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**SOME GEOMETRICAL PROPERTIES OF PACKINGS OF EQUAL
SPHERES IN CYLINDRICAL VESSELS**

PART 1 - EXPLORATORY STUDY OF RANDOM PACKINGS IN SMALL VESSELS

by

G.A. TINGATE

April 1970

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ABSTRACT

An experimental method is described for determining some of the mean geometrical properties of packings of equal spheres in cylindrical vessels. Essentially it involves determining the volumes of the sphere material in an outer region of thickness one sphere diameter, and the remaining central region. Related properties, including the mean void fractions of the two regions, can then be calculated.

Since the properties of the central region of random packings prepared by a particular method are almost independent of the cylinder-to-sphere diameter ratio it was possible to conduct exploratory experiments using small cylinders. The packings were prepared by a variety of methods.

General expressions are given for calculating related properties from the observations, and also for extending the results to larger vessels to an estimated accuracy of about ± 2 percent.

The accuracy of about ± 1 percent inherent in the method, may be achieved by refinements such as the use of vessels of larger diameter and the elimination of end effects.

PREFACE

This report is a part of a series on "Some Geometrical Properties of Packings of Equal Spheres in Cylindrical Vessels" as follows:

- Part I Exploratory Study of Random Packings in Small Vessels.
G. A. Tingate, this report, AAEC/E202.
- Part II The Cylindrically Ordered Packing.
F. A. Rocke, A.A.E.C. report in preparation.
- Part III Basic Model away from the Influence of Wall Effects.
N. W. Ridgway, G.A. Tingate, AAEC/E202.
- Part IV Extension of Model to Outer Region of Semi-infinite Vessel with Plane Wall.
G. A. Tingate, A.A.E.C. report in preparation.
- Part V Adaptation of Model to Packings in Cylindrical Vessels.
G. A. Tingate, A.A.E.C. report in preparation.
- Part VI Discussion and Conclusions.
N. W. Ridgway, F. A. Rocke, and G. A. Tingate,
A.A.E.C. report in preparation.

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1. INTRODUCTION

In 1966 the Australian Atomic Energy Commission completed a feasibility study of a high temperature gas-cooled reactor system based on a pebble bed concept, using spheres of fuelled beryllium oxide. A number of experimental engineering studies were made in support of this project, including a comprehensive pebble mechanics programme.

The largest experiment, known as the main pebble bed experiment, was designed to investigate the influence of a large number of design features on various pebble bed characteristics. (Szomanski 1967, Szomanski and Tingate 1967). Of particular interest were pebble velocity profiles during recirculation, and the structure after recirculation. Pebble properties, bed geometries and dynamic effects were investigated as well as the effects of pebble extraction devices, multiple outlets and control rods.

The physical features of the projected reactor were simulated as far as practicable, for example the pebble diameter (nominally 1 inch), the specific gravity of the pebble material (nominally 3.0), and the outlet diameter (nominally 8 inches) were duplicated, and full scale pebble extraction devices, developed for possible use in a reactor system, were used. For practical reasons the vessel diameter and packing height were limited to about 30 inches, compared with about 120 inches for the reactor. However, even this reduced size gives a substantial central region away from the influence of wall effects, so the results should be directly applicable to larger packings.

For two reasons concurrent subsidiary studies were also undertaken. The first concerns the interpretation and possible extension of the results of the main pebble bed experiment. It is desirable to know of any extreme effects which might mask or invalidate some of the conclusions if not taken into account. Further, since design features were of necessity closely tied to those of the projected reactor it was desirable to obtain results over a much wider range of parameters. The second reason concerns experimental methods generally. It was felt that some of the reactor features might have been duplicated unnecessarily in the main pebble bed experiment. For example it might turn out that pebble flow under gravity is independent of the specific gravity of the pebble material, or that an even smaller vessel could have been used.

In this report an account is given of a subsidiary study of the structure of small cylindrical packings of equal spheres prepared by a wide variety of methods, but not including pebble recirculation.

2. EXISTING INFORMATION ON PEBBLE BED STRUCTURE

2.1 Structure Without Pebble Recirculation

In the substantial literature on the structure of pebble beds prepared by various methods, whether of spheres and near spheres, or other shapes, the results are usually expressed in terms of void fraction, or as percentage voidage or porosity. In this report the terms void fraction and occupancy fraction (the sum of the two equals unity) will be used. Percentage errors are given in terms of the numerical value of the variable in question.* Of the literature, four papers, Lobo et al. (1945), Denton (1953), Roblee et al. (1958), and Benenati and Brosilow (1962) may be taken as typical.

Lobo et al. reported on beds of Raschig rings prepared by three methods — dumping dry, dropping into water, and shaking after dropping into water to obtain the most dense arrangement. Their results are in some detail, together with formulae for predicting the void fraction to an accuracy of about ± 2.5 percent. A sample calculation (for a bed of Raschig rings of 1 inch outer diameter, 0.8 inch inner diameter, and 1 inch axial length) indicates that shaking after dropping into water should increase the occupancy fraction by about 1.7 percent (that is, the void fraction should decrease from 0.82 to 0.79).

* For example, when a void fraction of 0.40 increases by 1 percent, it becomes 0.404 not 0.41.

Denton poured $\frac{1}{4}$ inch glass marbles into a 10 inch diameter cylindrical vessel from a small jug held about two or three inches above the surface of the marbles. Using the same marbles the experiment was repeated 116 times, and the most probable value of the void fraction was estimated to be 0.391 ± 0.0016 . Similar experiments were carried out using a mechanical charger, arranged to add marbles near the axis of the vessel without impact. The corresponding figure was 0.394 ± 0.001 . Vibration and jolting resulted in a decrease in void fraction to about 0.368, a decrease of 5.9 percent.

Roblee et al. investigated randomly packed beds of spheres, cylinders, Raschig rings and Berl saddles. Each bed was prepared by simple rapid pouring of the packing into a cardboard cylinder. Several methods of pouring were compared for reproducibility by measuring the overall void fraction. Without tamping, results were reproducible within about ± 1 percent and methods of pouring differed by approximately ± 2 percent. Tamping the side of the cylinder thoroughly until no further change occurred produced about a 5 percent decrease in the overall void fraction. They also investigated radial variations of the void fraction within the bed, whereas previous investigators had in most cases restricted their work to the overall void fraction. They froze the packing in wax, removed annular layers by machining, and determined the mean void fraction of each layer by weighing. For spheres and cylinders they found considerable radial variations in void fraction within several pebble diameters of the wall, as shown in Figure 1. For more irregular shapes, radial variations did not persist further than one pebble diameter from the wall.

Benenati and Brosilow used a similar machining and weighing method but restricted their work to packings of spheres in cylindrical vessels. They found essentially the same radial variations within about five sphere diameters of the wall, and gave results for cylinder-to-sphere diameter ratios (D_v/D_p) of 2.61, 5.6, 14.1, 20.3 and infinity. The packings were prepared by pouring the spheres into the container, a method which gave excellent reproducibility. It would appear that impact was avoided, though this is not stated, since the mean void fraction in the larger vessels closely approached 0.39.

It is thus reasonable to expect that "normal" methods of bed preparation, that is, methods which avoid extremes of drop height, vibration etc., will give a standard reproducible packing, and that the mean void fraction of such a packing of spheres or near spheres will be about 0.39 away from the influence of wall effects.

2.2 Structure During and After Recirculation

Denton found that the mean void fraction always reached a final steady value of about 0.395 under recirculation, although the initial value ranged from 0.375 to 0.395. He used glass marbles, with a diameter variation of ± 5 percent about their mean value.

Results from the German AVR pebble bed reactor project indicate an increase in packing density in the wall region under recirculation. Uniform spheres of high precision, such as ball bearings, have this behaviour to a marked degree.

It appears that no detailed comparative study has been made of as-poured and recirculated pebble structures. Of particular interest is the time taken to reach a steady state; if this were to be excessive, the initial packing used in the main pebble bed experiment would have to be selected to minimise it as far as possible. It would then be necessary to develop a reproducible packing method to give the desired structure without the need for time-consuming confirmation of its properties.

3. EXPERIMENTAL METHOD

3.1 Basic Concept

The essential object of the experiment was to determine the mean void fraction in a particular region of the packing and compare it with the corresponding void fraction in the densest possible packing in the same vessel. In practice this involves counting spheres and evaluating the volumes of a number of part spheres, so it is more convenient for observations and calculations to be expressed in terms of the occupancy fraction.

The two regions which lend themselves to such a comparative study are the outer region, of thickness equal to the sphere diameter, and the remainder, the central region. Figure 2 shows the three sphere categories contributing to the two occupancy fractions. The outer region contains whole spheres which touch the wall, and intruding segments of a number of spheres which do not touch the wall. The central region contains the other segments of the intruding spheres, and the balance of the whole spheres which are entirely within the central region.

An outer region of thickness equal to the sphere radius was considered but was rejected because it makes available only two categories, spheres touching the wall and those not touching the wall. Furthermore in practical random packings, segments of some spheres not touching the wall can intrude into this outer region, but not when there is a very dense packing of spheres touching the wall. This might possibly obviate a valid comparison of the two packings.

3.2 Relevant Geometrical Properties of Densest Possible Packing

The very complex calculation of the geometrical properties of the densest possible packing of equal spheres in a finite cylindrical vessel was not tried. Instead, the rhombohedral array, the densest possible packing in a semi-infinite vessel with a plane wall, was taken as standard. A practical vessel would be a cylinder with its axis vertical, and with a close packed stack of horizontal rings of spheres against the wall, to give a regular triangular packing. The rest of the packing would be a series of similar concentric cylindrical shells, the whole approaching a true rhombohedral packing as the diameter of the cylinder increased indefinitely.

The relevant properties are given in Table 1.

TABLE 1
MEAN OCCUPANCY FRACTIONS FOR RHOMBOHEDRAL ARRAY

Contributing Sphere Category	Occupancy Fraction	
	Outer Region	Central Region
Spheres touching wall	$\frac{\pi}{3\sqrt{3}} = 0.6046$	-
Intruding spheres	$\frac{\pi}{3\sqrt{3}} \left(\frac{4}{3} \sqrt{\frac{2}{3}} - 1 \right) = 0.0536$	-
Spheres entirely in central region	-	$\frac{\pi}{3\sqrt{2}} = 0.7405$
Total	$\frac{8\pi}{27\sqrt{2}} = 0.6582$	$\frac{\pi}{3\sqrt{2}} = 0.7405$

Two other geometrical properties of the rhombohedral array are of interest. The first is the cyclic nature of the occupancy fraction when plotted against distance from the wall (Figure 3). The slab within one sphere radius of the wall has different geometrical properties from all other modular slabs, whose width is also less, namely 0.8165 times the sphere radius. The corresponding curve for a random packing in a cylinder of infinite diameter (after Benenati and Brosilow) is plotted for comparison. Close preservation of the pitch of the cyclic variations near the wall is apparent. The other property is the number of sphere contact points at the wall per unit area, that is, in a square of side equal to the sphere diameter. This number is calculated to be 1.1547, and is in fact the ratio of the area of the circumscribed square to that of the unit cell, the circumscribed hexagon.

3.3 Applicability of Method to Packings of Equal Spheres in Cylindrical Vessels

3.3.1 Spheres touching cylindrical wall

The densest possible packing of spheres touching a cylindrical wall is similar to the rhombohedral array, but is always less dense even when an integral number of spheres fit exactly into each horizontal ring. This is because the contact points between adjacent spheres in the same horizontal ring do not lie on the pitch cylinder, leaving a slight gap at the pitch cylinder. This is partly offset by the fact that the axial spacing of the horizontal rings is slightly less than for the rhombohedral array. The derivation of the appropriate factor F is given in Appendix 1. The correction is small in practice, even for small D_v/D_p ratios. For example, if $D_v/D_p = 5$, $F = 0.9921$.

In general an integral number of spheres do not fit exactly into each horizontal ring. For example if $D_v/D_p = 5$, the "theoretical" number of spheres per ring is 12.43. The "theoretical" number is used when calculating the number of spheres in the densest possible packing, on the ground that the annular space corresponding to the unoccupied gap (in this case 0.43) is accessible to spheres in random packings, and only becomes inaccessible as the densest possible packing is approached.

3.3.2 Intruding spheres

No direct allowance for curvature is made in the calculation of the volume of intruding sphere material. It is taken as being 0.08866 times the volume of the spheres touching the wall, the same ratio as for the rhombohedral array. Even if this estimate were out by 10 percent, its effect on the calculated occupancy fraction of the wall region would be less than 1 percent.

3.3.3 Spheres in central region

The results of Benenati and Brosilow show that random packings with small D_v/D_p ratios are subject to gross radial variations in occupancy fraction throughout (Figure 4). Nevertheless, analysis of these results shows that the mean occupancy fraction of the central region is virtually constant for D_v/D_p ratios between 5.6 and infinity, as shown in Table 1. The volumes given in the table have been standardised to a sphere diameter of 1 inch and a packing height of 1 inch.

TABLE 2
PROPERTIES OF PACKINGS WITH VARIOUS D_v/D_p RATIOS

Property	D_v/D_p				
	2.61	5.6	14.1	20.3	Infinity
Diameter of central region, in	0.61	3.6	12.1	18.3	--
Volume of packing, in ³	5.350	24.63	156.1	323.6	--
Volume of central region, in ³	0.2922	10.18	115.0	263.0	--
Volume of outer annulus, in ³	5.058	14.45	41.15	60.63	--
Mean void fraction of outer annulus	0.540	0.475	0.435	0.430	0.410
Void volume of outer annulus, in ³	2.731	6.864	17.90	26.07	--
Mean void fraction of packing	0.550	0.445	0.410	0.395	0.385
Void volume of packing, in ³	2.943	10.96	64.02	127.8	--
Void volume of central region, in ³	0.2113	4.096	46.12	101.8	--
Mean void fraction of central region	0.723	0.402	0.401	0.387	0.385
Mean occupancy fraction of central region	0.277	0.598	0.599	0.613	0.615

The mean occupancy fraction decreases rapidly as D_v/D_p decreases below 5.6, but the idea of a central region becomes rather meaningless when $D_v/D_p = 2.61$, since its diameter is then less than the sphere diameter.

Hence for D_v/D_p ratios down to 5.6, the occupancy fraction of the densest possible packing in the central region is taken as being the same as for the rhombohedral array, namely 0.7405. Hence in practice, cylinders of comparatively small D_v/D_p ratio can be used for exploratory experiments without unduly sacrificing accuracy, and the results thus obtained can be compared directly with the rhombohedral array.

3.4 Application of Method

A random packing of equal spheres is prepared in a cylindrical vessel by the method under study, and the following properties are determined:

Diameter of cylindrical vessel	...		D_v
Diameter of sphere	D_p
Packing height	H
Total number of spheres in packing	...		N
Number of spheres touching cylindrical wall			n
Volume of intruding sphere material	...		V_I

The results are then expressed in terms of occupancy factor (OF), defined as:

$$OF = \frac{\text{observed volume of sphere material of a given category}}{\text{equivalent volume calculated for densest possible packing}}$$

The occupancy factor is calculated for each of the three sphere categories and for the total packing from the following equations:

(i) Occupancy factor of spheres touching wall (OFW)

$$\begin{aligned}
 OFW &= \frac{n}{\text{number of spheres touching wall with densest possible packing}} \\
 &= \frac{n}{F(\text{surface area of pitch cylinder}) / (\sqrt{3} D_p^2 / 2)} \\
 &= \frac{n}{\{F \pi (D_v - D_p)H\} / \{0.8660 D_p^2\}} \\
 &= \frac{n D_p^2}{3.628 FH (D_v - D_p)} \quad (1)
 \end{aligned}$$

(ii) Occupancy factor of intruding spheres (OFI)

$$\begin{aligned}
 \text{OFI} &= \frac{V_I}{0.08866 \text{ (volume of spheres touching wall with densest possible packing)}} \\
 &= \frac{V_I}{\{0.08866 F \pi (D_v - D_p) H \frac{\pi}{6} D_p^3\} / \{0.8660 D_p^2\}} \\
 &= \frac{V_I}{0.16840 F H D_p (D_v - D_p)} \quad (2)
 \end{aligned}$$

(iii) Occupancy factor of central region (OFC)

$$\begin{aligned}
 \text{OFC} &= \frac{(N - n) \frac{\pi}{6} D_p^3 - V_I}{\text{volume of sphere material in central region with densest possible packing}} \\
 &= \frac{(N - n) \frac{\pi}{6} D_p^3 - V_I}{0.7405 \text{ (volume of central region)}} \\
 &= \frac{(N - n) \frac{\pi}{6} D_p^3 - V_I}{0.7405 \frac{\pi}{4} (D_v - 2D_p)^2 H} \\
 &= \frac{(N - n) D_p^3 - 1.9099 V_I}{1.1107 H (D_v - 2D_p)^2} \quad (3)
 \end{aligned}$$

(iv) Occupancy factor of total packing (OFT)

$$\begin{aligned}
 \text{OFT} &= \frac{N \frac{\pi}{6} D_p^3}{\text{total volume of spheres in densest possible packing}} \\
 &= \frac{N \frac{\pi}{6} D_p^3}{\{1.0887 F \pi (D_v - D_p) H \frac{\pi}{6} D_p^3\} / \{0.8660 D_p^2\} + 0.7405 \frac{\pi}{4} (D_v - 2D_p)^2 H} \\
 &= \frac{0.2532 N D_p^3}{H F D_p (D_v - D_p) + 0.2813 H (D_v - 2D_p)^2} \quad (4)
 \end{aligned}$$

If the occupancy factors calculated from these equations are equal, the rhombohedral ratio is said to be preserved. This means that the abilities of the three sphere categories to fill the available space are equal.

3.5 Advantages and Limitations of Method

By selecting a transparent cylinder to accommodate, say, a few hundred spheres, H , N and n can be determined in a few minutes. H can be measured with good accuracy and N and n can be determined without error. The determination of V_I is more tedious but it can be readily accomplished with good accuracy. If the vessel has a plane base, the end effect will be a source of error. This can be overcome by leaving the disturbed region out of the experimental determinations.

The chief limitation of the method is that only the mean properties of the central region of the bed can be calculated; in particular no information is given on radial variations in the central region.

4. EXPERIMENTS

4.1 Purpose of Experiments

- (i) To determine whether the rhombohedral ratio is preserved, whatever the packing method.
- (ii) If not, to investigate other characteristics, for example the extent to which the wall effect is due to the geometrical properties of the pebbles.
- (iii) To determine whether the structure is independent of the packing method over a practical range.

4.2 Preliminary Experiments

4.2.1 Nature and scope

Before proceeding with detailed experiments to determine all the abovementioned properties of various packings of interest, two simplified variants of the basic method were used in a limited number of preliminary experiments.

The first quickly checks whether different preparation methods produce different structures. The same vessel and spheres are used throughout, and the only observation necessary is the total number of spheres N in the vessel in each packing. The second provides a convenient guide as to whether the rhombohedral ratio is preserved. All the properties listed at the beginning of Section 3.4 are determined except V_I . For each packing this takes only a few minutes and enables a wide range of preparation methods to be investigated in a day or two.

The results can be analysed by the following simplified method. If a cylinder contains N spheres, of which n touch the wall, then $(N - n)/n$ is a function of any radial variation in occupancy factor. It is in fact a more sensitive indicator of radial variations in small packings than say N/n .

An adaptation of this variant can be applied to a semi-infinite vessel, and this was in fact the first experiment of the series. Assuming the mean void fraction of the central region of a packing in a semi-infinite vessel to be 0.39, the occupancy factor would be $(1 - 0.39)/0.7404 = 0.824$. Assuming further that the rhombohedral ratio is preserved at the wall, the number of spheres touching the wall per unit area would be $0.824 \times 1.1547 = 0.9513$.

4.2.2 Preliminary experiment with rectangular vessel

Glass marbles of 0.67 inch nominal diameter were poured without impact into a rectangular vessel with a transparent vertical face about 18 inches wide by 12 inches high, to simulate, if possible, a semi-infinite vessel. An area 12 inches x 6 inches was marked for counting points of contact at the wall, allowing five diameters to avoid possible edge effects, and the effect of the irregular top surface of the packing. For the basis outlined in Section 4.2.1 to be valid, 156.4 marbles would touch the wall in the test area. Four successive runs gave 157.5, 154.5, 150.5 and 157.0, an average of 155.0, which corresponds to a mean void fraction in the central region of 0.396. This was encouraging evidence that the rhombohedral ratio might be preserved closely.

Experiments with the rectangular vessel were discontinued at this point because:

- (i) It would have been a long and tedious job to confirm that the mean void fraction was in fact 0.39 away from the influence of the wall.
- (ii) Several runs would be necessary to allow for the experimental scatter of the points of contact in the test area at the wall. An even larger vessel and test area might be necessary to confirm that it was in fact experimental scatter and not a significant difference in the packing.
- (iii) There was no guarantee that the structure was not sensitive to other effects, such as packing method and non-sphericity of marbles.

4.2.3 Preliminary experiments with cylindrical vessels

Points of contact at wall: The experiment was repeated with a 6.2 inch i.d. glass cylinder, using the same glass marbles and pouring method. Equivalent calculations showed that the number of marbles observed to touch the wall corresponded to a mean void fraction in the central region of 0.394. However, when 1.125 inch wooden spheres were poured into the same vessel the number of spheres touching the wall corresponded to a mean void fraction in the central region of 0.41. It was not considered worth while at that stage to determine whether the mean void fraction of the central region of the bed of glass marbles was in fact about 0.394 because:

- (i) The diameters of the glass marbles ranged ± 3.5 percent about a mean value, compared with ± 0.7 percent for the 1.125 inch wooden spheres. The discrepancy in the calculated mean void fractions might be due to this difference in shape.
- (ii) The effect of small departures from the intended packing method was not known.

The use of glass marbles was therefore discontinued, and all further experiments were conducted with spheres which were uniform to within ± 1 percent, with regard to both sphericity and range of mean diameters. Wooden spheres of several sizes were available, from 0.5 to 1.125 inch nominal diameter, also 1 inch zircon spheres of specific gravity 3.4 (manufactured by pressing and baking zircon sand mixed with a plastic binder).

Variations in total number of spheres in packings produced by similar methods: A Perspex cylinder 3.75 inch i.d. and 15 inch axial length was used with 0.75 inch wooden spheres for the next series of experiments. Two drop heights were chosen and three similar preparation methods were used for each. The results are given in Table 3. Each figure is the average of at least four packings.

TABLE 3

PRELIMINARY COMPARISON OF PACKINGS PRODUCED BY SIMILAR METHODS

Preparation Method	Total Number of Spheres, N	
	Dropped from top of vessel	Dropped from 5 ft above top of vessel
Dropping spheres one at a time	383	387
Uniform pouring	382	385
Dropping a beakerful (about 20 spheres) at a time	379	387
Average	381	387

The extra drop height increased the number of spheres in the packing by about 1.4 percent and halved the scatter of the results of the three methods.

Radial variations in structure of packings produced by various methods: Experiments were next carried out, using the same 3.75 inch cylinder and a range of sphere diameters and packing methods, and $(N-n)/n$ was determined for each. No systematic trend in the results could be discerned; certainly there was no general tendency for the rhombohedral ratio to be preserved, despite the encouraging signs at the start of the series.

4.2.4 The need for more detailed experiments

More detailed studies of the structure of individual packings were now considered to be worth while. The method involved the unpicking of several packings by hand to determine the radial positions of the intruding spheres. The use of wooden spheres was discontinued because they are readily dislodged during unpicking.

4.3 Main Experimental Investigation

4.3.1 Variations of $(N-n)/n$ over practical range of packing methods

For this investigation one-inch zircon spheres in the largest available transparent cylinder were used. The 6.2 inch glass cylinder, being slightly irregular, was replaced by a 5.75 inch Perspex cylinder, whose dimensional and shape variations were less than ± 0.005 inch. It had a flat base and a height of 19 inches.

Each packing was prepared with the axis of the vessel vertical. The bottom was covered by hand with the maximum number of spheres possible, and the rest of the packing was prepared by the particular method to be investigated. The vessel was slightly overfilled, and the surplus spheres were carefully picked off until no portion of any sphere projected above the top of the vessel. The bottom layer of spheres was left out of consideration, and the packing height was measured from the top of this layer. The spheres at the top of the packing lay randomly, not in a plane. The average height of the packing was determined by measuring to the tops of the same number of spheres as were in the bottom layer. The height was consistently found to be close to 17.75 inches, so the height measurements were soon discontinued and this figure was assumed for all other packings.

The practice of covering the base of the vessel with the densest possible layer of spheres was adopted primarily to facilitate the determination of N . By using a known number of spheres (in this case 530), N can be calculated by subtraction by merely counting the surplus spheres after levelling the top surface. It also provides a fixed plane of reference for measuring the packing height, and helps to eliminate the end effect due to the base of the vessel.

Typical results are given in Table 4. The loosest packing was obtained by placing a 4.75 inch load tube into the vessel, filling it with spheres, and allowing the spheres to "expand" out into the vessel by withdrawing the tube. The densest random packing was obtained by shaking the vessel and contents bodily up and down. This gave a structure at the wall with a much more regular appearance than for a vibrated packing. The last packing in the table was hand packed to give the densest possible concentric shells of balls, the outer shell being packed first. This did not result in the greatest value of N , due presumably to the relatively small and constricted central region.

These results indicate that only one point exists for which the wall-to-centre rhombohedral ratio is preserved.

4.3.2 Relationship between N and n

The values of N and n were next plotted for the various packings prepared, 26 in all (Figure 5). Most were prepared individually, and show considerable scatter. In particular n can vary over a significant range for a given N . Four of the packings were prepared by shaking down an initially loose packing in stages, in the hope of finding a central tendency for such packings as are free of radial bias. There

TABLE 4

(N-n)/n OBTAINED WITH VARIOUS PACKING METHODS

Packing			Number of Spheres			$\frac{N-n}{n}$
General Category	Ident No.	Preparation Method	N	n	N-n	
Loose	7	Placed singly near wall	495	223	272	1.22
	2	4.75 inch load tube	480	221	259	1.17
	6	Placed singly in centre	498	231	267	1.16
	8	Placed on top of packing a handful at a time	484	225	259	1.15
Medium	10	Dropped singly from top of vessel	504	237	267	1.13
	14	Bulk poured from top of vessel in 15 sec	508	239	269	1.13
	11	Dropped from top of vessel a handful at a time	506	248	258	1.04
	15	Bulk poured from top of vessel in 5 sec	493	244	249	1.02
Dense	19	Vibrated while placing singly	524	268	256	0.96
	26	Vessel shaken bodily after filling	533	279	254	0.91
	21	Handpacked dense concentric shells	529	296	233	0.79

was a tendency to follow the solid line of best fit in Figure 5. The departures of points 24 and 25 are probably due to axial variations in structure, resulting from the somewhat erratic manual shaking method used. The line of best fit is inclined to the rhombohedral line (along which the rhombohedral ratio is preserved), and intersects it at R. This point corresponds to an occupancy factor of 0.835.

The packings known to have significant radial bias fall well away from the line of best fit, while a reasonable proportion of the packings which had no deliberate radial bias lie on or close to it.

4.3.3 Determination of H, N, n and V_I for various packings

Four packings were prepared to cover the range, and to include one (No. 20) known to be strongly biased radially. These have been included in Figure 5 and particulars are given in the following table.

TABLE 5

PACKINGS SELECTED FOR DETAILED EXAMINATION

Packing Identification Number (see Figure 5)	General Category	Preparation Method
19	Dense random	Vibrated while placed singly
6	Medium random	Placed singly in centre
2	Dense Loose random	4.75 inch load tube
20	Hybrid	Hand packed outer shell, loose centre (3 inch load tube used for centre)

In each case D_v , D_p , H , N and n were recorded without disturbing the packing. Then the packing was unpicked sphere by sphere and a measurement made of the radial distance of each intruding sphere from the cylindrical wall. Details are given in Appendix 2. The intrusion distance was then calculated for each from the relationship:

$$\text{Intrusion distance} = D_p - \text{radial distance from wall.}$$

5. ANALYSIS AND INTERPRETATION OF RESULTS

5.1 Mean Properties of Structure of Packings

Several properties of the intruding spheres were calculated from these results, notably the volumes of sphere material in various radial zones of the outer region, the number of intruding spheres intersected by cylinders of various radii, and their sectional areas. These were then compared with various properties of the rest of the packing.

An excellent correlation was found between V_I and V_W , the volume of the spheres touching the wall (Figure 6). No correlation was found between any of the other features compared. The volume of the intruding sphere material increases linearly as the volume of the spheres touching the wall decreases, in accordance with the equation given in the figure. This means that the two compete for space in the outer annulus, so that the intruding sphere volume increases linearly with increase in the available space left by the spheres which touch the wall. Consequently the experimentally determined $V_I - V_W$ line is almost normal to the rhombohedral line, which is also shown in the figure, and is the line along which the two volumes would vary at the same rate.

A general expression connecting V_I and V_W (derived in Appendix 3) enables the mean properties of both the outer and central regions to be calculated directly from D_v , D_p , H , N and n . There can only be one point at which the rhombohedral ratio is preserved between the spheres touching the wall and the intruding spheres, and the corresponding occupancy factor is calculated to be 0.859 (see Appendix 4 for derivation). For this occupancy factor $N = 527$ and $n = 263$, giving a point some distance removed from point R in Figure 5. Neither point is considered to have any particular significance since both are dependent on the D_v/D_p ratio.

General trends over the range of random packings can be seen from Table 6, where various figures for the dense random and loose random packings are compared.

TABLE 6

COMPARISON OF PROPERTIES OF DENSE, MEDIUM AND LOOSE RANDOM
PACKINGS (STANDARDISED TO 17.75 INCH PACKING HEIGHT)

Property	Packing Type and Number				Percentage Change, Dense to Loose
	Dense Random (19)	Medium Random (6)	Top 12" of Medium Random (6)	Loose Random (2)	
Total volume of spheres in packing, V_T in ³	272.1	258.2	258.6	249.3	-8.4
Volume of spheres touching wall, V_W in ³	139.2	119.7	122.0	114.5	-17.7
Volume of spheres not touching wall, $(V_T - V_W)$ in ³	133.0	138.5	136.6	134.8	+ 1.4
Volume of intruding sphere material, V_I in ³	11.1	18.8	17.5	21.0	+89.0
V_I/V_W ratio	0.080	0.157	0.143	0.184	+130.0
Volume of sphere material in outer region, $(V_W + V_I)$ in ³	150.3	138.5	139.5	135.5	-9.8
Volume of sphere material in central region, $(V_T - V_W - V_I)$ in ³	121.9	119.7	119.2	113.7	-6.6
Mean occupancy fraction of outer region	0.569	0.524	0.528	0.513	-9.8
Mean void fraction of outer region	0.431	0.476	0.472	0.487	+11.3
Mean occupancy fraction of central region	0.619	0.608	0.606	0.578	-6.6
Mean void fraction of central region	0.381	0.391	0.394	0.422	+10.8
Mean occupancy fraction of total packing	0.590	0.560	0.561	0.541	-8.4
Mean void fraction of total packing	0.410	0.440	0.439	0.459	+12.0
Mean occupancy factor of central region, OFC	0.836	0.822	0.818	0.781	-6.6
Mean occupancy factor of spheres touching wall, OFW	0.877	0.753	0.768	0.721	-17.7
Mean occupancy factor of intruding spheres, OFI	0.789	1.334	1.239	1.494	+89.0
Mean occupancy factor of total packing, OFT	0.854	0.810	0.812	0.782	-8.4

If the rhombohedral ratio had been preserved, the sphere volumes, occupancy fractions and occupancy factors would all have decreased by the same percentage. Instead, the values for the spheres touching the wall decreased at about three times the rate for the central region. This means that the intruding spheres have been allowed to move further into the outer region, so much so that their values have increased at about thirteen times the rate for the central region. Although the number of spheres not touching the wall has increased slightly, they occupy a substantially larger effective region for this particular D_v/D_p ratio. This accounts for the decrease in occupancy fraction of the central region.

Not only is the rhombohedral ratio not preserved, but there does not appear to be a one-to-one correspondence between N and n . Both must be determined, particularly if there is any reason to suspect that the preparation method being investigated can give rise to radial bias in the structure. Even with severe radial bias (No. 20) the linear relationship between V_I and V_W holds.

5.2 Radial Variations in Properties of Packings

Radial variations in the occupancy fraction of the outer region can be calculated from the figures presented in Appendix 2. This was done for packings 19, 6 and 2, and it was found that the packing of Benenati and Brosilow plotted in Figure 3 lay between packings 19 and 6. The result of the appropriate interpolation is the dotted curve in Figure 3. A pronounced cusp occurs at a distance D_p from the wall, this being the cut-off point for the relatively large number of spheres which touch the wall. The close agreement between calculation and observation confirms that the radial variations in the vicinity of the wall are governed almost entirely by the pebble geometry.

6. ACCURACY OF RESULTS

The experiments were carried out with the smallest D_v/D_p ratio believed to give a reasonably accurate measure of the properties of packings with large D_v/D_p ratios. The only significant source of error in the present study is believed to be axial variations in the structure, and these are inherent in some preparation methods. For example when the spheres are poured or dropped from the top of the vessel, the packing is subjected to diminishing impact effects as its top surface rises. With the other methods any axial bias would result mainly from the end effect, since the practice of covering the base of the vessel with the densest possible layer of spheres cannot be expected to offset it entirely. This is because a dense layer presents a boundary to the rest of the packing which more closely resembles a plane than does the interlaced three-dimensional boundary resulting from a loose random layer. A measure of the residual effect was obtained for one packing (No. 6). The vessel was marked to give three sections each with an axial height of 6 inches, and N , n and V_I were determined for each section. The properties of the top 12 inches of the packing were calculated, and standardised to a height of 17.75 inches, and the figures are included in Table 6 for comparison with the complete packing. There is a 2 percent discrepancy in the occupancy factor of the spheres touching the wall, and a 7 percent discrepancy in the occupancy factor of the intruding spheres, though there is virtually no discrepancy in the occupancy factor for the central region or for the total packing.

Few of the points in Figure 5 lie on the line of best fit, but many of the 23 points which were not deliberately biased lie close to it. Using n as the datum, the value of N given by the line agrees with the observed value within 1 percent for 14 points, and within 2 percent for eight of the other nine, while the error for the remaining point (No. 18) is 2.4 percent. It is therefore not possible to state with better accuracy whether packings prepared by a given method are reproducible, or whether there is a close one-to-one correspondence between N and n for a variety of methods. Disagreement between packings prepared by similar methods is even greater, see for example packings 3, 4, 5, 6, 8 and 9 in Figure 5 which were prepared by "normal" methods. It is however possible that many of the departures are due to experimental error. Experiments with larger D_v/D_p ratios might show a better consistency for some of the methods. If so, it would only be necessary to determine N or n to check that the intended structure had been obtained.

There is no doubt, however, that some preparation methods give rise to radial bias, in which case both N and n have to be determined.

7. RANGE OF RANDOM PACKINGS

The literature provides little information to enable estimation of the range of random packings

possible with either large or small D_v/D_p ratios. At the dense end the mean void fraction of the central region, 0.378 (No. 19), falls well within Denton's figure. He observed an overall void fraction of 0.368 with a nominal D_v/D_p ratio of 40. The mean void fractions of the outer and central regions are estimated by the author to have been 0.437 and 0.362 respectively. No figure could be found at the loose end of the range, for comparison with the figure of 0.422 (No. 2).

An experiment was therefore carried out with $\frac{1}{4}$ inch diameter nylon spheres (specific gravity 1.3) in a 7.75 inch cylinder under water, to minimise the effect of drop height and other densifying effects. The spheres were dumped in bulk, and the packing took several seconds to form, due to swirling of the water as the spheres descended. This resulted in alternating downflow and upflow of the water borne spheres throughout the vessel. This gave the impression that the influence of drop height was virtually eliminated; at the same time the rate at which the spheres were deposited on the top of the packing was large enough to enable many to be trapped before they could roll to the lowest local equilibrium position. The experiment was repeated several times and the results averaged. The overall void fraction was 0.434 and the mean void fractions of the outer and central regions were calculated to be 0.510 and 0.420 respectively. Loose packings were next prepared by the load tube method with the same (dry) nylon spheres and the same structure was obtained, indicating that buoyancy and other effects of the water offer no advantage.

The ranges of values of the mean void fractions of the central regions were next examined, for both small and large D_v/D_p ratios (see Table 7). There is a difference of 0.016 at the dense end of the range, due to the constricted central region with a D_v/D_p ratio of 5.75, but this effect virtually disappears at the loose end of the range. The difference does not appear to have fallen off significantly at the intermediate packing of Benenati and Brosilow.

TABLE 7
CHANGE IN MEAN VOID FRACTION OF CENTRAL REGION
WITH D_v/D_p RATIO

Type of Random Packing	Mean Void Fraction		Difference
	Small D_v/D_p (≈ 5.75)	Large D_v/D_p (≥ 20)	
Densest known	0.378	0.362	0.016
Intermediate (Benenati and Brosilow)	0.402	0.387	0.015
Loosest known	0.422	0.420	0.002

This suggests an explanation of the result of the exploratory experiment with the rectangular vessel, reported in Section 4.2.2. The preparation method, pouring without deliberate impact, should have given a structure in the central region equivalent to packings 8 and 9 in Figure 5. The results for these two packings were averaged, giving a mean void fraction of the central region of 0.375. The corresponding figure for the rectangular vessel would be $0.375 \pm 0.015 = 0.390$, which is in reasonable agreement with the figure calculated in Section 4.2.2. On this basis the rhombohedral ratio was preserved closely for this, the first packing examined, but merely because it happened to lie close to the point R where the rhombohedral line crosses the line of best fit.

8. APPLICABILITY OF RESULTS TO LARGE D_v/D_p RATIOS

8.1 Spheres Touching Wall

The only allowance necessary is for the curvature of the wall. To obtain the occupancy factor

for the spheres touching the wall, the results for $D_v/D_p = 5.75$ are adjusted by multiplying by the ratio

$$\frac{\text{Factor F for } D_v/D_p = 5.75}{\text{Factor F for } D_v/D_p \text{ ratio of interest}} = \frac{0.9921}{\text{Factor F for } D_v/D_p \text{ ratio of interest}}$$

8.2 Intruding Spheres

The volume of the intruding sphere material follows directly by multiplying the adjusted V_W by the V_I/V_W ratio, which is independent of the D_v/D_p ratio.

8.3 Spheres in Central Region

From Table 7 it can be seen that the mean void fraction of the central region can be obtained for normal preparation methods by subtracting 0.015 from the results for $D_v/D_p = 5.75$, if the D_v/D_p ratio of interest is 20 or greater. For intermediate values of D_v/D_p a linear interpolation of the correction should be used until the appropriate investigation has been carried out, and a more accurate rule formulated.

9. FURTHER EXPERIMENTS WITH LARGER D_v/D_p RATIOS

Experience indicates that it should be possible to determine the mean properties of random packings to an accuracy of about ± 1 percent by the unpicking method. It is felt that better accuracy than this is unlikely to be attained by any method. For this accuracy to be achieved, errors in individual observations should not exceed about ± 0.3 percent. It would then be possible to resolve various details of pebble structure, for example whether there is a one-to-one correspondence between N and n for many preparation methods over a practical range, and whether a given method gives results reproducible to ± 1 percent.

To achieve this accuracy with 1 inch diameter spheres, nominal D_v/D_p ratios of about 8 and 12 and a packing height of 12 inches should be used. Unless irreconcilable differences between the two were found the smaller ratio would suffice for subsequent experiments. This would result typically in 250 spheres touching the wall, 200 intruding, and 500 in the central region, so that an error of one sphere in an individual observation could be tolerated. Any unknown effect of the curvature of the cylindrical wall should also be of the same order, since the factor F is 0.9974. The mean void fraction of the central region would be about 0.01* greater than for a D_v/D_p ratio of 20, representing a discrepancy of about 2.5 percent. However, the value of this discrepancy would be known with sufficient accuracy to enable a correction of the required standard to be made. Even the linear correction referred to under Section 8.3 would come close to meeting it.

The end effect can be eliminated by using a vessel height of 18 inches, but taking into consideration only the figures for the top 12 inches. Each half would be determined and axial bias eliminated by rejecting all packings where the numbers of spheres touching the wall in the two halves differ by more than say 2 percent.

10. COMMENTS ON EXPERIMENTAL METHODS

In practice the use of the unpicking method to extend the determination of radial variations in the structure of the central region would involve a large number of spheres, and would be tedious for large D_v/D_p ratios, but not necessarily more so than the methods used by other workers. Unfortunately

* The results of Benenati and Brosilow in Table 2 show virtually no difference between D_v/D_p ratios of 5.6 and 14.1, yet a significant difference between 14.1 and 20.3. There is no reason to expect such a trend, which is therefore attributed to experimental error.

the method has the failing that spheres in the central region are readily dislodged before their radial position can be recorded. This extension of the method is therefore not recommended.

In contrast the spheres touching the wall are comparatively stable; furthermore their number is recorded before the packing is disturbed. The intruding spheres have a tendency to roll as other spheres are removed, but this is readily overcome by recording their positions before disturbing their neighbours.

In determinations of the volume of the intruding sphere material by this method, a record should be kept of the numbers and radial positions of all spheres up to a distance of $1.25 D_p$ from the wall in a few typical cases. Because this was not done in the present study, it has only been possible to evaluate the dotted curve in Figure 3 to the left of the cusp; the dotted curve to the right was joined into the curve of Benenati and Brosilow by interpolation.

The unpicking method gives better accuracy than the machining and weighing method within this distance of the wall, but offers no advantage farther into the central region. Errors are inherent in the machining and weighing method unless a count is also made of the number of spheres touching the wall. As far as can be ascertained, the practice has always been to determine the volume of sphere material without differentiating between the spheres touching the wall and the intruding spheres. The effects of this omission can be estimated from the results of the present study. The volume of sphere material in the first thin layer (say $0.1 D_p$ thick) machined from the outside cannot give a reliable measure of the number of spheres touching the wall because of excessive experimental error. The effect of experimental error should however be negligible in the outer layer of $0.5 D_p$ thickness, and, if all sphere material weighed was assumed to have come from spheres touching the wall, the results and errors would be as given in Table 8.

TABLE 8
ERRORS INTRODUCED BY NOT ASCERTAINING NUMBER OF
SPHERES TOUCHING WALL

Property	Type of Random Packing		
	Dense	Medium	Loose
Total volume of sphere material removed (in ³)	73.5	62.7	60.7
Volume contributed by spheres touching wall (in ³)	72.3	61.6	59.2
Volume contributed by intruding spheres (in ³)	1.17	1.17	1.50
Calculated number of spheres touching wall	273.4	233.3	225.6
Actual number of spheres touching wall	269	229	220
Error	+1.6%	+1.9%	+2.5%

As the thickness of the removed annular layer is increased beyond $0.5 D_p$, the error increases rapidly due to the proportionately larger contribution of the intruding spheres. From Table 6 it can be seen that the mean occupancy fraction of the outer region has decreased by 9.8 percent, while the number of spheres touching the wall has decreased by 17.7 percent. Thus if variations in radial distribution were determined from the mean properties of the outer region, without differentiating between the two sphere categories, almost half the available sensitivity would be sacrificed.

The masking of the cusp at the boundary between the two regions is also inherent in the method, because of the finite thickness of the machined layers. A better approximation to the true shape could have been obtained by determining the exposed areas of the spheres after each machining operation, particularly at this boundary.

11. SUMMARY AND CONCLUSIONS

(i) The structure of random packings of equal spheres in a cylindrical vessel depends strongly on the preparation method.

(ii) Experimental and analytical studies of such packings are facilitated by considering two regions of the packing, one being the outer annular region of thickness equal to the sphere diameter and the other being the cylindrical central region. Three sphere categories are associated with the two regions, and their properties can be evaluated and compared experimentally and analytically.

(iii) Published literature indicates that the mean void fraction of the central region of random packings prepared by a given method is almost independent of the D_v/D_p ratio in the range 5.6 to infinity. Experimental results obtained with small D_v/D_p ratios are therefore applicable to large ratios with fair accuracy.

(iv) In general the rhombohedral ratio is not preserved throughout a random packing; in other words the three sphere categories do not have the same ability to fill the space available to them.

(v) The spheres touching the wall and the intruding spheres compete for space in the outer region. The volume of the intruding sphere material increases linearly with decrease in the number of spheres touching the wall.

(vi) From a typical dense to a typical loose random packing, the occupancy factor of the spheres touching the wall decreases at about three times the rate for the central region. Over the same range the occupancy factor of the intruding spheres increases at about thirteen times the rate for the central region.

(vii) The unpicking method adopted for this study gives better accuracy within a radial distance $1.25 D_p$ of the wall than the machining and weighing method commonly used by other workers. In particular any method will give rise to experimental error in the outer region if it does not include the counting of the number of spheres touching the wall and the determination of the mean void fraction at the boundary between the two regions of the packing.

(viii) The unpicking method could possibly be used for the determination of radial variations of void fraction further into the central region, but offers no advantage over the machining and weighing method.

(ix) Further experiments need to be carried out with a larger D_v/D_p ratio, which probably need not exceed 8, to clarify some outstanding points to the best attainable accuracy. The results should then be directly applicable to much larger ratios with errors not exceeding one percent. They should also show whether general equations based on results with a ratio of 5.75 can be used to this accuracy. The experimental method would also need to be improved to eliminate residual errors due to the end effect and axial variations in the rest of the packing.

(x) The mean void fractions of the central regions of the packings investigated with a D_v/D_p ratio of 5.75 covered the range 0.378 to 0.422. This range corresponds to 0.362 to 0.420 for ratios of 20 or greater. No change would be expected for larger ratios, and a systematic variation would be expected between 5.75 and 20. This is expected to be less pronounced for larger ratios; in the meantime a linear variation should be assumed when interpolating.

(xi) Attempts to produce looser packings than those recorded under (x) above were unsuccessful.

12. ACKNOWLEDGEMENTS

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13. NOTATION

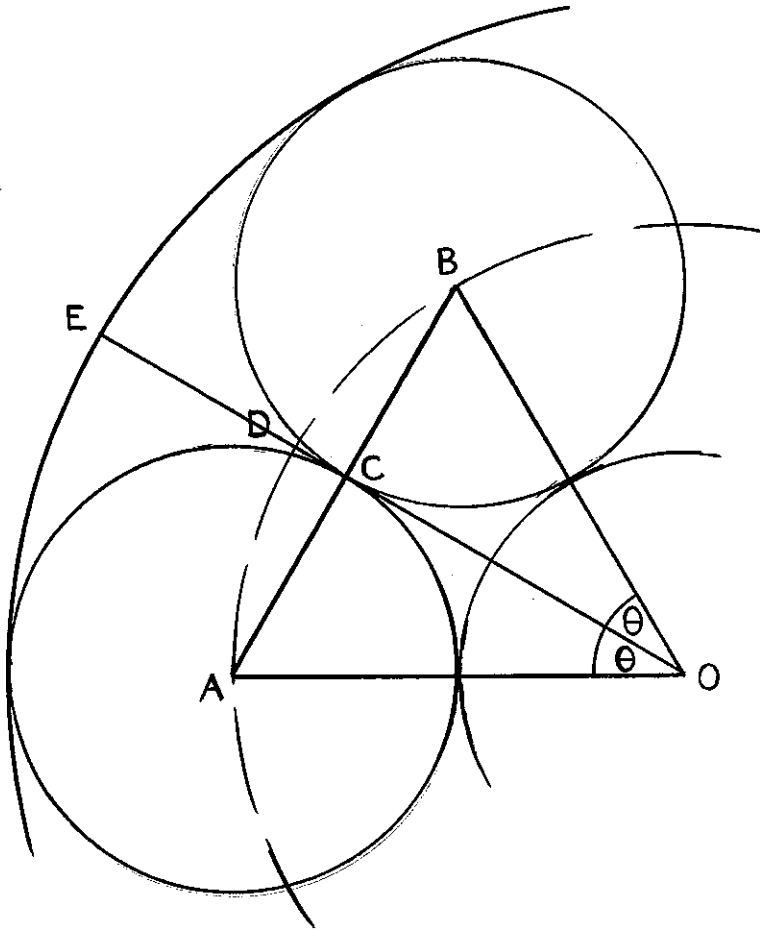
D_v	=	diameter of cylindrical vessel
D_p	=	diameter of sphere
F	=	factor for calculating number of spheres in densest possible packing against cylindrical wall
H	=	packing height
N	=	total number of spheres in packing
n	=	number of spheres touching cylindrical wall
V_w	=	volume of spheres touching cylindrical wall
V_I	=	volume of intruding sphere material
OFW	=	occupancy factor of spheres touching wall
OFI	=	occupancy factor of intruding spheres
OFC	=	occupancy factor of central region
OFT	=	occupancy factor of total packing.

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APPENDIX 1

DERIVATION OF FACTOR F FOR CALCULATING NUMBER OF SPHERES
IN DENSEST POSSIBLE PACKING AGAINST CYLINDRICAL WALL



$$OE = \frac{D_v}{2}$$

$$AC = CB = ED = \frac{D_p}{2}$$

Factor F1: accounts for gap at pitch cylinder between spheres in the same horizontal ring.

$$\begin{aligned}
 F1 &= \frac{\text{Diameter of sphere}}{\text{Length of arc between centres of spheres}} \\
 &= \frac{D_p}{\text{Arc ADB}} = \frac{D_p}{2 OB \theta} \\
 &= \frac{D_p}{(D_v - D_p) \arcsin \left(\frac{D_p}{D_v - D_p} \right)}
 \end{aligned}$$

Factor F2: accounts for reduced axial spacing of horizontal rings.

The centre of the sphere immediately above the two spheres shown in the figure is vertically above point D. It is therefore offset by the distance DC from the vertical plane through A and B.

APPENDIX 1 (continued)

We have

$$DC = OD - OC.$$

where $OD = \frac{D_v - D_p}{2}$

and $OC = \sqrt{OB^2 - CB^2} = \sqrt{\left(\frac{D_v - D_p}{2}\right)^2 - \left(\frac{D_p}{2}\right)^2}$

$$\text{Axial spacing for rhombohedral array} = \frac{\sqrt{3}}{2} D_p .$$

Hence $F2 = \frac{\frac{\sqrt{3}}{2} D_p}{\sqrt{\left(\frac{\sqrt{3}}{2} D_p\right)^2 - DC^2}}$

Factor F: product of F1 and F2

Example: for a D_v/D_p ratio of 5,

$$F1 = 0.9894$$

$$F2 = 1.0027$$

and $F = 0.9921$.

APPENDIX 2

OBSERVED FEATURES OF PACKINGS SELECTED FOR
DETAILED EXAMINATION

Feature	Packing Type and Number			
	Dense Random (No. 19)	Medium Random (No. 6)	Loose Random (No. 2)	Hybrid (No. 20)
Diameter of vessel, D_v in.	5.75	5.75	5.75	5.75
Diameter of spheres, D_p in.	0.996	0.997	0.997	0.997
Packing height, H in.	17.75	17.625	17.70	17.75
Total number of spheres in packing, N	526	494	479	516
Number of spheres touching wall, n	269	229	220	294
Number of spheres intruding a distance				
0.125 D_p	63	51	44	93
0.250 D_p	34	40	29	11
0.375 D_p	6	22	14	0
0.500 D_p	5	11	20	1
0.625 D_p	2	9	12	1
0.750 D_p	1	5	8	2
0.875 D_p	6	3	3	1
Total number of intruding spheres	117	141	130	109

APPENDIX 3

DERIVATION OF GENERAL EXPRESSION FOR VOLUME OF INTRUDING SPHERE MATERIAL

For 0.997 inch spheres in 5.75 inch cylinder 17.75 inch high,

$$V_I = 68.0 - 0.410 V_W .$$

For other diameters and heights only the first term varies. It increases by the product of the following factors

$$\frac{H}{17.75} \quad \text{due to height}$$

$$\frac{D_v - D_p}{5.75 - 0.997} \quad \text{due to diameter of pitch circle}$$

$$\frac{F}{0.9944} \quad \text{due to curvature of wall}$$

$$\frac{0.997^2}{D_p^2} \quad \text{due to area of circumscribed hexagon of sphere}$$

$$\frac{D_p^3}{0.997^3} \quad \text{due to volume of sphere.}$$

The product of these factors is

$$0.01196 FH D_p (D_v - D_p), \text{ whence}$$

$$V_I = 0.812 FH D_p (D_v - D_p) - 0.410 V_W .$$

APPENDIX 4

CALCULATION OF OCCUPANCY FACTOR FOR WHICH RHOMBOHEDRAL RATIO IS PRESERVED BETWEEN SPHERES TOUCHING WALL AND INTRUDING SPHERES

$$\text{OFW} = \text{OFI}$$

where

$$\text{OFW} = \frac{n D_p^2}{3.628 FH (D_v - D_p)}$$

and

$$\text{OFI} = \frac{0.8123 FH D_p (D_v - D_p) - 0.410 n \frac{\pi}{6} D_p^3}{0.1684 FH D_p (D_v - D_p)}$$

$$= 4.823 - \frac{1.275 n D_p^2}{FH (D_v - D_p)}$$

Hence

$$n D_p^2 = 17.50 FH (D_v - D_p) - 4.626 n D_p^2$$

or

$$n = \frac{17.50 FH (D_v - D_p)}{5.626 D_p^2}$$

$$= \frac{3.111 FH (D_v - D_p)}{D_p^2}$$

For 0.997 inch spheres in 5.75 inch cylinder 17.75 inch high,

$$n = \frac{3.111 \times 0.9944 \times 17.75 \times 4.753}{0.997^2}$$

$$= 262.6$$

This corresponds to an occupancy factor of

$$\frac{262.6}{305.6} = 0.859$$

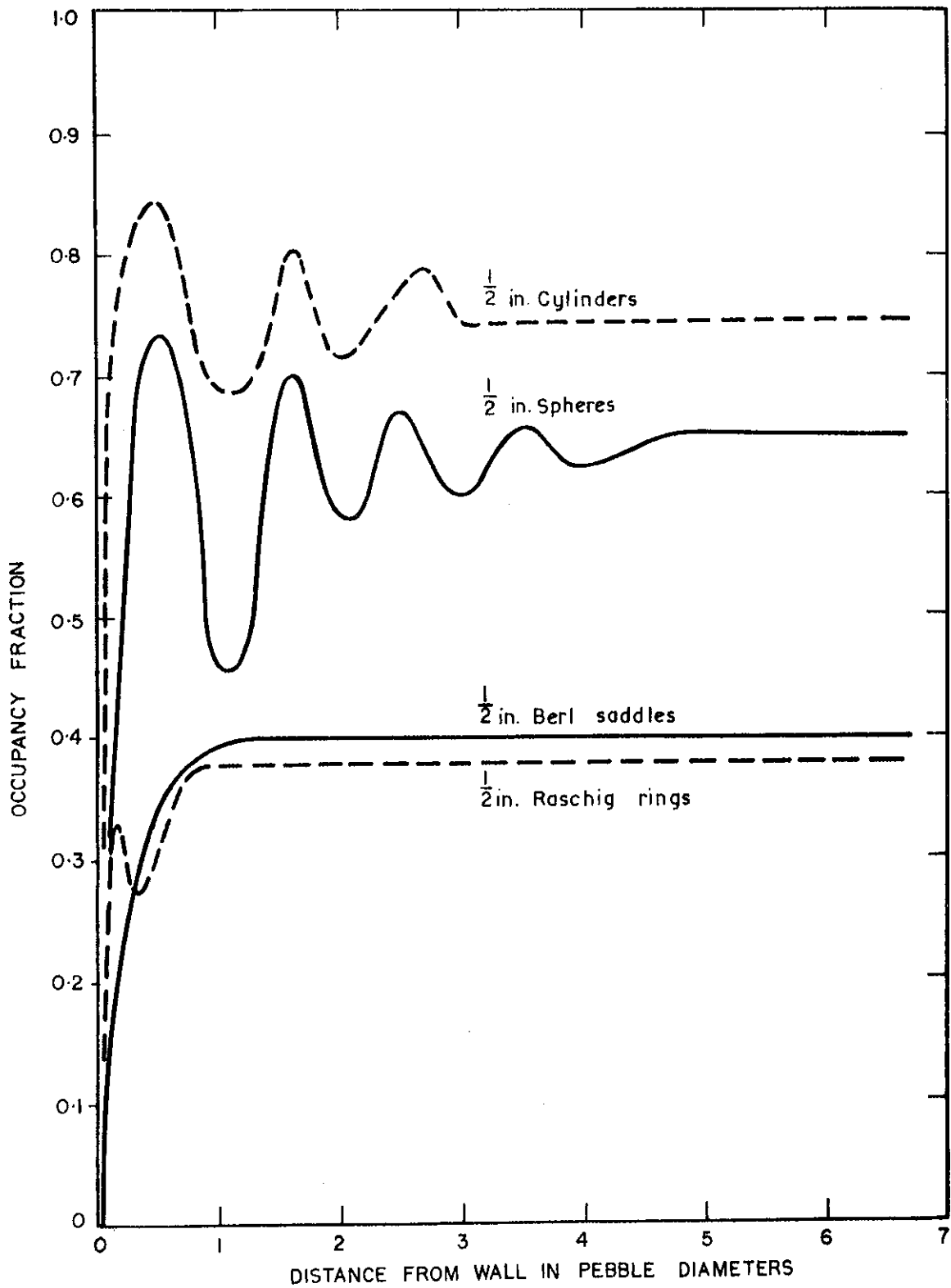
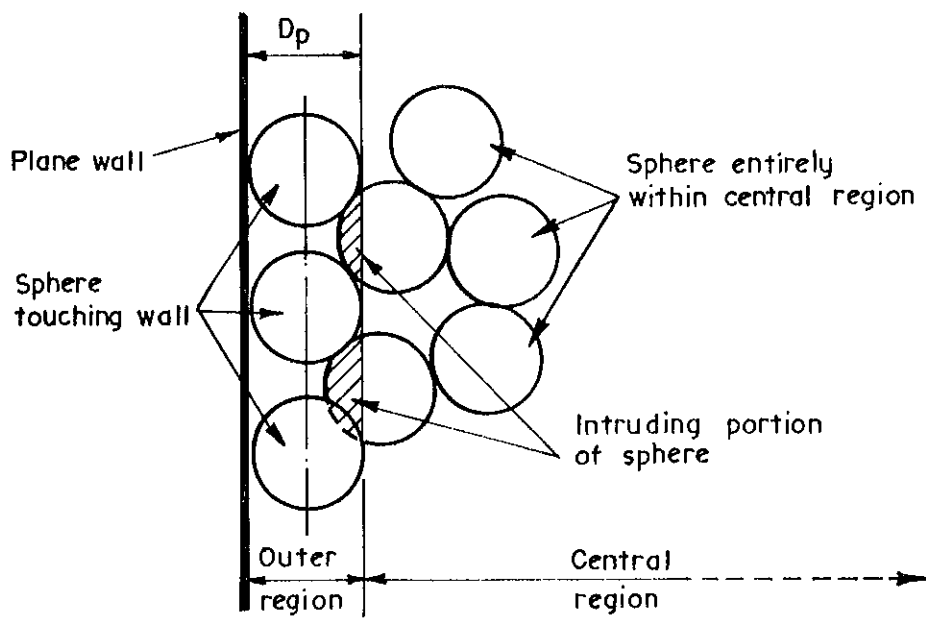
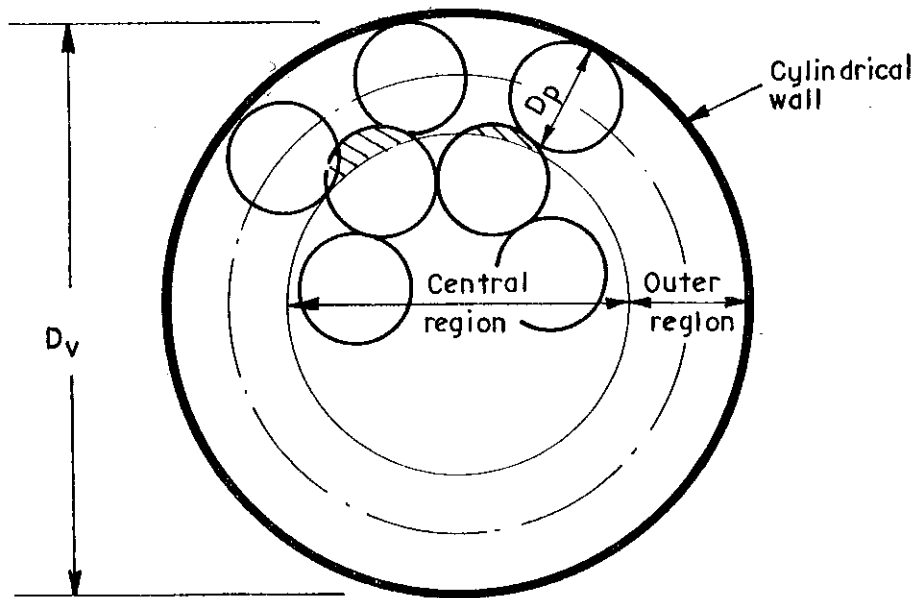


FIGURE 1. RADIAL VARIATIONS IN OCCUPANCY FRACTION OF RANDOM PACKINGS OF VARIOUS $\frac{1}{2}$ in. PEBBLES IN 6.7in. DIAMETER CYLINDRICAL VESSEL (After Roblee, Baird and Tierney 1958)



(a) SEMI - INFINITE VESSEL



(b) CYLINDRICAL VESSEL

FIGURE 2. PACKINGS OF EQUAL SPHERES IN SEMI-INFINITE AND CYLINDRICAL VESSELS (SHOWING TWO REGIONS AND THREE SPHERE CATEGORIES)

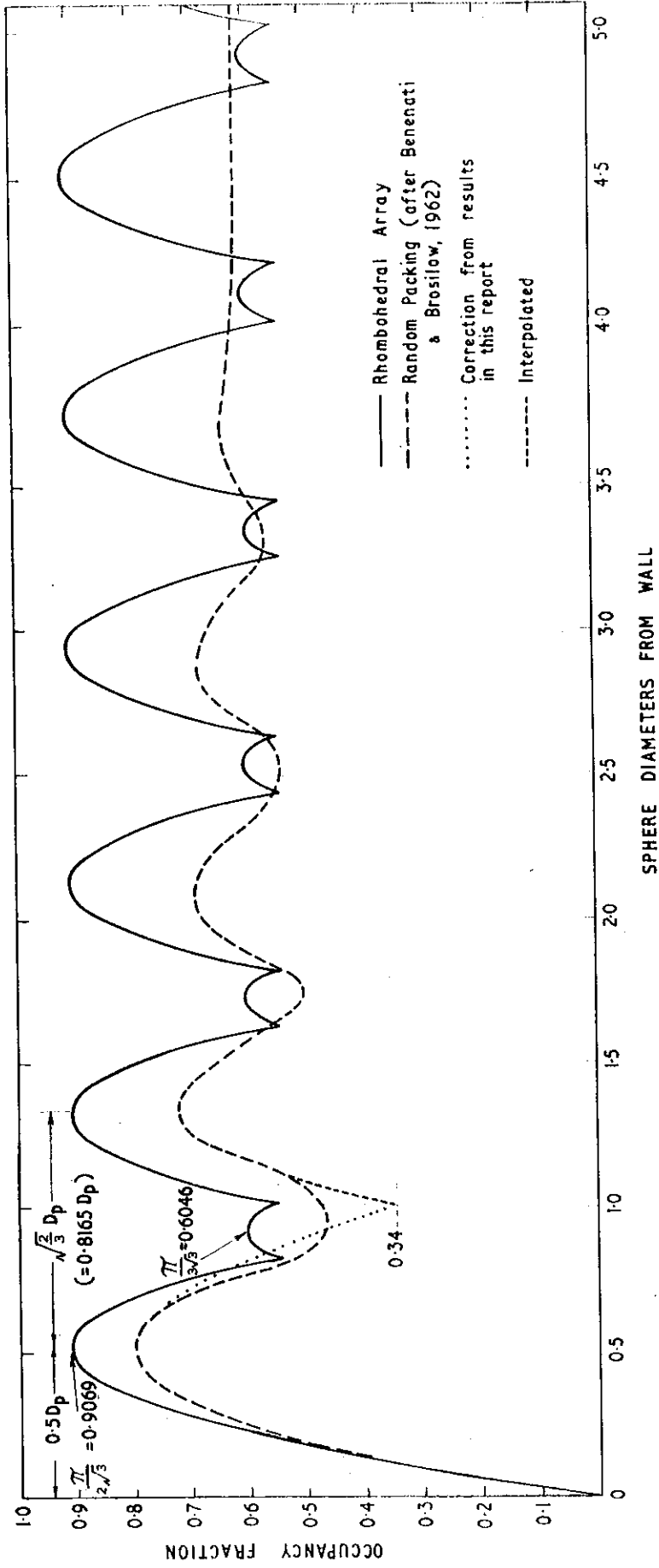


FIGURE 3. RADIAL VARIATIONS IN OCCUPANCY FRACTION OF PACKINGS OF EQUAL SPHERES IN SEMI-INFINITE VESSEL

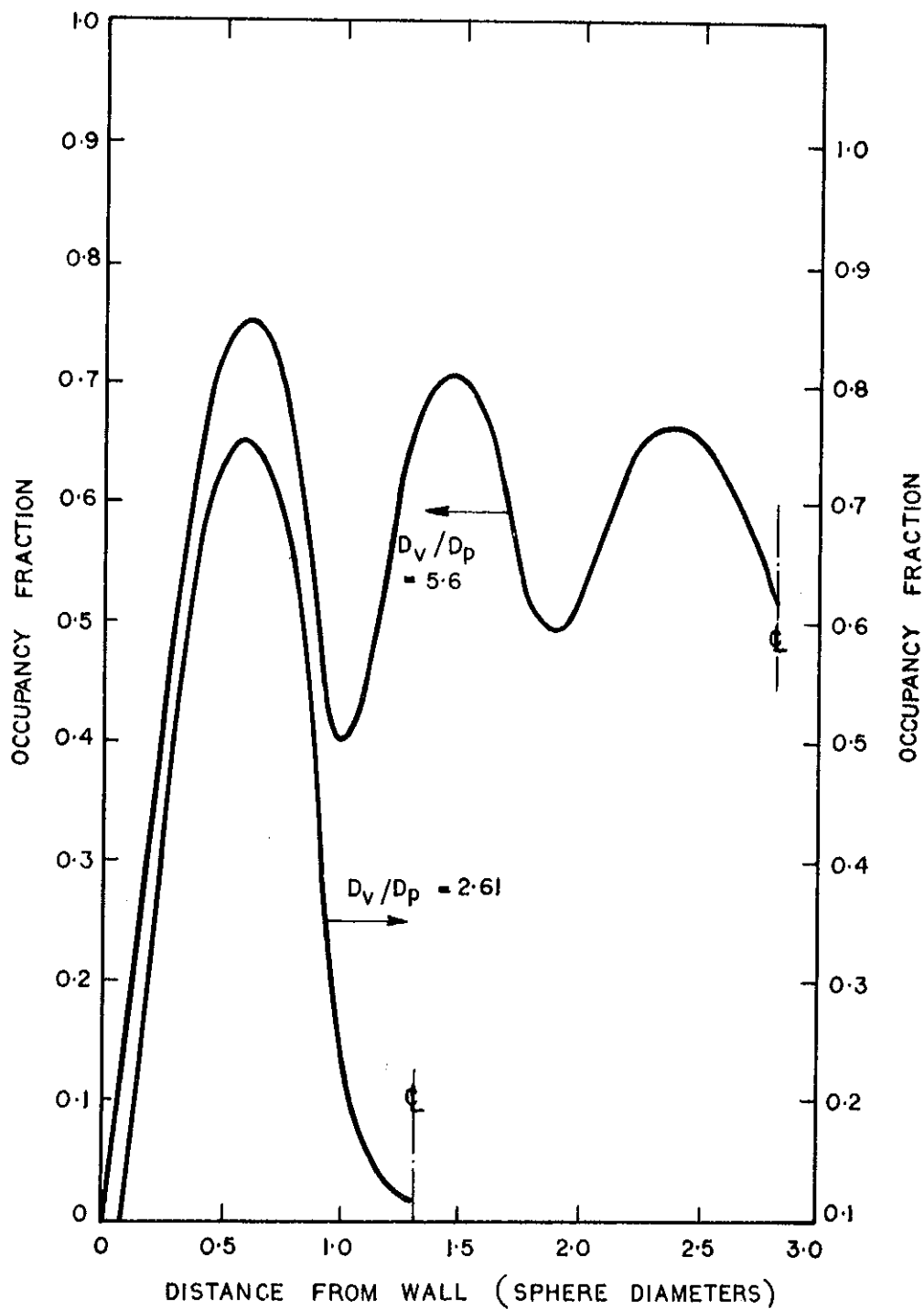


FIGURE 4. RADIAL VARIATIONS IN OCCUPANCY FRACTION OF PACKINGS OF EQUAL SPHERES IN CYLINDRICAL VESSEL

$D_v/D_p = 2.61$ and 5.6

(After Benenati and Brosilow 1962)

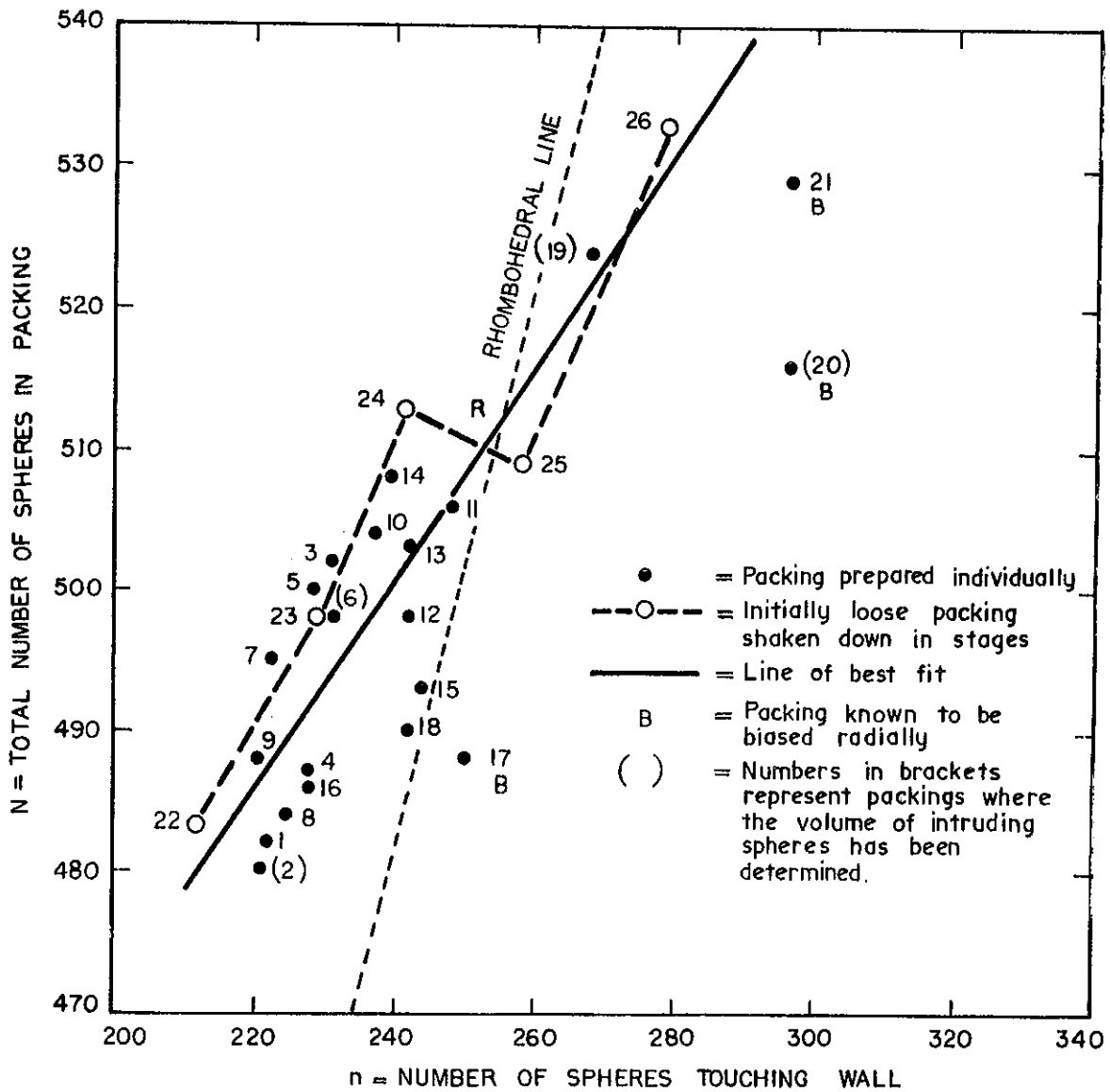


FIGURE 5. EXPERIMENTALLY DETERMINED VALUES OF N AND n

(1 in. spheres in 5.75 in. cylinder, standardised to 17.75 in. packing height)

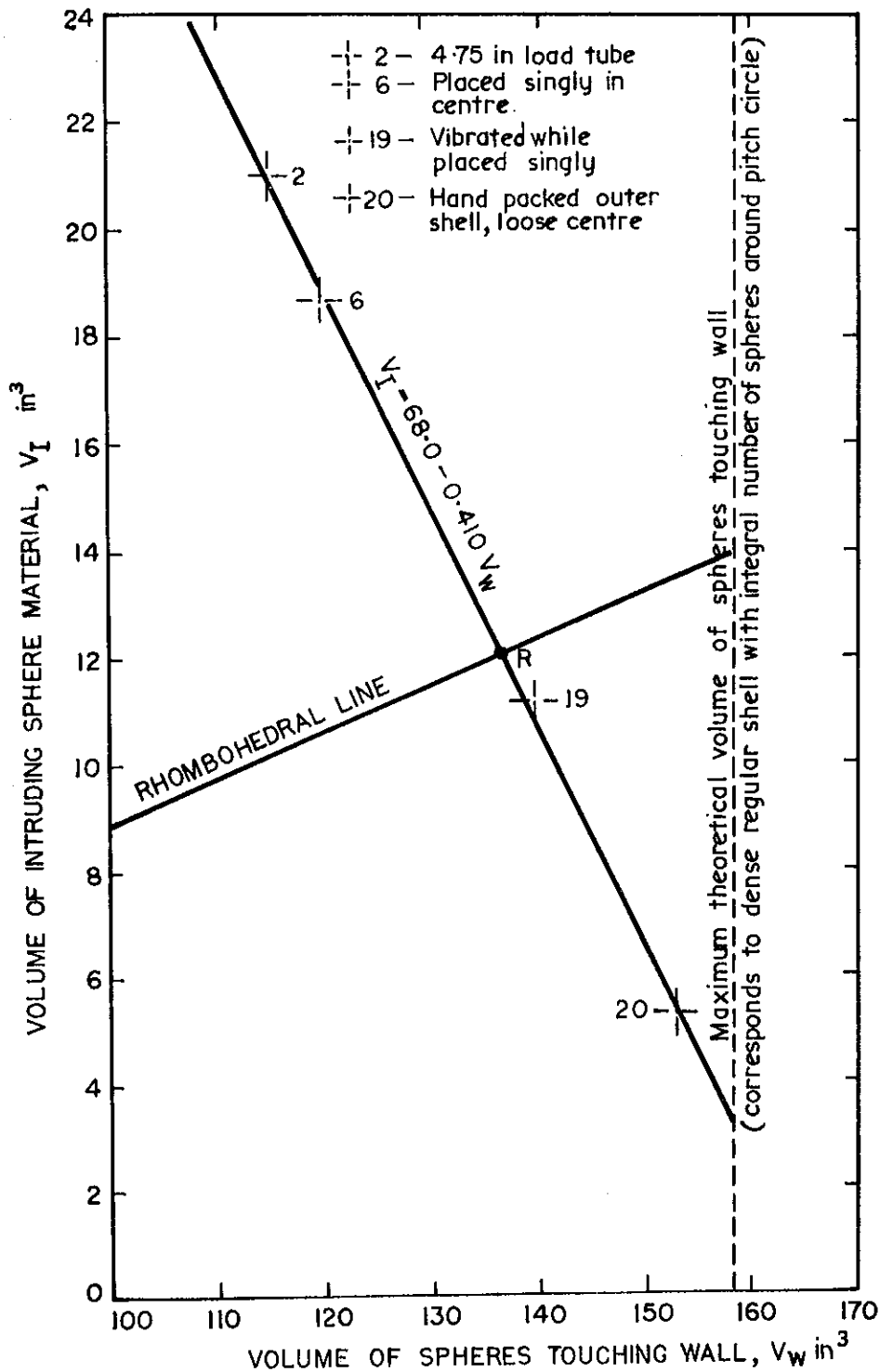


FIGURE 6. EXPERIMENTALLY DETERMINED RELATIONSHIP BETWEEN VOLUME OF INTRUDING MATERIAL AND VOLUME OF SPHERES TOUCHING WALL

(1 in. spheres in 5.75 in. cylinder, standardised to 17.75 in. packing height)