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A PRELIMINARY KINETIC STUDY OF A SODIUM–BERYLLIUM–URANIUM CIRCULATING FUEL REACTOR

by

J. J. Thompson

Summary

The initial response of a 1:100:2000 U–Na–Be Reactor, operating at a power density of 1200 cals/cc/sec, to a sudden change in the fuel concentration in the circulating carrier, has been calculated on the assumption of constant inlet temperature and no delayed neutrons.

The results confirm that the peak temperatures and power can be accurately calculated by ignoring the moderator temperature rise, and that the approach to the new steady state values is given by the solution of the linearized equations. The slow moderator heating implies that the departures from the steady state are still large when the carrier and fuel have completed one transit of the external circuit.

The assumption that the mean temperature is the arithmetic mean of inlet and outlet temperatures very nearly gives the true mean temperature peak but overestimates the maximum power and outlet temperature excursions by 15½% and 7% respectively.

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Notation

v	Flow velocity
L	Channel length
P	Power density
T	Temperature
Δ	Excess reactivity due to a change in fuel concentration
ρ	Density
Cp	Specific heat
S	$1/\rho Cp$
V/V'	Carrier/moderator volume ratio
p	Power excursion
$\mathcal{J}$	Temperature excursion
x	Fraction of power generated in moderator
k	Power transferred from moderator for unit volume of carrier per degree temperature difference
z	Distance along channel from inlet
t	Time
τ	Mean neutron lifetime
q	Complex variable occurring in the Laplace Transform solution of the linearized equations
q_i	i'th root of the secular equation for small disturbed motion
α	Temperature coefficient of reactivity
'	Denotes moderator

Suffix o Denotes initial steady state conditions

Suffix m Denotes mean w.r.t. z

Suffix ∞ Denotes new steady state values

Assumptions

(1) No delayed neutrons
(2) Constant power density along the channel
(3) Heat transfer coefficient and specific heats constant along the channel throughout the disturbance
(4) Density and temperature reactivity coefficients independent of position along the channel
(5) No axial heat conduction

Equations

Power equation:

$$\frac{dP}{dt} = \frac{P}{\tau}\left[\Delta - \alpha(T_m - T_{om}) - \alpha'(T_m' - T_{om}')\right]$$

Heat balance for carrier:

$$P + k(T' - T) = \rho C_p\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial z}\right)$$

Heat balance for moderator:

$$\chi P\frac{V}{V'} - k(T' - T)\frac{V}{V'} = \rho' C_p'\frac{dT'}{dt}$$

The initial steady state is defined by

$$P_o + k(T_o' - T_o) = \rho C_p v\frac{dT_o}{dz}$$

$$\chi P_o = k(T_o' - T_o)$$

i.e., $$T_o = T_{in} + \frac{SP_oL}{v}(1 + \chi)\frac{z}{L}$$

$$T_o' = T_{in} + \frac{\chi P_o}{k} + \frac{SP_oL}{v}(1 + \chi)\frac{z}{L}$$

In terms of the power and temperature excursions from the steady state,

$$\frac{dp}{dt} = \frac{P_o + p}{\tau}\left[\Delta - \alpha\mathcal{T}_m - \alpha'\mathcal{T}_m'\right] \tag{1}$$

$$S(p + k(\mathcal{T}' - \mathcal{T})) = \frac{\partial\mathcal{T}}{\partial t} + v\frac{\partial\mathcal{T}}{\partial z} \tag{2}$$

$$S'\frac{V}{V'}(\chi p - k(\mathcal{T}' - \mathcal{T})) = \frac{\partial\mathcal{T}'}{\partial t} \tag{3}$$

Approximate Solutions

By assuming $\mathcal{T}_{out} = 2\mathcal{T}_m$ the equations simplify to

$$\frac{dp}{dt} = \frac{P_o + p}{\tau}\left[\Delta - \alpha\mathcal{T}_m - \alpha'\mathcal{T}_m'\right] \tag{4}$$

$$\frac{d\mathcal{T}_m}{dt} = Sp - Sk(\mathcal{T}_m' - \mathcal{T}_m) - \frac{2v\mathcal{T}_m}{L} \tag{5}$$

$$\frac{d\mathcal{T}_m'}{dt} = \frac{S'V}{V'}(\chi p - k(\mathcal{T}_m' - \mathcal{T}_m)) \tag{6}$$

These equations have been solved numerically by step-by-step methods using the first two derivatives in a Taylor's Series Expansion with time intervals of .01 sec, for two cases.

(a) $\vartheta_m' = o, \; k = o$

(b) $x = .05, \; k = .5505$ cal/sec/cc/°C

Case (a) implies no heat transfer from moderator to carrier, while the value of k in case (b) was obtained by assuming a channel radius of 3 cms and a heat transfer coefficient of 6400 $BTU/ft^2/hr/^oF$, together with the following numerical values.

P_o = 1200 cal/cc/sec ($\approx$ 5 MW/litre)

τ = 2.5×10^{-4} sec

S = $3.3\,(cal/cc/^oC)^{-1}$

S' = $.85\,(cal/cc/^oC)^{-1}$

L = 80 cm

v = 800 cm/sec

V/V' = .287

α = 52×10^{-6} (a)

50×10^{-6} (b)

α' = 74×10^{-6}

For both (a) and (b)

$\Delta = .04t \quad o \leq t \leq .1$

$\Delta = .004 \quad t \geq .1$

corresponding to 2% change in concentration and a density coefficient of reactivity of 0.2

Case (a)

The results are shown in Figs. 1 and 2 from AAEC/ARC/P29. The new steady states are given by

$$(\vartheta_m)_\infty = \Delta_\infty/\alpha$$

$$(p)_\infty = 2v(\vartheta_m)_\infty / LS$$

The linearized equations give a quadratic whose roots imply a damped oscillatory approach to the new values. The damping factor 10 ($\equiv v/L$) and the half wavelength of .094 sec. calculated from these roots agree well with the results of the numerical calculation.

Case (b)

These results are shown in Figs. 1, 2 and 3 with Case (a) replotted on Figure 1 for comparison.

The new steady states are given by

$$(p)_\infty = \Delta_\infty / \left[\frac{\alpha' \chi}{k} + \frac{SL(1+\chi)}{2v}(\alpha + \alpha')\right]$$

$$(\mathcal{T}_m)_\infty = \frac{SL(1+\chi)}{2v}(p)_\infty$$

$$(\mathcal{T}_m')_\infty = \left(\frac{\chi}{k} + \frac{SL(1+\chi)}{2v}\right)(p)_\infty$$

The linearized equations give a cubic equation with one real and one pair of complex roots with the following values, calculated for several values of χ.

	q_1 sec^{-1}	$q_{2,3}$ sec^{-1}	½ wavelength sec
0	-.3278	$-10.81 \pm i\ 28.88$	.1083
.05	-.4354	$-10.75 \pm i\ 28.73$	.1044
.10	-.5684	$-10.69 \pm i\ 28.02$	.1121

The oscillation is damped fairly rapidly leaving a slow exponential approach to the new steady state, essentially due to the slower heating of the moderator. For Case (b) a moderator temperature disturbance falls to half its value in about 1.6 sec., indicating that the departures from the new steady state associated with the altered fuel concentration may still be large by the time the heated carrier returns to the inlet.

The discrepency between the maximum temperature excursions for (a) and (b) shown on Figure 1 is due to the fact that in (a) $k = o$, and the value of α is slightly different. The values of p at the time that $\mathcal{T}_m$ is a maximum are approximately the same, and the ratio of the maxima for (a) and (b) is very close to $(1 + SkL/2v)$ as would be expected if the difference is due to k.

It follows that in the approximate solution i.e. $\mathcal{T}_{out} = 2\,\mathcal{T}_m$, the initial response is calculated quite accurately by assuming constant moderator temperature and including the heat transferred from the carrier.

Exact Solution

To assess the accuracy of the assumption that $\mathcal{T}_{out} = 2\,\mathcal{T}_m$, Case (b) has been solved exactly assuming constant moderator temperature. The relevant equations are

$$\frac{dp}{dt} = \frac{P_o + p}{\tau}(\Delta - \alpha \mathcal{T}_m) \qquad (1a)$$

$$S(p - k\mathcal{T}) = \frac{\partial \mathcal{T}}{\partial t} + v \frac{\partial \mathcal{T}}{\partial z} \qquad (2a)$$

The solution of 2a in the general case is

$$\mathcal{T} = S \int_{t-\frac{z}{v}}^{t} p(\tau) \exp(-kS(t-\tau))\, d\tau + \mathcal{T}_{in}\left(t - \frac{z}{v}\right) \exp(-kSz/v)$$

For the problem in hand $\mathcal{T}_{in} = o$ and it follows that

$$\frac{d\mathcal{T}_{out}}{dt} = S(p - k\,\mathcal{T}_{out}) - Sp\left(t - \frac{L}{v}\right) \exp(-kSL/v)$$

$$\frac{d\mathcal{T}_m}{dt} = S(p - k\,\mathcal{T}_m) - \frac{v}{L}\,\mathcal{T}_{out}$$

Thus an identical step-by-step method can be used to determine $\mathcal{T}_{out}$ and p. The results are shown in Figs. 4 and 5, and compared with the solution of (b). The new steady state solutions - which have little significance as $\mathcal{T}'$ will not remain zero - are given by

$$(\mathcal{T}_m)_\infty = \Delta_\infty / \alpha$$

$$(\mathcal{T}_{out})_\infty = (p)_\infty (1 - \exp(-kSL/v))/k$$

$$(p)_\infty = k\,(\mathcal{T}_m)_\infty \Big/ \left(1 + \frac{v}{LSk}(\exp(-kSL/v) - 1)\right)$$

Both the maximum outlet temperature and the maximum power are overestimated by the approximate analysis. The linearized equations lead to an equation which has no real roots, so the new steady state is approached via a number of superimposed damped oscillations. These complex roots have not been evaluated as the example becomes artificial with increasing t, so the question of stability cannot be answered. If the oscillations are damped, the ratio of $\mathcal{T}_{out}/\mathcal{T}_m$ approaches a value close to 2, and it is inferred that in the exact solution of the problem in which the moderator temperature rise is included, there will occur a slow exponential approach to the new values practically identical with that given by the approximate solution. For future reference, the secular equation for this problem is as follows:

$$q + \frac{P_o \alpha'}{\tau} \cdot \frac{S' \frac{V}{V'} x}{q + kS' \frac{V}{V'}}$$

$$+ \frac{SP_o}{q\tau}\left(\alpha + \frac{\alpha' k S' \frac{V}{V'}}{q + kS'\frac{V}{V'}}\right) \frac{q + kS'\frac{V}{V'}(1 + x)}{q + k(S + S'\frac{V}{V'})} \left[1 + \frac{\frac{v}{Lq}\left(q + kS'\frac{V}{V'}\right)}{q + k(S + S'\frac{V}{V'})} \left\{ \exp\left(-\frac{qL}{v} \cdot \frac{q + k(S + S'\frac{V}{V'})}{q + kS'\frac{V}{V'}}\right) - 1 \right\}\right] = o$$

Conclusion

A similar investigation of the response following varying inlet temperature should be carried out to assess the reliability of the methods to be used at a later date in more detailed investigations of kinetics. In particular it should be determined if the approximate solutions lead to conservative results by overestimating the maximum excursions.

Acknowledgment

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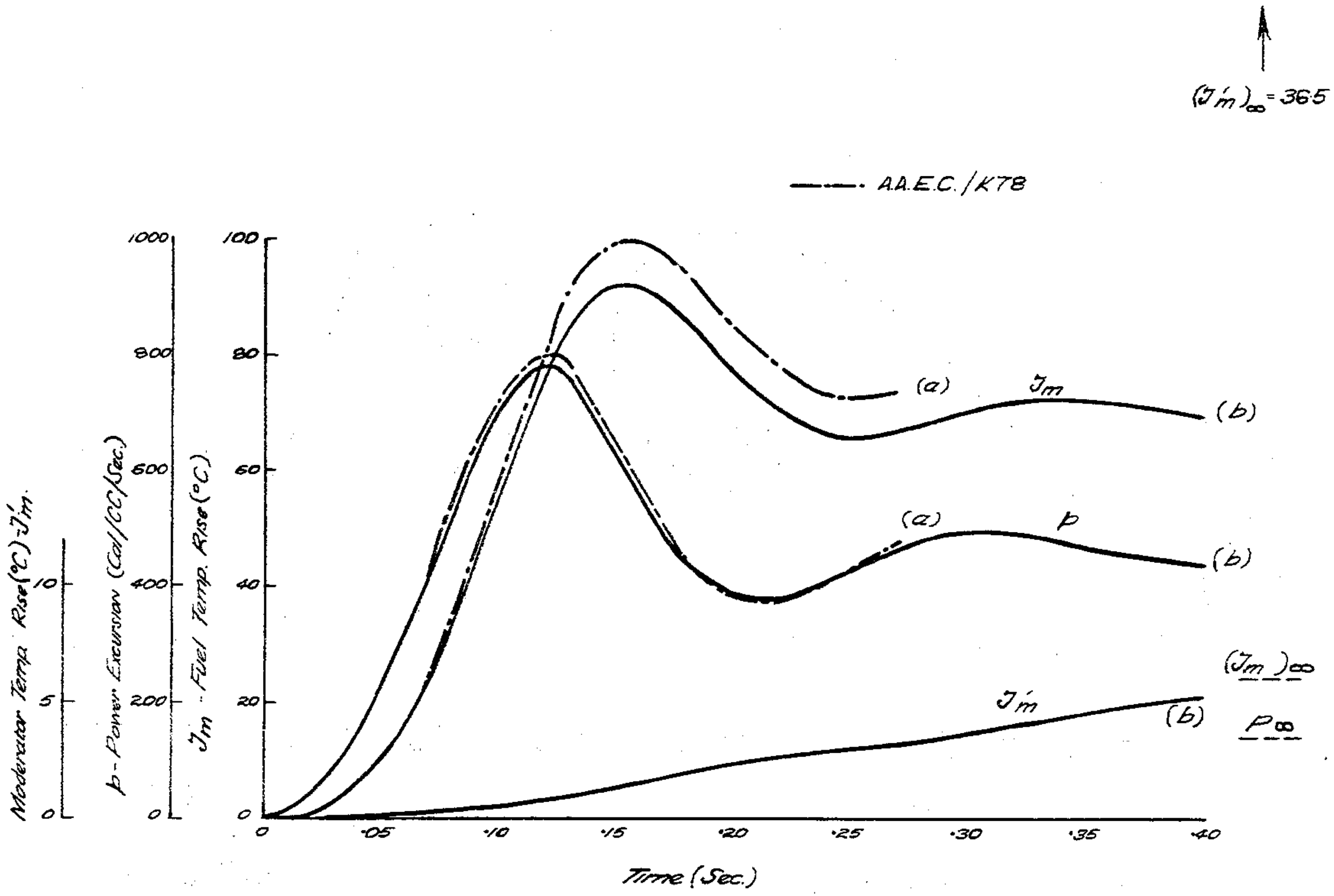

$(J'_m)_\infty = 36{\cdot}5$
AA.E.C./K78
Moderator Temp. Rise (°C) - J'_m.
p - Power Excursion (Cal/CC/Sec.)
J_m - Fuel Temp. Rise (°C)
(a)
J_m
(b)
p
$(J_m)_\infty$
J'_m
P_∞
Time (Sec.)

FIGURE. 1.

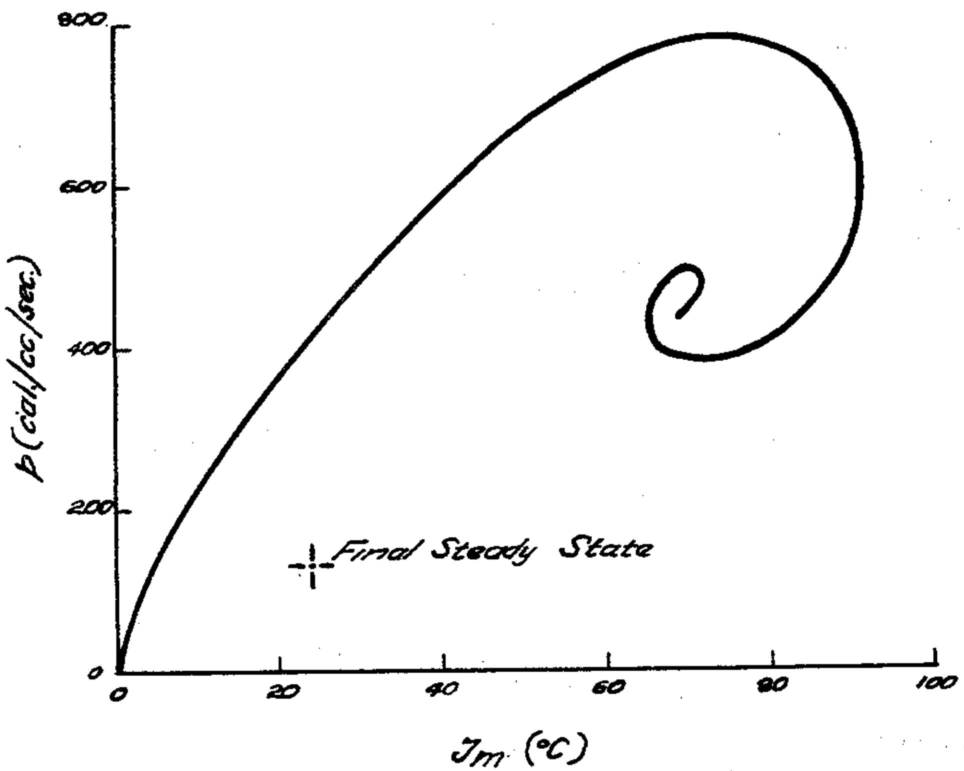

FIGURE 2

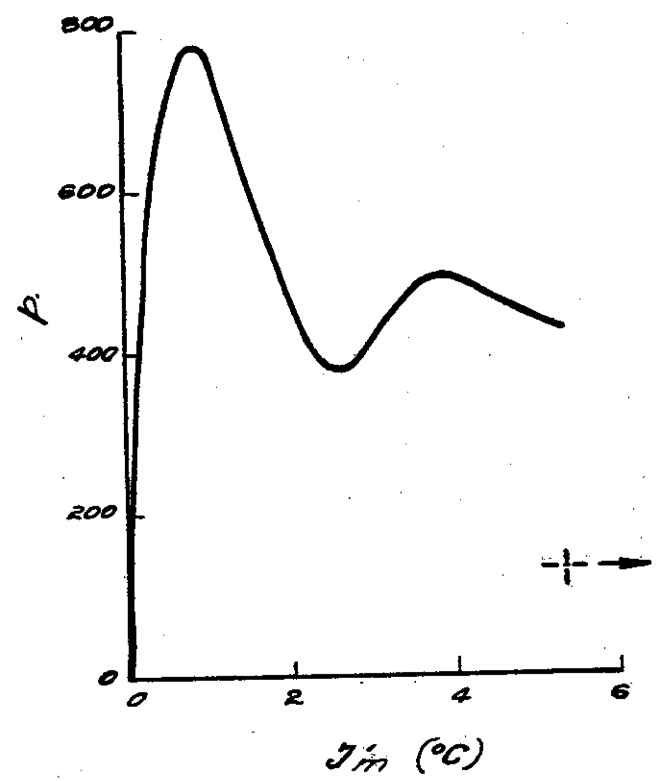

FIGURE.3

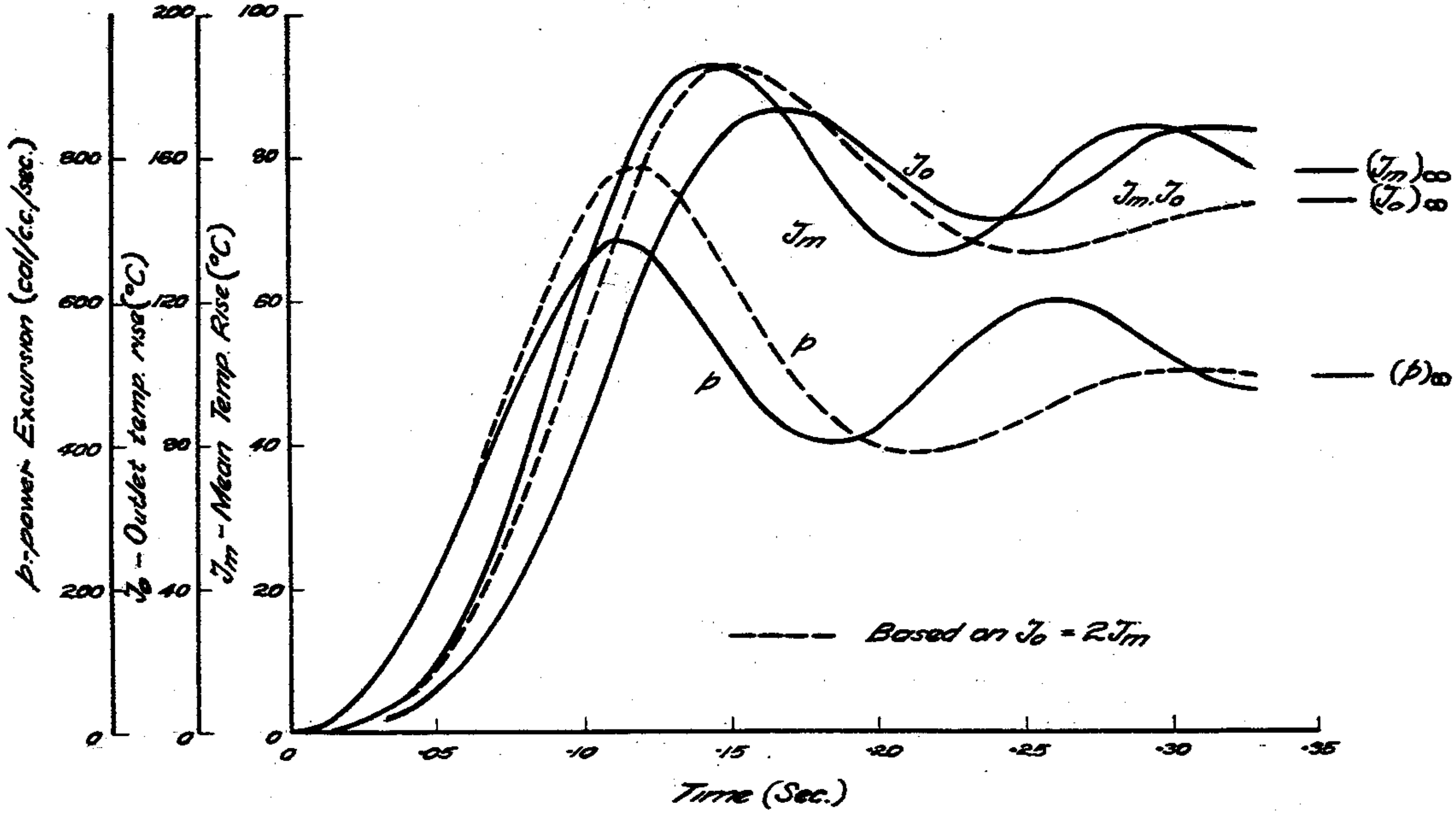
p-power Excursion (cal/c.c./sec.)
J_o - Outlet temp. rise (°C)
J_m - Mean Temp. Rise (°C)
0
200
400
600
800
0
40
80
120
160
200
0
20
40
60
80
100
0
·05
·10
·15
·20
·25
·30
·35
Time (Sec.)
J_o
J_m
$J_m . J_o$
p
p
$(J_m)_\infty$
$(J_o)_\infty$
$(p)_\infty$
Based on $J_o = 2J_m$

FIGURE 4

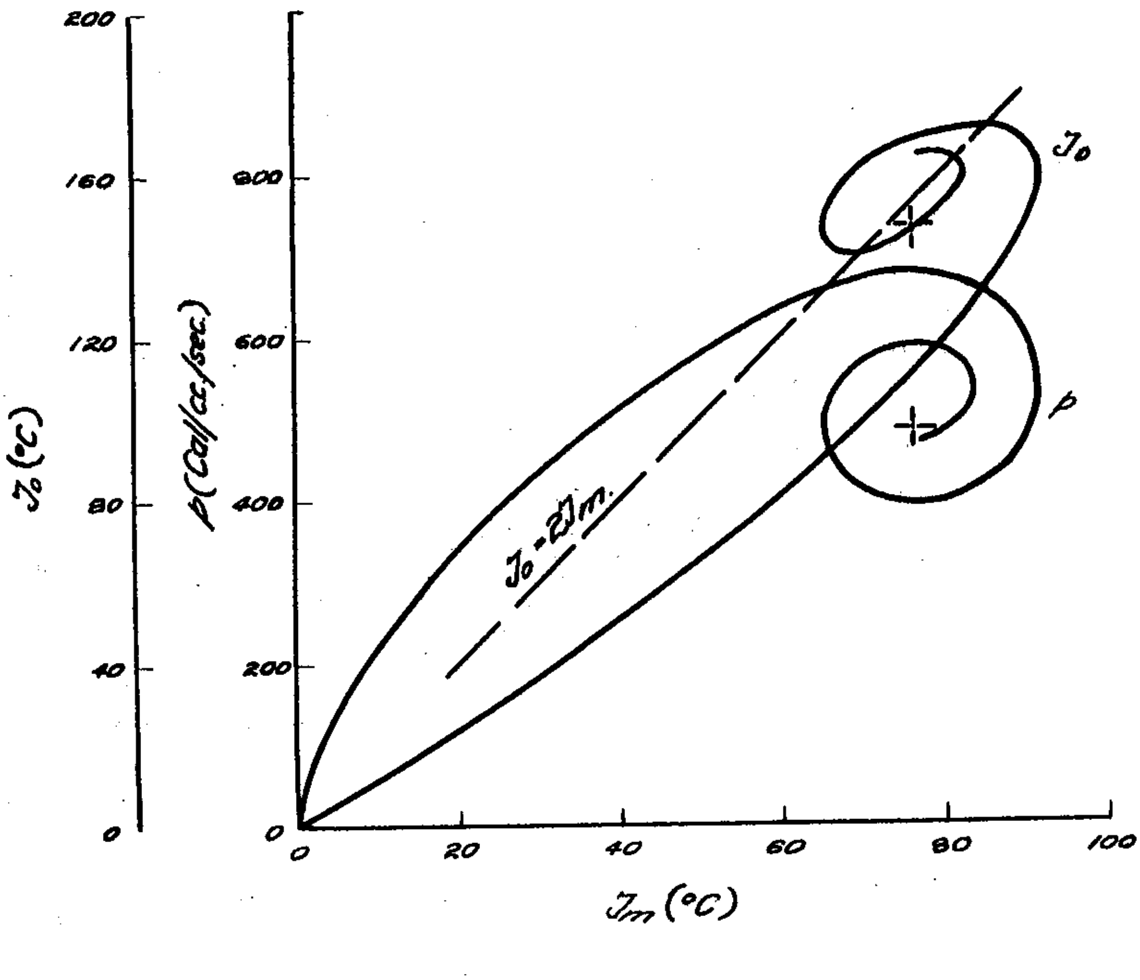
200
160
120
80
40
0
800
600
400
200
0
0
20
40
60
80
100
T_o (°C)
p(Cal/cc/sec.)
T_m (°C)
$T_o = 2T_m$
T_o
p

FIGURE 5.