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LUCAS HEIGHTS RESEARCH LABORATORIES

RESULTS OF PIPE BEND ANALYSIS
PART I: END EFFECT DECAY CONSTANTS

by

J.F. WHATHAM

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ABSTRACT

Numerical values of decay constants are given for end effects on curved and straight pipes subjected to both in-plane and out-of-plane loading. Calculations are based on linear thin shell theory.

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PIPES; EIGENVALUES; FLANGES

CONTENTS

1. INTRODUCTION

This report presents tabulated values of decay constants, or eigenvalues, for end effects on a range of curved pipes subjected to in-plane or out-of-plane loading. When used in conjunction with the system equations, the eigenvalues will enable solutions to be obtained to problems involving these pipe bends with any end conditions or, by extension, any surface loading. The calculations are based on the thin shell theory of Novozhilov [1970], this being a consistent linear theory in which equilibrium is satisfied and no forces or moments are generated by rigid body movements.

2. SOLUTION UNKNOWNS

The pipe is represented by its middle surface, that is, an imaginary surface mid-way between the inner and outer surfaces; Figure 1 shows a short segment of the middle surface of a curved pipe with one end attached to a flange.

The forces T_η, P, Q and moment M_η per unit length of circumference act on the cross-section, resulting in the displacements u, v, w and a rotation ψ. These are the eight unknowns to be determined; they may be grouped non-dimensionally as follows:

$$\xi_1 = \{u/r,\ w/r,\ M^*_\eta,\ T^*_\eta\}$$

$$\xi_2 = \{v/r,\ \psi\ ,P^*,\ Q^*\}$$

where

$$T^*_\eta,\ P^*,\ Q^* = (T_\eta,\ P,\ Q)/Et,$$

$$M^*_\eta = M_\eta/rEt, \text{ and}$$

$$E = \text{Young's modulus}$$

Now let

$$\xi_1 = \xi_1^e + \xi_1^f$$

$$\xi_2 = \xi_2^e + \xi_2^f \qquad (1)$$

where ξ_1^e and ξ_2^e are endless pipe bend solutions for a particular end loading [Whatham 1981] and ξ_1^f and ξ_2^f are the flange effects [Whatham 1982]. Then, by definition, flange effects represent self-equilibrating load distributions over the pipe cross-section and therefore, by Saint-Venant's principle, decay with distance from the end of the pipe.

3. FLANGE EFFECTS

The distributions of the eight components of the vectors ξ_1^f and ξ_2^f around the pipe cross-section are given in Appendix A for in-plane and out-of-plane loadings. The coefficients given in Appendix A are not constant but decay with distance $\ell(=\eta r)$ from a flanged end; to be more exact, each coefficient is the sum of a series of decaying coefficients. For example:

$$u_n = \sum_{j=1}^{J} C_j \, u_{nj} \, e^{-\Omega_j \eta} \qquad (2)$$

where

$$J = \begin{cases} 2N & \text{(in-plane)} \\ 2N-1 & \text{(out-of-plane)}, \end{cases}$$

$\Omega_j = a_j \pm ib_j$ = eigenvalue (decay constant),

$u_{nj} = u_{nj}^R \pm iu_{nj}^I$ = eigenvector component, and

$C_j = CA_j \pm iCB_j$ = eigenvector coefficient.

Substituting the complex forms of the eigenvalue, vector and coefficient into Equation (2),

$$u_n = \sum_{j=1}^{J} CA_j \, (u_{nj}^R \, c + u_{nj}^I \, s) + CB_j \, (u_{nj}^R \, s - u_{nj}^I \, c) \qquad (3)$$

where

$$c = e^{-a_j \eta} \cos b_j \eta, \text{ and}$$

$$s = e^{-a_j \eta} \sin b_j \eta$$

4. EIGENVALUE DETERMINATION

The eigenvalues and eigenvectors depend on the pipe bend curvature, diameter, wall thickness and Poisson's ratio, regardless of bend angle, end conditions and loading, apart from knowing whether the loading is in-plane or out-of-plane. They are determined from the following equations shown in matrix form:

$$[L_1] \left\{\xi_1^f\right\} - \frac{\partial}{\partial \eta}\left\{\xi_2^f\right\} = 0 \qquad (4)$$

$$[L_2] \left\{\xi_2^f\right\} - \frac{\partial}{\partial \eta}\left\{\xi_1^f\right\} = 0$$

where L_1, L_2 are presented in Table 1. Assuming that the flange effects decay as

$$\begin{Bmatrix} \xi_1^f \\ \xi_2^f \end{Bmatrix} = e^{\Omega \eta} \begin{Bmatrix} \xi_1^f(\theta) \\ \xi_2^f(\theta) \end{Bmatrix}$$

then

$$\begin{bmatrix} 0 & L_2 \\ L_1 & 0 \end{bmatrix} \begin{Bmatrix} \xi_1^f \\ \xi_2^f \end{Bmatrix} = \Omega \begin{Bmatrix} \xi_1^f \\ \xi_2^f \end{Bmatrix} \qquad (5)$$

Substituting the expansions from Appendix A and expanding by Fourier analysis give 8N + 3 equations (in-plane) or 8N - 1 equations (out-of-plane). The eigenvalues Ω were determined by the computer code EB06AD [Hopper 1973] and a typical eigenvalue distribution is plotted in Figure 2.

Because of symmetry, only the eigenvalues of one quadrant are plotted in Figure 3 which is for the in-plane loading case; eigenvalues for out-of-plane loading are identical except where the paths fork. Note the effect of series truncation (Appendix A) for curved pipes.

Eigenvalues are tabulated in Appendix B for curved pipes under in-plane and out-of-plane loading using sixteen term truncation and a Poisson's ratio of 0.3; the latter is a typical value for steel and the eigenvalues are insensitive to small variations in it. For straight pipes, the increasing eigenvalues, as listed, correspond to increasing wave number n in the simple modal shapes which vary as sin $n\theta$ or cos $n\theta$ for n = 0 to 16.

Using these eigenvalues, it is relatively easy to obtain the eigenvectors and hence the solution for any end effect on a pipe bend with the dimension ratios listed. The eigenvectors can also be used to generate Green's functions for the effect of concentrated loads applied around a pipe cross-section and hence to solve problems involving an arbitrary distribution of surface load.

For the straight pipes, note that the first and second mode shapes (n = 0 and 1) have approximately equal real and imaginary components of

$$(3(1-\nu^2)/\gamma)^{0.25}$$

whereas the next shape (n = 2), representing a flattened cross-section, has an additional eigenvalue with approximately equal components of

$$(3\gamma(1-\nu^2))^{0.25}$$

which indicates a significantly slower decay rate along the pipe.

A computer program package BENDPAC, written in FORTRAN IV and ASSEMBLER for an IBM3031 computer, was used to calculate the eigenvalues and is available from either the Australian Atomic Energy Commission or the National Energy Software Center, Argonne National Laboratory, USA.

5. CONCLUSION

Decay constants (eigenvalues) tabulated for a range of circular pipe bends will enable solutions to be derived for these pipe bends when subjected to any end condition or suface loading.

6. REFERENCES

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Whatham, J.F. [1981] - Thin shell equations for circular pipe bends. J. Nucl. Eng. Des., 65(1)77.

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TABLE 1
MATRIX OPERATORS L_1 AND L_2

$$[L_1]$$

	$\frac{u^f}{r}$	$\frac{w^f}{r}$	M_η^f	T_η^f
(i)	$\frac{\sin\theta}{\rho} - \nu\delta\frac{\partial}{\partial\theta}$	$-\frac{\cos\theta}{\rho} - \nu\delta$	0	$(1-\nu^2)\,\delta$
(ii)	$\frac{\sin\theta}{\rho} - \nu\delta\frac{\partial}{\partial\theta}$	$-\frac{\sin\theta}{\rho}\frac{\partial}{\partial\theta} + \nu\delta\frac{\partial^2}{\partial\theta^2}$	$\frac{12(1-\nu^2)}{\gamma}\,\delta$	0
(iii)	$-(1+\frac{\gamma}{12})\frac{\partial}{\partial\theta}\left(\delta\frac{\partial}{\partial\theta}\right)$	$-\frac{\partial}{\partial\theta}\left(\delta - \frac{\gamma}{12}\delta\frac{\partial^2}{\partial\theta^2}\right)$	$-\frac{\sin\theta}{\rho} - \nu\frac{\partial}{\partial\theta}\left(\delta\right)$	$-\frac{\sin\theta}{\rho} - \nu\frac{\partial}{\partial\theta}\left(\delta\right)$
(iv)	$\delta\frac{\partial}{\partial\theta} - \frac{\gamma}{12}\frac{\partial^2}{\partial\theta^2}\left(\delta\frac{\partial}{\partial\theta}\right)$	$\delta + \frac{\gamma}{12}\frac{\partial^2}{\partial\theta^2}\left(\delta\frac{\partial^2}{\partial\theta^2}\right)$	$-\frac{\partial}{\partial\theta}\left(\frac{\sin\theta}{\rho}\right) - \nu\frac{\partial^2}{\partial\theta^2}\left(\delta\right)$	$\frac{\cos\theta}{\rho} + \nu\delta$

$$[L_2]$$

	$\frac{v^f}{r}$	ψ^f	p^f	Q^f
(i)	$-\frac{\sin\theta}{\rho} - \frac{3}{3+\gamma}\delta\frac{\partial}{\partial\theta}$	$-\frac{\gamma}{3+\gamma}\delta\frac{\partial}{\partial\theta}$	$\frac{6(1+\nu)}{3+\gamma}\,\delta$	0
(ii)	$\frac{\cos\theta}{\rho}$	$-\delta$	0	0
(iii)	$\frac{\gamma}{2(1+\nu)(3+\gamma)}\frac{\partial}{\partial\theta}\left(\delta\frac{\partial}{\partial\theta}\right)$	$\frac{-\gamma}{2(1+\nu)(3+\gamma)}\frac{\partial}{\partial\theta}\left(\delta\frac{\partial}{\partial\theta}\right)$	$-\frac{\gamma}{3+\gamma}\frac{\partial}{\partial\theta}\left(\delta\right)$	δ
(iv)	$\frac{-\gamma}{2(1+\nu)(3+\gamma)}\frac{\partial}{\partial\theta}\left(\delta\frac{\partial}{\partial\theta}\right)$	$\frac{\gamma}{2(1+\nu)(3+\gamma)}\frac{\partial}{\partial\theta}\left(\delta\frac{\partial}{\partial\theta}\right)$	$\frac{\sin\theta}{\rho} - \frac{3}{3+\gamma}\frac{\partial}{\partial\theta}\left(\delta\right)$	$-\frac{\cos\theta}{\rho}$

$\rho = R/r$, $\delta = 1 + \frac{\cos\theta}{\rho}$, $\gamma = (t/r)^2$, ν = Poisson's ratio

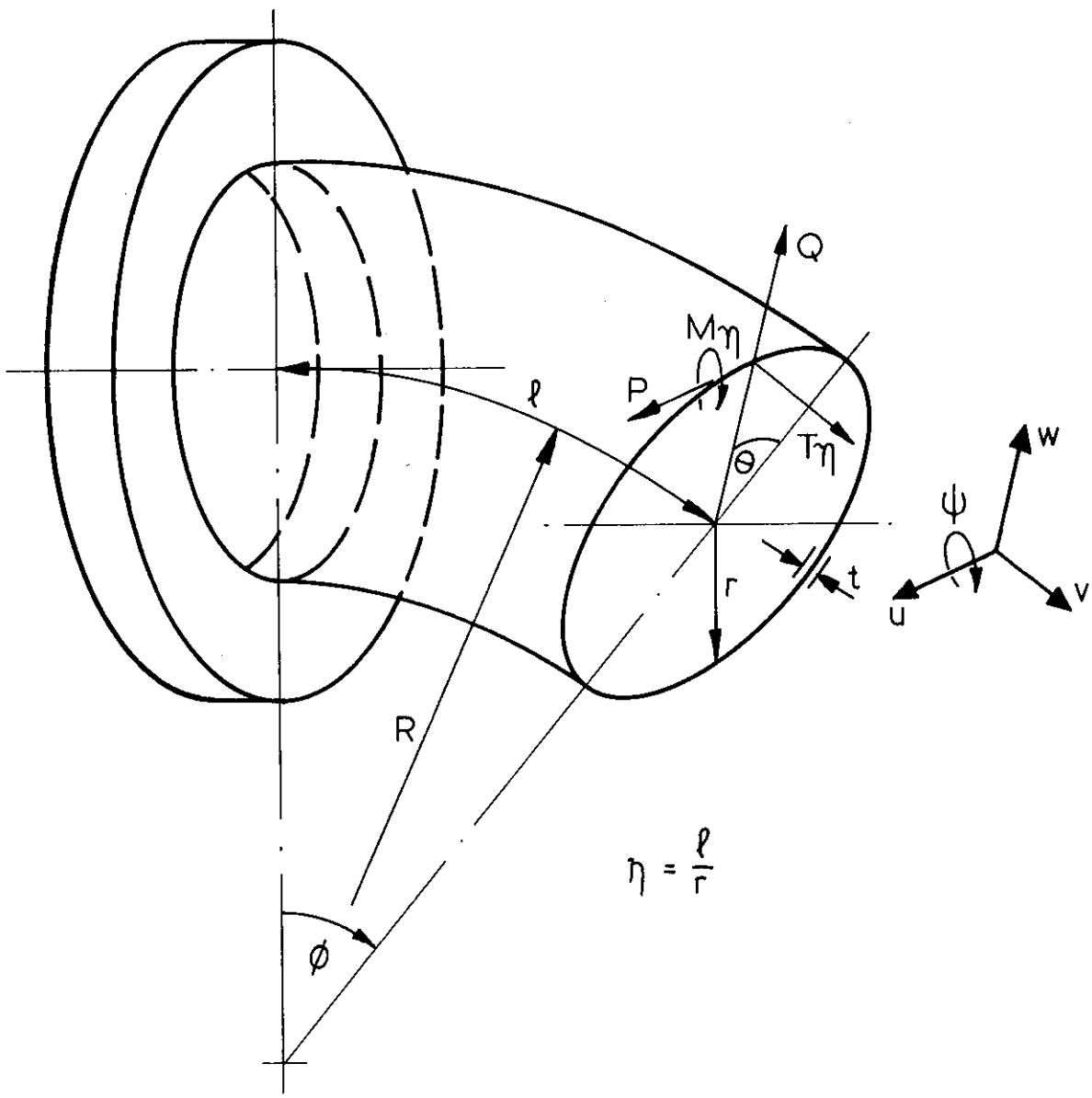

FIGURE 1. PIPE MIDDLE SURFACE

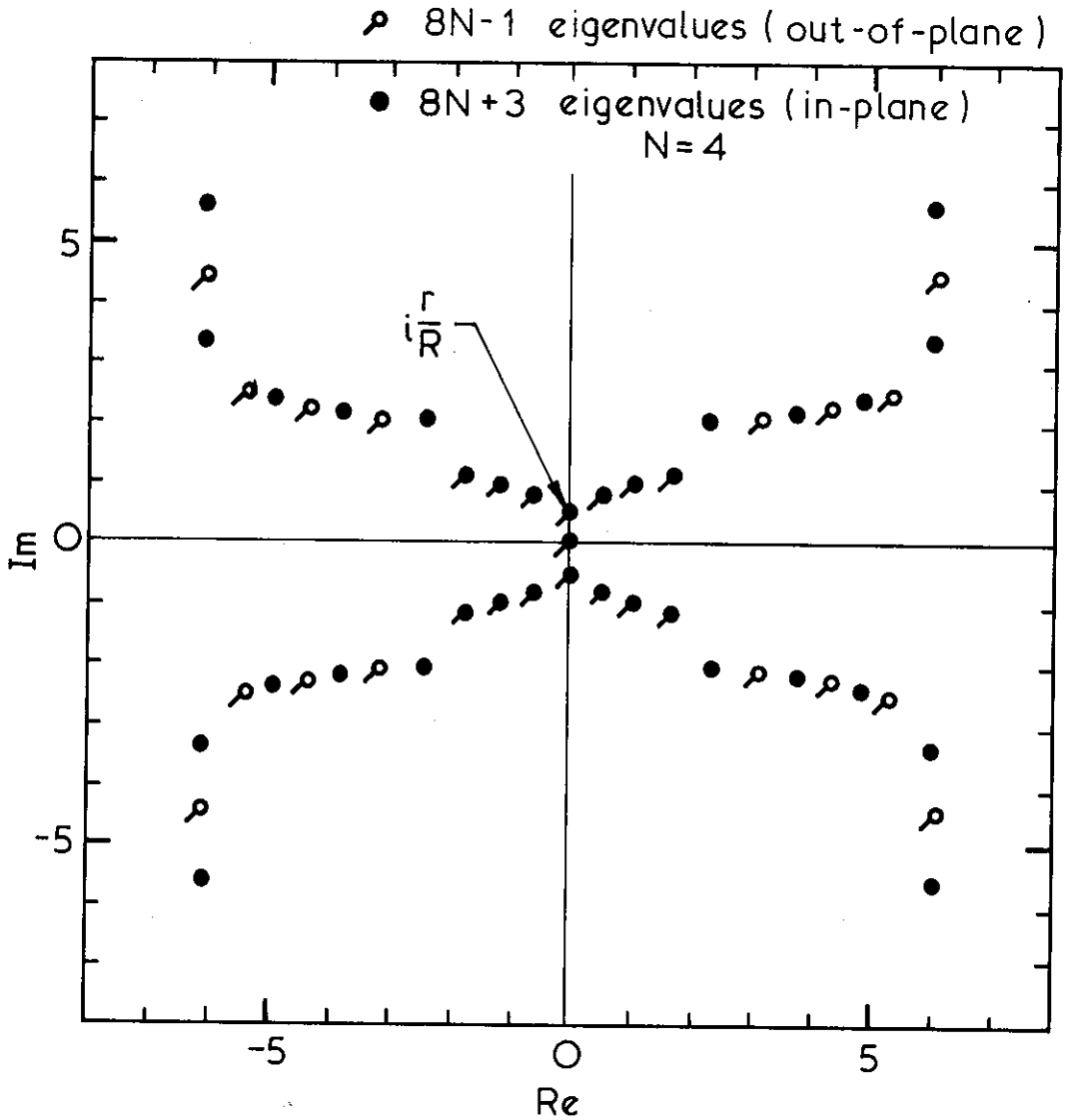

FIGURE 2. TYPICAL CURVED PIPE EIGENVALUES

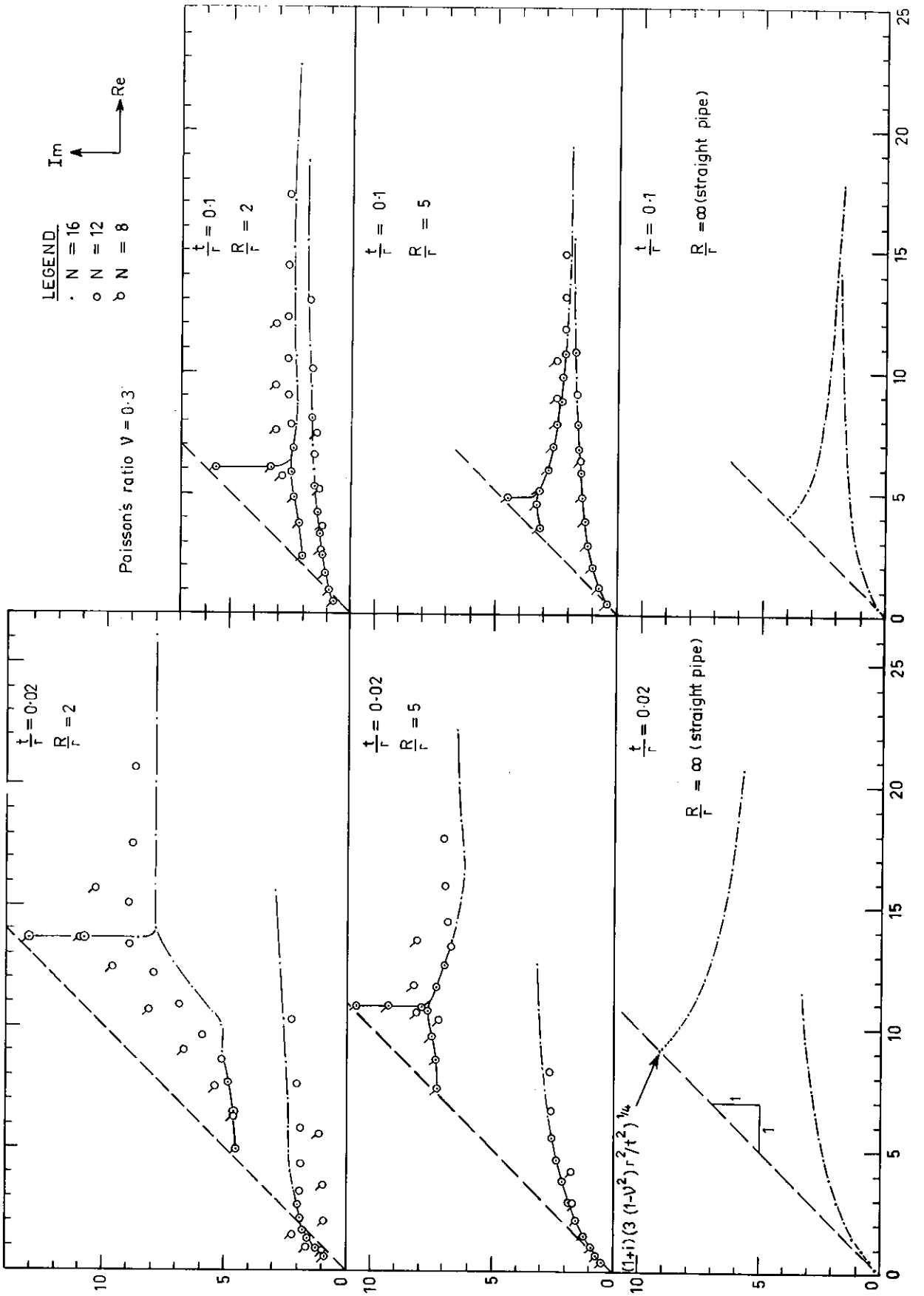

FIGURE 3. COMPLEX EIGENVALUES FOR IN-PLANE LOADING

APPENDIX A
SERIES EXPANSIONS FOR THE FLANGE EFFECT UNKNOWNS

In-Plane	Out-of-Plane
$u^f/r = u_1 \sin\theta + \sum_{n=2}^{N} u_n \sin n\theta$	$u^f/r = -u_o - u_1 \cos\theta - \sum_{n=2}^{N} u_n \cos n\theta$
$w^f/r = w_o + w_1 \cos\theta + \sum_{n=2}^{N} w_n \cos n\theta$	$w^f/r = w_1 \sin\theta + \sum_{n=2}^{N} w_n \sin n\theta$
$M_\eta^f = M_o + M_1 \cos\theta + \sum_{n=2}^{N} M_n \cos n\theta$	$M_\eta^f = M_1 \sin\theta + \sum_{n=2}^{N} M_n \sin n\theta$
$T_\eta^f = - M_1 \cos\theta - \sum_{n=2}^{N} T_n \cos n\theta$	$T_\eta^f = -M_1 \sin\theta - \sum_{n=2}^{N} T_n \sin n\theta$
$v^f/r = v_o + v_1 \cos\theta + \sum_{n=2}^{N} v_n \cos n\theta$	$v^f/r = v_1 \sin\theta + \sum_{n=2}^{N} v_n \sin n\theta$
$\psi^f = \psi_o + \psi_1 \cos\theta + \sum_{n=2}^{N} \psi_n \cos n\theta$	$\psi^f = \psi_1 \sin\theta + \sum_{n=2}^{N} \psi_n \sin n\theta$
$P^f = Q_1 \sin\theta + \sum_{n=2}^{N} P_n \sin n\theta$	$P^f = -Q_1 \cos\theta - \sum_{n=2}^{N} P_n \cos n\theta$
$Q^f = Q_o + Q_1 \cos\theta + \sum_{n=2}^{N} Q_n \cos n\theta$	$Q^f = Q_1 \sin\theta + \sum_{n=2}^{N} Q_n \sin n\theta$
8N+3 coefficients	8N-1 coefficients

Note: Coefficients are different for in-plane and out-of-plane expansions.

APPENDIX B
EIGENVALUES FOR CURVED AND STRAIGHT PIPES

Parameters of pipe bends considered:

R/r = 2, 3, 5, infinite (straight pipe)

t/r = 0.01, 0.02, 0.05, 0.1

ν = 0.3

TABLE B1

CURVED PIPE EIGENVALUES

R/r = 2.0 t/r = 0.01

IN-PLANE

REAL	IMAG	REAL	IMAG
0.5810	0.8708	0.9710	1.2986
1.3283	1.6905	1.6738	2.0311
1.9858	2.2927	2.3458	2.4969
2.8682	2.5816	3.2483	2.5932
3.7619	3.1995	4.3565	2.3994
5.6146	2.3394	7.0647	2.3879
8.8018	2.5133	6.8565	6.4285
10.9677	2.7062	8.4162	6.5427
9.8052	6.7214	13.9003	2.9778
11.1164	6.8113	11.6958	7.1187
12.7424	8.1307	13.9935	9.1399
15.3448	10.2117	16.7398	11.3272
18.1221	12.4592	19.9138	13.3381
19.2648	14.1947	22.3553	13.1287
19.2733	16.4890	19.2685	18.7336
25.1325	12.9486	28.6805	12.8263

OUT-OF-PLANE

REAL	IMAG	REAL	IMAG
0.5807	0.8708	0.9711	1.2986
1.3287	1.6900	1.6702	2.0315
1.9936	2.2976	2.3408	2.4844
2.8703	2.5963	3.2440	2.5868
3.7646	3.2002	4.3562	2.3980
5.6149	2.3395	7.0646	2.3879
8.8018	2.5133	7.6624	6.4725
10.9677	2.7062	9.1275	6.6276
10.4675	6.8130	11.4681	6.8035
13.9003	2.9778	12.1673	7.6406
13.3521	8.6286	14.6612	9.6692
16.0398	10.7642	17.4388	11.8932
18.7625	13.0932	20.0024	13.4123
19.2802	15.3616	22.3551	13.1292
19.2692	17.6127	25.1325	12.9485
28.6805	12.8263		

TABLE B2

CURVED PIPE EIGENVALUES

R/r = 2.0 t/r = 0.02

IN-PLANE

REAL	IMAG	REAL	IMAG
0.5782	0.8608	0.9603	1.2559
1.3056	1.5741	1.6672	1.7631
2.1452	1.8739	2.7231	2.0065
3.3823	2.1632	4.0941	2.3139
4.8713	2.3819	5.8467	2.3912
7.0590	2.4307	4.9726	4.5454
8.5310	2.5136	6.4755	4.6804
7.7755	4.8709	10.3261	2.6339
8.9562	5.0709	10.0359	5.1602
12.5748	2.7877	10.6118	5.5336
11.5415	6.3318	12.5987	7.1154
15.6142	2.9821	14.0032	7.8803
13.6481	8.5413	15.8833	7.9127
13.6202	10.8433	17.8441	7.8915
13.6153	13.0911	20.0679	7.8777
22.7119	7.8750	26.1433	7.8845

OUT-OF-PLANE

REAL	IMAG	REAL	IMAG
0.5776	0.8608	0.9605	1.2559
1.3055	1.5740	1.6672	1.7631
2.1451	1.8739	2.7231	2.0065
3.3823	2.1632	4.0941	2.3139
4.8713	2.3819	5.8467	2.3912
7.0590	2.4307	5.7568	4.5995
8.5310	2.5136	7.1446	4.7731
8.3767	4.9711	10.3261	2.6339
9.5251	5.1555	10.3411	5.2266
12.5748	2.7877	11.0453	5.9400
12.0656	6.7255	15.6142	2.9821
13.1030	7.5622	14.1594	7.9390
13.6316	9.7132	15.8845	7.9132
13.6150	11.9692	17.8441	7.8915
20.0679	7.8777	22.7119	7.8750
26.1433	7.8845		

TABLE B3

CURVED PIPE EIGENVALUES

R/r = 2.0 t/r = 0.05

IN-PLANE

REAL	IMAG	REAL	IMAG
0.5712	0.8304	0.9485	1.1045
1.4238	1.2432	2.0607	1.3937
2.7995	1.5448	3.5931	1.6758
4.4164	1.7844	3.2996	2.8718
5.2687	1.8688	6.1960	1.9302
4.7107	3.0276	7.2770	1.9871
5.8896	3.2226	6.9462	3.4218
8.5714	2.0543	7.9163	3.6377
8.6621	3.8065	10.1262	2.1329
9.2678	3.6592	10.1943	3.8009
8.5933	5.8370	12.0097	2.2199
11.3981	3.9239	12.7683	3.9990
8.5917	8.0900	14.3548	2.3134
14.3213	4.0485	16.1017	4.0826
17.5074	2.4142	18.1827	4.1053
20.7015	4.1173	24.0063	4.1148

OUT-OF-PLANE

REAL	IMAG	REAL	IMAG
0.5701	0.8302	0.9487	1.1046
1.4239	1.2431	2.0607	1.3937
2.7995	1.5448	3.5931	1.6758
4.4164	1.7844	5.2687	1.8688
4.0464	2.9377	6.1960	1.9302
5.3198	3.1241	7.2770	1.9871
6.4295	3.3209	8.5714	2.0543
7.4448	3.5135	8.4252	3.6474
10.1262	2.1329	9.2642	3.6171
8.6130	4.7023	10.1927	3.7965
12.0097	2.2199	11.3980	3.9237
8.5874	6.9663	12.7683	3.9990
14.3548	2.3134	14.3213	4.0485
16.1017	4.0826	17.5074	2.4142
18.1827	4.1053	20.7015	4.1173
24.0063	4.1148		

TABLE B4

CURVED PIPE EIGENVALUES

R/r = 2.0 t/r = 0.10

IN-PLANE

REAL	IMAG	REAL	IMAG
0.5628	0.7696	1.0587	0.9437
1.7558	1.0961	2.5505	1.2316
3.3845	1.3406	2.4551	2.0232
4.2362	1.4279	3.7876	2.1848
5.0994	1.4986	4.8807	2.3695
5.9888	1.5551	5.9153	2.5173
6.9623	1.6020	6.0467	3.3178
6.9145	2.4419	8.0988	1.6471
7.8064	2.3199	8.6708	2.2451
9.4541	1.6931	9.5986	2.2649
6.0506	5.5765	11.0758	1.7372
10.6890	2.3186	11.9581	2.3569
13.0360	1.7747	13.4225	2.3734
15.4762	1.7986	15.1189	2.3687
17.1120	2.3406	18.7649	1.7937
19.5274	2.2813	22.6878	2.1668

OUT-OF-PLANE

REAL	IMAG	REAL	IMAG
1.7558	1.0961	2.5505	1.2316
1.0586	0.9439	0.5616	0.7695
3.3845	1.3406	4.2362	1.4279
3.1678	2.0946	5.0994	1.4986
4.3500	2.2807	5.9888	1.5551
5.3521	2.4794	6.9623	1.6020
6.1427	2.5965	6.9336	2.4596
8.0988	1.6471	7.8071	2.3219
8.6708	2.2453	6.0389	4.4517
9.4541	1.6931	9.5986	2.2649
11.0758	1.7372	10.6890	2.3186
11.9581	2.3569	13.0360	1.7747
13.4225	2.3734	15.4762	1.7986
15.1189	2.3687	17.1120	2.3406
18.7649	1.7937	19.5274	2.2813
22.6878	2.1668		

TABLE B5

CURVED PIPE EIGENVALUES

R/r = 3.0 t/r = 0.01

IN-PLANE

REAL	IMAG	REAL	IMAG
0.5318	0.6851	0.8730	1.0401
1.1910	1.3608	1.4928	1.6364
1.8177	1.8126	2.2439	1.9590
2.7763	2.1696	3.4003	2.4199
4.1121	2.6671	4.8765	2.9392
5.5620	3.0387	6.6322	2.9511
8.0244	2.9754	9.7731	3.0819
12.1441	3.2687	8.9415	8.5687
10.3334	8.6435	11.6114	8.7787
12.7961	8.9482	13.8910	9.1654
15.0383	8.4144	14.9140	9.6714
15.8056	10.4538	16.7243	11.3451
17.5370	11.6347	17.1201	13.1745
19.2457	11.5411	17.1257	14.9416
21.1707	11.4861	17.1297	16.7059
23.4122	11.4495	26.2852	11.4300

OUT-OF-PLANE

REAL	IMAG	REAL	IMAG
0.5318	0.6850	0.8730	1.0401
1.1908	1.3610	1.4939	1.6358
1.8158	1.8133	2.2459	1.9588
2.7745	2.1697	3.4018	2.4194
4.1114	2.6676	4.8767	2.9390
5.5620	3.0388	6.6322	2.9509
8.0245	2.9755	9.7731	3.0819
12.1441	3.2687	9.6532	8.5957
10.9849	8.7059	12.2145	8.8606
13.3580	9.0433	14.4112	9.3553
15.0418	8.4190	15.3606	10.0565
16.2675	10.8669	17.4562	11.5424
17.1163	12.2650	19.2450	11.5393
17.1231	14.0589	21.1705	11.4865
17.1278	15.8238	23.4123	11.4495
26.2852	11.4300		

TABLE B6

CURVED PIPE EIGENVALUES

R/r = 3.0 t/r = 0.02

IN-PLANE

REAL	IMAG	REAL	IMAG
0.5300	0.6777	0.8666	1.0066
1.1808	1.2427	1.5894	1.4050
2.1499	1.6303	2.8367	1.8804
3.6098	2.1151	4.4383	2.3226
5.3018	2.5020	6.1896	2.6458
7.1445	2.7296	8.2798	2.7788
9.6871	2.8453	6.4314	6.0578
7.7868	6.1512	11.4731	2.9453
8.9982	6.3101	10.1047	6.5005
11.1271	6.7110	13.9072	3.0860
12.0145	6.8673	12.9072	6.7153
13.5700	6.2513	12.0844	8.1559
14.4258	6.6969	12.0973	9.9249
15.7858	6.8560	12.1057	11.6895
17.3086	6.9474	19.0672	7.0129
21.1808	7.0663	23.9431	7.1163

OUT-OF-PLANE

REAL	IMAG	REAL	IMAG
0.5299	0.6777	0.8667	1.0066
1.1808	1.2427	1.5894	1.4051
2.1499	1.6303	2.8367	1.8804
3.6098	2.1151	4.4383	2.3226
5.3018	2.5020	6.1896	2.6458
7.1445	2.7296	8.2798	2.7787
9.6871	2.8453	7.1300	6.0923
11.4731	2.9453	8.4077	6.2254
9.5627	6.4023	10.6283	6.6024
13.9072	3.0860	11.6587	6.8256
12.0997	7.2829	12.8868	6.7157
13.5703	6.2497	14.4262	6.6972
12.0917	9.0415	15.7858	6.8560
12.1017	10.8074	17.3086	6.9474
19.0672	7.0129	21.1808	7.0663
23.9431	7.1163		

TABLE B7

CURVED PIPE EIGENVALUES

R/r = 3.0 t/r = 0.05

IN-PLANE

REAL	IMAG	REAL	IMAG
0.5237	0.6499	0.8906	0.8758
1.4584	1.1018	2.2043	1.3419
3.0440	1.5458	3.9307	1.7094
4.8419	1.8403	5.7666	1.9462
6.7007	2.0328	4.2039	3.8275
7.6543	2.1018	5.4957	3.9453
8.6829	2.1560	6.5995	4.1296
7.5368	4.2636	9.8862	2.2076
8.4059	4.1138	9.3331	3.9046
7.6209	5.4770	11.3593	2.2669
10.2726	3.7377	11.2105	3.6026
7.6421	7.2423	12.1345	3.5127
13.2171	2.3356	13.1087	3.5272
14.2627	3.6017	15.7366	2.4154
15.6377	3.6697	17.2774	3.7201
19.2862	3.7547	21.9431	3.7738

OUT-OF-PLANE

REAL	IMAG	REAL	IMAG
0.5235	0.6498	0.8906	0.8759
1.4584	1.1018	2.2043	1.3419
3.0440	1.5458	3.9307	1.7094
4.8419	1.8403	5.7666	1.9462
6.7007	2.0328	4.8787	3.8725
7.6543	2.1018	6.0686	4.0325
8.6829	2.1560	7.1428	4.2487
9.8862	2.2076	7.6467	4.6196
8.3876	4.1173	9.3325	3.9042
11.3593	2.2669	10.2726	3.7377
7.6322	6.3604	11.2105	3.6026
12.1345	3.5127	13.2171	2.3356
13.1087	3.5272	14.2627	3.6017
15.7366	2.4154	15.6377	3.6697
17.2774	3.7201	19.2862	3.7547
21.9431	3.7738		

TABLE B8

CURVED PIPE EIGENVALUES

R/r = 3.0 t/r = 0.10

IN-PLANE

REAL	IMAG	REAL	IMAG
0.5305	0.6043	1.1093	0.8413
1.9091	1.0683	2.7973	1.2426
3.7196	1.3719	4.6572	1.4699
3.0810	2.6995	5.6027	1.5464
4.3185	2.8324	6.5530	1.6075
5.2715	3.2523	5.6516	2.9544
7.5089	1.6568	6.6253	2.6855
8.4874	1.6965	7.5654	2.5156
8.5033	2.3845	5.3856	5.0059
9.5530	1.7292	9.4389	2.2780
10.8059	1.7589	10.3716	2.1888
11.3041	2.1225	12.3386	1.7866
12.2773	2.1032	13.3848	2.1218
14.2693	1.8084	14.6960	2.1388
16.2661	2.1359	16.8900	1.8150
18.1944	2.1061	20.7432	2.0341

OUT-OF-PLANE

REAL	IMAG	REAL	IMAG
3.7196	1.3719	4.6572	1.4699
2.7973	1.2426	5.6027	1.5464
1.9091	1.0683	1.1093	0.8413
0.5302	0.6043	6.5530	1.6075
3.7317	2.7515	4.8229	2.9112
7.5089	1.6568	5.7195	2.8916
6.6285	2.6858	8.4874	1.6965
7.5655	2.5157	8.5033	2.3845
9.5530	1.7292	5.3660	4.1248
9.4389	2.2780	10.8059	1.7589
10.3716	2.1888	11.3041	2.1225
12.3386	1.7866	12.2773	2.1032
13.3848	2.1218	14.2693	1.8084
14.6960	2.1388	16.2661	2.1359
16.8900	1.8150	18.1944	2.1061
20.7432	2.0341		

TABLE B9

CURVED PIPE EIGENVALUES

R/r = 5.0 t/r = 0.01

IN-PLANE

REAL	IMAG	REAL	IMAG
0.4418	0.5107	0.7142	0.7830
0.9648	1.0174	1.2279	1.1827
1.6065	1.3995	2.1155	1.6908
2.7330	2.0050	3.4371	2.3148
4.2070	2.6075	5.0292	2.8755
5.8875	3.1191	6.7611	3.3163
7.7457	3.3904	9.0512	3.4311
10.8808	3.5347	10.5828	10.2830
11.7200	10.3350	12.7796	10.4399
13.7662	10.5868	14.7115	10.7771
15.7087	10.6122	15.3542	11.2142
16.6750	10.3186	17.5514	10.0740
18.0339	9.3744	15.3707	12.4684
19.0233	10.0857	15.4045	13.7807
20.4837	10.2019	15.4185	15.0988
22.2130	10.2750	24.4544	10.3280

OUT-OF-PLANE

REAL	IMAG	REAL	IMAG
0.4418	0.5107	0.7142	0.7830
0.9648	1.0173	1.2281	1.1830
1.6062	1.3993	2.1156	1.6908
2.7333	2.0049	3.4369	2.3149
4.2073	2.6073	5.0291	2.8756
5.8875	3.1193	6.7611	3.3161
7.7457	3.3905	9.0511	3.4310
10.8809	3.5347	12.2587	10.3823
11.1616	10.3010	13.2828	10.5094
14.2432	10.6721	15.0973	10.8410
15.8089	10.6573	16.6199	10.3154
15.3423	11.8189	17.5775	10.0704
18.0333	9.3778	15.3905	13.1253
19.0155	10.0848	15.4137	14.4384
20.4852	10.2026	22.2131	10.2747
24.4544	10.3280		

TABLE B10

CURVED PIPE EIGENVALUES

R/r = 5.0 t/r = 0.02

IN-PLANE

REAL	IMAG	REAL	IMAG
0.4403	0.5045	0.7021	0.7389
1.0168	0.9346	1.5083	1.2137
2.1506	1.5269	2.9021	1.8263
3.7288	2.0934	4.6057	2.3243
5.5154	2.5217	6.4465	2.6901
7.3924	2.8332	8.3625	2.9448
9.4387	3.0162	10.7866	3.0792
7.5707	7.2705	12.6580	3.1722
8.6834	7.3382	9.6940	7.4710
10.7101	7.6449	10.8293	7.9207
11.6911	7.2953	12.5646	6.9878
10.8749	9.2685	13.4720	6.7264
14.4028	6.5053	10.8977	10.5834
15.3458	6.3130	16.2551	6.1739
17.2442	6.2227	18.5198	6.3365
20.1248	6.4289	22.2659	6.5045

OUT-OF-PLANE

REAL	IMAG	REAL	IMAG
0.4403	0.5045	0.7021	0.7389
1.0168	0.9346	1.5083	1.2137
2.1506	1.5269	2.9021	1.8263
3.7288	2.0934	4.6057	2.3243
5.5154	2.5217	6.4465	2.6901
7.3924	2.8332	8.3625	2.9448
9.4387	3.0162	10.7866	3.0792
12.6580	3.1722	8.1410	7.2941
9.2001	7.3985	10.1595	7.5592
10.9274	7.6135	11.6934	7.2983
10.8509	8.6154	12.5646	6.9879
13.4720	6.7263	14.4029	6.5054
10.8897	9.9241	15.3457	6.3129
16.2551	6.1739	17.2442	6.2227
18.5198	6.3365	20.1248	6.4289
22.2659	6.5045		

TABLE B11

CURVED PIPE EIGENVALUES

R/r = 5.0 t/r = 0.05

IN-PLANE

REAL	IMAG	REAL	IMAG
0.4311	0.4736	0.8325	0.7227
1.4755	1.0322	2.2775	1.3166
3.1649	1.5457	4.0958	1.7249
5.0494	1.8659	6.0154	1.9787
6.9887	2.0706	7.9668	2.1465
8.9500	2.2098	4.8977	4.5960
5.9664	4.6853	9.9594	2.2617
6.9876	4.7046	7.8791	4.3974
6.8250	5.2696	8.7913	4.1354
11.0845	2.3061	9.7334	3.9321
10.6904	3.7717	6.8821	6.5794
12.4877	2.3536	11.6548	3.6419
12.6228	3.5342	14.4244	2.4127
13.5920	3.4437	14.5635	3.3788
15.5894	3.3734	16.7999	3.4172
18.3218	3.4638	20.3774	3.4965

OUT-OF-PLANE

REAL	IMAG	REAL	IMAG
0.4311	0.4736	0.8325	0.7227
1.4755	1.0322	2.2775	1.3166
3.1649	1.5457	4.0958	1.7249
5.0494	1.8659	6.0154	1.9787
6.9887	2.0706	7.9668	2.1465
8.9500	2.2098	5.4524	4.6287
6.4251	4.7842	7.0672	4.7491
9.9594	2.2617	7.8786	4.3990
8.7913	4.1354	11.0845	2.3061
9.7334	3.9321	6.8660	5.9227
10.6904	3.7717	12.4877	2.3536
11.6548	3.6419	12.6228	3.5342
14.4244	2.4127	13.5920	3.4437
14.5635	3.3788	15.5894	3.3734
16.7999	3.4172	18.3218	3.4638
20.3774	3.4965		

TABLE B12

CURVED PIPE EIGENVALUES

R/r = 5.0 t/r = 0.10

IN-PLANE

REAL	IMAG	REAL	IMAG
0.4882	0.4769	1.1337	0.7857
1.9846	1.0540	2.9174	1.2476
3.8822	1.3866	4.8612	1.4898
5.8474	1.5688	3.5505	3.2459
4.5380	3.3939	6.8382	1.6307
5.1105	3.2759	5.9728	2.9309
7.8324	1.6801	6.9110	2.7046
4.8533	4.5649	8.8293	1.7197
7.8679	2.5390	8.8310	2.4104
9.8317	1.7515	9.7955	2.3052
10.8657	1.7766	10.7597	2.2153
12.0291	1.7967	11.7226	2.1355
12.6839	2.0635	13.4859	1.8135
13.6485	2.0034	14.6595	1.9727
15.4972	1.8239	15.8294	1.9682
17.2919	1.9578	19.2674	1.9170

OUT-OF-PLANE

REAL	IMAG	REAL	IMAG
0.4881	0.4769	1.1337	0.7857
1.9846	1.0540	2.9174	1.2476
3.8822	1.3866	4.8612	1.4898
5.8474	1.5688	4.0877	3.2795
6.8382	1.6307	5.0781	3.2215
5.9736	2.9301	4.8209	3.9154
7.8324	1.6801	6.9111	2.7046
7.8679	2.5390	8.8293	1.7197
8.8310	2.4104	9.8317	1.7515
9.7955	2.3052	10.8657	1.7766
10.7597	2.2153	12.0291	1.7967
11.7226	2.1355	12.6839	2.0635
13.4859	1.8135	13.6485	2.0034
14.6595	1.9727	15.4972	1.8239
15.8294	1.9682	17.2919	1.9578
19.2674	1.9170		

TABLE B13

STRAIGHT PIPE EIGENVALUES

t/r = 0.01

N	REAL	IMAG	REAL	IMAG
0	12.8541	12.8541		
1	12.8929	12.8151		
2	13.0106	12.6995	0.1359	0.1335
3	13.2112	12.5118	0.3376	0.3217
4	13.5001	12.2604	0.6271	0.5730
5	13.8826	11.9575	1.0103	0.8755
6	14.3618	11.6195	1.4901	1.2131
7	14.9359	11.2646	2.0650	1.5672
8	15.5977	10.9108	2.7276	1.9203
9	16.3355	10.5720	3.4663	2.2582
10	17.1352	10.2573	4.2671	2.5719
11	17.9833	9.9710	5.1164	2.8571
12	18.8682	9.7137	6.0025	3.1132
13	19.7805	9.4838	6.9161	3.3417
14	20.7131	9.2789	7.8502	3.5452
15	21.6607	9.0961	8.7993	3.7265
16	22.6195	8.9326	9.7598	3.8884

TABLE B14

STRAIGHT PIPE EIGENVALUES

t/r = 0.02

N	REAL	IMAG	REAL	IMAG
0	9.0892	9.0892		
1	9.1439	9.0339		
2	9.3118	8.8720	0.1939	0.1870
3	9.6028	8.6172	0.4868	0.4422
4	10.0272	8.2944	0.9127	0.7643
5	10.5872	7.9382	1.4742	1.1194
6	11.2702	7.5841	2.1589	1.4719
7	12.0514	7.2578	2.9422	1.7963
8	12.9034	6.9710	3.7966	2.0809
9	13.8032	6.7248	4.6990	2.3246
10	14.7344	6.5150	5.6331	2.5316
11	15.6860	6.3359	6.5879	2.7075
12	16.6510	6.1822	7.5566	2.8578
13	17.6251	6.0492	8.5345	2.9870
14	18.6053	5.9331	9.5189	3.0991
15	19.5898	5.8309	10.5078	3.1970
16	20.5773	5.7401	11.5001	3.2831

TABLE B15

STRAIGHT PIPE EIGENVALUES

t/r = 0.05

N	REAL	IMAG	REAL	IMAG
0	5.7485	5.7485		
1	5.8342	5.6603		
2	6.1054	5.4127	0.3133	0.2864
3	6.5872	5.0636	0.8000	0.6345
4	7.2698	4.7005	1.4876	0.9941
5	8.0954	4.3872	2.3189	1.3025
6	9.0012	4.1381	3.2318	1.5454
7	9.9479	3.9429	4.1866	1.7331
8	10.9151	3.7877	5.1633	1.8797
9	11.8930	3.6613	6.1518	1.9962
10	12.8766	3.5559	7.1473	2.0906
11	13.8632	3.4660	8.1471	2.1683
12	14.8514	3.3876	9.1498	2.2332
13	15.8404	3.3182	10.1546	2.2880
14	16.8297	3.2557	11.1609	2.3347
15	17.8189	3.1985	12.1686	2.3748
16	18.8079	3.1456	13.1773	2.4094

TABLE B16

STRAIGHT PIPE EIGENVALUES

t/r = 0.10

N	REAL	IMAG	REAL	IMAG
0	4.0648	4.0648		
1	4.1841	3.9385		
2	4.5769	3.6096	0.4549	0.3816
3	5.2584	3.2317	1.1480	0.7543
4	6.1231	2.9299	2.0263	1.0460
5	7.0634	2.7119	2.9834	1.2502
6	8.0310	2.5503	3.9714	1.3946
7	9.0085	2.4238	4.9729	1.5005
8	9.9891	2.3195	5.9814	1.5809
9	10.9703	2.2298	6.9944	1.6433
10	11.9507	2.1498	8.0106	1.6925
11	12.9296	2.0764	9.0295	1.7316
12	13.9068	2.0075	10.0509	1.7625
13	14.8819	1.9416	11.0747	1.7868
14	15.8548	1.8775	12.1009	1.8054
15	16.8254	1.8145	13.1296	1.8188
16	17.7935	1.7518	14.1607	1.8278