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RESEARCH ESTABLISHMENT

LUCAS HEIGHTS

A BASIS FOR THE COMPUTATION OF THE ENERGY DEPENDENCE OF THE NEUTRON FLUX AND SLOWING DOWN DENSITY IN A BARE REACTOR

by

B. R. LAWRENCE

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A BASIS FOR THE COMPUTATION OF THE ENERGY DEPENDENCE OF THE NEUTRON FLUX AND SLOWING DOWN DENSITY IN A BARE REACTOR

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ABSTRACT

A method is described for the computation of the energy dependence of the neutron flux and slowing down density in a bare reactor to form the basis of a 7090 Fortran compute program for survey studies of power reactors. Allowance is made for resonance absorption and resonance fission, neutron thermalisation, fast neutron reactions including (n, 2n) and (n, n'), and anisotropic elastic scattering.

CONTENTS

INTRODUCTION

For survey studies of initial conversion ratios and sizes of power reactors whose neutron fl spectrum may be anything from thermal to fast it is desirable to have a method of computation which inherently accounts for the effect of reactor composition on the neutron flux spectrum and slowing c density and which avoids the need for sets of group averaged cross sections and transfer cross sect for different reactor compositions and temperatures.

To this end some effort has been directed towards the development of a method of spectrum calculation based essentially on the use of the heavy gas thermalisation model and the synthetic kernel slowing down model. Initial work along these lines for bare, uniform, homogeneous reactors has been described by Richardson (1960). Further work, described in this report, has led to numeric stable solutions and allows for resonance absorption and resonance fission in individual resonance Provision is made for the computation of the Be(n,2n) reaction rate for studies of reactors containing beryllium.

The method described forms the basis of a 7090 Fortran computer program, called SKATE, f the computation of neutron flux spectrum, slowing down density, initial conversion ratios and critic sizes of bare, uniform, homogeneous reactors. Details of the computer program will be given in a fo coming report.

The aim of future work will be to extend the method to cope with reflected reactors.

GENERAL PROCEDURE

The approach makes use of the neutron balance equation

$$-(\Sigma_a + DB^2)\,\phi(E) + \frac{dq(E)}{dE} + S(E) = 0 \quad ,$$

together with the relation between flux and slowing down density,

$$q = \bar{\xi}\Sigma_S\left[(E-T)\,\phi(E) + ET\,\frac{d\phi(E)}{dE}\right] + \bar{\epsilon}\,\bar{\xi}\left[(E-T)\,\{(\Sigma_a + DB^2)\phi(E) - S(E)\}\right.$$
$$\left. + ET\,\frac{d}{dE}\,\{(\Sigma_a + DB^2)\,\phi(E) - S(E)\}\right] \quad .$$

Equation 2, proposed by Thompson (1962), is based on the heavy gas thermalisation model and for $\frac{T}{E} << 1$ becomes the general synthetic kernel relation between ϕ and q for neutron slowing down:

$$q = \bar{\xi}\,[\Sigma_S + \bar{\epsilon}\,(\Sigma_a + DB^2)]\,E\,\phi(E) - \bar{\epsilon}\,\bar{\xi}\,E\,S(E) \quad .$$

Also,

$$S(E) = \frac{1}{k}\left(\chi(E)\int_0^\infty \nu\Sigma_f\,\phi(E)\,dE + 2\,\Omega(E)\int_0^\infty \Sigma_{n\,2n}\,\phi(E)\,dE\right)$$
$$+ \omega(E)\int_0^\infty \Sigma_{in}\,\phi(E)\,dE \quad .$$

In writing (4) allowance is made for emission of neutrons from (n,2n) and inelastic scattering reaction and the emission spectra χ, Ω, and ω are assumed to be independent of the incident neutron energy and identical for all nuclei undergoing that type of reaction. Ω is a composite emission spectrum fo both (n,2n) emitted neutrons, so that

$$\int_0^\infty 2\,\Omega(E)\,dE = \int_0^\infty \left(\Omega_1(E) + \Omega_2(E)\right)dE \quad .$$

Also, $\int_0^\infty \Omega_1(E)\,dE = \int_0^\infty \Omega_2(E)\,dE = \int_0^\infty \Omega(E)\,dE = \int_0^\infty \chi(E)\,dE = \int_0^\infty \omega(E)\,dE = 1$.

In (1) and (2), Σ_a includes Σ_{in}, $\Sigma_{n\,2n}$ and $\Sigma_{n\alpha}$.

For a specified reactor composition and k, (1) and (2) are solved numerically for B^2, ϕ For this purpose it is convenient to work in terms of lethargy and to obtain ϕ and q normalised fission neutron produced. The relations of interest thus become:

$$-(\Sigma_a + DB^2)\,\phi(u) - \frac{dq(u)}{du} + S(u) = 0 \ ,$$

$$\frac{dq(u)}{du} - \left(\frac{\frac{T}{E}}{1-2\frac{T}{E}}\right)\frac{d^2q(u)}{du^2} = \frac{\Sigma_S}{\bar{\epsilon}}\left[\phi(u) - \frac{\frac{T}{E}}{1-2\frac{T}{E}}\,\frac{d\phi(u)}{du}\right] - \frac{q(u)}{\bar{\epsilon}\bar{\xi}\left(1-2\frac{T}{E}\right)} \ ,$$

$$S(u) = \frac{1}{k}\left(\chi(u) + 2\,\Omega(u)\,\beta\right) + \omega(u)\,\alpha \ ,$$

$$\alpha = \int_0^\infty \Sigma_{in}\ \phi(u)\,du \ ,$$

$$\beta = \int_0^\infty \Sigma_{n\,2n}\ \phi(u)\,du \ ,$$

where, in writing (6), use has been made of (5). Starting with initial estimates of B^2 and α and ϕ and q are obtained by solution of (5), (6), and (7). $\int_0^\infty \nu\Sigma_f\ \phi du$ is then calculated and if relation

$$\left|\int_0^\infty \nu\Sigma_f\ \phi\,du - 1\right| < c \text{ , a small positive constant,}$$

is not satisfied, better estimates of α and β calculated from (8) and (9) are used with an impr value of B^2 to calculate ϕ and q again. The procedure is repeated until (10) is satisfied.

The energy range of interest from u = 0 (10 MeV) to u = 23 is divided into a high en region $u < u_c$ where $\frac{T}{E} << 1$ and $S \neq 0$ and a low energy region $u > u_c$ where S = 0 and E is the order of T. u_c is chosen such that S = 0 and $\frac{T}{E} << 1$ at u_c.

HIGH ENERGY REGION

The neutron flux and slowing down density in the high energy region are obtained by numerical solution of (5) and

$$q = \bar{\xi}\left[\Sigma_S + \bar{\epsilon}\,(\Sigma_a + DB^2)\right]\phi - \bar{\epsilon}\,\bar{\xi}\,S \ ,$$

which is obtained from (6) by putting $\frac{T}{E} = 0$ and rewriting in the form of (2).

Using (11) to eliminate ϕ in (5),

$$\frac{dq}{du} + \frac{\Sigma_a + DB^2}{\bar{\xi}\left[\Sigma_S + \bar{\epsilon}\,(\Sigma_a + DB^2)\right]}\,q - \frac{\Sigma_S\,S}{\Sigma_S + \bar{\epsilon}\,(\Sigma_a + DB^2)} = 0 \ .$$

Integrating (12) between u_n and u_{n+1} where $u_{n+1} - u_n = \Delta_n$ is a small lethargy interval, an assuming the integrands to be linearly varying over this interval,

$$q_{n+1} - q_n + (G_{n+1}\,q_{n+1} + G_n\,q_n)\,\frac{\Delta_n}{2} - (H_n + H_{n+1})\,\frac{\Delta_n}{2} = 0 \ ,$$

where $$G_n = \left. \frac{\Sigma_a + DB^2}{\bar{\xi}\,[\Sigma_S + \bar{\epsilon}\,(\Sigma_a + DB^2)]} \right|_n$$

and $$H_n = \left. \frac{\Sigma_S\, S}{\Sigma_S + \bar{\epsilon}\,(\Sigma_a + DB^2)} \right|_n .$$

Hence

$$q_{n+1} = \frac{\frac{2}{\Delta_n} - G_n}{\frac{2}{\Delta_n} + G_{n+1}} \cdot q_n + \frac{H_n + H_{n+1}}{\frac{2}{\Delta_n} + G_{n+1}} .$$

Also, from (11),

$$\phi_n = \frac{q_n + \bar{\epsilon}_n \bar{\xi}_n S_n}{\bar{\xi}_n\,[\Sigma_S + \bar{\epsilon}\,(\Sigma_a + DB^2)]_n} .$$

Starting at $u = 0$ with $q = 0$ and proceeding in the direction of increasing u, (14) is used to obtain q_{n+1} from q_n, terminating at $u = u_c$. Whenever a resolved resonance is encountered this procedure is temporarily interrupted to calculate the change in slowing down density in passing through the resonance. At each step ϕ_n is obtained from q_n using (15).

LOW ENERGY REGION

For $S = 0$, that is, $u \geqslant u_c$, Equation 6 and

$$\frac{dq}{du} = -\,(\Sigma_a + DB^2)\,\phi$$

are the equations of interest. Rearranging (6) and using (16) to eliminate $\frac{dq}{du}$,

$$\frac{d\phi}{du} = \left(\frac{E}{T} - 2\right)\left[1 + \frac{\bar{\epsilon}}{\Sigma_S}\,(\Sigma_a + DB^2)\right]\phi + \frac{\bar{\epsilon}}{\Sigma_S}\,\frac{d^2q}{du^2} - \frac{q\,E}{\bar{\xi}\,\Sigma_S\,T} .$$

Integrating (17) between u_n and u_{n+1} and again assuming integrands to be linearly varying,

$$\phi_{n+1} - \phi_n = \left\{\left(\frac{E}{T} - 2\right)_{n+1}\left[1 + \frac{\bar{\epsilon}}{\Sigma_S}\,(\Sigma_a + DB^2)\right]_{n+1}\phi_{n+1} + \left(\frac{E}{T} - 2\right)_n\left[1 + \frac{\bar{\epsilon}}{\Sigma_S}\,(\Sigma_a + DB^2)\right]_n \phi_n\right\}\frac{\Delta_n}{2}$$

$$+ \left\{\left(\frac{\bar{\epsilon}}{\Sigma_S}\right)_{n+1} \left.\frac{d^2q}{du^2}\right|_{n+1} + \left(\frac{\bar{\epsilon}}{\Sigma_S}\right)_n \left.\frac{d^2q}{du^2}\right|_n\right\}\frac{\Delta_n}{2} - \left\{\left(\frac{q\,E}{\bar{\xi}\Sigma_S}\right)_{n+1} + \left(\frac{q\,E}{\bar{\xi}\Sigma_S}\right)_n\right\}\frac{\Delta_n}{2\,T} .$$

Using Taylor series approximations together with (16), we can write

$$\left.\frac{d^2q}{du^2}\right|_n = \frac{2}{\Delta_n^2}\,\{\,q_{n+1} - q_n + \Delta_n\,(\Sigma_a + DB^2)_n\,\phi_n\,\} ,$$

$$\left.\frac{d^2q}{du^2}\right|_{n+1} = \frac{2}{\Delta_n^2}\,\{\,q_n - q_{n+1} - \Delta_n\,(\Sigma_a + DB^2)_{n+1}\phi_{n+1}\,\} .$$

Substituting (19) and (20) into (18),

$$A_{n,n+1}\ \phi_{n+1} - B_n\ \phi_n = C_{n,n+1}\ q_n + D_{n,n+1}\ q_{n+1}\ ,$$

where

$$A_{n,n+1} = 1 - \frac{\Delta_n}{2}\left(\frac{E}{T}-2\right)_{n+1}\left[1+\frac{\bar{\epsilon}}{\Sigma_S}(\Sigma_a + DB^2)\right]_{n+1} + \left(\frac{\bar{\epsilon}}{\Sigma_S}\right)_{n+1}(\Sigma_a + DB^2)_{n+1}\ ,$$

$$B_n = 1 + \frac{\Delta_n}{2}\left(\frac{E}{T}-2\right)_n\left[1+\frac{\bar{\epsilon}}{\Sigma_S}(\Sigma_a + DB^2)\right]_n + \left(\frac{\bar{\epsilon}}{\Sigma_S}\right)_n(\Sigma_a + DB^2)_n\ ,$$

$$C_{n,n+1} = \left(\frac{\bar{\epsilon}}{\Sigma_S}\right)_{n+1}\cdot\frac{1}{\Delta_n} - \left(\frac{\bar{\epsilon}}{\Sigma_S}\right)_n\cdot\frac{1}{\Delta_n} - \frac{\Delta_n}{2T}\left(\frac{E}{\xi\Sigma_S}\right)_n\ ,$$

$$D_{n,n+1} = \left(\frac{\bar{\epsilon}}{\Sigma_S}\right)_n\cdot\frac{1}{\Delta_n} - \left(\frac{\bar{\epsilon}}{\Sigma_S}\right)_{n+1}\cdot\frac{1}{\Delta_n} - \frac{\Delta_n}{2T}\left(\frac{E}{\xi\Sigma_S}\right)_{n+1}\ .$$

Integration of (16) leads to

$$q_n = q_{n+1} + P_{n,n+1}\ \phi_{n+1} + Q_n\ \phi_n\ ,$$

where $P_{n,n+1} = (\Sigma_a + DB^2)_{n+1}\cdot\dfrac{\Delta_n}{2}$,

$$Q_n = (\Sigma_a + DB^2)_n\cdot\frac{\Delta_n}{2}\ .$$

Eliminating q_n in (21) using (22),

$$\phi_n = \frac{A_{n,n+1} - C_{n,n+1}\cdot P_{n,n+1}}{B_n + C_{n,n+1}\cdot Q_n}\cdot\phi_{n+1} - \frac{C_{n,n+1} + D_{n,n+1}}{B_n + C_{n,n+1}\cdot Q_n}\cdot q_{n+1}\ .$$

Knowing ϕ_{n+1} and q_{n+1}, (23) can be used to calculate ϕ_n, and q_n may be obtained fr
Starting at $u = 23$ with $q(23) = 0$ and an arbitrary value a for $\phi(23)$ and moving in the direction
decreasing u, ϕ and q may thus be determined at each point, terminating at u_c. Again, whene
resolved resonance is encountered this procedure is temporarily interrupted to calculate the cha
slowing down density in passing through the resonance.

The other boundary condition for the low energy region is made

$$q_h(u_c) = q_l(u_c)\ ,$$

to achieve continuity of slowing down density at u_c. Subscripts h and l distinguish values calculated from the high and low energy treatments respectively. To satisfy (24) the low energy region solutions $q_l(u)$ and $\phi_l(u)$ are obtained from

$$q_l(u) = b\ q'_l(u) \qquad u \geq u_c\ ,$$

$$\phi_l(u) = b\ \phi'_l(u) \qquad u \geq u_c\ ,$$

where the dash denotes solutions obtained in the low energy region with $\phi(23) = a$. Since (16
(17) are homogeneous, q and ϕ as given by (25) and (26) are solutions of (16) and (17). The co
b is determined by

In practice $S = 0$ for $u \geq u_c$ but $\frac{T}{E} \neq 0$ at u_c so that

$$\phi_h(u_c) \approx \phi_l(u_c) \quad .$$

The magnitude of this discrepancy depends on the values of T and u_c and can be made small.

B^2 ITERATION

A simple regular falsi scheme may be used to obtain B^2. Two trial values of B^2 may be us initially to calculate two values of each of $f_R = \int_0^\infty \nu \Sigma_f \phi(u)\, du$, α and β, and B^2 may thereaf be changed in accordance with

$$B^{2(n+2)} = \frac{B^{2(n)}\left(f_R^{(n+1)} - 1\right) + B^{2(n+1)}\left(1 - f_R^{(n)}\right)}{f_R^{(n+1)} - f_R^{(n)}} \quad ,$$

until (10) is satisfied. It is desirable also to require that

$$\left| \alpha^{(n+2)} - \alpha^{(n+1)} \right| < c \quad ,$$

and

$$\left| \beta^{(n+2)} - \beta^{(n+1)} \right| < c \quad ,$$

before terminating the procedure.

TREATMENT OF RESONANCES

In the resonance region the cross sections are assumed to be;

$$\Sigma_S(E) = \sum_j N^j \left(\sigma_h^j + \sum_i \frac{\Gamma_{n_i}^j}{\Gamma_i^j} \sigma_{o_i}^j \psi_i^j \right) \quad ;$$

$$\Sigma_a(E) = \sum_j N^j \left(\sigma_{a_b}^j(E) + \sum_i \frac{\Gamma_{\gamma_i}^j + \Gamma_{f_i}^j}{\Gamma_i^j} \sigma_{o_i}^j \psi_i^j \right) \quad ,$$

$$\Sigma_f(E) = \sum_j N^j \left(\sigma_{f_b}^j(E) + \sum_i \frac{\Gamma_{f_i}^j}{\Gamma_i^j} \sigma_{o_i}^j \psi_i^j \right) \quad .$$

This representation results from resonances assumed to have symmetric single level Breit Wigner Doppler broadened contours superimposed on a background cross section. Interference scattering is neglected. The background cross sections $\sigma_{a_b}(E)$ and $\sigma_{f_b}(E)$ consist of the tails of all resonances, including negative energy resonances, as well as all of the extremely wide Pu 239 0.3 eV and U 233 1eV resonances, and are assumed to be temperature independent. In the unresolved region the same representation may be used with resonance parameters obtained from a separate statistical analysis.

Starting at $u = 0$ the solution for ϕ and q is commenced using the high energy region treatment until the lethargy corresponding to the E_o of the first resonance is reached. Knowing $\bar{\xi}$, Σ_{S_b} $\left(= \sum_j N^j \sigma_h^j\right)$ and Σ_{a_b} at E_o and the resonance parameters for that resonance, its resona escape probability p may be calculated and the change in slowing down density Δq due to absorp in that resonance may be obtained from

$$\Delta q_i = (1 - p_i)\, q_{i-} \quad ,$$

This is allowed for by a step function decrease in q at the resonance energy in question

$$q_{i+} = q_{i-} - \Delta q_i \quad .$$

The change in ϕ is also allowed for by a step function decrease at the resonance energy given

$$\frac{\phi_{i+}}{q_{i+}} = \frac{\phi_{i-}}{q_{i-}}$$

Between the resonance just treated and the next, the high energy region treatment is again appli with $\Sigma_a(E)$, $\Sigma_S(E)$, and $\Sigma_f(E)$ given by $\Sigma_{a_b}(E)$, $\Sigma_{S_b}(E)$, and $\Sigma_{f_b}(E)$.

In the low energy region the same procedure is used, care being taken to compute

$$\Delta q_i = \left(1 - \frac{1}{P_i}\right) q_{i+} \quad ,$$

$$q_{i-} = q_{i+} + \Delta q_i \quad ,$$

as the direction of solution is reversed.

When computing the resonance escape probability for an individual resonance, the reso is assumed sufficiently far removed from adjacent resonances for $\Sigma_a(E)$ to be given by

$$\Sigma_a(E) = N^J \frac{\Gamma{\gamma_I}^J + {\Gamma_{f_I}}^J}{{\Gamma_I}^J} \sigma_{o_I}^J \, {\psi_I}^J \quad ,$$

the resonance in question being resonance I in nuclide J. In the energy region not too close thermal to be invalidated by up-scattering, the resonance escape probability may be computed the method of Goldstein and Cohen (1962) for the calculation of effective resonance integrals. thermalisation region a procedure which accounts for up-scattering into resonances within the work of the heavy gas thermalisation model (for example, Lawrence 1962) would be appropriate

Resonance reaction rates for a given nuclide are obtained from $\sum_i \Delta q_i$ (absorption), $\sum_i \frac{\Gamma_{f_i}}{\Gamma_{\gamma_i} + \Gamma_{f_i}} \Delta q_i$ (fission) and the resonance fission neutron release rate from $\sum_i \nu_i \frac{\Gamma_{f_i}}{\Gamma_{\gamma_i} + \Gamma_{f_i}} \Delta q_i$ where ν_i is the interpolated value of ν at the resonance energy E_{o_i} for the particular fissile r of interest. Thus, $\int_0^\infty \Sigma_a \phi \, du$ is computed as $\int_0^\infty \Sigma_{a_b} \phi du + \sum_j \sum_i \Delta q_i^j$.

ANISOTROPIC ELASTIC SCATTERING

For centre of mass anisotropic scattering for a mixture of nuclides in an infinite mediu

$$(\Sigma_a + \Sigma_S) \, \phi(u) = \sum_j \int_{u-\beta_j}^{u} g_j(u,u') \, \Sigma_{S_j}(u') \, \phi(u') \, du' + S(u) \quad ,$$

$$q(u) = \sum_j \int_{u-\beta_j}^{u} G_j(u,u') \, \Sigma_{S_j}(u') \, \phi(u') \, du' \quad ;$$

where $$g_j(u,u') \, du = 2 p_j(\mu_j) \frac{e^{u'-u}}{1-\alpha_j} \, du \quad (u' < u < u' + \beta_j) \quad ,$$

$$G_j(u,u') = \int_u^{u'+\beta_j} g_j(u'',u') \, du'' \quad ,$$

$$\mu_j = \frac{2}{1-\alpha_j} e^{u'-u} - \frac{1+\alpha_j}{1-\alpha_j} \quad ;$$

$$\beta_j = \log_e \frac{1}{\alpha_j} \quad ,$$

and $p_j(\mu_j)$ is the scattering law for nuclide j such that

$$\int_{-1}^{1} p_j(\mu_j)\, d\mu_j = 1 \ .$$

Defining $v = u - u'$ and using the first two Taylor series terms,

$$\Sigma_{S_j}(u')\,\phi(u') \approx \Sigma_{S_j}(u)\,\phi(u) - v\,[\Sigma_{S_j}(u)\,\phi(u)]' \quad ;$$

where $\quad [\Sigma_{S_j}(u)\,\phi(u)]' \equiv \frac{d}{du}[\Sigma_{S_j}(u)\,\phi(u)] \quad ,$

Equations 28 and 29 may be written:

$$(\Sigma_S + \Sigma_a)\,\phi = \sum_j \Sigma_{S_j}\phi \int_0^{\beta_j} \frac{2}{1-\alpha_j} p_j(\mu_j)\, e^{-v}\, dv - \sum_j [\Sigma_{S_j}\phi]' \int_0^{\beta_j} \frac{2}{1-\alpha_j} p_j(\mu_j)\, v\, e^{-v}\, dv + S \quad ,$$

$$q = \sum_j \Sigma_{S_j}\phi \int_0^{\beta_j} G_j(v)\, dv - \sum_j [\Sigma_{S_j}\phi]' \int_0^{\beta_j} v\, G_j(v)\, dv \quad .$$

Define

$$\xi_j = \int_0^{\beta_j} 2\, p_j(\mu_j) \frac{v\, e^{-v}}{1-\alpha_j}\, dv \quad ,$$

$$\epsilon_j\, \xi_j^2 = \int_0^{\beta_j} p_j(\mu_j) \frac{v^2\, e^{-v}}{1-\alpha_j}\, dv \quad .$$

Then, for isotropic scattering in the centre of mass system, $p_j(\mu_j)\, d\mu_j = \frac{1}{2}\, d\mu_j$, these becon

$$\xi_j = \int_0^{\beta_j} \frac{v\, e^{-v}}{1-\alpha_j}\, dv = 1 + \frac{\alpha_j \log_e \alpha_j}{1-\alpha_j} \quad ,$$

$$\epsilon_j\, \xi_j^2 = \int_0^{\beta_j} \frac{v^2}{2} \frac{e^{-v}}{1-\alpha_j}\, dv = \xi_j - \frac{\alpha_j (\log_e \alpha_j)^2}{2(1-\alpha_j)} \quad .$$

Using (32) in (30),

$$(\Sigma_S + \Sigma_a)\,\phi = \Sigma_S\,\phi - \sum_j \xi_j\, [\Sigma_{S_j}\phi]' + S \ .$$

Also,

$$\frac{\partial G_j}{\partial v} = -g_j(v) \quad ,$$

so that

$$q = \sum_j \Sigma_{S_j} \phi \int_0^{\beta_j} v\, g_j(v)\, dv - \sum_j [\Sigma_{S_j} \phi]' \int_0^{\beta_j} \frac{v^2}{2}\, g_j(v)\, dv ,$$

$$= \sum_j \Sigma_{S_j} \phi\, \xi_j - \sum_j [\Sigma_{S_j} \phi]' \epsilon_j \xi_j^2 .$$

Note that (32) and (33) are equivalent to

$$\xi_j = - \int_{-1}^{1} \log_e \left[1 - \frac{1-\alpha_j}{2} (1-\mu_j)\right] p_j(\mu_j)\, d\mu_j ,$$

$$\epsilon_j \xi_j^2 = \frac{1}{2} \int_{-1}^{1} \left\{\log_e \left[1 - \frac{1-\alpha_j}{2} (1-\mu_j)\right]\right\}^2 p_j(\mu_j)\, d\mu_j ,$$

which are the same results as obtained by Keane and Pollard (1962) by a different approach.

Defining

$$\bar{\xi} = \frac{\sum_j \Sigma_{S_j} \xi_j}{\Sigma_S} ,$$

$$\overline{\epsilon\xi} = \frac{\sum_j [\Sigma_{S_j} \phi]' \epsilon_j \xi_j^2}{\sum_j [\Sigma_{S_j} \phi]' \xi_j} ,$$

and

$$\bar{\epsilon} = \frac{\overline{\epsilon\xi}}{\bar{\xi}} ,$$

in the usual way, (34) and (35) give

$$q = \bar{\xi} (\Sigma_S + \bar{\epsilon}\, \Sigma_a) \phi - \bar{\epsilon}\, \bar{\xi}\, S$$

upon elimination of $\sum_j [\Sigma_{S_j} \phi]' \xi_j$.

As Σ_{S_j} is slowly varying,

$$\bar{\epsilon} \approx \frac{\sum_j \Sigma_{S_j} \epsilon_j \xi_j^2}{\Sigma_S\, \bar{\xi}^2} .$$

Thus, provided $\bar{\epsilon}$ and $\bar{\xi}$ are defined by (32), (33), (36), (37), and (38), anisotropic elast[ic] scattering may be accounted for with the formulation (2).

CRYSTAL BINDING

As the preceding work has been aimed at studies of reactors operating at temperature the[re] has been no emphasis on crystal binding. However, at the expense of providing ξ as a function [of] temperature, the effect of crystal binding on neutron thermalisation may be approximately accoun[ted] for by a further modification of ξ_j for the moderator, thereby enabling the method to be used for [studies] of zero energy assemblies.

From (2), for the moderator only,

$$q = L(\phi) = \xi_j \Sigma_S M(\phi) + \epsilon_j \xi_j M \{ (\Sigma_a + DB^2)\phi - S \}, \qquad (4$$

where M is the heavy gas operator $(E - T) + E T \frac{d}{dE}$.

Horowitz (1962) has suggested the use of a factor f (E,T) to modify M. Applying this to (4

$$q = L(\phi) = \xi_j f_j(E,T) \Sigma_S M(\phi) + \epsilon_j \xi_j f_j (E,T) M \{ (\Sigma_a + DB^2) \phi - S \}.$$

Thus, by modification of ξ_j for the moderator by the factor f_j (E,T), the effect may be allowed for within the formulation (2). $\bar{\epsilon}$ and $\bar{\xi}$ for the mixture are then obtained from (36), (37), and (38). f (E,T) is already available for graphite at room temperature from experimental measurements (Leslie 1962).

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* Not referred to in the text.

NOTATION

ϕ	Neutron flux
q	Slowing down density
E	Neutron energy
T	Temperature in eV
ϵ	Greuling – Goertzel parameter
χ	Fission spectrum

ω Inelastic scattering emission spectrum

u $\log_e (10^7/E)$, E in eV

ξ Mean lethargy increase per scattering collision

k Effective multiplication constant

f_R Fission neutron release rate

N Nuclide concentration

σ_o $2.6 \times 10^6 \; \Gamma_n g/E_o \Gamma$ (in barns)

ψ Doppler broadened line shape function

σ_h Constant potential elastic scattering cross section

Γ Resonance total width

Γ_γ Resonance capture width

Γ_f Resonance fission width

Γ_n Resonance elastic scattering width

E_o Resonance energy

p Resonance escape probability

Δq Change in slowing down density in passing through a resonance

μ Cosine of scattering angle in the centre of mass system

α Inelastic scattering rate, also $(A-1/A+1)^2$

A Mass number

g Statistical weight factor, also elastic scattering kernel

Subscripts

i i^{th} resonance

f Fission

S Elastic scattering

in Inelastic scattering

a Absorption

i− Immediately before resonance (in lethargy)

i+ Immediately after resonance (in lethargy)

Superscript

j Nuclide type

$\sum_i$ and $\sum_j$ denote summation over all values of i and all values of j respectively.