFIER - SCA COMBINATION OF SYSTEM 1

Consider the combination when the SCA delay is less than the width of the output pulse from the overloaded β amplifier, as shown in figure 5. Suppose that the first pulse is immediately followed by a second. The first SCA delay interval finishes at time I_1, and an SCA output pulse is generated. The second delay interval starts at I_2. Thus the intrinsic dead time of the combination is the width of the β-amplifier output pulse.

Now consider the combination when, as shown in figure 6, the SCA delay, D, is greater than the output pulse width, W, of the β amplifier. Suppose that the gap between the two successive SCA delays is infinitesimal. The first pulse is recognised by the SCA at time I_2 and the delay circuit is triggered. Immediately after the delay circuit has recovered, the second pulse is recognised at time I_4. If the second pulse had started before time I_3, it would have decayed through the 50% level before the delay circuit had recovered, and would not have been recognised by the SCA. Thus the β channel is dead between times I_1 and I_3. The intrinsic dead time of the combination, T_1, is equal to I_3-I_1, which is equal to D.

Summarising, the intrinsic dead time of the components of the β channel which precede the paralysis unit is either the output pulse width of the amplifier, or the delay in the timing single-channel analyser, depending on which is the greater.

6. DIAGRAMMATIC REPRESENTATION OF THE INTRINSIC AND SET DEAD TIMES OF SYSTEM 1

It is now shown that the set dead time imposed by the paralysis unit effectively starts at the same instant as the intrinsic dead time. Suppose that in figures 7 and 8 the gap between the two dead times of the paralysis unit is infinitesimal. In figure 7, the intrinsic dead time, T_I, is identical with the β-amplifier output pulse width because the latter is greater than the SCA delay, D. The paralysis unit dead time is set at a value, T, which is greater than T_I.

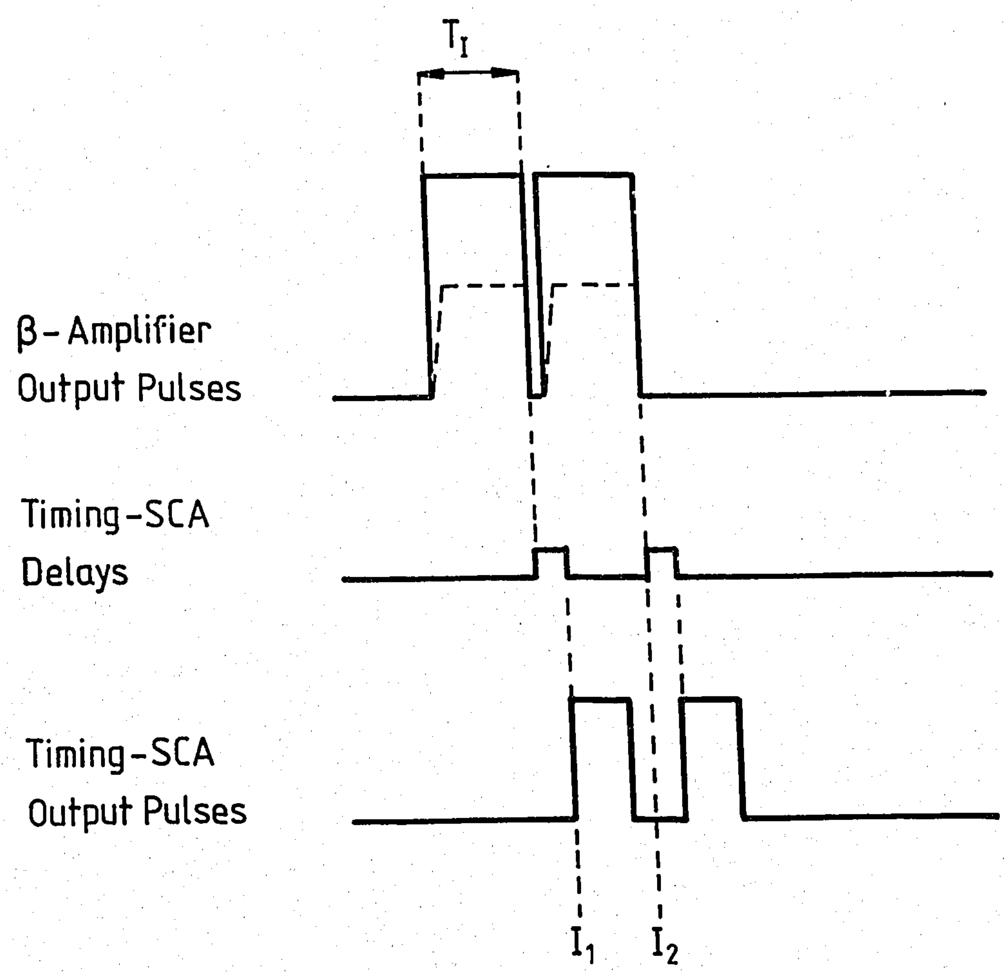


Figure 5. Two successive output pulses from the β amplifier of system 1. The widths of the pulses are greater than the timing-SCA delay.

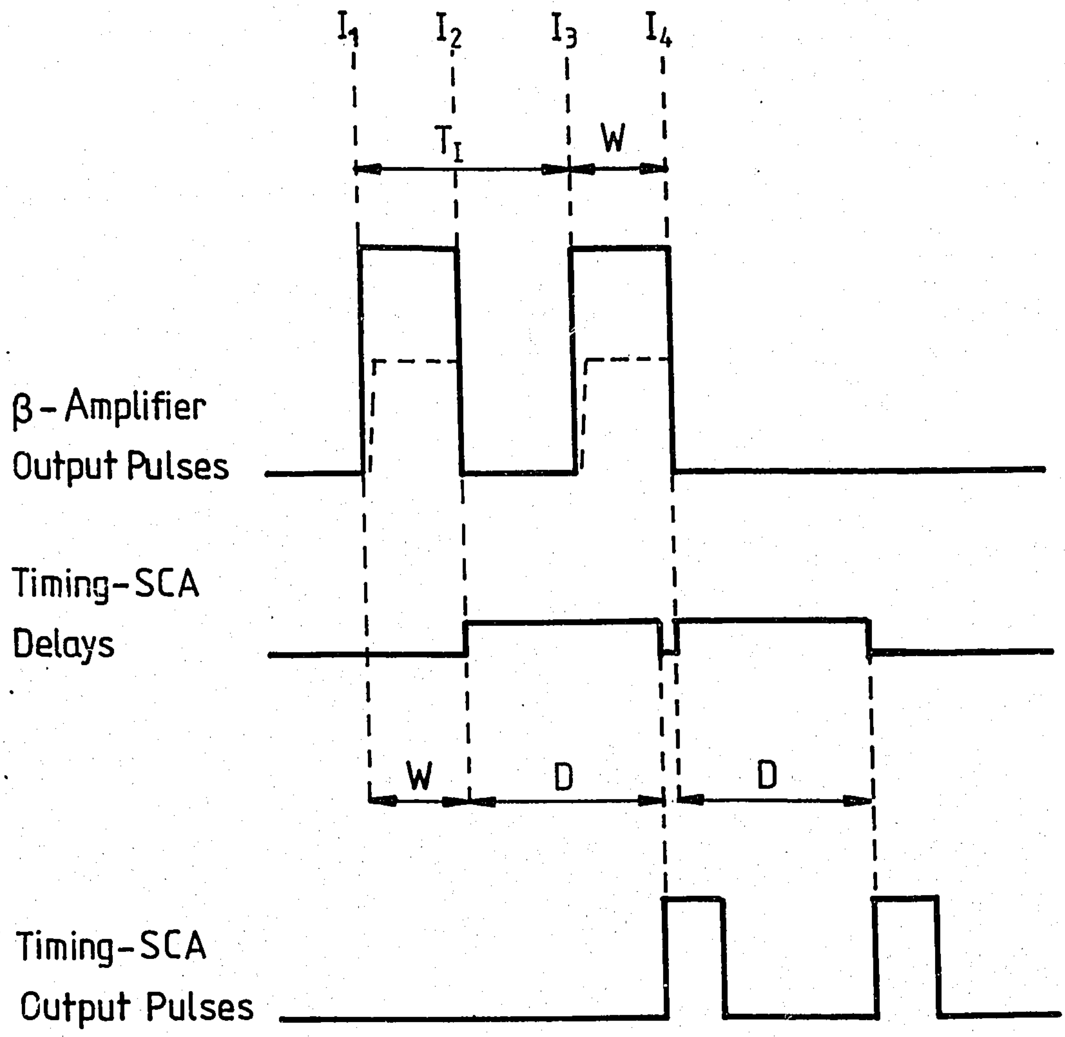


Figure 6. Two successive output pulses from the β amplifier of system 1. The widths of the pulses are less than the timing-SCA delay, D.

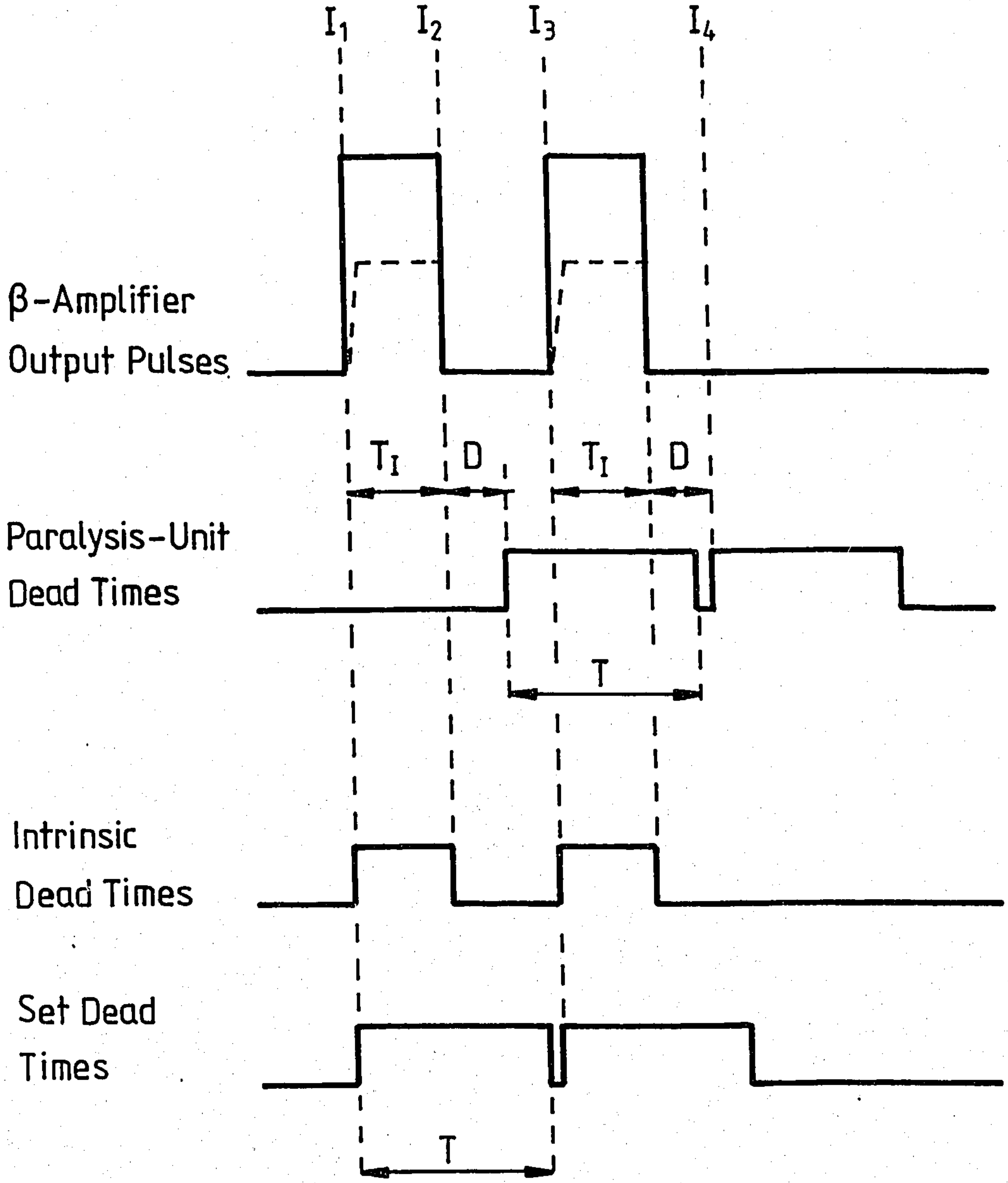


Figure 7. The bottom line shows the set dead time of system 1 starting at the same instant as the intrinsic dead time. In this case, the intrinsic dead time, T_I, is equal to the β-amplifier output pulse width because the latter is greater than the SCA delay, D.

The β-amplifier output pulse which starts at time I_1 is recognised by the SCA at I_2. After the SCA delay, D, the paralysis unit goes dead for the interval T. The earliest time at which the paralysis unit can go dead again is I_4. It will go dead at I_4 if a β-amplifier output pulse starts at I_3, where $I_4-I_3 = T_I + D$. Thus the channel is dead between I_1 and I_3, and the interval I_3-I_1, which is the set dead time, is equal to T.

In figure 8, the intrinsic dead time, T_I, is equal to the SCA delay, D, because D is greater than the β-amplifier output pulse width, W. The paralysis unit dead time, T, is set to be greater than T_I. The pulse which starts at I_1 is recognised by the SCA at I_2. After the delay D, the paralysis unit goes dead for the interval T. The earliest time at which the paralysis unit can go dead again is I_5. It will go dead at I_5 if another pulse is recognised by the SCA at I_4, where $I_5-I_4 = D$. Thus the channel is dead between I_1, the start of the first pulse, and I_3, the start of the second pulse shown. The interval I_3-I_1 is the set dead time for the channel, and it is equal to T.

7. INTRINSIC AND SET DEAD TIMES OF SYSTEM 2

The intrinsic and set dead times of system 2 are shown in figure 9 with the β-amplifier output pulse width, W, greater than the discriminator output pulse width, L, and greater than the timing SCA delay, D. Suppose that there is an infinitesimal gap at time I_2 between the first two output pulses from the β-amplifier, and at time I_4 between the two dead times of the paralysis unit. The β-amplifier output pulse which starts at time I_1 is recognised when its amplitude rises through the level set in the discriminator. As the output pulse of the discriminator enters the SCA it generates a signal, indicated in figure 9 by the dotted line, which rises to 50% of the SCA input peak amplitude. The SCA delay, D, starts when the SCA input pulse decays through the 50% level. At the end of the SCA delay, the paralysis unit goes dead for the interval T.

It can be seen in figure 9 that the β-amplifier pulse which starts at I_2 finds the discriminator live, and the discriminator finds the SCA live. After I_1 and before I_2 the β-amplifier output cannot rise through the discriminator level. Therefore, an input pulse to the amplifier between I_1 and I_2 will not be noticed by the discriminator. Thus the intrinsic dead time, T_I, of the combination of amplifier, discriminator and SCA is equal to W.

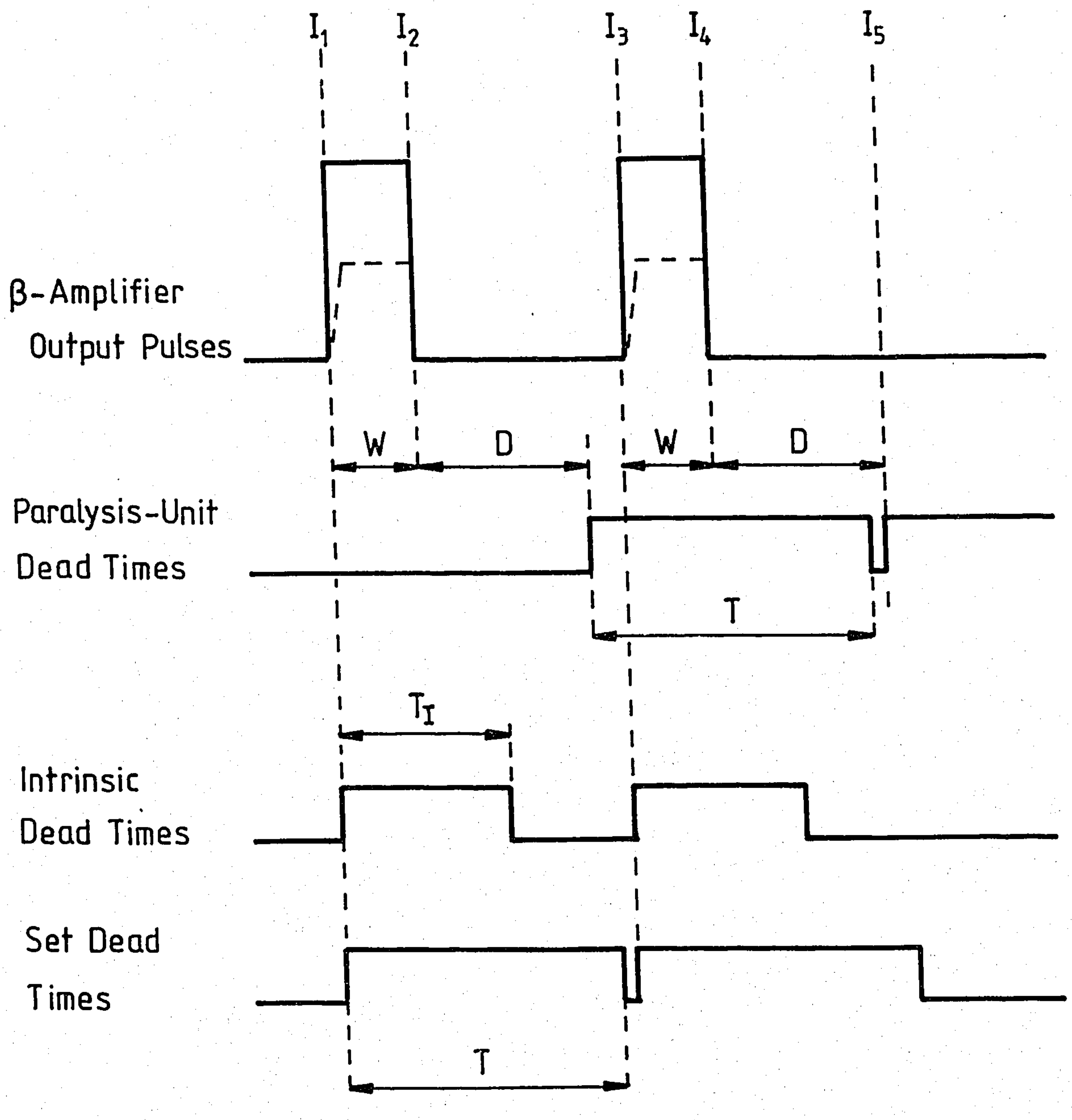


Figure 8. The bottom line shows the set dead time of system 1 starting at the same instant as the intrinsic dead time. In this case, the intrinsic dead time, T_I, is equal to the SCA delay, D, because the latter is greater than the width of the β-amplifier output pulse, W.

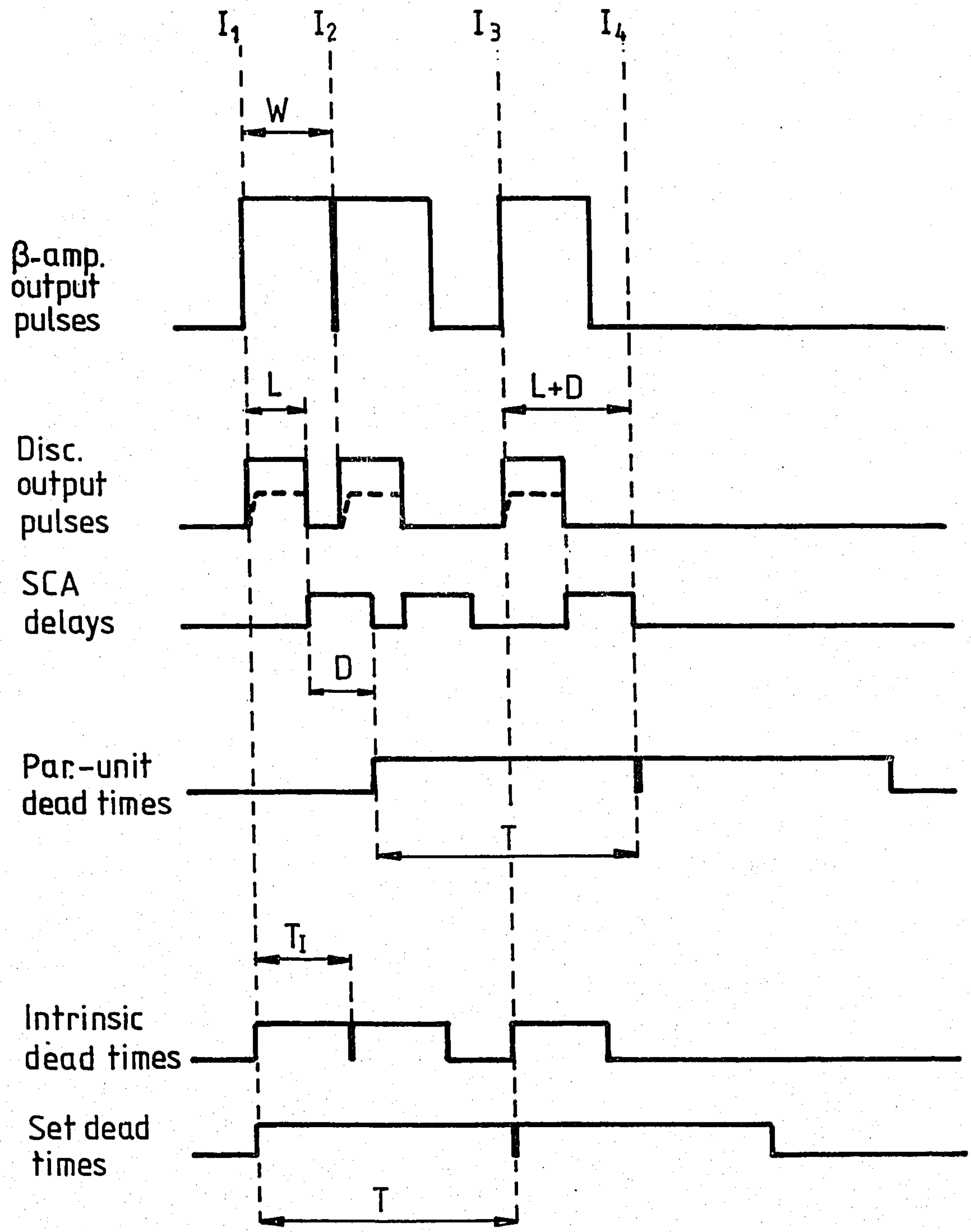


Figure 9. The intrinsic and set dead times of system 2, with the β-amplifier output pulse width, W, greater than the discriminator output pulse width, L, and greater than the timing SCA delay, D.

The earliest time at which the paralysis unit can go dead a second time is I_4. This occurs if a β-amplifier output pulse starts at I_3, where $I_4 - I_3 = L + D$. Thus the channel is dead between I_1 and I_3. The interval $I_3 - I_1$ is the set dead time, and it can be seen from figure 9 that it is equal to T.

The intrinsic and set dead times of system 2 are shown in figure 10 with W greater than L but less than D. Suppose that there is an infinitesimal gap at time I_4 between the first two SCA delays, and at time I_6 between the two dead times of the paralysis unit. The second SCA delay, starting at I_4, is a consequence of the β-amplifier output pulse which starts at I_3, where $I_4 - I_3 = L$. If the second β-amplfifier output pulse had started between I_2 and I_3, the SCA input pulse would have decayed through the 50% level before the SCA delay circuit had recovered. Thus, the combination of amplifier, discriminator and SCA is dead between I_1 and I_3. The interval between I_1 and I_3 is the intrinsic dead time, T_I, of the combination, and it can be seen from the diagram that $I_3 - I_1 = D$.

The second dead time of the paralysis unit, starting at I_6, is a consequence of the β-amplifier output pulse which starts at I_5, where $I_6 - I_5 = L + D$. The β-amplifier output pulse which starts at I_3 gives rise to an SCA delay which ends during the first dead time of the paralysis unit. It can be seen that the channel is dead between I_1 and I_5, and that the interval $I_5 - I_1$, which is the set dead time, is equal to T.

8. CORRECTION OF COUNT RATE FOR DEAD-TIME LOSSES, NEGLECTING INTRINSIC DEAD TIME

A count rate which is corrected for dead-time losses is the count rate which would be obtained if there was no dead time. The usual equation [NCRP 1985:62] for a count rate corrected for the loss of counts during the set dead time is now derived.

The count rate, including background, which is recorded by scaler A in figures 1 and 2 is denoted by A″, and the background count rate if there were no dead time is denoted by A_b. Let N be the disintegration rate of the source. Let E_A be the efficiency of the counter, i.e. the probability that the counter, when live, will detect a particular type of radiation.

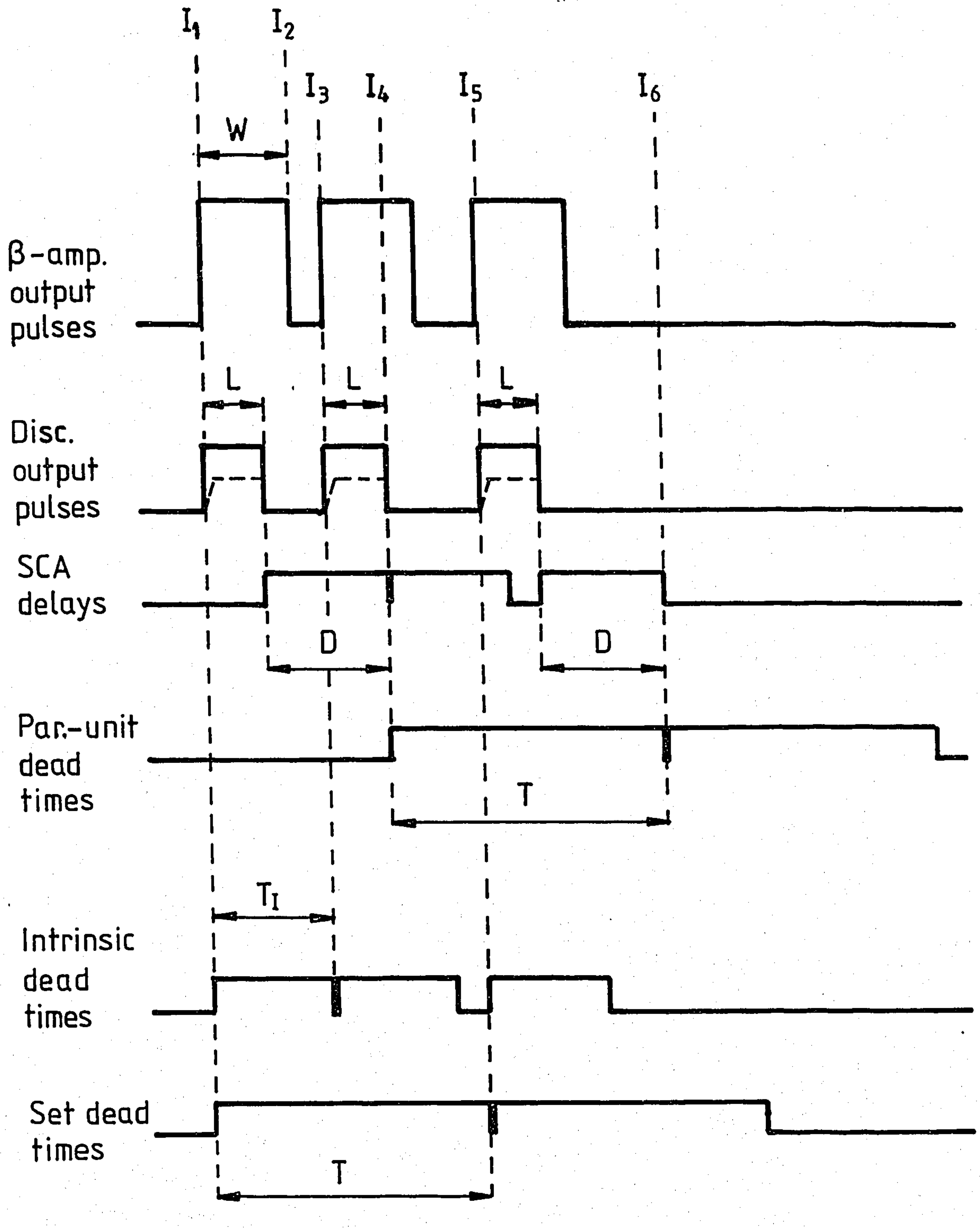


Figure 10. The intrinsic and set dead times of system 2, with the β-amplifier output pulse width, W, greater than the discriminator output pulse width, L, but less than the timing SCA delay, D.

The number of disintegrations detected by the counter per unit time would be NE_A if there was no dead time, and the count rate recorded by scaler A would be $NE_A + A_b$.

The total dead time of the channel per unit time is $A''T$. During this time, the average disintegration rate is assumed to be N. If the channel were live during the time $A''T$, the number of events recorded in that time would be $(NE_A + A_b)\,A''T$. This is the number of counts lost per unit time while the paralysis unit is dead. The number of lost counts per unit time is also equal to $NE_A + A_b - A''$; i.e.,

$$(NE_A + A_b)\,A''T = NE_A + A_b - A''.$$

Rearranging,

$$(NE_A + A_b)(A''T - 1) = -A'',$$

$$NE_A + A_b = \frac{A''}{1 - A''T}. \qquad (1)$$

When the background is counted, N = 0, and equation (1) becomes

$$A_b = \frac{A''}{1 - A''T},$$

where A'' is then the experimentally-observed background count rate and $A''T$ is usually negligible. Thus A_b is taken as being equal to the observed background count rate.

9. CORRECTION OF COUNT RATE I″

The width of the output pulse of the β amplifier is variable. Thus the intrinsic dead time is variable if it is equal to the β-amplifier output pulse width. Let T_p denote the intrinsic dead time started by one particular disintegration. Let I'' be the count rate, including background, which is recorded by scaler I in figures 1 and 2. The total intrinsic dead time per unit time will be equal to

$$\sum_{p=1}^{I''} T_p.$$

The number of counts lost per unit time during the intrinsic dead time will be

$$(NE_A + A_b) \sum_{p=1}^{I''} T_p .$$

The number of counts lost per unit time during the intrinsic dead time will also be equal to $NE_A + A_b - I''$. Thus,

$$(NE_A + A_b) \sum_{p=1}^{I''} T_p = NE_A + A_b - I''.$$

Rearranging,

$$NE_A + A_b = \frac{I''}{1 - \sum_{p=1}^{I''} T_p} .$$

Let T_I henceforth denote the average intrinsic dead time; i.e.

$$T_I = \frac{\sum_{p=1}^{I''} T_p}{I''} .$$

Then

$$NE_A + A_b = \frac{I''}{1 - I''T_I} . \qquad (2)$$

10. EQUATIONS FOR THE DETERMINATION OF INTRINSIC DEAD TIME

10.1 General Equation for the Corrected Count Rate

To derive a general equation for the corrected count rate, $NE_A + A_b$, consider an event which is detected by the proportional counter at instant I_1 in figure 11. The paralysis unit A goes dead for time T, and the preceding components for time T_I. An event which is detected at instant I_2 extends the set dead time, T, by an interval $T_I - x$. Let X be the sum of all the intervals $T_I - x$ during unit time, i.e. X is the total extension of the intervals T during unit time. The total channel dead time per unit time is then $A''T + X$. The number of counts lost per unit time to scaler A during the extended dead time is equal to $(NE_A + A_b)(A''T + X)$, and it is also equal to $NE_A + A_b - A''$.

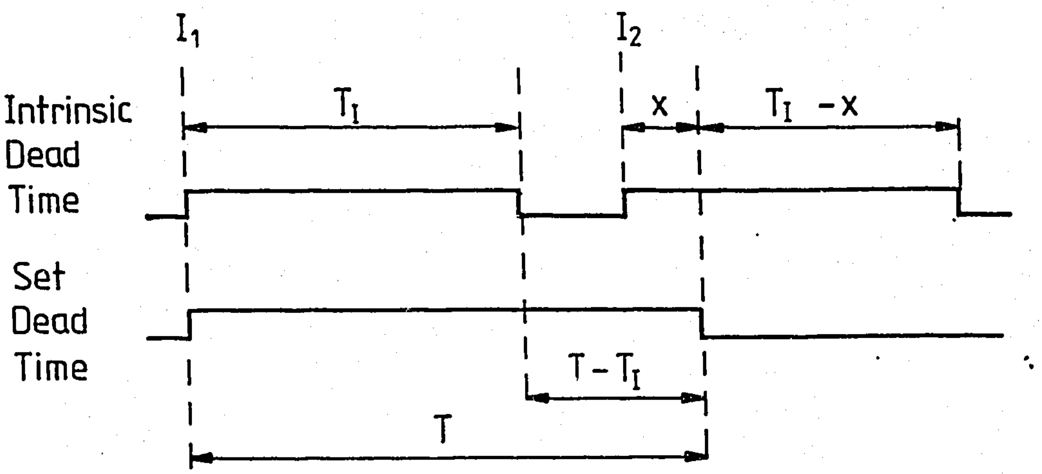


Figure 11. The extension, T_I-x, of the set dead time, T, by the intrinsic dead time, T_I. In the case shown, $T_I > \frac{1}{2}$ T.

Thus,

$$(NE_A + A_b)\ (A''T + X) = NE_A + A_b - A''.$$

Rearranging,

$$NE_A + A_b = \frac{A''}{1 - (A''T + X)}. \qquad (3)$$

Figure 11 shows $T_I > \frac{1}{2}T$. The derivation of equation (3) is the same when $T_I < \frac{1}{2}T$ (figure 12).

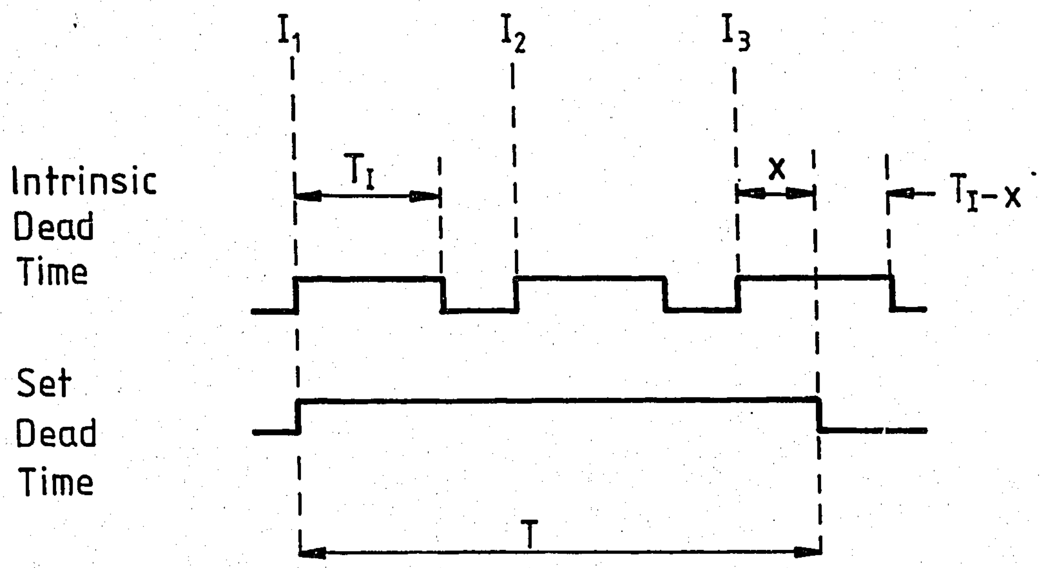


Figure 12. The extension, T_I - x, of the set dead time, T, by the intrinsic dead time, T_I. In this case, $T_I < \frac{1}{2}T$.

10.2 Formulae for T_I

There are two cases to consider; in case 1, T_I is greater than ½T, and in case 2, T_I is less than ½T.

10.2.1 Case 1, $T_I > ½T$

The event detected at I_1 (figure 11) is recorded by scalers I and A. The event detected at I_2, during the set dead time, is recorded by scaler I only. Thus, the number of events per unit time which are detected in all of the intervals $T-T_I$ is equal to $I''-A''$. It is assumed that all values of x between 0 and $T-T_I$ are approximately equally probable. Thus, of the events which are detected in the intervals $T-T_I$, the number per unit time which are detected at instants between x and $x + \delta x$ is

$$(I'' - A'') \frac{\delta x}{T - T_I} .$$

The resulting extension, per unit time, of the dead times T is

$$(T_I - x)(I'' - A'') \frac{\delta x}{T - T_I} .$$

Summing for all the intervals δx,

$$X = \frac{I'' - A''}{T - T_I} \int_0^{T-T_I} (T_I - x)\, dx$$

$$= \frac{I'' - A''}{T - T_I} \left[T_I (T - T_I) - ½(T - T_I)^2 \right]$$

$$= ½(I'' - A'')(3T_I - T).$$

Substituting for X in equation (3),

$$NE_A + A_b = \frac{A''}{1 - A''T - ½(I'' - A'')(3T_I - T)} . \qquad (4)$$

Equations (2) and (4) are simultaneous equations in which the two unknowns are (NE_A) and T_I. Equating the right-hand sides of equations (2) and (4),

$$\frac{I''}{1 - I''T_I} = \frac{A''}{1 - A''T - \frac{1}{2}(I'' - A'')\ (3T_I - T)}.$$

Multiplying out and rearranging,

$$I'' - I''A''T - \tfrac{1}{2}I''(3T_I I'' - TI'' - 3T_I A'' + TA'') = A'' - A''I''T_I$$

$$I''T_I\ (-\frac{3}{2} I'' + \frac{3}{2} A'' + A'') = A'' - I'' + I''A''T - \tfrac{1}{2}TI''^2 + \tfrac{1}{2}I''TA''$$

$$I''T_I\ \frac{5A'' - 3I''}{2} = A'' + I''(- 1 + \frac{3}{2} A''T - \tfrac{1}{2}TI'')$$

$$T_I = \frac{2A'' + I''\ [T\ (3A'' - I'') - 2]}{I''(5A'' - 3I'')} \qquad (5)$$

10.2.2 Case 2, $T_I < \frac{1}{2}T$

As in case 1, the set dead time starts when an event is detected at instant I_1 (in figure 12). The event which is detected at I_2 is recorded by scaler I only, but does not extend the set dead time. The event which is detected at I_3 is recorded by scaler I only and extends the set dead time by the interval T_I - x. Such an extension occurs when x is between 0 and T_I. Thus, again assuming that all values of x between 0 and $T\text{-}T_I$ are approximately equally probable,

$$X = \frac{I'' - A''}{T - T_I} \int_0^{T_I} (T_I - x)\ dx$$

$$= \frac{\frac{1}{2}(I'' - A'')\ T_I^{\ 2}}{T - T_I}$$

Substituting for X in equation (3),

$$NE_A + A_b = \frac{A''}{1 - A''T - \dfrac{\frac{1}{2}(I'' - A'')\, T_I^{\,2}}{T - T_I}} \qquad (6)$$

Equating the right-hand sides of equations (2) and (6),

$$\frac{I''}{1 - I''T_I} = \frac{A''}{1 - A''T - \dfrac{\frac{1}{2}(I'' - A'')\, T_I^{\,2}}{T - T_I}}$$

Multiplying out and rearranging,

$$I'' = \frac{(T - T_I)\,(A'' - A''I''T_I)}{T - T_I - A''T^2 + A''TT_I - \frac{1}{2}(I'' - A'')\, T_I^{\,2}} ,$$

$$I''(T - A''T^2) + I''T_I\,(A''T - 1) - \tfrac{1}{2}I''(I'' - A'')T_I^{\,2}$$

$$= A''T - T_I(A'' + A''I''T) + A''I''T_I^{\,2} ,$$

$$(\tfrac{1}{2} A''I'' + \tfrac{1}{2}I''^2)\, T_I^{\,2} + [I''(1 - A''T) - (A'' + A''I''T)]\, T_I$$

$$+ A''T + I''(A''T^2 - T) = 0 .$$

The above equation can be written

$$a\, T_I^{\,2} + bT_I + c = 0, \qquad (7)$$

where $a = \frac{1}{2} I''(A'' + I'')$,
$b = I'' - A''(2I''T + 1)$, and
$c = T[A''(I''T + 1) - I'']$.

The required root of equation (7) is given by

$$T_I = \frac{- b - (b^2 - 4ac)^{\frac{1}{2}}}{2a} , \qquad (8)$$

where $(\quad)^{\frac{1}{2}}$ signifies the positive square root.

A procedure is now given for deciding which root of equation (7) is applicable. Consider the situation when $T_I = \frac{1}{2}T$. Equation (7) becomes

$$\tfrac{1}{2} I''(A'' + I'') \frac{T^2}{4} + \left[I'' - A''(2I''T + 1)\right] \frac{T}{2} + T[A''(I''T + 1) - I''] = 0 .$$

Dividing by T, and collecting terms containing A'',

$$A'' \left[\frac{1}{8} I''T - \tfrac{1}{2}(2I''T + 1) + I''T + 1\right] = -\frac{1}{8} I''^2 T - \tfrac{1}{2}I'' + I'',$$

$$A'' \left[\frac{1}{8} I''T + \tfrac{1}{2}\right] = I''(\tfrac{1}{2} - \frac{1}{8} I''T), \text{ and}$$

$$A'' = \frac{I''(4 - I''T)}{4 + I''T} . \tag{9}$$

$$a = \tfrac{1}{2}I'' \left\{ I'' \left[\frac{4 - I''T}{4 + I''T} + 1\right] \right\}$$

$$= \tfrac{1}{2}I''^2 \frac{4 - I''T + 4 + I''T}{4 + I''T}$$

$$= \frac{4I''^2}{4 + I''T} .$$

$$b = I'' - \frac{I''(4 - I''T)(2I''T + 1)}{4 + I''T}$$

$$= I'' \frac{4 + I''T - [8I''T + 4 - 2(I''T)^2 - I''T]}{4 + I''T}$$

$$= 2I''^2 T \frac{I''T - 3}{4 + I''T} .$$

$$\frac{-b}{2a} = \frac{2I''^2 T (3 - I''T)}{8I''^2}$$

$$= \frac{T(3 - I''T)}{4} .$$

Let $T = \frac{10}{10^6}$ and $I'' = 20{,}000$, these being typical values.

$$\frac{-b}{2a} = \frac{10(3 - 0.2)}{10^6 \times 4} = \frac{7}{10^6} .$$

$$T_I = \tfrac{1}{2}T = \frac{5}{10^6} .$$

Substituting into the formula for the roots of equation (7),

$$\frac{5}{10^6} = \frac{7}{10^6} \pm \frac{(b^2 - 4ac)^{\frac{1}{2}}}{2a} .$$

Since a is positive, $\frac{(b^2 - 4ac)^{\frac{1}{2}}}{2a}$ is positive, and

$\frac{5}{10^6} < \frac{7}{10^6} + \frac{(b^2 - 4ac)^{\frac{1}{2}}}{2a}$. Thus the choice of + from ± is incorrect, and equation (8) must give the correct root of equation (7).

The choice of the appropriate root of equation (7) is discussed further in Section 12.

10.3 A Check on the Derivations of T_I for Cases 1 and 2

When $T_I = ½T$, the count rate A″ is given by equation (9), which is obtained from equation (7) in the derivation of T_I for case 2. Equation (9) can also be obtained by substituting ½T for T_I in equation (5) which gives the formula for T_I in case 1. Thus,

$$\frac{T}{2} = \frac{2A'' + 3I''TA'' - I''^2T - 2I''}{5I''A'' - 3I''^2} ,$$

$$5I''TA'' - 3I''^2T = (4 + 6I''T)A'' - 2I''^2T - 4I'' ,$$

$$A''(- I''T - 4) = I''^2T - 4I'' ,$$

$$A'' = \frac{I''(4 - I''T)}{4 + I''T} .$$

This shows that the derivations of T_I for cases 1 and 2 are consistent.

10.4 Choosing between the Values of T_I Calculated by Equations (5) and (8)

To calculate the intrinsic dead time, T_I, of a counting system, the experimental values of I″, A″ and T are inserted into both of the equations (5) and (8). It has been found that there are two possibilities:

(a) The two calculated values of T_I are both less than ½T. In this case the correct value is that obtained from equation (8) which has been derived for $0 < T_I < ½T$.

(b) The value of T_I calculated by equation (5) is greater than ½T, and the value calculated by equation (8) is greater than ½T or is imaginary. In this case the correct value is that obtained from equation (5) which has been derived for $½T < T_I < T$.

It is shown below in Section 13 that the above criteria are valid over a wide range of experimental data.

11. CORRECTION FOR INTRINSIC AND SET DEAD TIME IN A SINGLE-CHANNEL SYSTEM

The second unknown in the simultaneous equations of Section 10 is NE_A. The known quantities are I″, A″, T and A_b. The count rate corrected for dead-time losses, $NE_A + A_b$, is obtained by completing the solution of the equations, as described below in Section 11.1.

In Section 11.2, the intrinsic dead time, T_I, is regarded as being a known quantity in the simultaneous equations, with NE_A and I″ as the unknowns. Formulae for $NE_A + A_b$ are derived which are functions of T_I, A″ and T.

11.1 Correction Formulae which are Functions of I″, A″ and T

The count rate corrected for dead-time losses ($NE_A + A_b$) is obtained by calculating T_I from equations (5) or (8), and then inserting it into equation (2). The same result is obtained by inserting the calculated value of T_I into the appropriate alternative simultaneous equation, viz, (4) when $T_I > ½T$ or (6) when $T_I < ½T$.

11.2 Correction Equations of the Form: $NE_A + A_b = f(T_I, A'', T)$

There are two cases to consider; in case 1, T_I is greater than ½T, and in case 2, T_I is less than ½T.

11.2.1 Case 1, $T_I > ½T$

Equations (2) and (4) are now regarded as simultaneous equations in which (NE_A) and I″ are the unknowns. Rearranging equation (2),

$$NE_A + A_b - (NE_A + A_b)\, I''T_I = I'',$$

$$I'' = \frac{NE_A + A_b}{1 + (NE_A + A_b)T_I}. \qquad (10)$$

For brevity, let $NE_A + A_b$ be denoted by n. Substituting the right-hand side of equation (10) for I″ in equation (4) and multiplying out,

$$n \left\{ 1 - A''T - \tfrac{1}{2} \left[\frac{n}{1 + nT_I} - A'' \right] (3T_I - T) \right\} = A'' ,$$

$$n \left\{ 1 - A''T + nT_I (1 - A''T) - \tfrac{1}{2} \left[n - A'' - A''nT_I \right] (3T_I - T) \right\}$$

$$= A'' + A'' nT_I ,$$

$$n^2 \left\{ T_I(1 - A''T) - \tfrac{1}{2}(1 - A''T_I)(3T_I - T) \right\}$$

$$+ n \left\{ 1 - A''T + \tfrac{1}{2}A''(3T_I - T) - A''T_I \right\} - A'' = 0.$$

Multiplying out and multiplying by -2,

$$n^2 (2T_I A''T - 2T_I + 3T_I - T - 3A''T_I^{\,2} + A''T_I T)$$

$$+ n (2A''T - 2 - 3A''T_I + A''T + 2A''T_I) + 2A'' = 0,$$

$$n^2 (3T_I A''T + T_I - T - 3A''T_I^{\,2}) + n (3A''T - 2 - A''T_I) + 2A'' = 0,$$

$$n^2 (T - T_I)(3A''T_I - 1) + n \left[A''(3T - T_I) - 2 \right] + 2A'' = 0. \qquad (11)$$

[If $T_I = T$, equation (11) reduces to equation (1).]

Then, for the upper range of T_I,

$$NE_A + A_b = \frac{-b_u - (b_u^{\,2} - 4a_u c_u)^{\frac{1}{2}}}{2a_u} , \qquad (12)$$

where $a_u = (T - T_I)(3A''T_I - 1),$

$b_u = A''(3T - T_I) - 2,$ and

$c_u = 2A''.$

The sign which prefixes the square root in equation (12) is chosen as follows:

$$\text{Let} \quad T = \frac{10}{10^6},$$

$$T_I = \frac{5}{10^6},$$

$$A'' = 20{,}000.$$

$$\text{Then,} \quad -b_u = 2 - (20{,}000 \times \frac{25}{10^6})$$

$$= 1.5 ,$$

$$2a_u = \frac{10}{10^6} \left[(3 \times 20{,}000 \times \frac{5}{10^6}) - 1 \right]$$

$$= -7 \times 10^{-6} .$$

$$\text{Hence,} \quad NE_A + A_b = -\frac{1.5}{7 \times 10^{-6}} \pm \frac{(b_u{}^2 - 4a_u c_u)^{\frac{1}{2}}}{-7 \times 10^{-6}} .$$

Because $NE_A + A_b$ is positive, the square root must be prefixed by a minus sign. It is confirmed in further discussion in Section 11.2.3 that the minus sign is the correct choice in equation (12).

11.2.2 Case 2, $T_I < \frac{1}{2}T$

In this case, the equation for $NE_A + A_b$ is obtained by solving the simultaneous equations (10) and (6). Multiplying out equation (6) and substituting the right-hand side of equation (10) for I″,

$$n \left\{ (T - T_I)(1 - A''T) - \tfrac{1}{2} \left[\frac{n}{1 + nT_I} - A'' \right] T_I{}^2 \right\} = A'' (T - T_I),$$

$$n \left\{ (T - T_I)(1 - A''T) + nT_I (T - T_I)(1 - A''T) \right.$$

$$\left. - \tfrac{1}{2}(n - A'' - A''nT_I)T_I{}^2 \right\} = A''(T - T_I) + A''(T - T_I)nT_I ,$$

$$n^2T_I(T - A''T^2 - T_I + T_IA''T - \tfrac{1}{2}T_I + \tfrac{1}{2}A''T_I^{\,2})$$

$$+ n(T - A''T^2 - T_I + T_IA''T + \tfrac{1}{2}A''T_I^{\,2} - A''TT_I + A''T_I^{\,2})$$

$$- A''(T - T_I) = 0.$$

Collecting terms together, and multiplying by -2,

$$n^2T_I\left[A''(2T^2 - 2TT_I - T_I^{\,2}) - 2T + 3T_I\right]$$

$$+ n\left[A''(2T^2 - 3T_I^{\,2}) - 2(T-T_I)\right] + 2A''(T - T_I) = 0. \qquad (13)$$

[If $T_I = 0$, equation (13) reduces to equation (1).]

Then for the lower range of T_I,

$$NE_A + A_b = \frac{-b_L - (b_L^{\,2} - 4a_Lc_L)^{\frac{1}{2}}}{2a_L}, \qquad (14)$$

where $a_L = T_I\left[A''(2T^2 - 2TT_I - T_I^{\,2}) - 2T + 3T_I\right]$,

$b_L = A''(2T^2 - 3T_I^{\,2}) - 2(T - T_I)$, and

$c_L = 2A''(T - T_I)$.

The choice of the minus sign to prefix the square root in equation (14) is justified as follows. Let $T_I = \frac{1}{2}T$ in equation (12). Then,

$$- b_u = 2 - 2\tfrac{1}{2}A''T;$$

$$b_u^{\,2} - 4a_uc_u = 4 - 10A''T + \frac{25}{4}A''^2T^2 - 8A''\frac{T}{2}(\frac{3}{2}A''T - 1)$$

$$= 4 - 6A''T + \tfrac{1}{4}A''^2T^2;$$

$2a_u = T(1\frac{1}{2}A''T - 1)$; and

$$NE_A + A_b = \frac{2 - 2\frac{1}{2}A''T - (4 - 6A''T + \frac{1}{4}A''^2T^2)^{\frac{1}{2}}}{T(1\frac{1}{4}A''T - 1)} \quad (15)$$

Let $T_I = \frac{1}{2}T$ in equation (14). Then,

$$-b_L = T - 1\frac{1}{4}A''T^2 = T(1 - 1\frac{1}{4}A''T);$$

$$b_L{}^2 - 4a_Lc_L = T^2(1 - 2\frac{1}{2}A''T + \frac{25}{16}A''^2T^2)$$

$$- 2A''T^2\left[\frac{3}{4}A''T^2 - \frac{1}{2}T\right]$$

$$= T^2(1 - 2\frac{1}{2}A''T + 1\frac{9}{16}A''^2T^2$$

$$- 1\frac{1}{2}A''^2T^2 + A''T)$$

$$= T^2(1 - 1\frac{1}{2}A''T + \frac{1}{16}A''^2T^2);$$

$$2a_L = T(\frac{3}{4}A''T^2 - \frac{1}{2}T); \text{ and}$$

$$NE_A + A_b = \frac{T(1 - 1\frac{1}{4}A''T) - T(1 - 1\frac{1}{2}A''T + \frac{1}{16}A''^2T^2)^{\frac{1}{2}}}{T^2(\frac{3}{4}A''T - \frac{1}{2})}$$

Multiplying top and bottom by $\frac{2}{T}$,

$$NE_A + A_b = \frac{2 - 2\frac{1}{2}A''T - [4(1 - 1\frac{1}{2}A''T + \frac{1}{16}A''^2T^2)]^{\frac{1}{2}}}{T(1\frac{1}{2}A''T - 1)}.$$

The above equation is identical with equation (15), thus showing that the minus sign which prefixes the square root in equation (14) has been correctly chosen.

11.2.3 The Sign of the Square Root in Equation (12)

Calculations are now given which confirm that a minus sign should prefix the square root in equation (12). Rearranging equation (11),

$$A'' \, [3n^2 T_I (T - T_I) + n(3T - T_I) + 2] = n^2 (T - T_I) + 2n.$$

Let A'' now be denoted by A''_u when T_I is in its upper range ($T_I > \frac{1}{2}T$). Rearranging the above equation,

$$A''_u = \frac{n^2 (T - T_I) + 2n}{3n^2 T_I (T - T_I) + n(3T - T_I) + 2} . \qquad (16)$$

The count rate A''_u, calculated from values of n, T and T_I given below, is used to calculate the other root of equation (11), denoted by n_p, and given by

$$n_p = \frac{- b_u + (b_u^2 - 4a_u c_u)^{\frac{1}{2}}}{2a_u} ,$$

where $a_u = (T - T_I)(3A''_u T_I - 1)$,

$b_u = A''_u (3T - T_I) - 2$, and

$c_u = 2A''_u$.

Values of n_p have been calculated for the parameters:

n = 500, 5,000, 20,000 and 50,000 s^{-1},
T = 5×10^{-6} and 10×10^{-6} s, and with
T_I varying from $\frac{1}{2}T$ to T.

The values of n_p over this range are negative, thus showing that n_p is the incorrect root. This can be seen in figure 13 where $\frac{n_p}{n}$ is plotted against $\frac{T_I}{T}$ with the latter varying from 0.5 to 1.

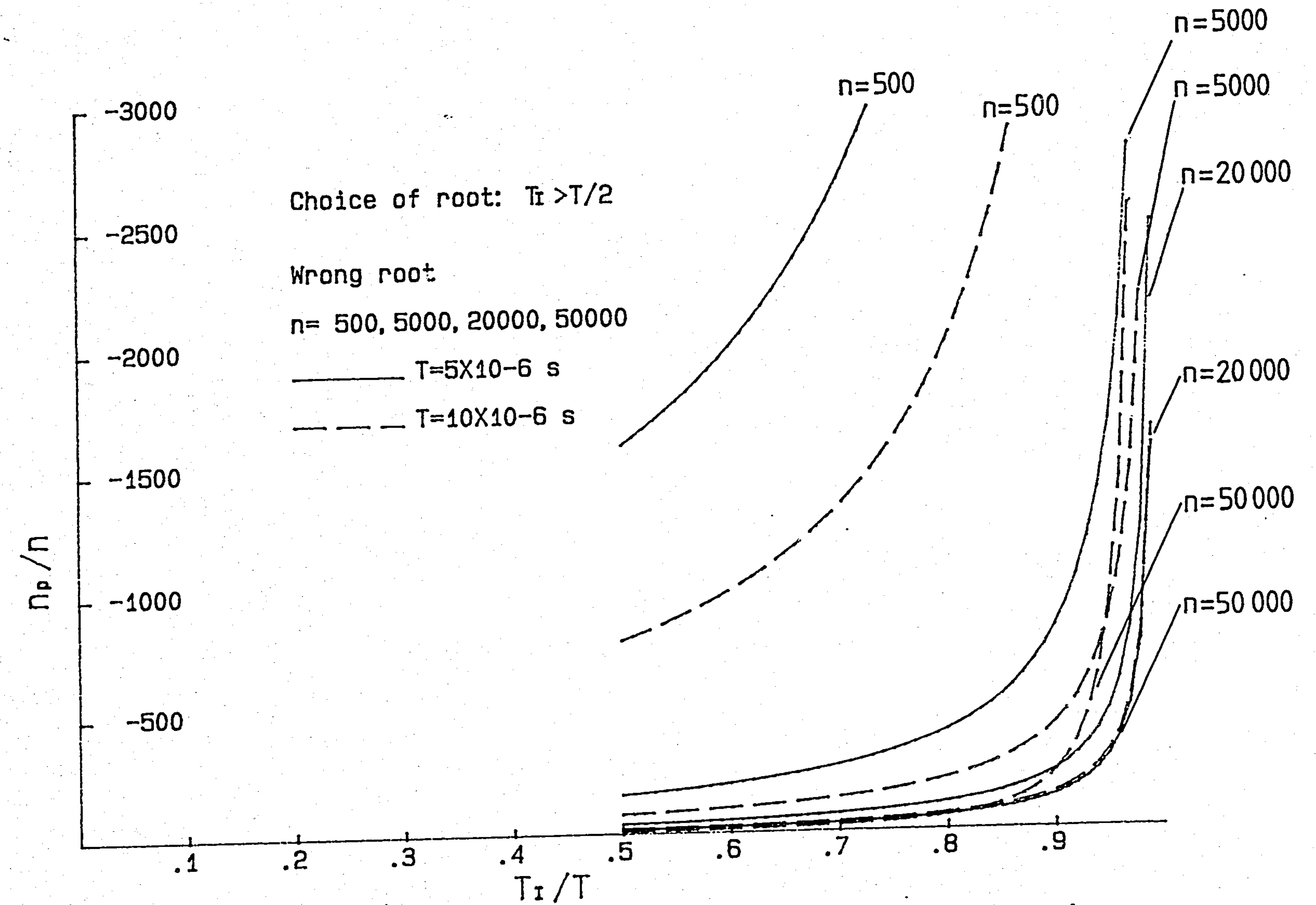


Figure 13. Choice of the correct root in dead-time correction equation (12). The graphs show that the other root of equation (11), denoted by n_p, is negative over the range of parameters considered in the diagram.

12. <u>THE SIGN OF THE SQUARE ROOT IN EQUATION (8)</u>

Calculations are now given which confirm that a minus sign should prefix the square root in equation (8). Rearranging equation (13),

$$A''[n^2T_I(2T^2 - 2TT_I - T_I^{\,2}) + n(2T^2 - 3T_I^{\,2}) + 2(T - T_I)]$$

$$= n^2T_I(2T - 3T_I) + 2n(T - T_I).$$

Let A'' now be denoted by A''_L when T_I is in its lower range ($T_I < \frac{1}{2}T$). Rearranging the above equation,

$$A''_L = \frac{n^2T_I(2T - 3T_I) + 2n(T - T_I)}{n^2T_I(2T^2 - 2TT_I - T_I^{\,2}) + n(2T^2 - 3T_I^{\,2}) + 2(T - T_I)} . \qquad (17)$$

The corresponding value of I'' is given by equation (10). The other root of equation (7) is denoted by T_{IP}, and is given by

$$T_{IP} = \frac{-b + (b^2 - 4ac)^{\frac{1}{2}}}{2a} ,$$

where $a = \frac{1}{2}I''(A''_L + I'')$,

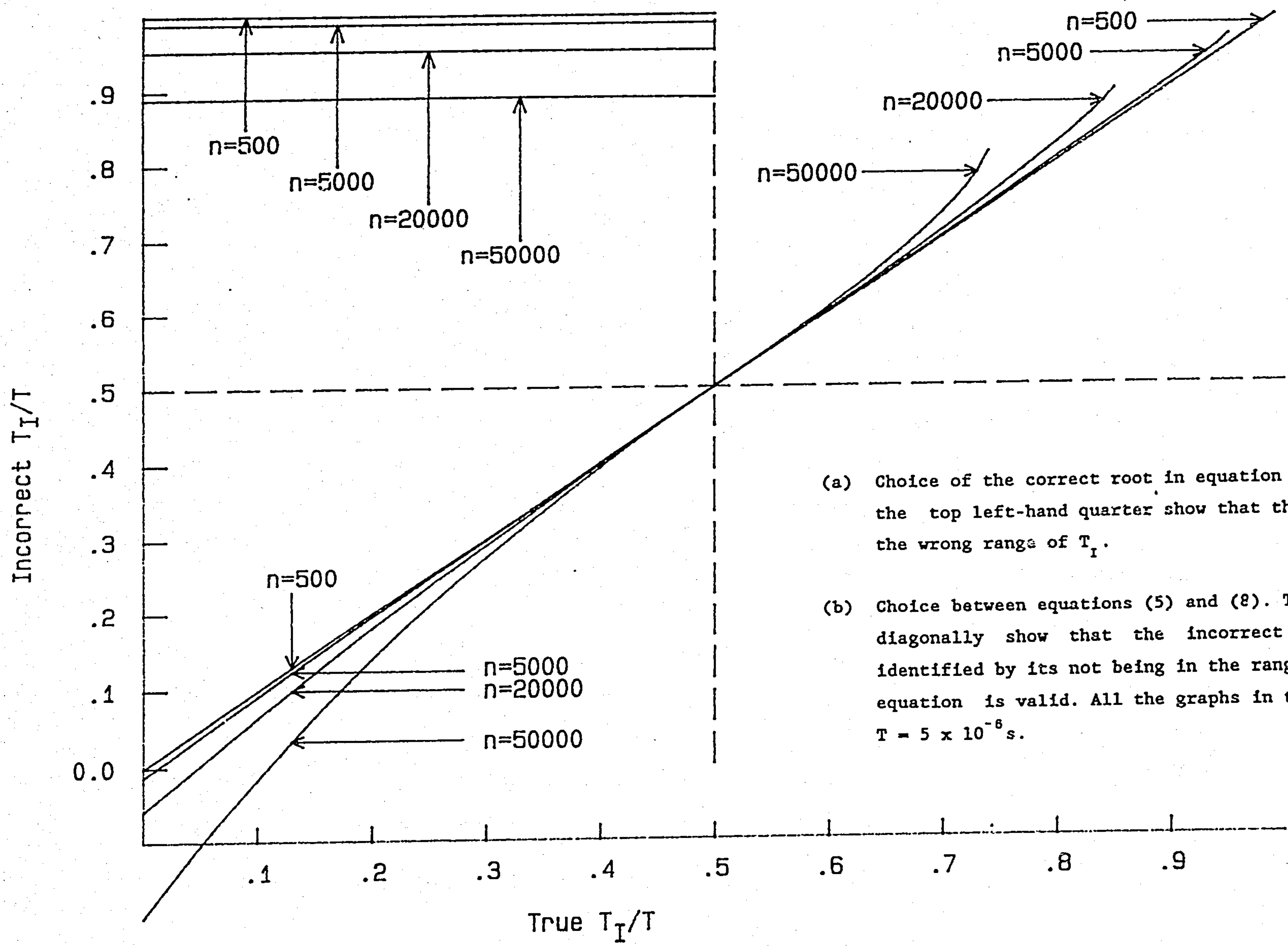
$b = I'' - A''_L(2I''T + 1)$, and

$c = T[A''_L(I''T + 1) - I'']$.

The count rates A''_L and I'' are calculated for the parameters:

$n = 500, 5{,}000, 20{,}000$ and $50{,}000\ s^{-1}$,
$T = 5 \times 10^{-6}$ and 10×10^{-6} s, and with
T_I varying from 0 to $\frac{1}{2}T$.

The ratio $\frac{T_{IP}}{T}$ is then calculated, and is shown graphed against $\frac{T_I}{T}$ in figures 14 and 15 (top left-hand quarters). It can be seen that T_{IP} is the wrong root of equation (7) because, as $\frac{T_I}{T}$ varies from 0 to $\frac{1}{2}$, $\frac{T_{IP}}{T}$ is always greater than $\frac{1}{2}$. In figure 14, $T = 5 \times 10^{-6}$ s, and in figure 15, $T = 10 \times 10^{-6}$ s.



(a) Choice of the correct root in equation (8). The graphs in the top left-hand quarter show that the other root is in the wrong range of T_I.

(b) Choice between equations (5) and (8). The graphs running diagonally show that the incorrect value of T_I can be identified by its not being in the range for which the equation is valid. All the graphs in this figure are for $T = 5 \times 10^{-6}$ s.

Figure 14.

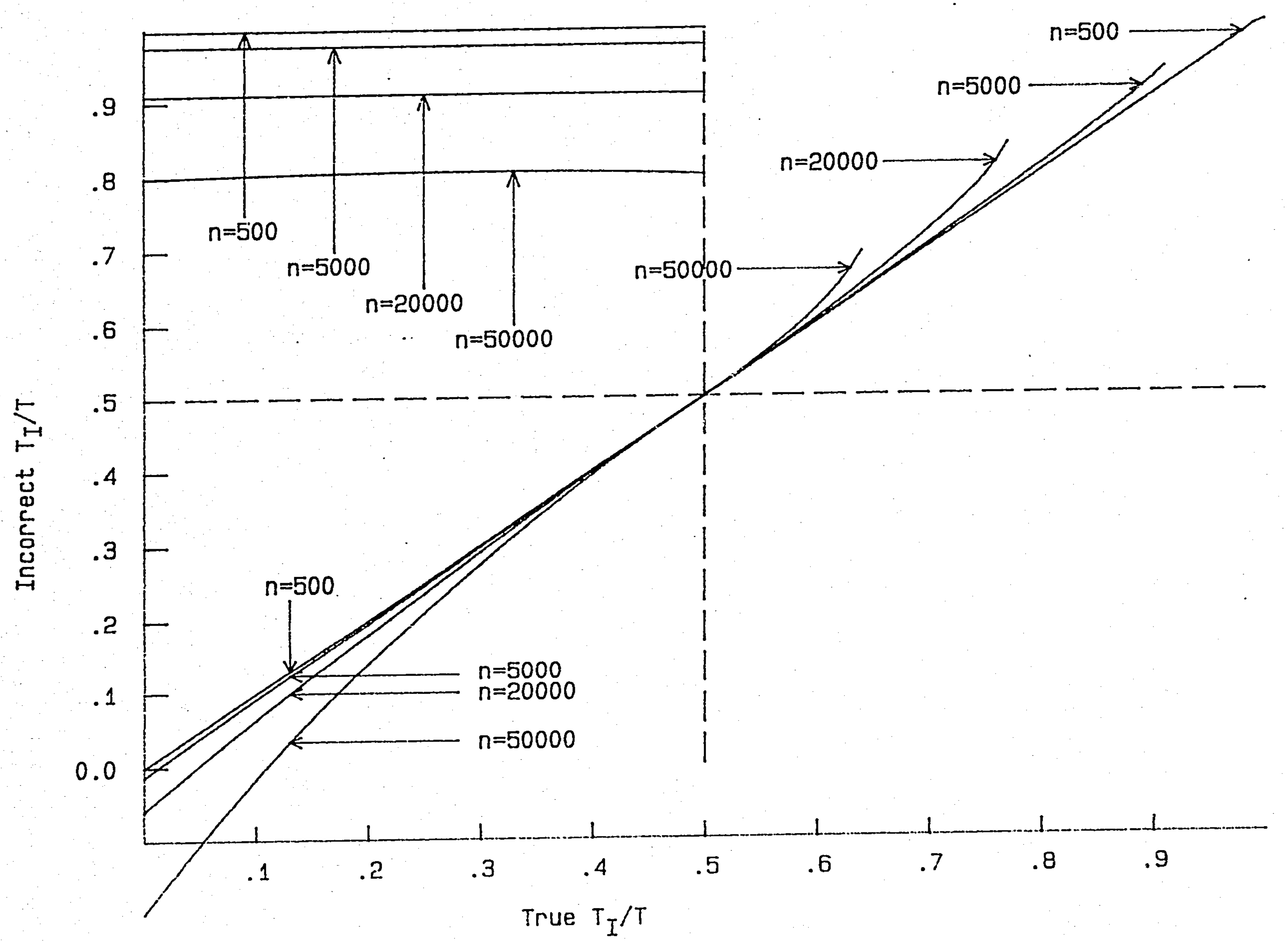


Figure 15. The same as Figure 14, except that $T = 10 \times 10^{-6}$ s.

13. THE CHOICE BETWEEN EQUATIONS (5) AND (8) OVER A RANGE OF PARAMETERS

After experimental data have been obtained, calculations are carried out using both of the equations (5) and (8), and two values of T_I are thus obtained. It is shown below that it will be obvious which value is the correct one.

Case 1, $T_I > \frac{1}{2}T$

The count rates A''_u and I'' are calculated by equations (16) and (10), respectively, for the parameters:

$n = 500, 5,000, 20,000$ and $50,000\ s^{-1}$,
$T = 5 \times 10^{-6}$ and 10×10^{-6} s, and with
T_I varying from $\frac{1}{2}T$ to T.

The values of A''_u, I'' and T are inserted into equation (8), which is not the appropriate equation for calculating T_I in the upper range ($T_I > \frac{1}{2}T$). The results are denoted by T_{IL}. In figures 14 and 15 (top right-hand quarters), $\frac{T_{IL}}{T}$ has been plotted against $\frac{T_I}{T}$. It can be seen from the graphs that the use of equation (8), which is valid when $T_I < \frac{1}{2}T$, has given a value, T_{IL}, which is greater than $\frac{1}{2}T$ and is therefore incorrect. This shows that the correct value of T_I is that obtained by equation (5). Beyond the upper ends of the graphs the values of T_{IL} are imaginary.

Case 2, $T_I < \frac{1}{2}T$

The count rates A''_L and I'' are calculated by equations (17) and (10), respectively, using the same values of n and T as for Case 1, and with T_I varying from 0 to $\frac{1}{2}T$. The values of A''_L, I'' and T are inserted into equation (5), which is not the appropriate equation for calculating T_I in the lower range ($T_I < \frac{1}{2}T$). The results are denoted by T_{IU}. In figures 14 and 15 (bottom left-hand quarters), $\frac{T_{IU}}{T}$ has been plotted against $\frac{T_I}{T}$. It can be seen that the use of equation (5), which is valid when $T_I > \frac{1}{2}T$, has given a value, T_{IU}, which is less than $\frac{1}{2}T$ and is therefore incorrect. This shows that the correct value of T_I is that obtained by equation (8).

14. ERROR INCURRED BY USING THE CORRECTION EQUATION FOR NON-EXTENDABLE DEAD TIME (1)

14.1 Graphical Representation of the Error

The correction equation for non-extendable dead time (1) gives a value for $NE_A + A_b$ which is too low. This is because the value of $NE_A + A_b$ so calculated does not include the counts which are lost during the extension of the set dead time. The resultant error is illustrated in the following manner. Let n_s denote the value of $NE_A + A_b$ calculated by equation (1), and, as before, let n denote the value of $NE_A + A_b$ which is obtained when account is taken of the extension of the set dead time. The percentage $\frac{n_s}{n} \times 100$ is graphed against $\frac{T_I}{T}$ for various values of n and T. There are two cases to consider.

Case 1, $T_I > \frac{1}{2}T$

The count rate A''_u is calculated from equation (16) for the parameters:

$n = 10{,}000,\ 20{,}000,\ 30{,}000,\ 40{,}000,\ 50{,}000\ s^{-1}$,

$T = 5 \times 10^{-6}$ and 10×10^{-6} s, and with

T_I varying from $\frac{1}{2}T$ to T.

Substituting A''_u for A'' in equation (1), n_s is calculated and then $\frac{n_s}{n} \times 100$. The latter is plotted against $\frac{T_I}{T}$ in the right-hand halves of figures 16 and 17. In figure 16, $T = 5 \times 10^{-6}$ s, and in figure 17, $T = 10 \times 10^{-6}$ s.

Case 2, $T_I < \frac{1}{2}T$

The count rate A''_L is calculated from equation (17) for the same values of n and T, and with T_I varying from 0 to $\frac{1}{2}T$. The percentage $\frac{n_s}{n} \times 100$ is calculated and plotted against $\frac{T_I}{T}$ in the left-hand halves of figures 16 and 17.

14.2 The Ratio $\frac{T_I}{T}$ at Which the Error is Greatest

As can be seen in figures 16 and 17, $\frac{n_s}{n} \times 100$ passes through a minimum where the error incurred by using equation (1) is greatest. The value of $\frac{T_I}{T}$ at the minimum point is calculated as follows.

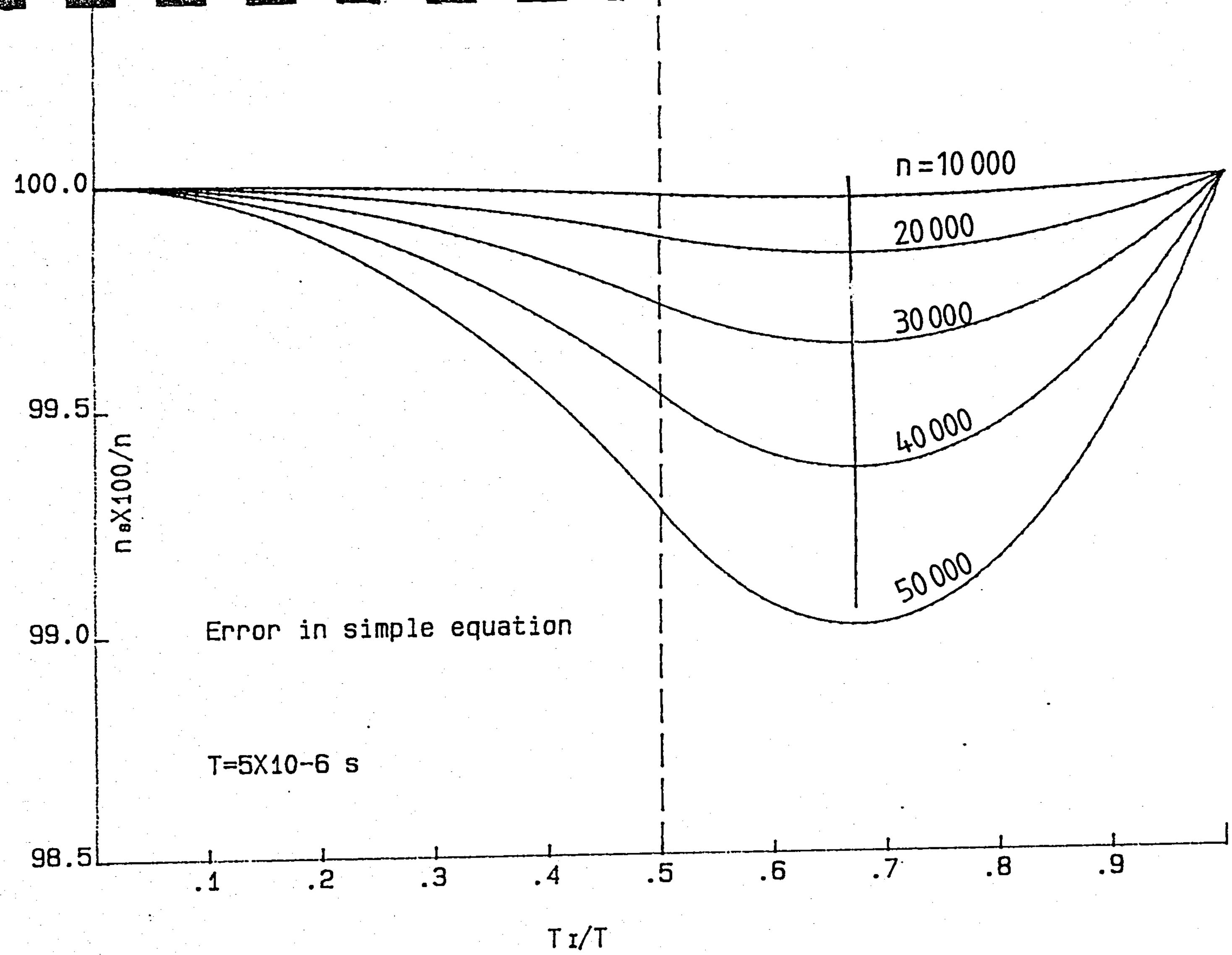


Figure 16. Count rate corrected by the simple equation (1) shown as a percentage of the count rate corrected by equation (2). The graphs are for T = 5 x 10^{-6}s. The locus of the minima of the family of curves is the approximately vertical graph.

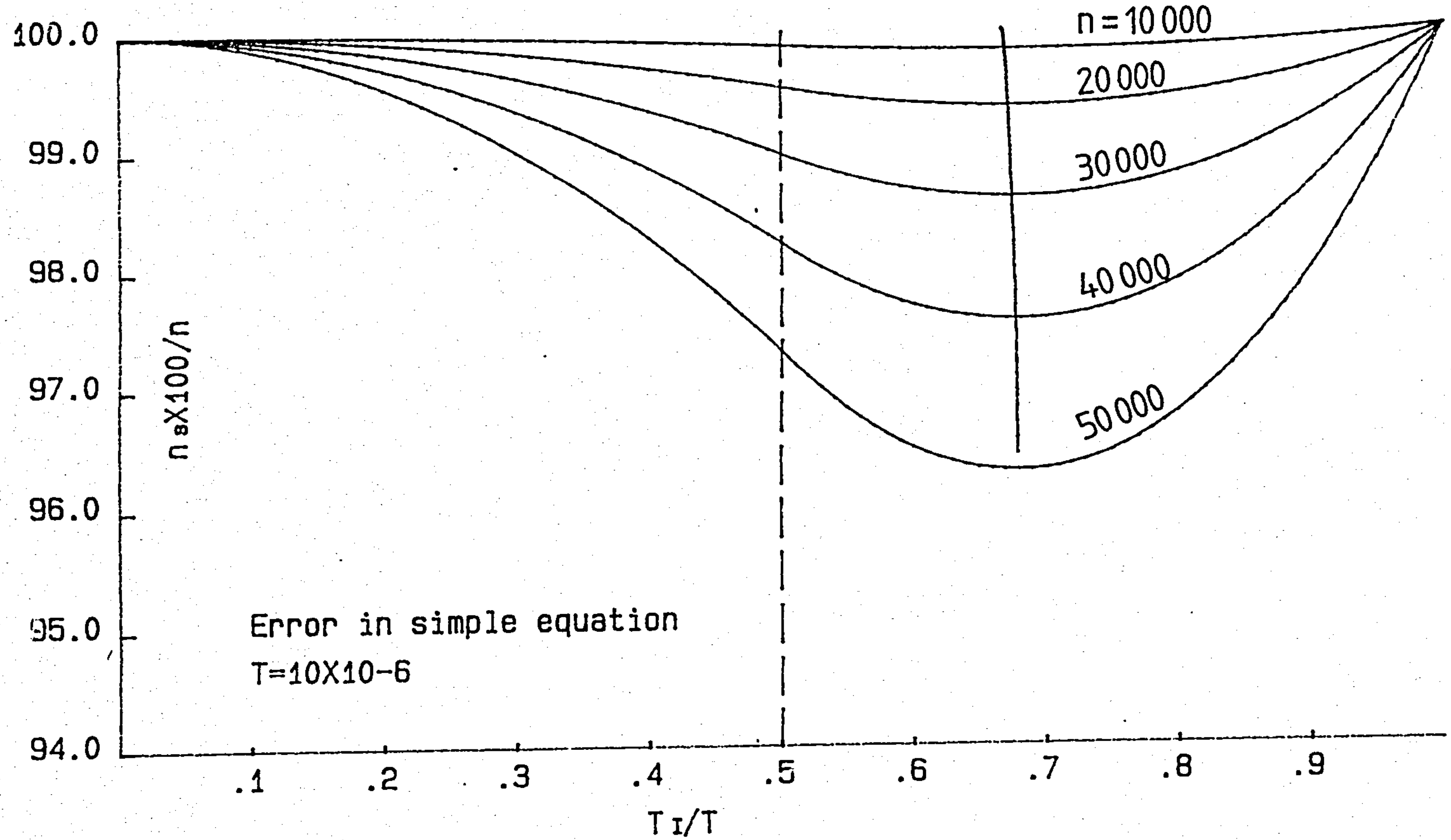


Figure 17. The same as Figure 16, except that $T = 10 \times 10^{-6}$ s.

$$\frac{d}{dT_I}\left(\frac{100\ n_s}{n}\right) = \frac{d\left(\frac{100\ n_s}{n}\right)}{d\left(\frac{T_I}{T}\right)} \times \frac{d}{dT_I}\left(\frac{T_I}{T}\right),$$

$$\frac{100}{n}\ \frac{dn_s}{dT_I} = \frac{d\left(\frac{100\ n_s}{n}\right)}{d\left(\frac{T_I}{T}\right)} \times \frac{1}{T}.$$

At the minimum, $$\frac{d\left(\frac{100\ n_s}{n}\right)}{d\left(\frac{T_I}{T}\right)} = 0,$$

and thus, $$\frac{dn_s}{dT_I} = 0;$$

and since $$\frac{dn_s}{dT_I} = \frac{dn_s}{dA''_u} \times \frac{dA''_u}{dT_I},$$

then, $$\frac{dn_s}{dA''_u} \times \frac{dA''_u}{dT_I} = 0. \qquad (18)$$

Differentiating equation (1),

$$\frac{dn_s}{dA''_u} = \frac{d}{dA''_u}\left\{\frac{A''_u}{1 - A''_u T}\right\}$$

$$= \frac{(1 - A''_u T) - A''_u(-T)}{(1 - A''_u T)^2}$$

$$= \frac{1}{(1 - A''_u T)^2}. \qquad (19)$$

Because the highest value which A''_u could attain would be $\frac{1}{T}$, then in practice,

$$A''_u < \frac{1}{T},$$

$$A''_u T < 1,$$

$$1 - A''_u T > 0,$$

and it follows from equation (19) that $\frac{dn_s}{dA''_u}$ is never zero. It can then be seen from equation (18) that at the minimum,

$$\frac{dA''_u}{dT_I} = 0. \qquad (20)$$

Differentiating equation (16),

$$\frac{dA''_u}{dT_I} = \left\{ \left[3n^2 T_I (T - T_I) + n(3T - T_I) + 2\right] (- n^2) \right.$$

$$\left. - \left[n^2 (T - T_I) + 2n\right] (3n^2 T - 6n^2 T_I - n) \right\}$$

$$\div \left[3n^2 T_I (T - T_I) + n(3T - T_I) + 2\right]^2 . \qquad (21)$$

Equating the right-hand sides of equations (20) and (21),

$$- (3n^4 T_I T) + \left\{3n^4 T_I{}^2\right\} - \left[3n^3 T\right] + \left\langle n^3 T_I \right\rangle - \overline{2n^2}$$

$$- 3n^4 T^2 + (3n^4 T_I T) - \left[6n^3 T\right] + 6n^4 T T_I - \left\{6n^4 T_I{}^2\right\}$$

$$+ 12n^3 T_I + \left[n^3 T\right] - \left\langle n^3 T_I \right\rangle + \overline{2n^2} = 0.$$

Dividing by $-n^3$,

$$3nT_I^2 - (12 + 6nT)\,T_I + (8 + 3nT)\,T = 0.$$

$$T_I = \frac{12 + 6nT \pm (144 + 144nT + 36n^2T^2 - 96nT - 36n^2T^2)^{\frac{1}{2}}}{6n}$$

$$= \frac{2 + nT \pm \frac{1}{6}\left[144\,(1 + \frac{1}{3}\,nT)\right]^{\frac{1}{2}}}{n}.$$

Since $T_I < T < \frac{2}{n} + T$, then

$$T_I < \frac{2 + nT}{n},$$

and therefore $T_I = \dfrac{2 + nT - 2(1 + \frac{1}{3}\,nT)^{\frac{1}{2}}}{n}$. (22)

Thus, on the graph of $\frac{n_s}{n}$ x 100 versus $\frac{T_I}{T}$, the abscissa of the minimum point is given by

$$\frac{T_I}{T} = \frac{2 + nT - 2(1 + \frac{1}{3}\,nT)^{\frac{1}{2}}}{nT}. \qquad (23)$$

Since for the values of n and T considered $\left|\frac{1}{3}\,nT\right| < 1$, equation (23) can be written:

$$\frac{T_I}{T} = \frac{2}{nT} + 1 - \frac{2}{nT}\,(1 + \frac{1}{6}\,nT - \frac{1}{72}\,n^2T^2 + \cdots\cdots) = \frac{2}{3} + \frac{1}{36}\,nT - \cdots\cdots.$$

Thus, $\lim_{n\to o} \left(\frac{T_I}{T}\right) = \frac{2}{3}.$

The locus of the minima is plotted in figures 16 and 17 in the following manner. Values of T_I are calculated from equation (22) with T equal to 5×10^{-6} s (figure 16) and 10×10^{-6} s (figure 17), and with n varying from 0 to 50,000 s^{-1}. The calculated values of T_I are inserted into equation (16) to obtain A''_u. Substituting A''_u for A″ in equation (1) gives n_s. The coordinates of the minimum points, $\frac{T_I}{T}$ and $\frac{n_s}{n} \times 100$, are then calculated, and the points are plotted to give an approximately vertical graph.

15. COINCIDENCE-COUNTING PARAMETERS

The β channel of the coincidence-counting system considered is shown in figure 2. The γ channel consists of a thallium-activated sodium iodide crystal, electron-multiplier phototube, pre-amplifier, amplifier, timing single-channel analyser, gate and delay generator, paralysis unit and scaler. Coincidences are detected by a coincidence mixer connected to the outputs of the β and γ paralysis units, and are recorded by a coincidence scaler.

15.1 The Required Magnitude of the Set Dead Time for Coincidence Counting

As stated in the Introduction, the dead time of the paralysis unit is set to be greater than the intrinsic dead time of the counting channel. In coincidence counting there is a further requirement that the set dead time, T, of the β and γ channels be greater than twice the resolving time, T_R, of the coincidence mixer. As explained below, this prevents the occurrence of a type of coincidence, represented in figure 19, which is not dealt with in the simplified model used for deriving the coincidence-counting correction equation for dead-time losses and accidental coincidences [Wyllie 1987]. The undesirable type of coincidence is accidental, and one of the pair of disintegrations involved is recorded by both channels; i.e. one of the pair gives rise to a true coincidence also.

15.1.1 The Coincidence Mixer

The detection of true and accidental coincidences by the coincidence mixer is illustrated in figure 18. Even though a coincidence is true, the two input pulses to the mixer do not arrive at the mixer simultaneously because of variable delays in the $4\pi\beta$ proportional counter. An input pulse to channel A of the mixer at instant I_1 gives rise to a shaped, square pulse whose width is the

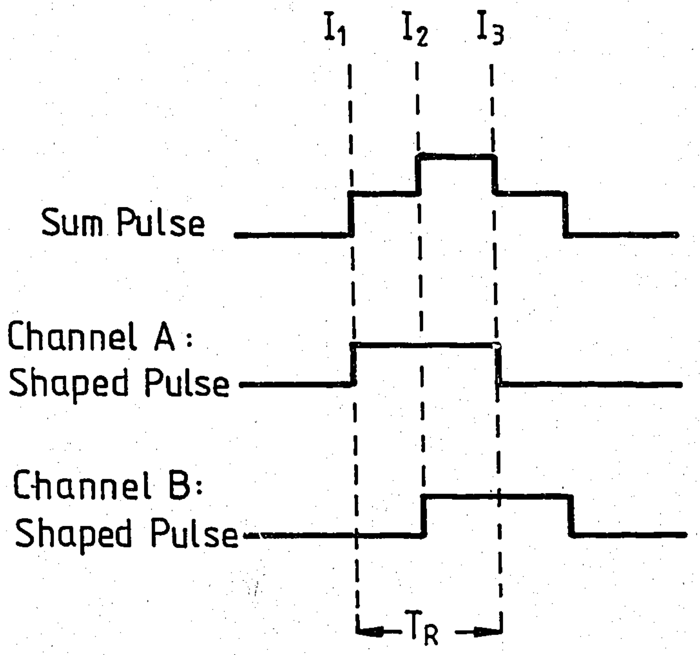


Figure 18. Detection of true and accidental coincidences by the coincidence mixer.

resolving time, T_R. An input pulse to channel B at instant I_2 also gives rise to a shaped pulse. The widths of both pulses can be adjusted, and are set to be equal. If the interval I_1I_2 is less than the width of the shaped pulse, T_R, a sum pulse with double the amplitude of the shaped pulse is produced, and as a result, a coincidence is recorded. If both input pulses arise from the disintegration of the same atom, the coincidence is a true one. If the second input pulse arises from the disintegration of another atom, the coincidence is classified as accidental.

The width of the shaped pulses must be large enough to ensure that when the coincidence is true, the end of the first shaped pulse at instant I_3 always occurs after instant I_2. The counting equipment is adjusted so that at the mixer input, half of the channel A pulses arrive first, and half of the channel B pulses arrive first.

15.1.2 Two Coincidences with One Pulse in Common

In figure 19, the set dead time, T, in each channel, is greater than the resolving time, T_R, as required in the derivation of the coincidence-counting correction formula [Wyllie 1987]. In addition, $T < 2T_R$, which allows the shaped pulse of channel B to be involved in coincidences at instants I_2 and I_3. The second input pulse to channel A arrives at instant I_3, just after the end of the first set dead time of channel A, and just before the end of the shaped pulse of channel B.

In the derivation of the coincidence-counting correction formula, it is assumed that the two pulses from a true coincidence send the paralysis units dead simultaneously, and these, with infinitesimal delay, enter the coincidence mixer. This implies that a shaped pulse which is involved in an accidental coincidence cannot also be involved in a true coincidence.

Consider case (a), when the pulses starting at I_1 and I_2 in figure 19 constitute a true coincidence, and those starting at I_2 and I_3 constitute an accidental coincidence. The first pulse of the accidental coincidence is the second pulse of a true coincidence. Now consider case (b), when the pulses starting at I_2 and I_3

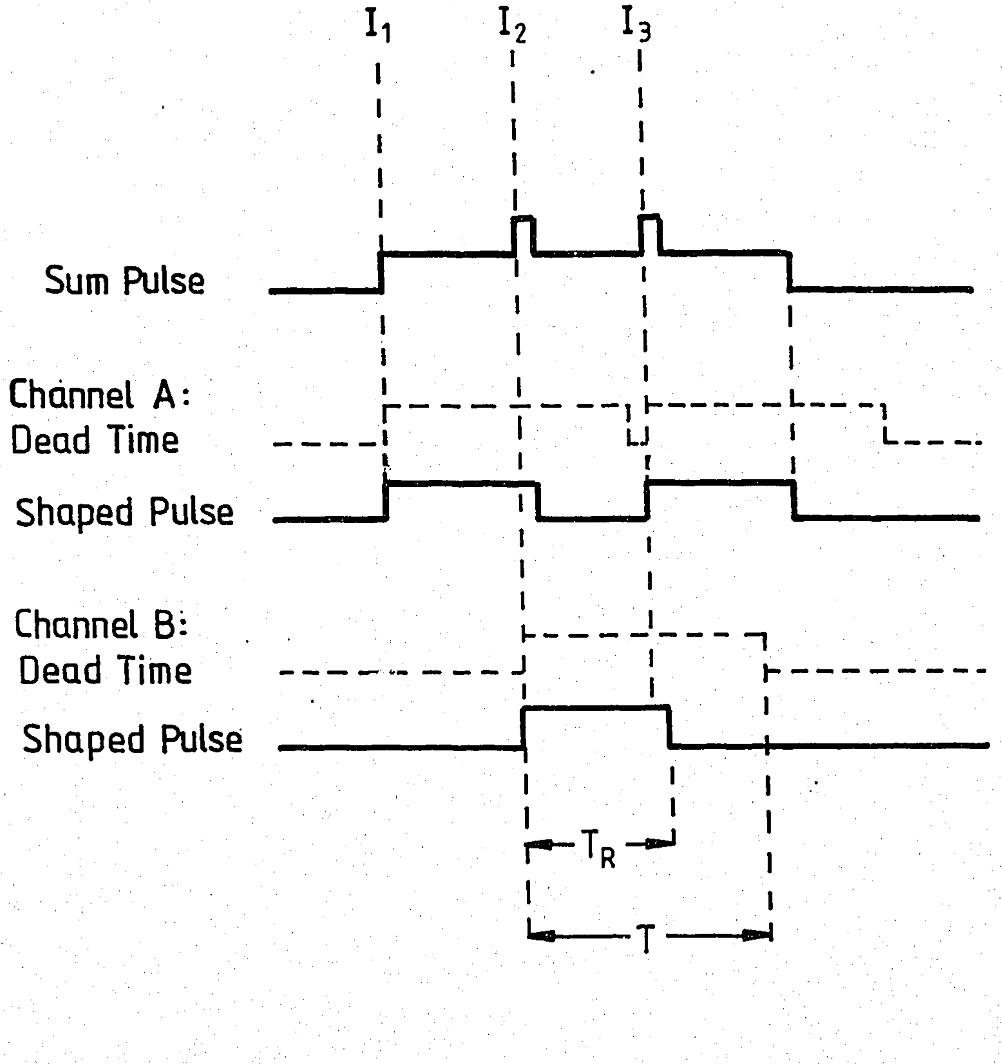


Figure 19. The shaped pulse in channel B of the coincidence mixer can overlap two shaped pulses in channel A when $T < 2T_R$.

constitute a true coincidence, and those starting at I_1 and I_2 constitute an accidental coincidence. The second pulse of the accidental coincidence is the first pulse of a true coincidence.

The accidental coincidences of cases (a) and (b) will be detected by the mixer, but because they are omitted from the model used for the correction formula, their rates are not included in the equation which gives the total rate of true and accidental coincidences. It is desirable therefore to prevent the occurrence of the above type of accidental coincidence. They occur as a result of a shaped pulse in one channel (B) overlapping two shaped pulses in the other channel (A). This possibility is eliminated, as shown below, by increasing the paralysis-unit dead times so that $T > 2T_R$.

15.1.3 Preventing a Pulse's Inclusion in Two Coincidences

In figure 20, the set dead time has been increased, so that $T > 2T_R$. Consider the extreme case of the coincidence shown in the figure, where the shaped pulses starting at instants I_1 and I_2, respectively, overlap for an infinitesimal interval at I_2. There is an infinitesimal gap between the dead times which start in channel A at I_1 and I_4, respectively. The interval I_3I_4 between the end of the pulse in channel B and the start of the second pulse in channel A has a minimum value of $T - 2T_R$. Thus, there is no possibility of the pulse in channel B overlapping two pulses in channel A.

15.2 The Delay in the $4\pi\beta$-Proportional Counter

When a disintegration is detected in the $4\pi\beta$-proportional counter, there is a variable delay between the instant of disintegration and the emission of a pulse from the counter. The distribution curve at the top of figure 21 shows the number of β pulses versus delay. The mean delay, H, is greater than the emission delays of half the pulses, and less than the delays of the other half.

The delay control of the gate and delay generator in the γ channel is adjusted so that in the case of true coincidences, half the β pulses arrive at the β paralysis unit before the corresponding γ pulses arrive at the γ paralysis unit, and the other half arrive after.

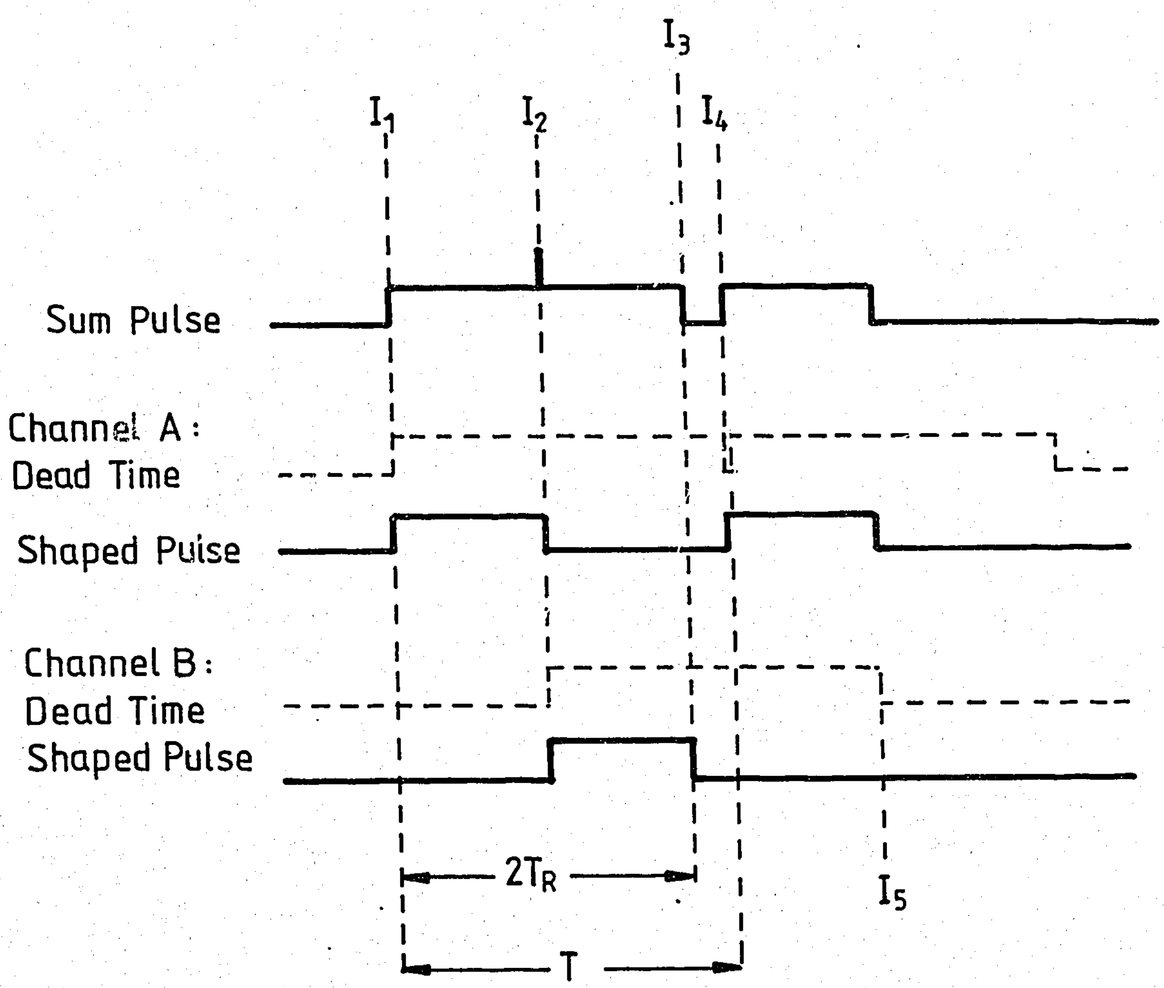


Figure 20. When $T > 2T_R$, the shaped pulse in channel B of the coincidence mixer can overlap only one shaped pulse in channel A; there is no possibility of two coincidences having one pulse in common.

In the model used for determining the disintegration rate, it is assumed that the delay in the proportional counter is constant and equal to the mean delay, H. In figure 21, a disintegration is detected in the β and γ channels at instant I_1. After the delay, H, output pulses emerge from the β amplifier and discriminator at instant I_2. The discriminator output-pulse's trailing edge is recognised by the timing single-channel analyser (the dotted lines within the discriminator and γ-amplifier pulses represent the Timing-SCA internally-generated signals). At the end of the SCA delay, the β paralysis unit goes dead for time T. Not until I_3 can another disintegration occur which will cause a pulse to arrive at the input of the β paralysis unit. The β channel intrinsic dead time is equal to the interval between I_1 and I_3, which is equal to the β amplifier pulse width, W.

After instant I_1, it is not until I_4 that a disintegration can occur which will result in the β paralysis unit going dead a second time. The β channel set dead time is equal to the interval between I_1 and I_4, which is equal to the β and γ paralysis units' dead time, T.

It is assumed in the coincidence-counting model that the delay in the γ gate and delay generator has been set so that the disintegrations at I_1 and I_4, which are detected by both channels, result in both paralysis units going dead simultaneously. In each channel there is the same delay, J, between these disintegrations and the start of the resultant paralysis-unit dead times. Thus, an accidental coincidence is represented by set dead times overlapping to the same extent as the paralysis-unit dead times.

15.3 <u>The Input Pulse to the Coincidence Scaler</u>

As shown in figure 20, the usual procedure is to have $T > 2T_R$. The sum pulse doubles in amplitude at instant I_2, i.e. at the start of the second pulse of the coincidence. The latter pulse, which in this example is in channel B, cannot overlap a second pulse in channel A. The next doubling of the sum pulse amplitude must involve a shaped pulse in channel B which starts after instant I_5. An output pulse from the mixer to the coincidence scaler commences at instant I_2. In order that another output pulse can be generated immediately after instant I_5, it is necessary that the width of the output pulse be less than the interval I_2I_5, i.e. less than T.

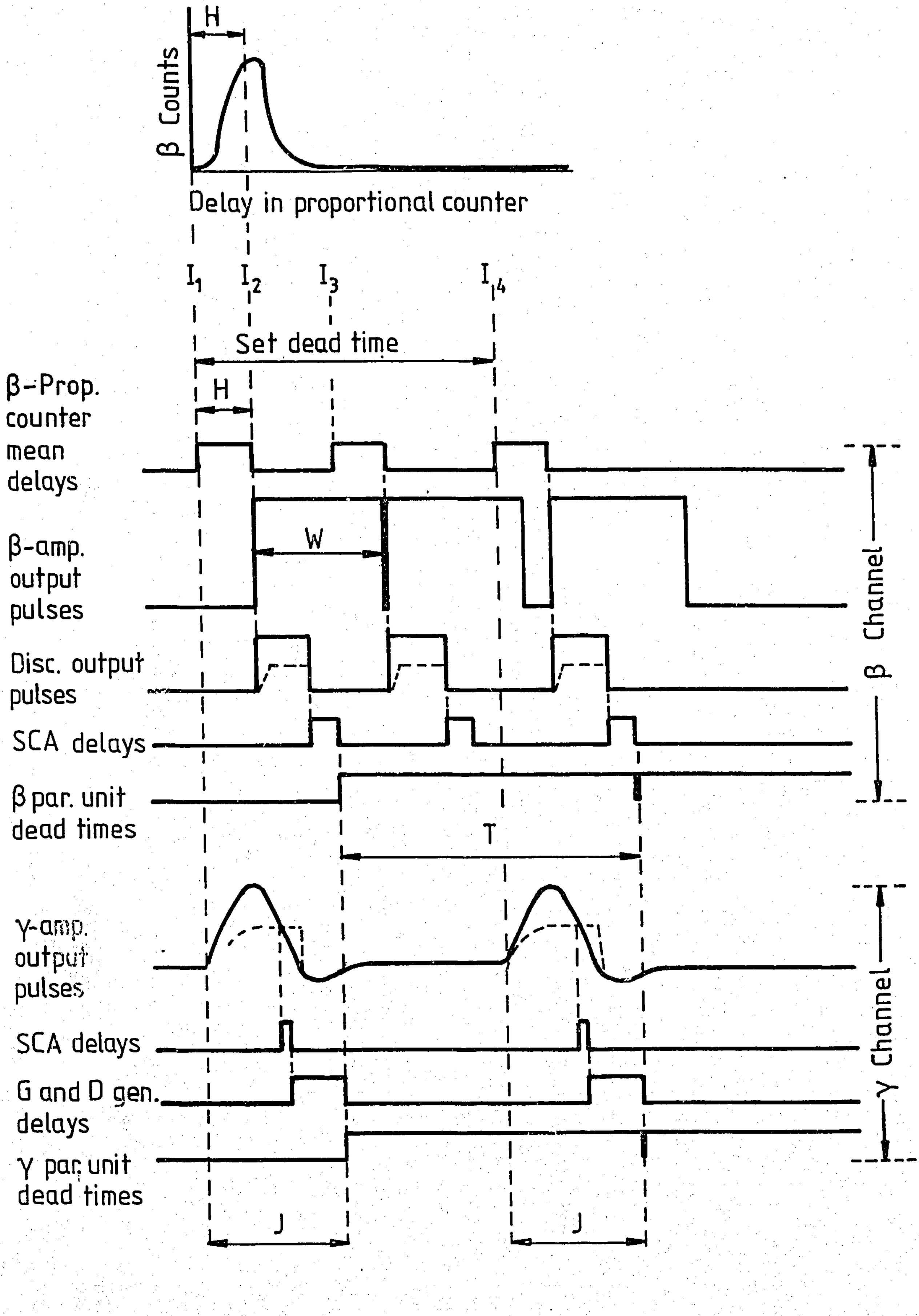


Figure 21. The delays and dead times of the coincidence-counting system.

16. EXPERIMENTAL PROCEDURES

16.1 Measurement of Delay in the Timing Single-Channel Analyser

As outlined in Section 4, the delay in the timing single-channel analyser of system 1 was measured by means of a Tektronix 468 Digital Storage Oscilloscope and a Systron Donner 100A Pulse Generator which was connected to the input of the β amplifier. The oscilloscope was triggered from the "Trigger Output Reference" of the pulse generator. The input and output of the SCA were connected, respectively, to channels 1 and 2 of the oscilloscope. The SCA input and output pulses appeared on the oscilloscope screen as shown in figure 4.

The "Time per Division" control on the oscilloscope was adjusted to make the distance CB appear as large as possible. The AVERAGE button was pressed to obtain the average image from 256 sweeps. The SAVE button was pressed, the two cursors were set on points A and B, respectively, and the SCA delay in microseconds appeared in the oscilloscope window.

16.2 Measurement of the Width of the β-Amplifier's Output Pulse

The width of the β-amplifier's output pulse is defined as the time interval between the start of the leading edge of the pulse and the point on the trailing edge of the pulse which corresponds with the lower level of the SCA in the case of system 1, and with the discriminator level in the case of system 2. The time interval was measured by setting the two cursors of the oscilloscope on these two points as follows. By adjusting the "Time per Division" on the oscilloscope, the image of the pulse on the screen was made as wide as possible. The AVERAGE button was pressed to obtain 256 sweeps. The SAVE button was pressed, the two cursors were then set in position, and the time interval in the window was noted.

16.3 Measurement of the Paralysis-Unit Dead Times by the Double-Pulse Method

The β pre-amplifier was disconnected from the amplifier, and the pulse-generator output was connected to the amplifier input in order to provide double pulses of 15 mV amplitude and 0.5 μs width. The oscilloscope input was connected to the output of the paralysis unit whose dead time was to be measured. The pulse-generator "Delay" was adjusted so that the image of the second of the pair of pulses on the oscilloscope screen was just on the point of disappearing. To improve the accuracy of the measurement, the vertical scale of the oscilloscope was set at 0.2

volts per division to give a steep slope to the leading edges of the pulses on the screen, and the "Time per Division" was adjusted to set the pulses as far apart as possible. The average image of 256 sweeps was obtained, and the dead time was measured by setting the cursors at the start of the leading edges of the pulses.

16.4 <u>Measurement of Dead Time by Baerg's Method</u>

The measurement of dead time by Baerg's [1965] method requires a pulse generator (AAEC Type 144) which provides tailing pulses. The output of the pulse generator is connected to an extra input on the β pre-amplifier. The method requires one scaler (I or A in figures 1 and 2) at the end of that section whose dead time is to be measured. With the pulse generator turned off, the count rate of a source, n_r, is measured. The pulse generator is turned on, and the count rate of the source and pulse generator in combination, n_{rp}, is measured. The E.H.T. to the proportional counter is then turned off, and the pulse generator's repetition rate, n_p^o, is measured.

Baerg's equation for calculating a dead time, τ_o, is as follows:

$$\tau_o = \frac{1}{n_r}\left[1 - \left(\frac{n_{rp} - n_r}{n_p^o}\right)^{1/2}\right] .$$

Müller [1976] concludes that the accuracy of the above equation depends on the magnitudes of τ_o, n_r and n_p^o. He gives a procedure for determining the ratio $\frac{\tau_o}{\tau}$, where τ is the dead time as calculated by an equation which he has derived.

In the experiments described in this report, the dead times measured were less than 20 μs, and the values of n_r and n_p^o were about 12,000 and 4,500, respectively. Carrying out Müller's procedure for ascertaining the value of $\frac{\tau_o}{\tau}$:

$$\rho' = \frac{n_r}{n_p^o} = \frac{12,000}{4,500} \simeq 2.7,$$

$$\tau' = n_p^o \, \tau < 4,500 \times 20 \times 10^{-6} = 0.09.$$

In Müller's figure 2(a), there are 3 plots of $\frac{\tau_o}{\tau}$ versus τ', corresponding to 3 values of ρ', viz. 1, 2 and 5. It can be seen from this diagram that if $\rho' = 2.7$, and $\tau' < 0.09$, $\frac{\tau_o}{\tau}$ differs from 1.0000 by less than 0.0001. Thus, according to Müller, Baerg's equation is quite adequate for dealing with the data of these experiments.

17. EXPERIMENTAL RESULTS

The method described in Section 10 was used for measuring the intrinsic dead time of the components upstream from paralysis unit A in system 1 which is shown in figure 1. The intrinsic dead times were calculated from equations (5) and (8). The results, together with those obtained by Baerg's method, are given in Table 1. A ^{60}Co source in the counter gave $I'' \simeq 12{,}000\ s^{-1}$. Three values of T were selected. The intrinsic dead time was varied by setting different values on the delay control of the timing single-channel analyser. The delay was measured by the method described in Section 16.1.

The average width of the amplifier's output pulses, measured as described in Section 16.2, was 0.57 (± 0.01) μs. It can be seen from Table 1 that for a delay less than the pulse width, the intrinsic dead time was approximately equal to the pulse width, as explained in Section 5 and figure 5. For a delay greater than the pulse width, the intrinsic dead time was equal to the delay, as explained in Section 5 and figure 6.

TABLE 1

Measurement in system 1 of the intrinsic dead time, T_I, of the $4\pi\beta$ proportional counter, pre-amplifier, amplifier and timing single-channel analyser. The uncertainties of the delays and the standard errors of the means of T_I are given in brackets.

Delay in timing SCA (μs)	Intrinsic dead time, T_I (μs), calculated by			
	Equation (5) or (8), with T equal to			Baerg's method
	2.34	5.08	8.96	
0.21(0.01)	0.51(0.02)	0.61(0.01)	0.53(0.02)	0.67(0.02)
1.18(0.01)	1.22(0.01)	1.30(0.01)	1.18(0.01)	1.24(0.01)
2.18(0.01)	2.17(0.01)	2.29(0.01)	2.20(0.01)	2.18(0.01)
3.16(0.01)		3.31(0.02)	3.17(0.02)	3.20(0.02)

The method was checked (see figure 1) by measuring three settings of paralysis unit A as intrinsic dead times relative to paralysis unit W, using the extra scaler W. The extension of the dead time of paralysis unit A by the dead time of the earlier components in the channel was neglected. With a delay in the timing SCA of 0.21 μs, the dead time of the components preceding paralysis unit A was 0.6 μs, as shown in Table 1.

Three dead time settings of paralysis unit W were used; these were measured by the double-pulse method. Table 2 gives the dead times of paralysis unit A obtained by equations (5) and (8), and also by the double-pulse method and Baerg's method.

TABLE 2

The dead time of paralysis unit A in system 1 determined by (a) the double pulse method, (b) equation (5) or (8), and (c) Baerg's method. The uncertainties of the double pulse results, and the standard errors of the means of the other results are given in brackets.

Dead time (μs) set by paralysis unit A, and determined by				
Double pulse method	Equation (5) or (8), with T (of paralysis unit W) equal to			Baerg's method
	4.76	8.85	18.25	
2.34(0.01) 5.08(0.02) 8.96(0.02)	2.30(0.09)	2.40(0.08) 4.95(0.03)	2.39(0.07) 4.93(0.01) 8.83(0.01)	2.44(0.04) 5.04(0.01) 9.07(0.02)

18. CONCLUSIONS

A method for the determination of the intrinsic dead time, T_I, of a counting system, using equations (5) and (8), gives results which agree with those obtained by Baerg's method. Paralysis-unit dead times determined by the intrinsic dead-time method agree with those obtained by the double-pulse method and Baerg's method.

The dead-time correction for the count rate A″ is usually obtained by using equation (1), i.e.

$$NE_A + A_b = \frac{A''}{1 - A''T} .$$

A more accurate correction is obtained by substituting the experimentally determined value of T_I in equation (2), i.e.

$$NE_A + A_b = \frac{I''}{1 - I''T_I} .$$

The same correction is obtained by substituting the value of T_I in the simultaneous equation (4) if $T_I > ½T$, or in the simultaneous equation (6) if $T_I < ½T$.

If I'' is not measured, but the value of T_I is known, the dead-time correction may be obtained by substituting the value of T_I in equation (12) if $T_I > ½T$, or in equation (14) if $T_I < ½ T$.

19. ACKNOWLEDGEMENT

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20. REFERENCES

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APPENDIX

GLOSSARY OF SYMBOLS

A'' : The count rate, including background, which is recorded by scaler A.

A_b : The background count rate which is recorded by scaler A.

E_A : Efficiency of the counter.

I'' : The count rate, including background, which is recorded by scaler I.

N : The disintegration rate of the source.

n : An abbreviation for $NE_A + A_b$.

T : The paralysis-unit dead time.

T_I : The intrinsic dead time of the combined components preceding the paralysis unit.

T_R : The resolving time of the coincidence mixer.

X : The total extension of the intervals T during unit time.