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RESEARCH ESTABLISHMENT

LUCAS HEIGHTS RESEARCH LABORATORIES

RESULTS OF PIPE BEND ANALYSIS
PART III: FLEXIBILITY FACTORS OF FLANGED-PIPE BENDS

by

J.F. WHATHAM

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ABSTRACT

A matrix of flexibility factors is defined for pipe bends under in-plane and out-of-plane loading, and numerical values are presented for a wide range of 90 and 180° flange-ended pipe bends; the flexibility factors for the 90° bends are plotted against the pipe-bend characteristic.

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PIPES; FLANGES; FLEXIBILITY

CONTENTS

1. INTRODUCTION

The analysis by thin shell theory of flanged pipe elbows under in-plane and out-of-plane loading has been published [Whatham 1982]. The objective of the present report is to give flexibility factors from that analysis for force or moment loading in the three coordinate directions, both to assist pipework designers and to provide bench mark solutions for comparison with solutions by numerical methods.

2. FLEXIBILITY MATRIX

A flanged pipe bend is shown schematically in Figure 1, where ϕ' is the bend angle, R is the radius of curvature, t is the wall thickness, and r is the mean pipe radius. Forces F_x, F_y, F_z and moments M_x, M_y, M_z may act on the bend, causing translations δ_x, δ_y, δ_z and rotations γ_x, γ_y, γ_z.

The following assumptions are made in the analysis:

(i) the pipe wall is thin ($t/r < 0.3$);

(ii) normal stresses through the wall are negligible;

(iii) fibres through the wall which are normal to the wall surface before loading remain normal after loading and unchanged in length; and

(iv) the flanges are infinitely stiff.

As the pipe cross-section has lateral symmetry, in-plane and out-of-plane loading are decoupled systems; the only deflections produced by F_x, F_y, M_z are δ_x, δ_y, γ_z, and the only deflections produced by M_x, M_y, F_z are γ_x, γ_y, δ_z.

The displacements from each force or moment acting alone were calculated by the linear shell theory of Novozhilov [1970], giving a matrix of flexibilities from which the total deflection for combined loading could be obtained by superposition. For example, in in-plane loading, the total deflection δ_x produced by the combined action of forces F_x, F_y and moment M_z is

$$\delta_x = K_1 F_x + K_2 F_y + K_3 M_z \quad ,$$

where K_1, K_2, K_3 are flexibilities calculated when F_x, F_y, M_z are applied separately.

Flexibilities were first calculated from beam theory, omitting displacements due to shear or tension; the thin shell theory flexibility matrices of Table 1 were then derived by the application of appropriate flexibility factors f_{xx}, f_{xy}, etc. Note (a) that the matrices for in-plane and out-of-plane loading are symmetrical about the leading diagonal, thus bearing out the reciprocal theorem, and (b) that for out-of-plane loading the beam theory gives displacements by both bending and torsion, but that the flexibility factors are only applied to the bending component. In the case of 180° bends, it was found that

$$\gamma_x/M_y = 0, \text{ and}$$

$$\gamma_y/F_z = (\Delta - 2(1 + \nu))R^2/EI \quad ,$$

where Δ is a small discrepancy owing to the exclusion of tension and shear from the beam theory deflections; for bend angles from 179 to 180°, the following relationship was adopted:

$$f_{\overline{yz}} = \Delta \quad .$$

Flexibility factors are plotted against bend characteristic $h(=Rt/r^2)$ in Figures 2 and 3 for 90° flange-ended pipe bends, and numerical values of these factors are presented in Appendix A to enable the curves to be reconstructed; flexibility factors for 180° flange-ended bends are also tabulated in Appendix A. The pipe bends considered here range from $h = 1.25t/r$ to $h = 2$, when all flexibility factors approximately equal those for in-plane bending without end effects (Table A20)

$$f^e_{zz} = 1.17 \text{ for } \nu = 0.3 \quad .$$

For $h > 2$, the following relationship derived from the work of von Kármán [1911] applies:

$$f^e_{zz} = 1 + 3(1 - \nu^2)/(4h^2) \quad ,$$

$$f^e_{zz} = 1 + 0.68/h^2 \text{ for } \nu = 0.3$$

The computer program package BENDPAC used in the analysis is written in FORTRAN IV and Assembler for an IBM3031 computer and is available from either the Australian Atomic Energy Commission or the National Energy Software Center, Argonne, USA.

3. REFERENCES

Kármán, Th. von [1911] - Ueber die Formaenderung duennwandiger Rohre. Z Ver. Dtsch. Ing., 55(45)1889

Novozhilov, V.V. [1970] - Thin Shell Theory. 2nd Augmented and Revised Edition, Wolters-Noordhoff, Gröningen, The Netherlands.

Whatham, J.F. [1982] - Analysis of Circular Pipe Bends with Flanged Ends. J. Nucl. Eng. Des., 71(3)1.

TABLE 1
FLEXIBILITY MATRICES

	In-plane		
	F_xR^2/EI	F_yR^2/EI	M_zR/EI
δ_x/R =	$f_{xx}(\phi'+B-2S)$	$f_{xy}(1-C-D)$	$f_{xz}(S-\phi')$
δ_y/R =	$f_{yx}(1-C-D)$	$f_{yy}A$	$f_{yz}(C-1)$
γ_z =	$f_{zx}(S-\phi')$	$f_{zy}(C-1)$	$f_{zz}\phi'$
	Out-of-plane		
	F_zR^2/EI	M_yR/EI	M_xR/EI
δ_z/R =	$f_{\overline{zz}}A+P(\phi'+B-2S)$	$f_{\overline{zy}}D+P(1-C-D)$	$f_{\overline{zx}}A+P(B-S)$
γ_y =	$f_{\overline{yz}}D+P(1-C-D)$	$f_{\overline{yy}}B+PA$	$f_{\overline{yx}}D-PD$
γ_x =	$f_{\overline{xz}}A+P(B-S)$	$f_{\overline{xy}}D-PD$	$f_{\overline{xx}}A+PB$

$S = \sin\phi'$, $A = (\phi'-SC)/2$, $D = SS/2$, $P = 1+\nu$,

$C = \cos\phi'$, $B = (\phi'+SC)/2$, ν = Poisson's ratio,

E = Young's modulus, $I = \pi r^3 t$.

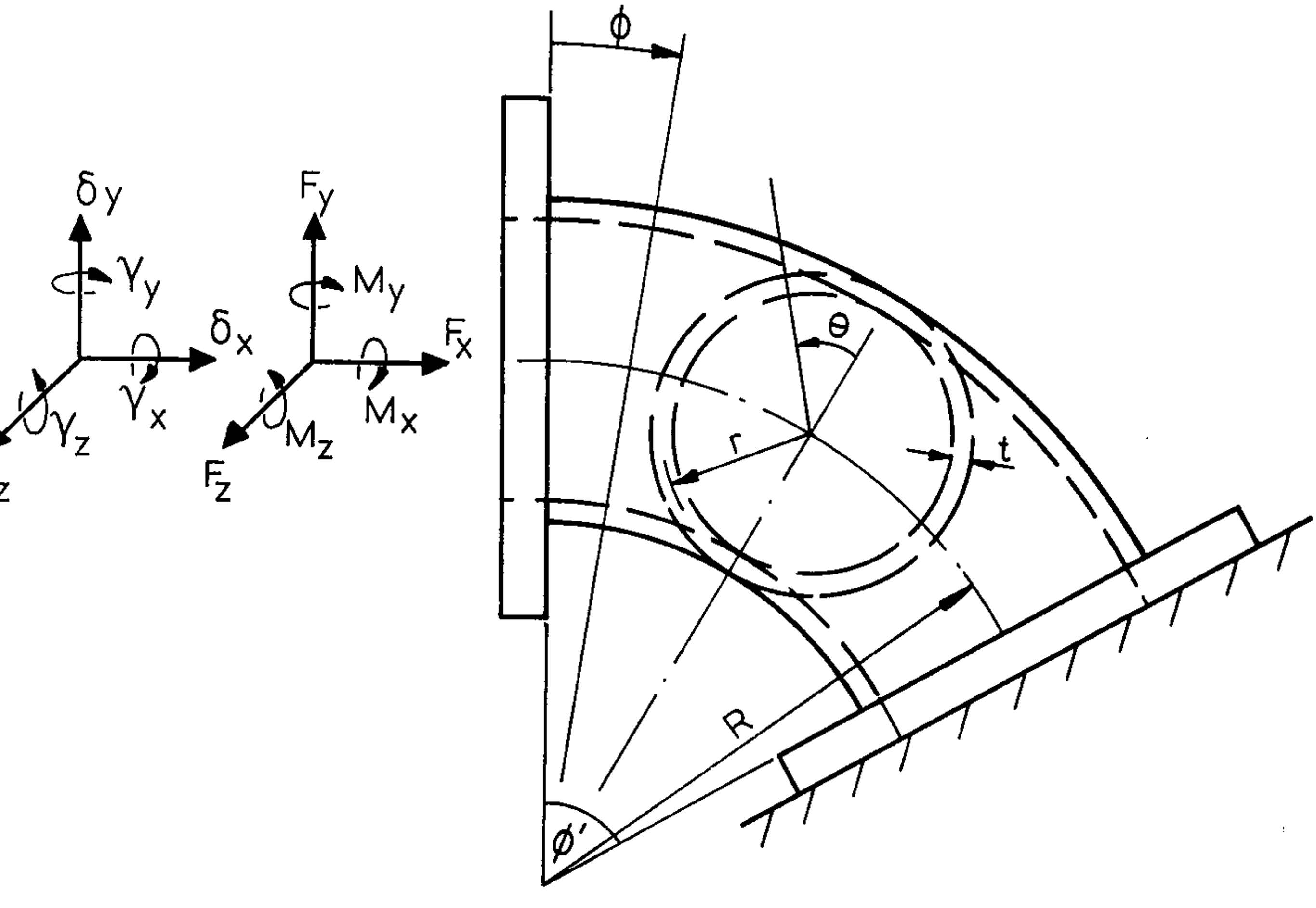

FIGURE 1. PIPE BEND CONFIGURATION

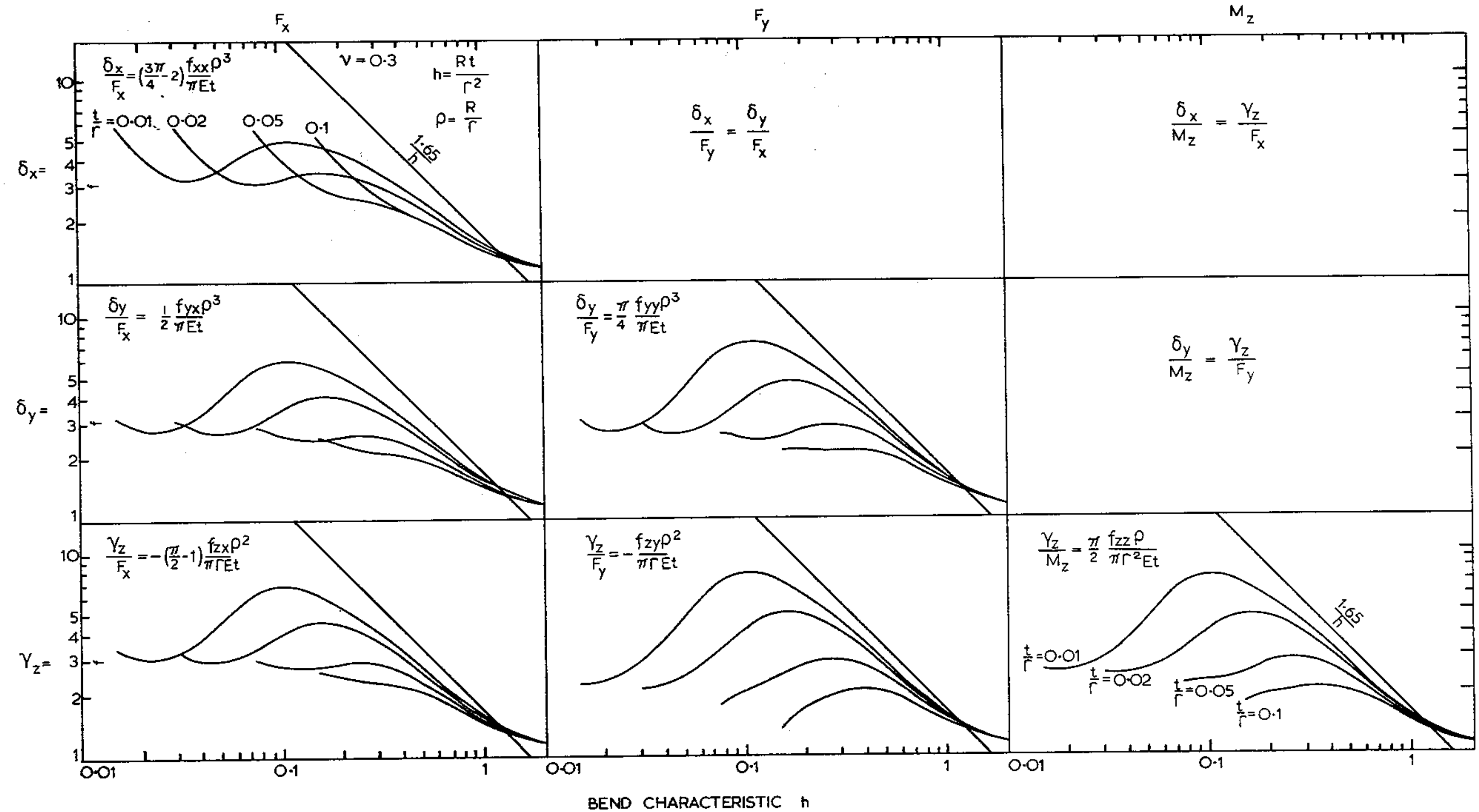

FIGURE 2. IN-PLANE FLEXIBILITY FACTORS FOR 90° FLANGED BENDS

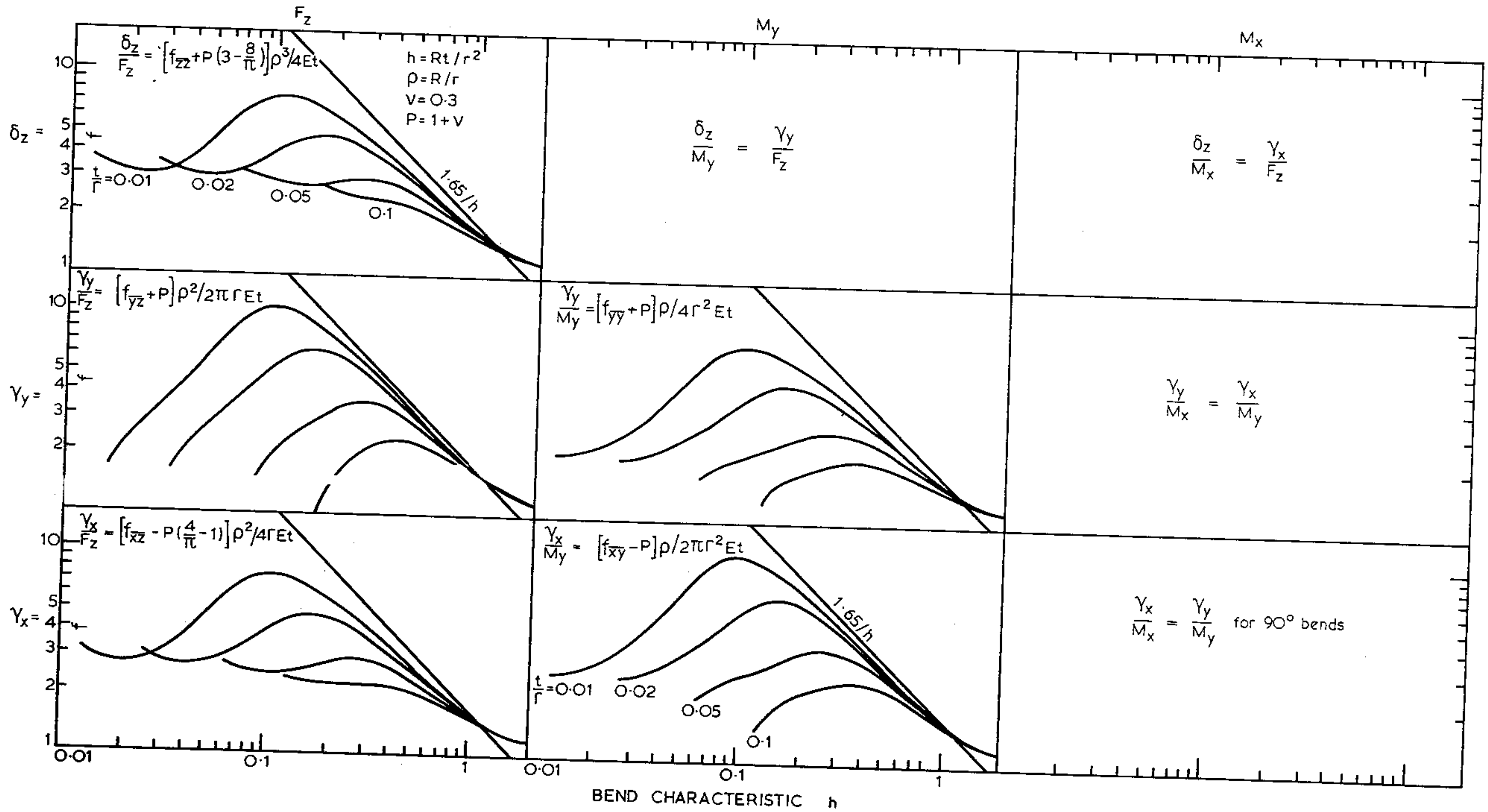

FIGURE 3. OUT-OF-PLANE FLEXIBILITY FACTORS FOR 90° FLANGED BENDS

APPENDIX A
FLEXIBILITY FACTORS FOR FLANGED PIPE BENDS

Parameters of pipe bends considered:

R/r = 1.25 to $2r/t$

t/r = 0.01, 0.02, 0.05, 0.1

ϕ' = 90° and 180°

ν = 0.3

Flexibility factors for in-plane bending, without end effects, are also given.

TABLE A1

$$\delta_x = f_{xx} \frac{0.113R^3}{r^3Et} F_x$$

$\phi' = 90°$

		f_{xx}		
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	7.23	7.04	6.59	5.92
1.5	5.74	5.64	5.39	5.03
2	4.16	4.09	3.92	3.66
2.5	3.49	3.42	3.24	2.98
3	3.22	3.13	2.90	2.60
4	3.22	3.03	2.62	2.20
5	3.53	3.15	2.50	1.97
6.5	4.13	3.33	2.34	1.73
8	4.57	3.38	2.17	1.56
10	4.81	3.31	1.98	1.42
13	4.74	3.08	1.74	1.28
16	4.51	2.84	1.58	1.20
20	4.18	2.54	1.42	1.14
26	3.71	2.18	1.28	
32	3.30	1.91	1.20	
40	2.86	1.67	1.14	
50	2.44	1.48		
65	2.01	1.31		
80	1.74	1.22		
100	1.52	1.15		
130	1.33			
160	1.23			
200	1.15			

TABLE A2

$$\delta_y = f_{yx} \frac{0.159R^3}{r^3Et} F_x$$

$\phi' = 90°$

	f_{yx}			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	3.45	3.24	2.78	2.34
1.5	3.14	3.05	2.82	2.50
2	2.79	2.73	2.56	2.31
2.5	2.73	2.67	2.47	2.19
3	2.83	2.74	2.47	2.12
4	3.31	3.07	2.55	2.02
5	3.98	3.47	2.60	1.91
6.5	5.01	3.90	2.53	1.72
8	5.72	4.03	2.37	1.57
10	6.05	3.94	2.13	1.42
13	5.87	3.60	1.85	1.28
16	5.47	3.24	1.64	1.20
20	4.93	2.82	1.46	1.14
26	4.23	2.35	1.30	
32	3.68	2.03	1.21	
40	3.11	1.74	1.14	
50	2.60	1.52		
65	2.10	1.33		
80	1.80	1.23		
100	1.55	1.15		
130	1.35			
160	1.24			
200	1.16			

TABLE A3

$$\gamma_z = -f_{zx} \frac{0.182R^2}{r^3ET} F_x$$

$\phi' = 90°$

	f_{zx}			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	3.77	3.57	3.11	2.76
1.5	3.38	3.28	3.00	2.62
2	3.09	3.01	2.78	2.44
2.5	3.09	2.99	2.73	2.36
3	3.25	3.12	2.76	2.31
4	3.87	3.56	2.88	2.20
5	4.70	4.04	2.92	2.05
6.5	5.94	4.52	2.80	1.81
8	6.74	4.63	2.58	1.62
10	7.02	4.43	2.27	1.45
13	6.64	3.94	1.92	1.29
16	6.05	3.48	1.68	1.21
20	5.33	2.97	1.48	1.14
26	4.49	2.43	1.31	
32	3.85	2.08	1.22	
40	3.22	1.77	1.15	
50	2.67	1.53		
65	2.14	1.34		
80	1.82	1.23		
100	1.56	1.15		
130	1.35			
160	1.24			
200	1.16			

TABLE A4

$$\delta_y = f_{yy} \frac{0.25R^3}{r^3Et} F_y$$

$\phi' = 90°$

		f_{yy}		
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	3.69	3.28	2.36	1.75
1.5	3.04	2.93	2.60	2.13
2	2.66	2.61	2.43	2.16
2.5	2.70	2.63	2.43	2.13
3	2.90	2.80	2.51	2.14
4	3.60	3.33	2.72	2.11
5	4.53	3.91	2.84	2.02
6.5	5.95	4.52	2.81	1.82
8	6.91	4.73	2.62	1.64
10	7.36	4.61	2.33	1.47
13	7.07	4.15	1.98	1.31
16	6.48	3.67	1.73	1.22
20	5.71	3.12	1.51	1.15
26	4.78	2.53	1.33	
32	4.07	2.14	1.23	
40	3.37	1.81	1.15	
50	2.76	1.55		
65	2.19	1.35		
80	1.85	1.24		
100	1.58	1.16		
130	1.36			
160	1.24			
200	1.16			

TABLE A5

$$\gamma_z = -f_{zy} \frac{0.318R^2}{r^3Et} F_y$$

$\phi' = 90°$

	f_{zy}			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	2.48	2.11	1.28	0.84
1.5	2.27	2.16	1.83	1.38
2	2.37	2.30	2.10	1.78
2.5	2.64	2.56	2.31	1.96
3	2.98	2.86	2.51	2.07
4	3.89	3.57	2.85	2.13
5	4.99	4.26	3.01	2.05
6.5	6.59	4.95	2.97	1.85
8	7.65	5.15	2.76	1.66
10	8.08	4.97	2.43	1.48
13	7.65	4.41	2.03	1.31
16	6.93	3.85	1.76	1.22
20	6.04	3.25	1.53	1.14
26	5.00	2.60	1.34	
32	4.22	2.19	1.23	
40	3.47	1.83	1.15	
50	2.82	1.57		
65	2.22	1.36		
80	1.87	1.24		
100	1.59	1.16		
130	1.36			
160	1.25			
200	1.16			

TABLE A6

$$\gamma_z = f_{zz} \frac{0.5R}{r^3Et} M_z$$

$\phi' = 90°$

R/r	f_{zz} t/r = 0.01	0.02	0.05	0.1
1.25	2.95	2.64	1.95	1.54
1.5	2.68	2.57	2.25	1.83
2	2.63	2.56	2.34	2.02
2.5	2.80	2.71	2.46	2.10
3	3.08	2.95	2.60	2.15
4	3.88	3.56	2.86	2.16
5	4.89	4.18	2.98	2.05
6.5	6.36	4.79	2.91	1.84
8	7.32	4.96	2.69	1.65
10	7.69	4.78	2.37	1.47
13	7.28	4.24	1.99	1.31
16	6.61	3.72	1.73	1.21
20	5.78	3.15	1.51	1.14
26	4.81	2.54	1.33	
32	4.09	2.15	1.23	
40	3.38	1.81	1.15	
50	2.77	1.55		
65	2.19	1.35		
80	1.85	1.24		
100	1.58	1.16		
130	1.36			
160	1.24			
200	1.16			

TABLE A7

$$\gamma_z = (f_{zz} + 0.59)\,\frac{0.25R^3}{r^3Et}\,F_z$$

$\phi' = 90°$

	f_{zz}			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	3.63	3.52	3.17	2.73
1.5	3.37	3.30	3.08	2.75
2	3.07	3.01	2.82	2.54
2.5	3.04	2.96	2.74	2.41
3	3.19	3.07	2.76	2.35
4	3.82	3.53	2.88	2.24
5	4.71	4.06	2.96	2.10
6.5	6.09	4.63	2.87	1.87
8	7.02	4.81	2.67	1.67
10	7.44	4.66	2.36	1.48
13	7.11	4.17	1.99	1.32
16	6.50	3.68	1.73	1.22
20	5.73	3.13	1.52	1.15
26	4.79	2.54	1.33	
32	4.08	2.14	1.23	
40	3.37	1.81	1.15	
50	2.76	1.56		
65	2.19	1.35		
80	1.85	1.24		
100	1.58	1.16		
130	1.36			
160	1.24			
200	1.16			

TABLE A8

$$\gamma_y = (f_{\overline{yz}} + 1.3)\ \frac{0.159R^2}{r^3Et}\ F_z$$

$\phi' = 90°$

	$f_{\overline{yz}}$			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	0.92	0.78	0.31	-0.21
1.5	1.51	1.43	1.14	0.70
2	2.30	2.21	1.95	1.56
2.5	2.87	2.76	2.43	1.96
3	3.43	3.26	2.79	2.19
4	4.72	4.28	3.30	2.35
5	6.21	5.22	3.54	2.27
6.5	8.35	6.12	3.49	2.01
8	9.72	6.36	3.19	1.77
10	10.20	6.07	2.74	1.54
13	9.49	5.26	2.21	1.34
16	8.43	4.48	1.87	1.23
20	7.18	3.68	1.60	1.15
26	5.77	2.86	1.37	
32	4.77	2.35	1.25	
40	3.83	1.92	1.16	
50	3.05	1.62		
65	2.35	1.38		
80	1.94	1.26		
100	1.63	1.17		
130	1.38			
160	1.26			
200	1.17			

TABLE A9

$$\gamma_x = (f_{\overline{xz}} - 0.36)\,\frac{0.25R^2}{r^3Et}\,F_z$$

$\phi' = 90°$

	$f_{\overline{xz}}$			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	3.18	3.08	2.74	2.33
1.5	2.89	2.82	2.62	2.30
2	2.73	2.67	2.49	2.21
2.5	2.81	2.73	2.50	2.19
3	3.02	2.90	2.59	2.18
4	3.72	3.43	2.79	2.15
5	4.65	4.00	2.89	2.04
6.5	6.05	4.59	2.84	1.83
8	7.00	4.78	2.64	1.65
10	7.42	4.64	2.34	1.47
13	7.10	4.17	1.98	1.31
16	6.50	3.67	1.73	1.22
20	5.72	3.13	1.51	1.15
26	4.79	2.54	1.33	
32	4.08	2.14	1.23	
40	3.37	1.81	1.15	
50	2.76	1.55		
65	2.19	1.35		
80	1.85	1.24		
100	1.58	1.16		
130	1.36			
160	1.24			
200	1.16			

TABLE A10

$$\gamma_y = (f_{\overline{yy}} + 1.3)\ \frac{0.25R}{r^3Et}\ M_y$$

$$\gamma_x = (f_{\overline{xx}} + 1.3)\ \frac{0.25R}{r^3Et}\ M_x$$

$\phi' = 90°$

	$f_{\overline{yy}}$, $f_{\overline{xx}}$			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	2.01	2.00	1.66	1.26
1.5	2.14	2.08	1.87	1.55
2	2.32	2.25	2.07	1.79
2.5	2.55	2.47	2.24	1.92
3	2.84	2.72	2.41	2.00
4	3.62	3.33	2.69	2.04
5	4.59	3.94	2.83	1.97
6.5	6.02	4.56	2.80	1.79
8	6.97	4.76	2.62	1.62
10	7.40	4.63	2.33	1.45
13	7.09	4.16	1.97	1.30
16	6.49	3.67	1.72	1.21
20	5.72	3.12	1.51	1.14
26	4.79	2.53	1.33	
32	4.07	2.14	1.22	
40	3.37	1.81	1.15	
50	2.76	1.55		
65	2.19	1.35		
80	1.85	1.24		
100	1.58	1.16		
130	1.36			
160	1.24			
200	1.16			

TABLE A11

$$\gamma_x = (f_{\overline{xy}} - 1.3)\,\frac{0.159R}{r^3Et}\,M_y$$

$\phi' = 90°$

	$f_{\overline{xy}}$			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	2.61	2.48	2.01	1.47
1.5	2.68	2.60	2.31	1.87
2	2.95	2.86	2.61	2.21
2.5	3.28	3.17	2.85	2.39
3	3.71	3.54	3.08	2.48
4	4.87	4.44	3.47	2.51
5	6.31	5.32	3.65	2.38
6.5	8.41	6.18	3.55	2.08
8	9.76	6.40	3.23	1.82
10	10.23	6.09	2.77	1.58
13	9.51	5.27	2.23	1.37
16	8.44	4.49	1.88	1.25
20	7.18	3.68	1.60	1.17
26	5.78	2.86	1.38	
32	4.77	2.35	1.25	
40	3.83	1.93	1.17	
50	3.05	1.62		
65	2.35	1.38		
80	1.94	1.26		
100	1.63	1.17		
130	1.38			
160	1.26			
200	1.17			

TABLE A12

$$\delta_x = f_{xx} \frac{1.5R^3}{r^3Et} F_x$$

$\phi' = 180°$

	f_{xx}			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	7.56	6.77	5.12	3.64
1.5	7.92	7.09	5.43	3.95
2	11.1	8.94	5.86	3.93
2.5	14.7	10.3	5.87	3.69
3	16.7	10.6	5.62	3.40
4	17.0	10.0	4.97	2.87
5	15.9	9.12	4.39	2.45
6.5	14.1	7.93	3.68	2.03
8	12.6	6.99	3.15	1.75
10	11.0	6.01	2.62	1.53
13	9.16	4.95	2.12	1.33
16	7.87	4.17	1.81	1.23
20	6.62	3.43	1.55	1.15
26	5.32	2.70	1.35	
32	4.42	2.24	1.24	
40	3.58	1.86	1.16	
50	2.89	1.58		
65	2.25	1.36		
80	1.88	1.25		
100	1.60	1.16		
130	1.37			
160	1.25			
200	1.16			

TABLE A13

$$\delta_y = f_{yx} \frac{0.637R^3}{r^3Et} F_x$$

$$\gamma_z = -f_{zy} \frac{0.637R^2}{r^3Et} F_y$$

$\phi' = 180°$

	f_{yx}, f_{zy}			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	8.47	7.46	5.21	3.07
1.5	9.84	8.67	6.33	4.25
2	16.5	12.9	7.86	4.84
2.5	23.7	15.8	8.24	4.69
3	27.6	16.6	7.92	4.31
4	27.8	15.3	6.78	3.50
5	25.0	13.4	5.76	2.88
6.5	21.0	11.0	4.59	2.27
8	17.9	9.32	3.77	1.90
10	14.9	7.69	3.02	1.61
13	11.9	6.04	2.33	1.38
16	9.82	4.92	1.93	1.25
20	7.98	3.91	1.62	1.16
26	6.19	2.96	1.38	
32	5.00	2.40	1.26	
40	3.95	1.95	1.17	
50	3.11	1.63		
65	2.37	1.39		
80	1.95	1.26		
100	1.63	1.17		
130	1.39			
160	1.26			
200	1.17			

TABLE A14

$$\gamma_z = -f_{zx} \frac{R^2}{r^3 Et} F_x$$

$$\gamma_z = f_{zz} \frac{R}{r^3 Et} M_z$$

$\phi' = 180°$

	f_{zx}, f_{zz}			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	9.35	8.34	6.21	4.27
1.5	10.3	9.08	6.75	4.73
2	15.1	11.9	7.41	4.71
2.5	20.0	13.6	7.31	4.33
3	22.4	13.7	6.83	3.89
4	21.8	12.3	5.77	3.15
5	19.6	10.8	4.93	2.62
6.5	16.6	9.06	4.01	2.12
8	14.4	7.79	3.36	1.80
10	12.3	6.56	2.75	1.55
13	10.0	5.29	2.18	1.35
16	8.47	4.40	1.84	1.24
20	7.02	3.57	1.57	1.16
26	5.57	2.77	1.36	
32	4.59	2.28	1.24	
40	3.69	1.88	1.16	
50	2.95	1.59		
65	2.29	1.37		
80	1.90	1.25		
100	1.61	1.16		
130	1.37			
160	1.25			
200	1.16			

TABLE A15

$$\delta_y = f_{yy} \frac{0.5R^3}{r^3Et} F_y$$

$\phi' = 180°$

	f_{yy}			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	8.60	7.69	5.60	3.42
1.5	9.23	8.27	6.32	4.50
2	15.6	12.3	7.75	4.98
2.5	23.5	15.8	8.32	4.86
3	28.4	17.0	8.12	4.48
4	29.5	16.0	7.05	3.65
5	26.7	14.1	5.98	2.99
6.5	22.4	11.6	4.75	2.34
8	19.0	9.73	3.88	1.95
10	15.6	7.97	3.08	1.64
13	12.3	6.20	2.37	1.39
16	10.1	5.02	1.95	1.26
20	8.17	3.96	1.64	1.17
26	6.29	2.99	1.39	
32	5.06	2.41	1.26	
40	3.98	1.96	1.17	
50	3.12	1.64		
65	2.38	1.39		
80	1.96	1.26		
100	1.64	1.17		
130	1.39			
160	1.26			
200	1.17			

TABLE A16

$$\delta_z = (f_{\overline{zz}} + 3.9)\,\frac{0.5R^3}{r^3Et}\,F_z$$

$\phi' = 180°$

	$f_{\overline{zz}}$			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	9.47	8.38	6.00	3.92
1.5	11.8	10.3	7.43	4.98
2	19.2	14.9	8.98	5.43
2.5	27.2	18.0	9.21	5.13
3	31.5	18.7	8.70	4.62
4	31.1	16.9	7.30	3.67
5	27.7	14.5	6.10	2.98
6.5	22.8	11.8	4.79	2.31
8	19.2	9.83	3.89	1.92
10	15.8	8.02	3.08	1.61
13	12.4	6.32	2.36	1.37
16	10.2	5.03	1.95	1.24
20	8.18	3.97	1.63	1.15
26	6.29	2.99	1.38	
32	5.07	2.41	1.25	
40	3.98	1.95	1.16	
50	3.12	1.63		
65	2.38	1.39		
80	1.96	1.26		
100	1.63	1.17		
130	1.39			
160	1.26			
200	1.17			

TABLE A17

$$\gamma_y = (f_{\overline{yz}} + 2.6)\ \frac{0.318R^2}{r^3Et}\ F_z$$

$\phi' = 180°$

	$f_{\overline{yz}}$			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	-1.57	-1.60	-1.67	-1.75
1.5	-1.01	-1.03	-1.07	-1.13
2	-0.54	-0.54	-0.57	-0.61
2.5	-0.34	-0.34	-0.36	-0.39
3	-0.23	-0.24	-0.26	-0.27
4	-0.14	-0.14	-0.15	-0.16
5	-0.09	-0.09	-0.10	-0.11
6.5	-0.05	-0.06	-0.06	-0.07
8	-0.04	-0.04	-0.04	-0.05
10	-0.02	-0.03	-0.03	-0.03
13	-0.01	-0.02	-0.02	-0.02
16	-0.01	-0.01	-0.01	-0.02
20	-0.01	-0.01	-0.01	-0.02
26			-0.01	
32				
40				
50				
65				
80				
100				
130				
160				
200				

TABLE A18

$$\gamma_x = (f_{\overline{xz}} + 1.3)\,\frac{0.5R^2}{r^3Et}\,F_z$$

$$\gamma_x = (f_{\overline{xx}} + 1.3)\,\frac{0.5R}{r^3Et}\,M_x$$

$\phi' = 180°$

	$f_{\overline{xz}}$, $f_{\overline{xx}}$			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	10.0	8.94	6.75	4.61
1.5	12.0	10.5	7.67	5.24
2	19.3	15.0	9.05	5.51
2.5	27.3	18.1	9.24	5.17
3	31.5	18.7	8.72	4.65
4	31.1	16.9	7.30	3.68
5	27.7	14.5	6.10	2.99
6.5	22.8	11.8	4.79	2.32
8	19.2	9.83	3.89	1.93
10	15.8	8.02	3.08	1.62
13	12.4	6.22	2.36	1.38
16	10.2	5.03	1.95	1.25
20	8.18	3.97	1.63	1.16
26	6.29	2.99	1.38	
32	5.07	2.41	1.26	
40	3.98	1.95	1.17	
50	3.12	1.63		
65	2.38	1.39		
80	1.96	1.26		
100	1.63	1.17		
130	1.39			
160	1.26			
200	1.17			

TABLE A19

$$\gamma_y = (f_{\overline{yy}} + 1.3)\,\frac{0.5R}{r^3Et}\,M_y$$

$\phi' = 180°$

	$f_{\overline{yy}}$			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	2.43	2.28	1.90	1.49
1.5	2.58	2.46	2.15	1.74
2	3.23	3.05	2.60	2.09
2.5	4.27	3.80	2.97	2.27
3	5.48	4.52	3.23	2.33
4	7.60	5.51	3.40	2.25
5	8.78	5.85	3.32	2.08
6.5	9.23	5.75	3.03	1.82
8	9.00	5.42	2.72	1.63
10	8.41	4.93	2.36	1.45
13	7.48	4.27	1.98	1.30
16	6.68	3.72	1.72	1.21
20	5.81	3.15	1.51	1.14
26	4.82	2.54	1.33	
32	4.09	2.14	1.22	
40	3.38	1.81	1.15	
50	2.77	1.55		
65	2.19	1.35		
80	1.85	1.24		
100	1.58	1.16		
130	1.36			
160	1.24			
200	1.16			

TABLE A20

$$\gamma_z = f^e_{zz} \frac{\phi' R}{180 r^3 E t} M_z$$

For ϕ' degree bend without end effects

	f^e_{zz}			
R/r	t/r = 0.01	0.02	0.05	0.1
1.25	132	65.8	26.1	12.9
1.5	110	54.9	21.9	10.9
2	82.6	41.3	16.5	8.21
2.5	66.1	33.0	13.2	6.55
3	55.1	27.5	11.0	5.43
4	41.3	20.7	8.26	3.99
5	33.0	16.5	6.59	3.14
6.5	25.4	12.7	5.00	2.39
8	20.7	10.3	3.99	1.96
10	16.5	8.27	3.13	1.64
13	12.7	6.33	2.38	1.39
16	10.3	5.09	1.96	1.26
20	8.27	3.99	1.64	1.17
26	6.33	3.00	1.39	
32	5.09	2.42	1.26	
40	3.99	1.96	1.17	
50	3.13	1.64		
65	2.38	1.39		
80	1.96	1.26		
100	1.64	1.17		
130	1.39			
160	1.26			
200	1.17			

$$f^e_{zz} = 1 + 0.68/h^2 \text{ for } h > 2$$