AAEC/E456

AAEC/E456

(2)

AUSTRALIAN ATOMIC ENERGY COMMISSION

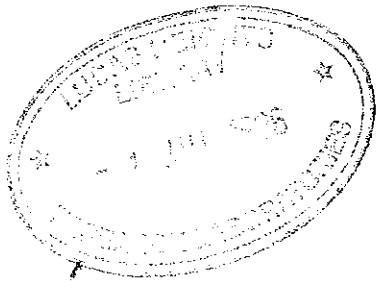
RESEARCH ESTABLISHMENT

LUCAS HEIGHTS

INVERSE KINETICS REACTIVITY MEASUREMENTS ON THE MATERIALS TESTING REACTOR HIFAR

by

J.R. HARRIES



December 1978

AUSTRALIAN ATOMIC ENERGY COMMISSION
RESEARCH ESTABLISHMENT
LUCAS HEIGHTS

INVERSE KINETICS REACTIVITY MEASUREMENTS ON THE MATERIALS TESTING REACTOR HIFAR

by

J.R. HARRIES

ABSTRACT

The inverse kinetics method of reactivity determination is compared with the asymptotic doubling time method and found to give more accurate results for the materials testing reactor HIFAR. The differential reactivity worth profile of the HIFAR coarse control arms is measured using the inverse kinetics method and the reactivity worth is found to be proportional to $M^{-0.75\pm0.10}$, where M is the mass of fissile material in the reactor. The fuel mass dependence is shown by perturbation theory to be a function of the relative flux depression at the fuel. The temperature coefficient of reactivity is found to be proportional to $M^{-1.01\pm0.16}$.

National Library of Australia card number and ISBN 0 642 59660 3

The following descriptors have been selected from the INIS Thesaurus to describe the subject content of this report for information retrieval purposes. For further details please refer to IAEA-INIS-12 (INIS: Manual for Indexing) and IAEA-INIS-13 (INIS: Thesaurus) published in Vienna by the International Atomic Energy Agency.

REACTIVITY; REACTOR KINETICS; HIFAR REACTOR; REACTIVITY WORTHS; TEMPERATURE COEFFICIENTS; SHIM RODS

CONTENTS

1. INTRODUCTION

The reactivity effect of a change in reactor properties depends on the mass of fissile material. In the materials testing reactor HIFAR, the fissile mass has varied between 2.4 and 3.2 kg and a mass correction factor must be used to adjust the standard coarse control arm (CCA) reactivity worth. Previously, the reactivities were assumed to be inversely proportional to the fuel mass and all reactivities were referred back to a 3.2 kg core [Connolly *et al.* 1968, Duerden 1973]. A review of the CCA worths has raised doubts about this mass correction factor and highlighted some of the difficulties in calibrating the CCAs using the standard asymptotic doubling time measurements.

The inverse kinetics procedure developed in this report removes many of the uncertainties inherent in the asymptotic doubling time measurement and enables the mass correction factor to be measured. Inverse kinetics methods have been used previously on HIFAR to analyse temperature and flow transients [Harries & Wilson 1978].

2. INVERSE KINETICS

2.1 Theory

The kinetic equations for a point reactor are:

$$\frac{dn(t)}{dt} = \frac{\rho(t)-\beta}{\Lambda} n(t) + \Sigma_1^N \lambda_i C_i(t) + S(t) \qquad (1)$$

and

$$\frac{dC_i(t)}{dt} = \frac{\beta_i n(t)}{\Lambda} - \lambda_i C_i(t) \quad , \qquad (2)$$

where $n(t)$ is the neutron density or power,

$\rho(t)$ is the reactivity,

Λ is the prompt neutron generation time,

$S(t)$ is is the neutron source,

$C_i(t)$ is the delayed neutron precursor concentration for group i,

β_i and λ_i are the yield and decay constants for group i,

β is the total delayed neutron fraction, and

N is the total number of delayed neutron groups.

Equation (2) can be integrated to give:

$$C_i(t) = C_i(0)e^{-\lambda_i t} + \int_0^t \frac{\beta_i n(\tau)}{\Lambda} e^{-\lambda_i (t-\tau)} d\tau \quad , \qquad (3)$$

where $C_i(0)$ is the delayed neutron precursor concentration at time

$t = 0$. If the initial delayed neutron precursors are at the equilibrium concentration, then,

$$C_i(0) = \beta_i n(0)/\lambda_i \Lambda \qquad (4)$$

Equations (3) and (4) are then substituted in Equation (1) to give

$$\rho(t) = \beta + \frac{\Lambda}{n(t)} \frac{dn(t)}{dt} - \frac{\Lambda}{n(t)} S(t) -$$

$$- \frac{1}{n(t)} \sum_{n=1}^{N} \beta_i \left\{ n(0) e^{-\lambda_i t} + \int_0^t n(\tau) \lambda_i e^{-\lambda_i (t-\tau)} d\tau \right\} \qquad (5)$$

This is known as the inverse kinetics equation and it enables the reactivity to be calculated from variations of the reactor power with time.

The photoneutron source is important in heavy water reactors. The gamma radiation emitted by fission products produces neutrons by photoneutron interaction with the deuterium in heavy water (D_2O). The threshold for photoneutron production is 2.23 MeV and the cross section is of the order of millibarns. The photoneutron groups are added to the standard delayed neutron groups used in the reactor kinetics calculation (Table 1). The standard photoneutron groups [Keepin 1965] correspond to an infinite volume of D_2O, and the yields need to be corrected by the photoneutron effectiveness factor, γ_p, to allow for absorption and leakage effects. For HIFAR, a photoneutron effectiveness of 0.54 has been used [Connolly *et al.* 1968].

The longest lived photoneutron group has a half-life of 12.8 days. This presents a problem when applying the inverse kinetics equation to HIFAR, even many weeks after 10 MW operation. The photoneutron concentration will depend on the time since shutdown and the refuelling program. It is convenient to include such residual photoneutron activity in the source term, and to determine the source strength that best fits the data.

2.2 Data Acquisition and Analysis

The inverse kinetics method requires a complete record of the reactor power during the experiment and it is useful also to record the D_2O temperature. The reactor power is registered on chart recorders in the reactor control room, but these are not very suitable for inverse kinetics analysis. For the inverse kinetics measurements, the input to

the single range linear recorder was taken to a Keithley picoammeter. The amplified signal was then fed to the HIFAR data acquisition system (DAS) for digitisation and recording. The DAS also received the signals from the thermocouples at the D_2O output of twenty-four of the twenty-five fuel elements. Finally a signal was taken from the relay which controls the 'fast rise' mode of the CCAs. This signal was useful for indicating the point at which the CCA movement stopped as the power was raised.

The DAS sampled and digitised the twenty-six signals at 2 s intervals and stored them on magnetic tape. After the experiment, the data were transferred to a disk file on the IBM360/65 computer for analysis.

A computer program was written to determine the reactivity using the inverse kinetics equation, and to determine the mean temperature of the D_2O at the fuel element outlets. It was assumed that the power changed exponentially during the 2 s interval between sampling points.

3. CCA SENSITIVITY MEASUREMENT

3.1 Evaluation of Asymptotic Doubling Time Method

The measurement of reactivity by the asymptotic doubling time method is a standard procedure in reactor physics; it is mainly suitable for low power reactors with small background neutron sources. The asymptotic doubling time is related to reactivity by the inhour equation. Previously, the standard HIFAR procedure used to determine the differential reactivity worth of the CCAs was to determine the critical CCA angle and use an asymptotic doubling time measurement to obtain the reactivity change to a second angle. The strong photoneutron source remaining after 10 MW operation complicates both the doubling time measurement and the determination of the critical angle.

The doubling time measurement was carried out with the reactor in low-power mode and the trip level on the shutdown amplifiers less than 150 kW. The corresponding reading on the multirange linear (MRL) recorder was about 6.3×10^{-7} A. Initially, the reactor power was stabilised at an MRL reading of 5×10^{-9} A. The CCAs were then raised, by pressing the 'fast raise' button, to achieve an asymptotic doubling time of about 20 s. The high source strength, which is multiplied as the reactor approaches critical, meant that the MRL signal was already about 8×10^{-8} A by the time the CCA movement was complete. Sometimes the operators continued to move the CCAs until the MRL reading was at 1×10^{-7} A.

TABLE 1

DELAYED AND PHOTONEUTRON DATA

Delayed neutrons, $\beta = 0.0065$

Group	Half-life	Decay Constant (s^{-1})	β_i
1	55.72 s	0.0124	0.000215
2	22.72 s	0.305	0.001424
3	6.22 s	0.111	0.001274
4	2.30 s	0.301	0.002568
5	0.61 s	1.14	0.000748
6	0.23 s	3.01	0.000273
			0.0065

Photoneutrons, $\beta_\gamma = 0.0010$, $\gamma_p = 0.54$

Group	Half-life	Decay Constant (s^{-1})	$\gamma_p \beta_i$
7	12.8 d	6.26×10^{-7}	2.65×10^{-7}
8	53 h	3.63×10^{-6}	5.51×10^{-7}
9	4.4 h	4.37×10^{-5}	1.72×10^{-6}
10	1.65 h	1.17×10^{-4}	1.25×10^{-5}
11	27 m	4.28×10^{-4}	1.11×10^{-5}
12	7.7 m	1.50×10^{-3}	1.80×10^{-5}
13	2.4 m	4.81×10^{-3}	3.76×10^{-5}
14	41 s	1.69×10^{-2}	1.09×10^{-4}
15	2.5 s	2.77×10^{-1}	3.49×10^{-4}
			5.40×10^{-4}

γ_p is the photoneutron effectiveness

The times were measured by stopwatch as the MRL reading passed 1.5, 2, 2.5, 3, 4, 5 and 6 x 10^{-7} A.

The time between the end of the CCA movement and the doubling time measurement is not sufficient to ensure that the asymptotic doubling time has been achieved. Even if the 'low-power' operational constraints were modified, it is likely that thermal effects would limit the accuracy of the method. Furthermore, it would not help to start the doubling time from a lower initial power, because the strong photoneutron source means that a large CCA movement is required to further reduce the power. Even with the CCAs fully in, the MRL reading is still ~ 2 x 10^{-10} A.

TABLE 2

DOUBLING TIMES DURING DOUBLING TIME MEASUREMENT

FOR CRITICAL ANGLE OF 21.1°

Run	1	2	3	4	5
MRL reading (A)	Instantaneous doubling time (s)				
1 x 10^{-7}	15.4	20.1	26.5	37.7	61
2 x 10^{-7}	17.1	22.5	30.7	44.6	82
3 x 10^{-7}	18.2	24.5	34.1	53.7	93
4 x 10^{-7}	19.0	25.6	35.4	56.7	110
5 x 10^{-7}	-	26.5	37.5	60.0	120
(a) Asymptotic DT (s)	20.2	28.5	39.5	65.0	130
Inverse kinetics reactivity (10^{-2} $\Delta k/k$)	0.170	0.139	0.113	0.081	0.049
Temp. rise (°C)	0.10	0.11	0.16	0.20	0.25
(b) Reactivity change 10^{-2} $\Delta k/k$	-0.003	-0.003	-0.005	-0.007	-0.009

(a) Asymptotic doubling time (DT) determined from reactivity calculated by inverse kinetics program.

(b) Reactivity change at 6 x 10^{-7} A due to the increase in temperature $\delta p = (\delta T - 0.02) \times 3.7 \times 10^{-4}$ $\Delta k/k$ (see text).

The inverse kinetics program has been used to analyse some asymptotic doubling time measurements. These were carried out about 70 hours after shutdown from 10 MW operation and before any fuel change. Table 2 shows the variation of doubling time as the MRL signal reached various levels. In all cases, the instantaneous doubling times overestimate the actual reactivity derived from the inverse kinetics. Fortuitously, the transient effects for doubling times of ~ 20 s were less than for longer doubling times. This occurred because the power was increasing by subcritical multiplication, with a doubling time of between 11 and 15 s while the CCAs were still moving. When the CCA movement stopped, the change to a 20 s doubling time introduced fewer transients than did a change to a longer doubling time.

The temperature rise measured at the output of the fuel elements during each of the doubling time runs is shown in Table 2. At 120 kW, the temperature rise due to heating is 0.072°C. Hence, the increase in the mean temperature of D_2O within the fuel elements will be 0.036°C less than the increase measured at the outlets of the fuel elements. The mean temperature rise of all the D_2O in the core will be greater than this because of the contribution from the warmer D_2O in the reactor aluminium tank (RAT) between the fuel elements. If the mean core temperature rise is taken to be 0.02°C less than the rise in the fuel element outlet temperature, then the estimated reactivity effect shown in Table 2 is obtained. The temperature contribution becomes significant at slower doubling times.

The temperature effects were further investigated by allowing two doubling time runs to continue to higher power. This meant changing to the 'full power' mode of operation and using doubling times that were longer than 30 s. The negative temperature feedback stabilised the power at an MRL signal of 4×10^{-6} for a run which initially had an asymptotic doubling time of 41 s. The temperature rise had reduced the reactivity by 10^{-4} $\Delta k/k$ by the time the MRL signal reached 1.4×10^{-6} A.

The determination of the critical CCA angle by the source multiplication method can also introduce errors. The critical angle is obtained by plotting the reciprocal of the power signal against the CCA angle for two or three power levels and extrapolating the line to zero reciprocal power. The main difficulty with this procedure is that the critical angle depends on both the D_2O temperature and the decay of xenon poisoning. Any variations in the reactor temperature, either

within a set of subcritical balances or between the balances and the doubling time measurement, requires correction of the critical angle. A change of 0.2°C, which is barely measurable on the installed instrumentation, would cause a reactivity change of 7×10^{-5} $\Delta k/k$. The decay of xenon poisoning also causes changes in the critical angle with time. Seventy hours after shutdown from 10 MW operation, the xenon poison reactivity is decreasing at about 3.5×10^{-4} $\Delta k/k\ h^{-1}$.

The reactivity calculated from the inverse kinetics relation is compared in Figure 1 with the CCA angle during the run. The calculated reactivity varies from -0.035 to +0.00157 $\Delta k/k$ and closely matches the variation in the CCA angle. Unfortunately, reactivities less than about -0.01 $\Delta k/k$ cannot be used for CCA calibration because of the strong dependence of the derived reactivity on the estimated photoneutron source strength. Possibly if the source strength were better understood, a single measurement like this could provide a whole range of differential CCA worths.

3.2 CCA Sensitivity by Inverse Kinetics

The inverse kinetics method provides a more accurate method of determining reactivity worth than the doubling time method. Figures 2 and 3 show the power level and the derived reactivity during a measurement of the differential worth of the CCAs.

Initially, the power was held at 1×10^{-8} A on the MRL for twenty minutes to remove most of the transients. The first power increase was produced when the operator raised the CCAs to 23.40°. The power increased with a doubling time of about 20 s. When the MRL signal reached 6×10^{-7} A, the operator moved the CCAs to 22.78 degrees to make the power decrease. The CCA movement was completed in less than 10 s and the change in reactivity is obtained directly from the inverse kinetics calculation.

When the MRL signal had decreased below 1×10^{-7} A, the operator again raised power by moving the CCAs to 23.43°. Again the power was allowed to increase until the MRL signal reached 6×10^{-7} A. The operator then quickly moved the CCAs to 22.92° to reduce the power. After about three minutes, he moved the CCAs to 22.49° to further reduce the power. This procedure was repeated four times and then the MRL signal was reduced to about 1×10^{-8} A by lowering the CCAs in a series of steps.

The calculated reactivity (Figure 3) shows that the inverse kinetics

calculation gives a constant reactivity at those times when the CCAs are not moving. The calculation required an estimate of the photoneutron source remaining from the 10 MW operation which finished 65 hours before the beginning of the experiment. The source strength was initially obtained by subcritical multiplication measurements, but it was then adjusted on the basis of the inverse kinetics results so as to make the subcritical reactivities constant. The subcritical reactivities are much more sensitive to changes in the source strength than reactivities near or above critical. Figure 4 shows the effect of increasing the source strength by 17 per cent.

During the experiment, the reactivity was also changing due to the decay of xenon poisoning and the increase in D_2O temperature. The xenon poisoning was reducing at about 6.3×10^{-4} $\Delta k/k\ h^{-1}$ and, over the 2500 s of the experiment, this increased the reactivity by 4.4×10^{-4} $\Delta k/k$. The power generated during the experiment caused the D_2O temperature to increase by about 1.3°C which decreased the reactivity by about 5.6×10^{-4} $\Delta k/k$.

The differential worth of the CCAs was determined for each power increase from the difference in the reactivity before and after the CCA movement that terminated the power rise. Each reactivity was averaged over 5 samples (10 s) to decrease the random errors. Although only 24 to 30 s elapsed between the two reactivity determinations, the average D_2O temperature was observed to rise, typically by 0.04°C. The calculated reactivity difference was corrected for the temperature change and the decay of xenon poisoning. The corrections were small because the time interval between the reactivity determinations was small. The differential worth, or sensitivity, of the CCA is obtained by dividing the change in reactivity by the change in CCA angle.

The temperature coefficient used in the analysis was -0.037×10^{-2} $\Delta k/k\ °C^{-1}$. The use of the previous standard HIFAR temperature coefficient of -0.043×10^{-2} $\Delta k/k\ °C^{-1}$ [Wilson 1972, Duerden 1973] reduced the CCA sensitivities by about 0.2 per cent.

The error in each estimate of the CCA sensitivity is made up of two components - the error in the CCA angle and the error in the reactivity determination. The CCA angle measurement has a reading uncertainty of ±0.005° which becomes an error of ±0.007° on the difference between two CCA angles. At small angles, where the sensitivity was measured over about 0.30°, the CCA error becomes dominant. It is estimated that the

effect of errors in the temperature correction, random fluctuations of the calculated reactivity, source uncertainties and other errors produces a standard deviation of 1.5×10^{-5} $\Delta k/k$ in the reactivity change.

3.3 Determination of Sensitivity Curve and Mass Correction Factor

A set of experiments to calibrate the CCAs was carried out at the beginning of Program No.231 during the decay of xenon produced in the preceding 23 days of 10 MW operation. The first run was taken 37 hours after shutdown when the xenon had decayed sufficiently for the reactor to be critical with a CCA angle of 30°. Seven runs were taken over the next 40 h as the xenon decayed and the critical angle reduced to 20°. These runs were carried out with a 2.553 kg fuel loading remaining from the previous program.

Further inverse kinetics runs were carried out as the fuel was changed, the final run being taken with a CCA angle of 14.7° and a fuel loading of 2.822 kg. To construct the CCA differential worth curve, the reactivities measured in these runs required correction for the changing mass of uranium-235 in the core; normal HIFAR practice has been to assume that the reactivity is inversely proportional to the fuel mass and to refer all such measurements to a standard core loading of 3.2 kg uranium-235.

A second series of runs was carried out at the beginning of program No.232. The first run was taken, before any fuel change, at a CCA angle of 24.9° and with a mass of 2.542 kg. The fuel was then changed to give a total mass in the core of 3.186 kg with a CCA angle of 14.4°. Seven runs were taken with this fuel mass but at different CCA angles achieved by the introduction of stainless steel liners and dummy rigs into the fuel elements. The largest CCA angle was 22.4°. The reactor was then prepared for normal operation and a final inverse kinetic run was taken with a mass of 2.899 kg and a CCA angle of 14.9°.

Each run consisted usually of three separate power increases and hence three estimates of the CCA sensitivity. There was a total of seventy estimates, each associated with a mean CCA angle and a fuel mass. The reactivity differences were each corrected for temperature change and xenon decay.

The CCA sensitivity as a function of angle and fuel mass was determined by least squares fitting all the data to the function:

$$W = (a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4)\ M^{a_5} \quad , \qquad (6)$$

where W is the CCA sensitivity at angle x for a fuel loading of M kg and $a_1,\ldots,a_5$ are the parameters to be fitted. The observed sensitivities were weighted by the squares of the estimated errors. The exponential form of the mass correction was chosen as the simplest functional form which could cover the complete range from no mass dependence ($a_5 = 0$) to the inverse mass relation ($a_5 = -1$) which had previously been used on HIFAR [Connolly *et al.* 1968]. The mass dependence obtained from the least squares fit was:

$$W \propto M^{-0.68} \tag{7}$$

The fuel elements in the core are at varying stages of burnup and their fuel content varies between 60 and 150 g of uranium-235. The distribution of the fuel mass varies between core loadings and could effect the mass correction factor. The analysis of the CCA sensitivities was repeated using an 'effective mass' instead of the measured mass of fuel. The effective mass was determined by weighting the mass of each fuel element by the relative statistical weight which is proportional to the square of the flux. The relative statistical weight was normalised so that the effective mass would equal the actual mass for a uniform fuel distribution across the 25 fuel elements.

The least squares analysis of the data using effective masses gave a mass dependence of

$$W \propto M^{-0.75 \pm 0.10} \tag{8}$$

This is probably more correct than Equation (7) because the analysis makes some allowance for the variations in the fuel distribution. The error is the combination of ± 0.06, which is the one standard deviation given by the least squares fit, and ± 0.04 for uncertainties in the correction for the fuel distribution.

The observed data corrected to the standard 3.2 kg core are compared with a fitted curve of Equation (6) in Figure 5. The CCA sensitivities obtained by the least squares fit are listed in Table 3.

3.4 Comparison with Previous Results

The CCA sensitivities over the range of the present experiment (14 to 30°) are compared with previous estimates of sensitivity in Figure 6. The latter are based on stopwatch asymptotic doubling time measurements obtained from the difference in CCA angle for either two different doubling times, or one doubling time and a previously measured critical setting. No corrections were made for temperature change, although the

measurements would not have been made until the reactor temperature was steady on the installed instrumentation.

Wilson [1972] measured the CCA sensitivity for a 3.75 kg core consisting of twenty four new Mark IV/5A fuel elements, and one partially burnt-up fuel element to provide a neutron source; it was heavily poisoned by rigs and stainless steel liners. Two curves are given for Wilson's results in Figure 6, one using the old inverse mass correction to correct the reactivity to 3.2 kg, and the other using the presently derived correction. His results are considerably below the inverse kinetics results at high angles and above them at low angles, but the integrated worth over the range is about the same. The difference probably reflects the major changes in the core.

TABLE 3

CCA SENSITIVITY FOR 3.2 kg CORE

CCA Angle (°)	Sensitivity 10^{-2} $\Delta k/k$ $°C^{-1}$	CCA Angle (°)	Sensitivity 10^{-2} $\Delta k/k$ $°C^{-1}$
14	0.717 ± 0.012	23	0.325 ± 0.004
15	0.674 ± 0.007	24	0.289 ± 0.004
16	0.629 ± 0.006	25	0.256 ± 0.003
17	0.584 ± 0.006	26	0.225 ± 0.003
18	0.538 ± 0.005	27	0.197 ± 0.003
19	0.492 ± 0.005	28	0.172 ± 0.003
20	0.447 ± 0.004	29	0.149 ± 0.003
21	0.405 ± 0.004	30	0.127 ± 0.003
22	0.364 ± 0.004	31	0.108 ± 0.006

Duerden [1973] evaluated the previous doubling time and temperature calibration data and made some measurements at CCA angles greater than 24°. His curve (Figure 6) is in reasonable agreement with the inverse kinetics results at CCA angles greater than 20°. At smaller angles, this curve merges with that of Wilson [1972].

Earlier data from Connolly *et al.* [1968] are in very good agreement with the inverse kinetics results. Their data are given for a core of 3.0 kg and these have been adjusted using the correction factor (Equation 7) before plotting the appropriate curve on Figure 6. The correction is only small and the agreement would not be materially altered if an 'inverse mass' correction was used. The Connolly *et al.* results were

obtained when Mark II and Mark III fuel elements were in use, but with the introduction of the Mark IV/5A fuel elements it was not known whether they were still applicable.

The comparison of all the data shows good agreement between the inverse kinetics results of Connolly *et al.* and the higher angle results of Duerden. Wilson's results are definitely different and it seems most likely that this is due to the atypical core loading.

4. TEMPERATURE COEFFICIENT OF REACTIVITY

Several times in each operating program, the reactor temperature is reduced by about 10°C and the CCA movement required to balance the reactor power is recorded. The temperature decrease and subsequent return to the normal operating temperature takes over two hours. This procedure is normally used to monitor the CCA reactivity worth, but the data can also be used to determine the temperature coefficient of reactivity if the CCA sensitivity is known. Each temperature balance provides the change in the CCA angle, $\Delta\theta$, required to balance a change in temperature, ΔT, for a given CCA angle and fuel loading M. The function given in Equation (6) was least squares fitted to the ratios $\Delta T_i/\Delta\theta_i$ (Figure 7). The mass dependence was found to be

$$\left(\frac{\Delta T}{\Delta\theta}\right)_M = \left(\frac{\Delta T}{\Delta\theta}\right)_{3.2} \left(\frac{M}{3.2}\right)^{0.26\pm0.05} , \qquad (9)$$

where M is given in kg. The error shown is one standard deviation assuming that the individual data points are randomly distributed. Six years' accumulation of data was used in the fit when the fuel mass had varied from 3.15 to 2.44 kg. Unfortunately, the fuel mass has tended to decrease over the years, so that any change in the reactor properties or operating procedures may not be separable from the mass effect. In view of this uncertainty, the error in the exponent is arbitrarily increased to ± 0.13.

The temperature coefficient of reactivity, α, is given by

$$\alpha = \left(\frac{d\rho}{d\theta}\right) \Big/ \left(\frac{dT}{d\theta}\right) , \qquad (10)$$

where $d\rho/d\theta$ is the differential worth of the CCAs, $dT/d\theta$ is the differential obtained from the temperature balances and both differentials are taken at the same CCA angle and fuel mass. Substituting from Equations (7) and (8) gives

$$\alpha = \left[\left(\frac{d\rho}{d\theta}\right)_{3.2}\left(\frac{3.2}{M}\right)^{0.75\pm0.10}\right] \Big/ \left[\left(\frac{dT}{d\theta}\right)_{3.2}\left(\frac{M}{3.2}\right)^{0.26\pm0.13}\right]$$

$$= (3.64 \pm 0.13) \times 10^{-4} \ (3.2/M)^{1.01\pm0.16} \ \Delta k/k \ {}^{\circ}C^{-1} \ , \qquad (11)$$

where the coefficient, $\alpha_{3.2}$, was obtained from the ratio of the temperature balance fits to the differential worths of the CCAs at angles between 20 and 25°.

5. DISCUSSION OF MASS CORRECTION FACTOR

Perturbation theory [Foell 1972] provides a formalism for determining the change in reactivity produced by small changes in the reactor properties. In the CCA sensitivity measurement, the reactivity worth of a small movement of the CCAs is measured. This is equivalent to a small change in absorption at the edges of the CCAs. The reactivity change due to a small change in absorption is given to the first order by

$$\rho = k_{eff} \frac{\int dV \sum_{i=1}^{N} \delta\Sigma_a^i \phi_i^+ \phi_i}{\int dV \sum_{i=1}^{N} \sum_{j=1}^{N} \chi_{ij} \nu_i \Sigma_f^i \phi_j^+ \phi_i} \qquad (12)$$

where i and j refer to the energy groups,

$\delta\Sigma_a^i$ is the change in the macroscopic absorption cross section of group i,

ϕ_i is the perturbed neutron flux in group i. In the first order perturbation theory this is replaced by the unperturbed flux.

ϕ_i^+ is the unperturbed adjoint neutron flux in group i,

χ_{ij} is the fraction of neutrons released into group j by a fission in group i,

ν_i is the number of neutrons emitted in a fission caused by a neutron in group i,

Σ_f^i is the macroscopic fission cross section for group i,

k_{eff} is the effective multiplication in the core; $k_{eff} = 1$ if the reactor is critical.

The integration in the numerator is performed only over the local volume where the absorption has changed and the integration in the denominator is over the fuel in the reactor.

Only the thermal group contributes to the numerator because the

CCAs contain cadmium which is a thermal absorber. The denominator can also be simplified because the bulk of the fissions are caused by thermal neutrons, and fission produces neutrons in the fast group. Hence the perturbation equation can be approximated by

$$\rho = \frac{\phi^{+}_{th}(\text{CCA})\phi_{th}(\text{CCA}) \int \delta\Sigma_a^{th}\, dV}{\phi^{+}_{f}(\text{fuel})\phi_{th}(\text{fuel})\chi\ \nu \int \Sigma_f dV} \quad , \tag{13}$$

where ϕ(CCA) and ϕ(fuel) are the mean neutron fluxes at the CCA and the fuel respectively, and the subscripts f and th refer to the fast and the thermal fluxes respectively.

Now consider two cores loaded with different masses of uranium-235 but critical at the same CCA settings. The neutron production of the extra fissile material is balanced by the absorption of extra rigs mainly within the fuel element liners. The value of χ, ν and $\delta\Sigma_a$ will be the same for each core. Further, the ratio $\phi^{+}_{th}(\text{CCA})/\phi^{+}_{f}(\text{fuel})$ will remain approximately constant because there is no signficant difference in the neutron leakage from the cores. The integral $\int\Sigma_f dV$ is proportional to the mass (M) of uranium-235 in the core. Hence, the ratio of the reactivity worths in the two cores for the same change in absorber is approximately:

$$\frac{\rho_1}{\rho_2} = \frac{D_2 M_2}{D_1 M_1} \tag{14}$$

where D is the thermal flux ratio between the fuel and the CCAs, *i.e.* $D = \phi_{th}(\text{fuel})/\phi_{th}(\text{CCA})$. If the fluxes in the two core loadings were identical, the reactivity would be inversely proportional to the mass of uranium-235. However, the addition of extra fissile material and the compensating absorbers depresses the flux in the fuel elements and this reduces the mass dependence.

The experimentally derived mass correction factor for CCA sensitivity data was:

$$W_1/W_2 = (M_2/M_1)^{0.75 \pm 0.10} \quad . \tag{15}$$

Combining Equations (14) and (15) gives an estimate of the change in the flux ratio:

$$D_2/D_1 = (M_1/M_2)^{0.25 \pm 0.10} \quad . \tag{16}$$

A core mass increase from 2.7 to 3.2 kg would be associated with a decrease of 4 per cent in the fuel thermal flux relative to that at the

CCAs. This small flux depression is sufficient to account for the observed mass dependence exponent of 0.75.

The main contribution to the temperature coefficient of reactivity is the spectrum change due to the temperature of the D_2O within the fuel elements. The ratio of the flux in the D_2O to that in the fuel will not be greatly changed by changes in the fuel mass. Hence the perturbation analysis would suggest that the temperature coefficient would have a mass dependence which would be closer to the inverse relationship than that of the CCA worth. This is in agreement with the measured temperature coefficient, Equation (11), which showed that mass correction factor is inversely proportional to the fuel mass.

6. CONCLUSIONS

The inverse kinetics method has been shown to be a valuable technique for reactivity measurements in the reactor HIFAR and to give more accurate results than the previously used asymptotic doubling time method. The high inherent neutron source of HIFAR, which is directly included in the inverse kinetics method, generally precludes the establishment of a true asymptotic doubling time before temperature effects become important.

Coarse control arm sensitivity data have been obtained by the inverse kinetics method and show that the standard HIFAR tabulation overestimates the CCA sensitivity by about 20 per cent at CCA angles between 17 and 14°. At angles greater than 23°, the inverse kinetics results are in good agreement with the standard curve.

The CCA sensitivities vary as $M^{-0.75\pm0.10}$ where M is the mass of uranium-235 in the core. This mass dependence has been shown to be reasonable, on the basis of perturbation theory, if an increase in fuel mass results in a small decrease in the flux at the fuel relative to that at the CCAs. An exponent of -1 would only be observed if the thermal flux ratio between the fuel and the CCAs was independent of fuel mass.

7. ACKNOWLEDGEMENTS

The author is grateful to Messrs R. Hicks and R. Sanders for programming the HIFAR data acquisition system to record the data and to Mr J. Bezimienny for his assistance in data collection.

8. REFERENCES

Connolly, J., Culley, D. & McCulloch, D.B. [1968] - AAEC unpublished report.

Duerden, P. [1973] - AAEC unpublished report.

Foell, W.K. [1972] - Small sample reactivity measurements in nuclear reactors. American Nuclear Society Monograph, Hinsdale, Illinois.

Harries, J.R. & Wilson, D.J. [1978] - Measurement of the dynamic response of the materials testing reactor HIFAR. AAEC/E428.

Keepin, G.R. [1951] - Physics of nuclear kinetics. Addison-Wesley, Mass.

Wilson, D.J. [1972] - AAEC unpublished report.

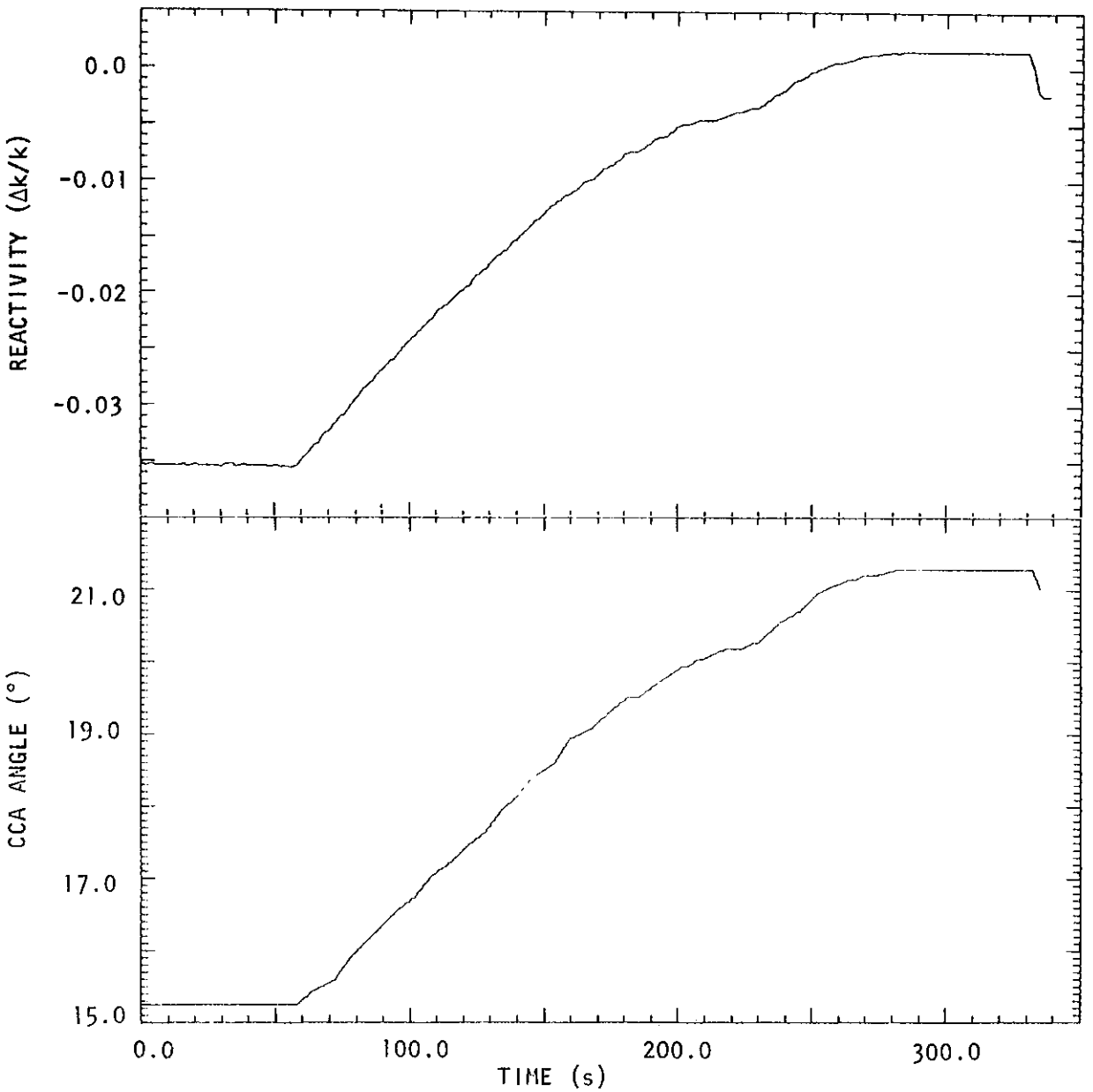


FIGURE 1. REACTIVITY CALCULATED BY THE INVERSE KINETICS METHOD IS COMPARED WITH THE CCA MOVEMENT DURING A DOUBLING TIME MEASUREMENT

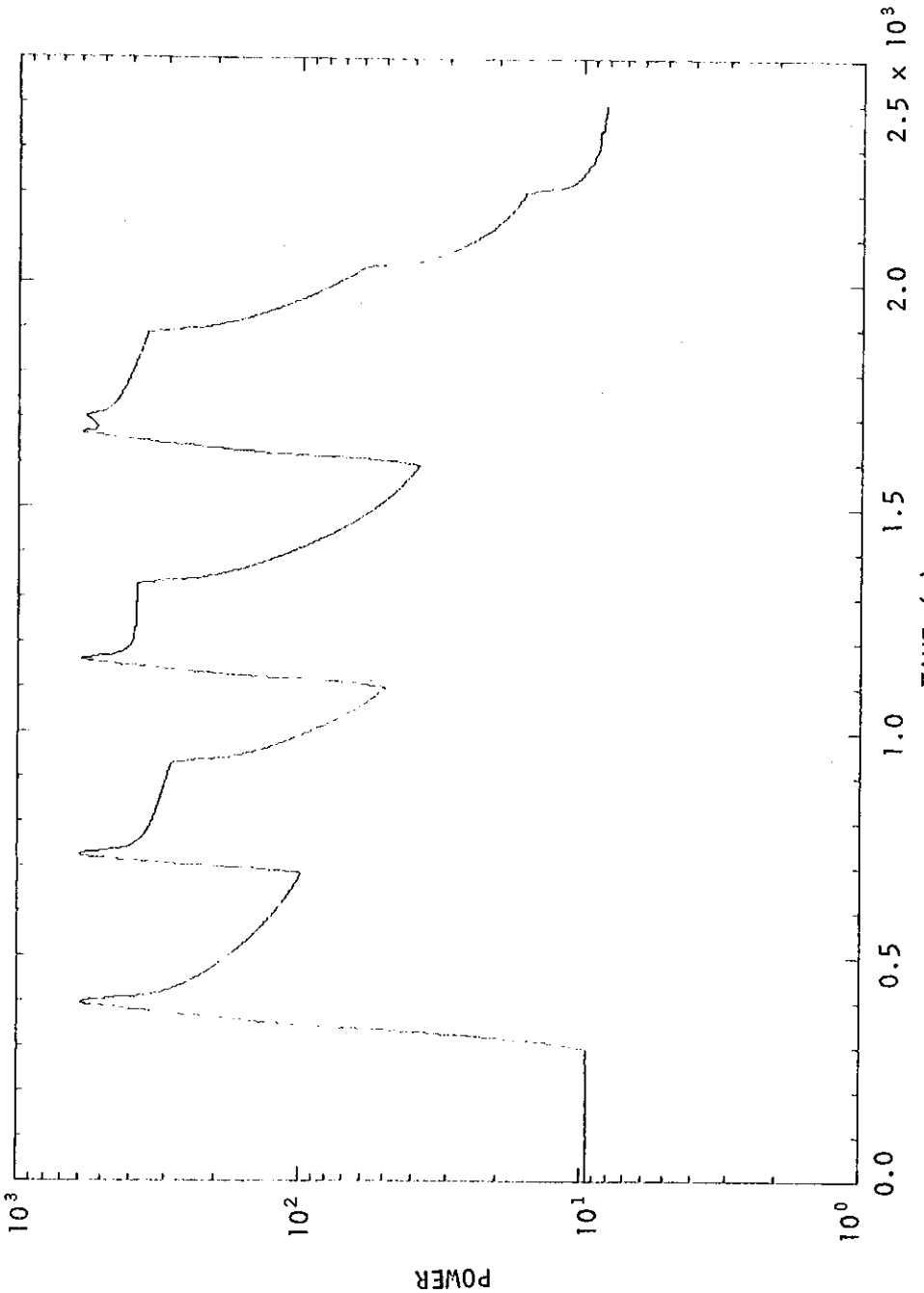


FIGURE 2. POWER PROFILE DURING AN INVERSE KINETICS CALIBRATION OF THE CCAS

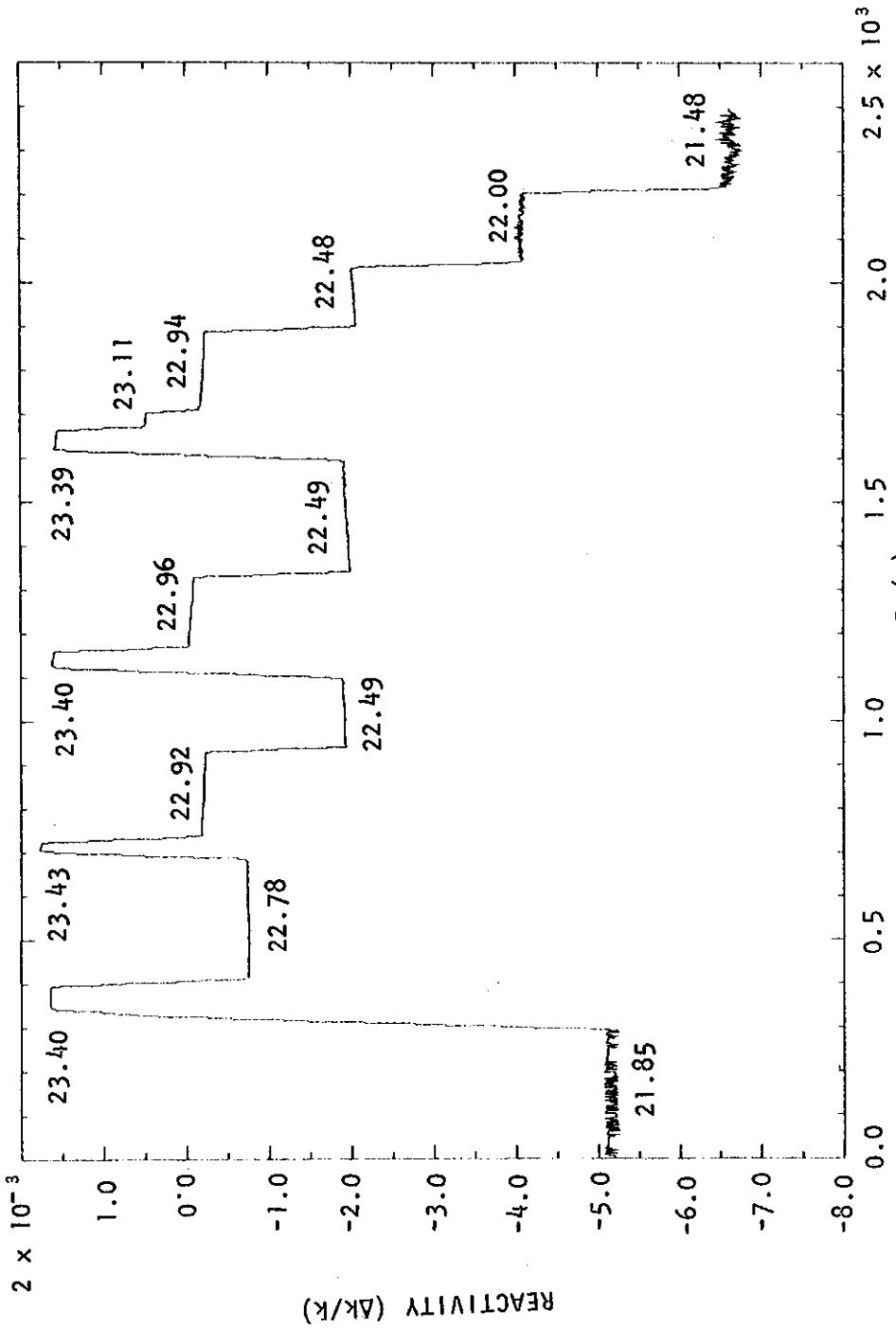


FIGURE 3. REACTIVITY CALCULATED FROM THE POWER PROFILE OF FIGURE 2. THE NUMBERS ON THE GRAPH ARE THE CCA ANGLES DURING THE EXPERIMENT

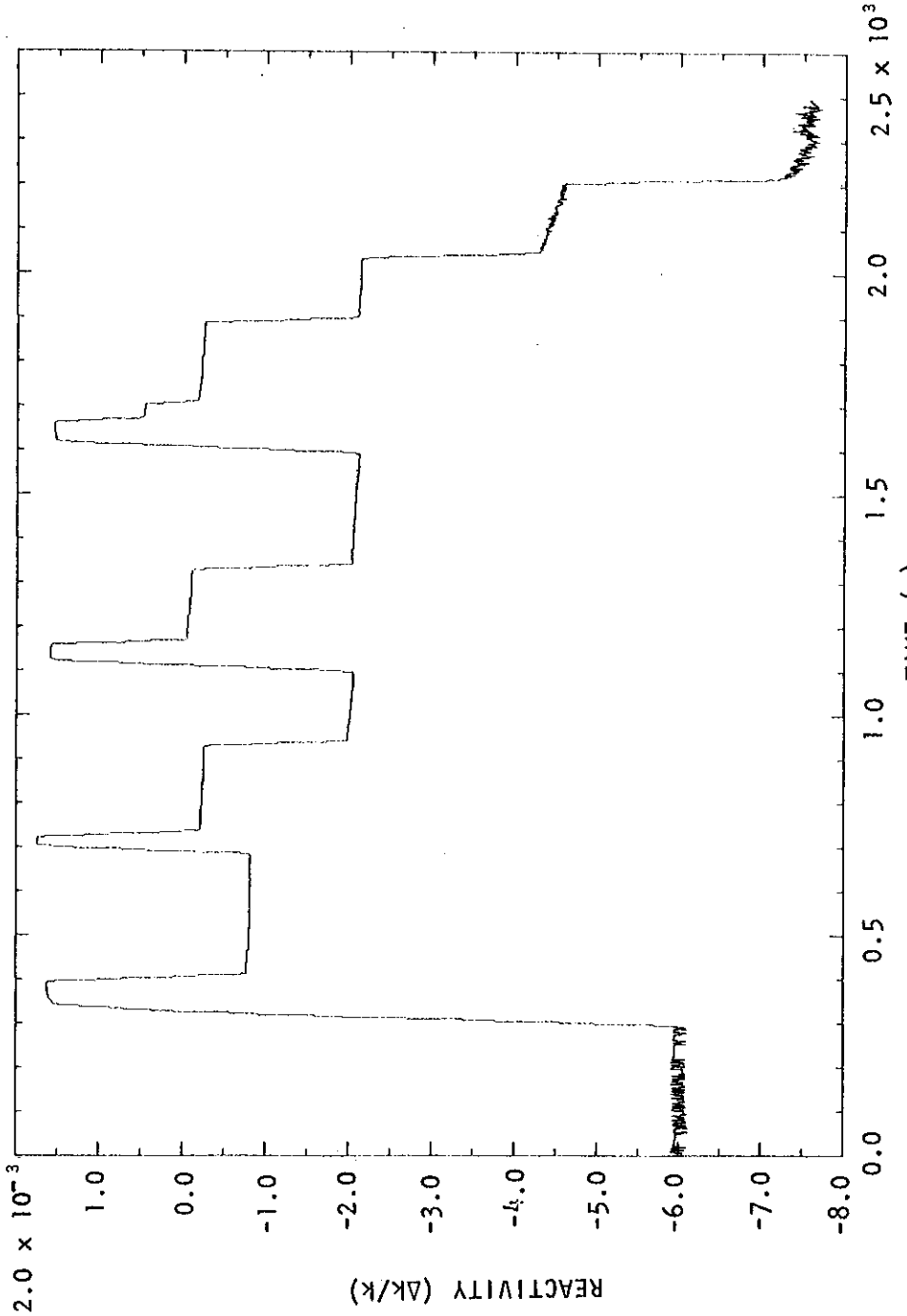


FIGURE 4. REACTIVITY CALCULATED FROM THE POWER PROFILE OF FIGURE 2 BUT WITH A SOURCE STRENGTH 17% GREATER THAN THAT USED IN FIGURE 3.

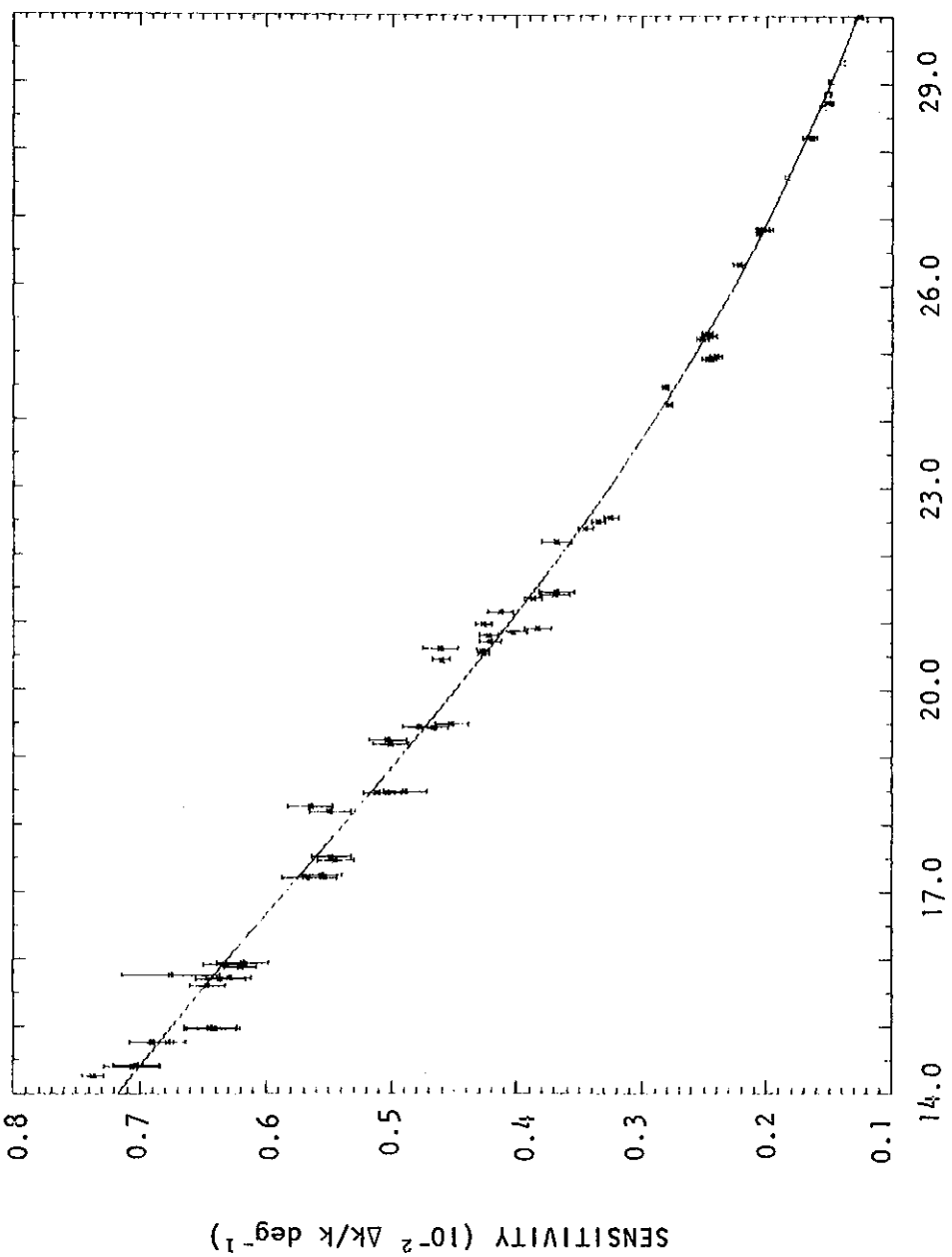


FIGURE 5. RESULTS OF THE INVERSE KINETICS DETERMINATION OF THE CCA SENSITIVITY. ALL DATA HAVE BEEN ADJUSTED TO A CORE MASS OF 3.2 kg USING THE DERIVED MASS CORRECTION FACTOR. THE LINE IS THE CCA SENSITIVITY OBTAINED BY THE LEAST SQUARES

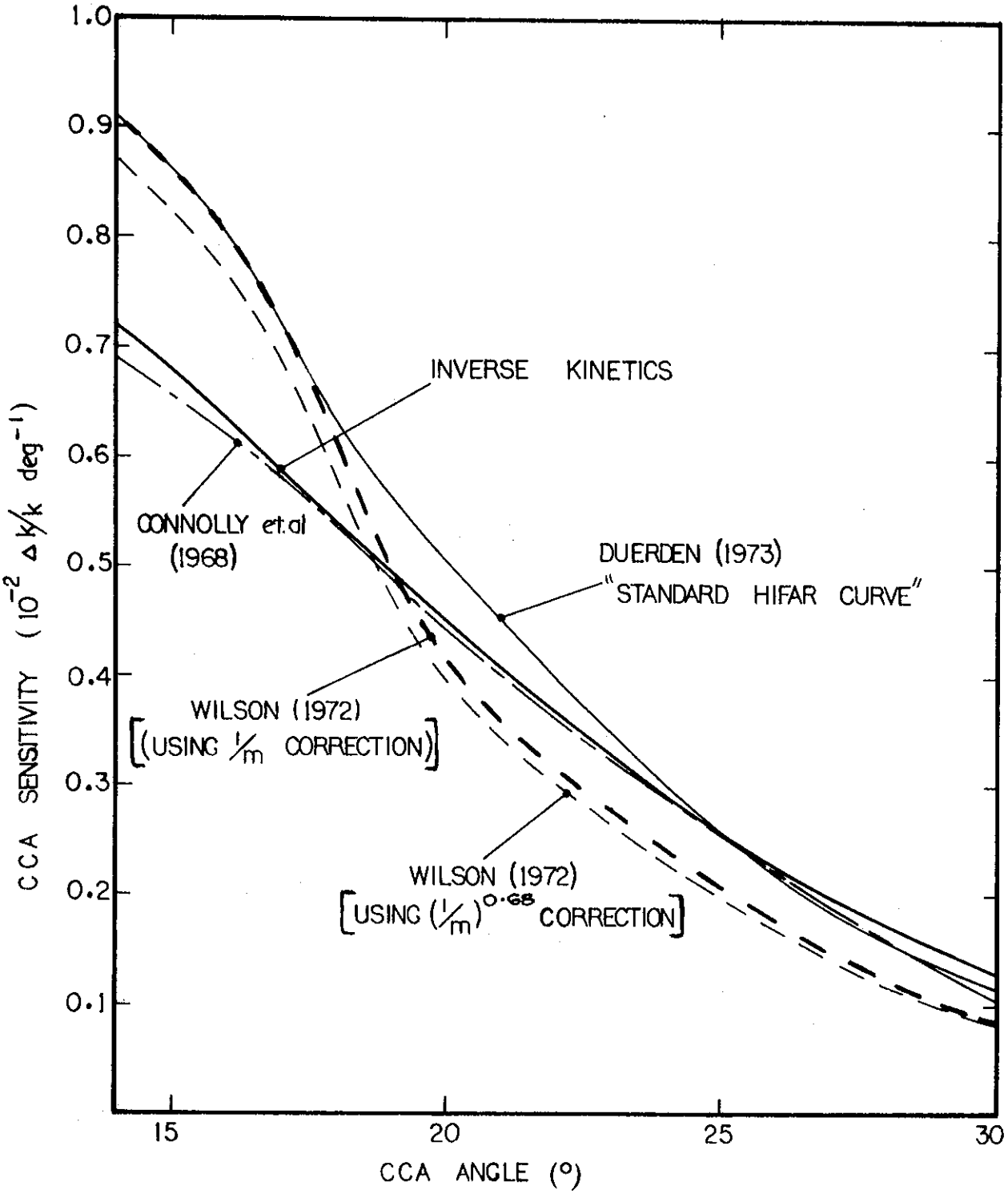


FIGURE 6. CCA SENSITIVITY MEASURED BY INVERSE KINETICS COMPARED WITH PREVIOUS RESULTS

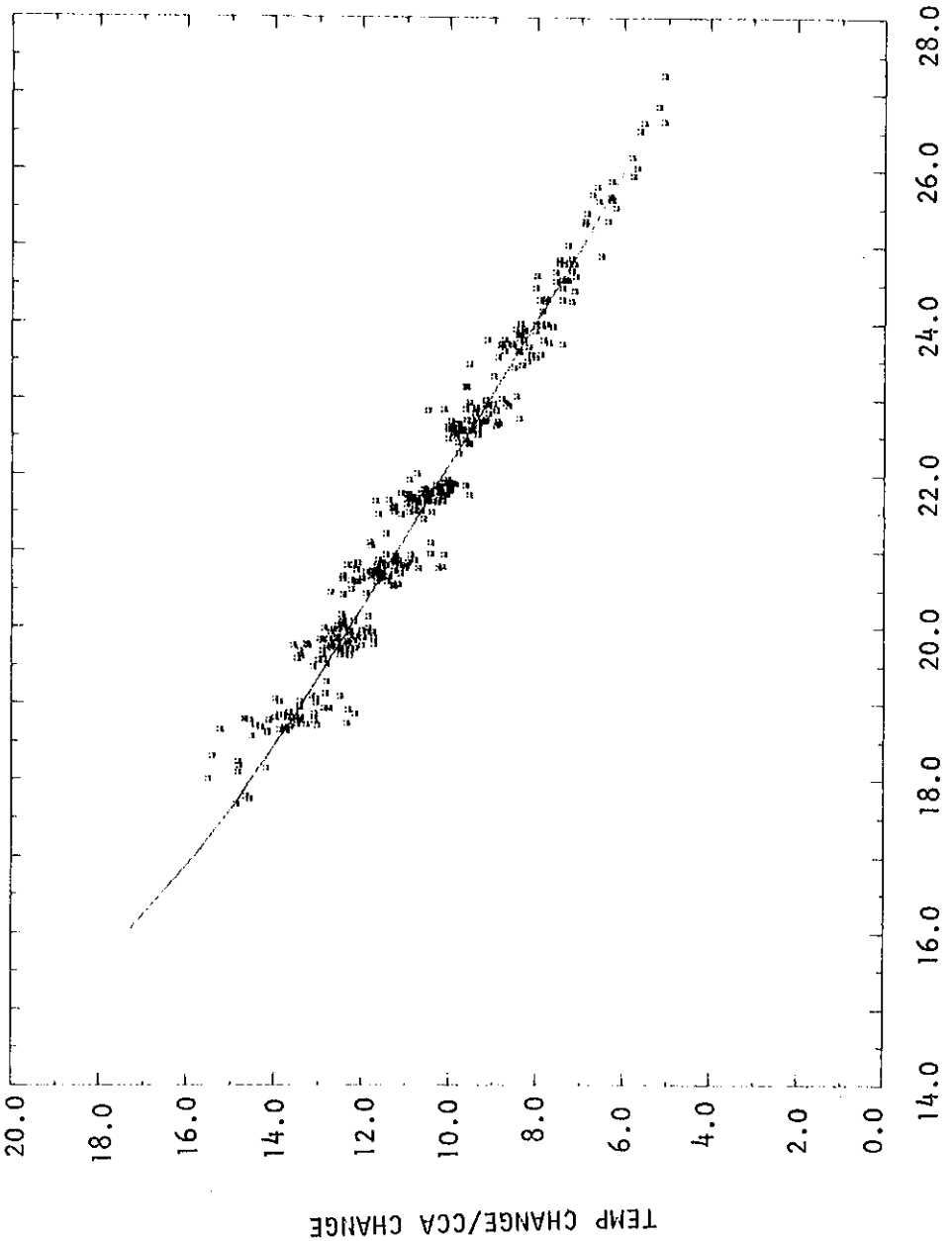


FIGURE 7. TEMPERATURE CALIBRATION DATA FROM THE PREVIOUS SIX YEARS. THE RATIO OF THE CHANGE IN TEMPERATURE TO THE BALANCING CCA MOVEMENT IS PLOTTED AGAINST CCA ANGLE