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RESEARCH ESTABLISHMENT

LUCAS HEIGHTS

TABLES RELATING TO TEXTURE IN MATERIALS WITH CUBIC CRYSTAL SYMMETRY

by

C.J. BALL

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ABSTRACT

Tables have been prepared giving

1. the coefficients of augmented Jacobi polynomials to order 22;
2. the relations between coefficients of associated Legendre polynomials used to represent pole figures and the coefficients of the augmented Jacobi polynomials that represent the corresponding crystallite orientation distribution function, for various sets of lattice planes;
3. the relations between coefficients of augmented Jacobi polynomials representing crystallite orientation distribution functions before and after transformation according to three common transformation laws.

CONTENTS

1. INTRODUCTION

Roe [1965] has shown how the crystallite orientation distribution function of a textured material may be determined from several pole figures. In his method the distribution $q_j(\mu,\lambda)$ of the reciprocal lattice vector r_j is expanded in a series of spherical harmonics

$$q_j(\mu,\lambda) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} Q_{\ell m}^{j} P_{\ell m}(\cos \mu) \exp(-im\lambda)$$

where $P_{\ell m}$ are associated Legendre polynomials and the $Q_{\ell m}^{j}$ are series coefficients, and μ and λ are polar and azimuthal orientation coordinates with respect to specimen axes.

The orientation of an individual crystallite is specified by the Euler angles ψ, θ and ϕ with respect to the same specimen axes. The crystallite orientation distribution function, which is a density function in (ψ, θ, ϕ) space, is expressed as a series of generalised spherical harmonics in the form

$$w(\psi,\theta,\phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \sum_{n=-\ell}^{\ell} W_{\ell mn} Z_{\ell mn}(\cos\theta) \exp(-im\psi) \exp(-in\phi) .$$

$Z_{\ell mn}$ are augmented Jacobi polynomials and the $W_{\ell mn}$ are series coefficients. The two sets of coefficients are related by the simultaneous equations

$$Q_{\ell m}^{j} = 2\pi \left(\frac{2}{2\ell+1}\right)^{\frac{1}{2}} \sum_{n=-\ell}^{\ell} W_{\ell mn} P_{\ell n}(\cos \zeta_j) \exp(in \eta_j)$$

where ζ_j and η_j are the polar and azimuthal angles of r_j with respect to the crystal axes.

Since there are $(2\ell+1)$ coefficients $W_{\ell mn}$ for given ℓ and m, it might appear that measurement of as many pole figures would be required to determine them. Fortunately the $W_{\ell mn}$ are not all independent. For materials with orthotropic physical symmetry and cubic crystal symmetry the coefficients $W_{\ell mn}$ are all real, $W_{\ell mn} = W_{\ell \bar{m} n} = W_{\ell m \bar{n}} = W_{\ell \bar{m} \bar{n}}$, and $W_{\ell mn} = 0$ unless ℓ and m are even and n is an integer multiple of 4. Roe [1966] has published a table showing additional relations between the non-zero $W_{\ell mn}$: in principle, one needs to determine only two pole figures to obtain all $W_{\ell mn}$ for ℓ up to 22. Clearly the only polynomials required are those for which $W_{\ell mn}$ is non-zero. Morris & Heckler [1968] have published a table of $Z_{\ell mn}$ polynomials in the form

$$Z_{\ell mn} = (D_{\ell mn})^{\frac{1}{2}} \; 2^{-k} \sum_{s=0}^{\ell} E_{\ell mns}(\cos s\theta)$$

for ℓ up to 16. Table 1 extends the compilation up to $\ell = 22$, which more than doubles the number of polynomials tabulated. For convenience the polynomials up to $\ell = 16$ have been recalculated and are included here. In practice, for a given crystallite orientation distribution, many of the polynomial coefficients with $\ell > 16$ will not be significantly different from zero (as, of course, are many of those with $\ell \leqslant 16$), but it seems preferable to evaluate them all and not arbitrarily truncate the series at $\ell = 16$. The rationale behind truncating the series at $\ell = 22$ is that, for $\ell > 22$, more than two pole figures are required to determine the coefficients.

The coefficients $Q^j_{\ell m}$ can be obtained by numerical integration of experimental data. The relations between $Q^j_{\ell m}$ and the independent coefficients $W_{\ell m0}$ and, if appropriate, $W_{\ell m4}$ depend only on the reciprocal lattice vector r_j, i.e. the set of planes (H K L) for which the pole figure was determined. These relations have been evaluated for all the sets of planes likely to be used and are given in Table 2 in terms of $A_{\ell m} \equiv Q_{\ell m}/2\pi$. In this form the simplicity of the relations is not obscured by the irrationality of π. The numbers are simple fractions in which the denominators are powers of $(H^2+K^2+L^2)$, as is easily seen for the low order coefficients in each set.

Roe [1965] also showed how the coefficients $W_{\ell mn}$ are related to a new set $W'_{\ell mn}$ referred to new crystallite reference axes. Sargent [1974] has shown that this relation is the key to examining texture transformations. If a material transforms in such a way that the orientation of each crystallite after transformation is related to its orientation before by one of a number of sets of Euler angles, then the texture coefficients after transformation $W'_{\ell mn}$, are related to $W_{\ell mn}$, the coefficients before transformation, by the equation

$$W'_{\ell mn} = \left(\frac{2}{2\ell+1}\right)^{\frac{1}{2}} \sum_{p=-\ell}^{\ell} \sum_{j} f_j \, W_{\ell mp} \, Z_{\ell pn}(\cos \beta_j) \exp(ip\,\alpha_j) \exp(in\,\gamma_j)$$

where f_j is the relative frequency of variant j with Euler angles α_j, β_j, γ_j. The variants may, but need not be, crystallographically related. Most interest, of course, centres around transformations where the variants are crystallographically related, in particular where they are

variants of the Kurdjumov-Sachs, Nishiyama or Bain orientation relations. The relations between the coefficients for these transformations are given in Table 3, for cases in which all crystallographically equivalent variants of the transformation are equally probable.

2. TABLE 1 - AUGMENTED JACOBI POLYNOMIALS

The polynomials are given in multiple-angle form

$$Z_{\ell mn} = (D_{\ell mn})^{\frac{1}{2}} 2^{-k} \sum_{s=0}^{\ell} E_{\ell mns} (\cos s\theta) \quad .$$

The coefficients have been evaluated from the following expressions [Roe 1965]

$$Z_{\ell mn}(\zeta) = N\, t^{(m-n)/2}\, (1 - t)^{(m+n)/2}\, f(t)$$

where $t = (1 - \zeta)/2$

$f(t) = {}_2F_1(-\ell+m, \ell+m+1; m-n+1; t) \qquad m \geqslant n$

where ${}_2F_1$ is the hypergeometric function defined by

$${}_2F_1(\alpha,\beta;\gamma;t) = 1 + \frac{\alpha.\beta}{1.\gamma}\, t + \frac{\alpha(\alpha+1)\beta(\beta+1)}{2!\gamma(\gamma+1)}\, t^2 + \ldots.$$

N is determined from the normalisation condition

$$\int_{-1}^{1} Z^2_{\ell mn}(\zeta)\, d\zeta = 1 \quad ,$$

so that

$$N^2_{\ell mn} = \left(\frac{2\ell+1}{2}\right) \frac{(\ell+m)!}{(\ell-m)!} \frac{(\ell-n)!}{(\ell+n)!} \left(\frac{1}{(m-n)!}\right)^2 \quad .$$

Polynomials not tabulated can be found from the relations [Morris & Heckler 1968],

$$\begin{aligned} Z_{\ell mn} &= (-1)^{m+n} Z_{\ell nm} \\ &= (-1)^{m+n} Z_{\ell \bar{m}\bar{n}} \\ &= Z_{\ell \bar{m}\bar{n}} \end{aligned}$$

$$Z_{\ell \bar{m} n}(\zeta) = Z_{\ell m \bar{n}}(\zeta) = Z_{\ell mn}(-\zeta) \quad .$$

	L= 0 M= 0	L= 2 M= 0	L= 2 M= 2	L= 4 M= 0	L= 4 M= 2	L= 4 M= 2	L= 4 M= 4	L= 4 M= 4	L= 6 M= 0	L= 6 M= 2
	N= 0 K= 1	N= 0 K= 3	N= 0 K= 3	N= 0 K= 7	N= 0 K= 6	N= 4 K= 7	N= 0 K= 7	N= 4 K= 8	N= 0 K=10	N= 0 K=11
D=	2	10	15	18	45	126	315	18	26	2730
S	E(L,M,N,S)									
0	1	1	1	9	3	5	3	35	50	10
1		0	0	0	0	4	0	56	0	0
2		3	-1	20	4	-4	-4	28	105	17
3				0	0	-4	0	8	0	0
4				35	-7	-1	1	1	126	6
5									0	0
6									231	-33

	L= 6 M= 2	L= 6 M= 4	L= 6 M= 4	L= 6 M= 6	L= 6 M= 6	L= 8 M= 0	L= 8 M= 2	L= 8 M= 2	L= 8 M= 2	L= 8 M= 4
	N= 4 K=11	N= 0 K=10	N= 4 K=11	N= 0 K=11	N= 4 K=11	N= 0 K=15	N= 0 K=14	N= 4 K=13	N= 8 K=14	N= 0 K=14
D=	195	819	26	6006	429	34	10710	935	17017	11781
S	E(L,M,N,S)									
0	42	10	126	10	42	1225	35	63	45	35
1	16	0	96	0	48	0	0	14	20	0
2	17	5	15	-15	-15	2520	64	64	-64	40
3	72	0	240	0	-40	0	0	90	-36	0
4	-26	-26	338	6	-26	2772	44	-36	20	-36
5	-88	0	176	0	-8	0	0	78	20	0
6	-33	11	33	-1	-1	3432	0	0	0	-104
7						0	0	-182	-4	0
8						6435	-143	-91	-1	65

	L= 8 M= 4	L= 8 M= 4	L= 8 M= 6	L= 8 M= 6	L= 8 M= 6	L= 8 M= 8	L= 8 M= 8	L=10 M= 0	L=10 M= 2	L=10 M= 2
	N= 4 K=14	N= 8 K=15	N= 0 K=14	N= 4 K=13	N= 8 K=14	N= 0 K=15	N= 8 K=16	N= 0 K=18	N= 0 K=18	N= 4 K=18
D=	34	15470	14586	4641	255	109395	34	42	3465	546
S	E(L,M,N,S)									
0	693	99	35	33	429	35	6435	7938	882	2310
1	308	88	0	22	572	0	11440	0	0	336
2	440	-88	0	0	0	-56	8008	16170	1666	3094
3	1500	-120	0	50	-364	0	4368	0	0	2464
4	324	-36	-84	36	-364	28	1820	17160	1352	-104
5	468	24	0	-42	-196	0	560	0	0	4000
6	2184	24	64	-64	-64	-8	120	19305	741	-1577
7	1820	8	0	-30	-12	0	16	0	0	952
8	455	1	-15	-5	-1	1	1	24310	-442	1122
9								0	0	-7752
10								46189	-4199	-4845

	L=10 M= 2	L=10 M= 4	L=10 M= 4	L=10 M= 4	L=10 M= 6	L=10 M= 6	L=10 M= 6	L=10 M= 8	L=10 M= 8	L=10 M=10
	N= 8 K=19	N= 0 K=17	N= 4 K=17	N= 8 K=18	N= 0 K=19	N= 4 K=18	N= 8 K=19	N= 0 K=18	N= 8 K=19	N= 0 K=19
D=	27846	45045	42	2142	90090	21	1071	255255	42	1939938
S	E(L,M,N,S)									
0	770	126	4290	1430	378	12870	4290	126	24310	126
1	224	0	1248	832	0	5616	3744	0	28288	0
2	-238	182	4394	-338	266	6422	-494	-42	1326	-210
3	448	0	7744	1408	0	24288	4416	0	13056	0
4	-1144	-8	8	88	-664	664	7304	-264	49368	120
5	-1600	0	8000	-3200	0	-800	320	0	65280	0
6	589	-249	6889	-2573	-711	19671	-7347	279	49011	-45
7	1232	0	272	352	0	-5848	-7568	0	23232	0
8	118	-374	12342	1298	1054	-34782	-3658	-118	6962	10
9	-304	0	15504	608	0	-23256	-912	0	1216	0
10	-95	323	4845	95	-323	-4845	-95	19	95	-1

	L=10 M=10 N= 4 K=18	L=10 M=10 N= 8 K=19	L=12 M= 0 N= 0 K=22	L=12 M= 2 N= 0 K=21	L=12 M= 2 N= 4 K=22	L=12 M= 2 N= 8 K=22	L=12 M= 2 N=12 K=22	L=12 M= 4 N= 0 K=23	L=12 M= 4 N= 4 K=24	L=12 M= 4 N= 8 K=23
D=	101745	1995	50	75075	75	113050	6128925	450450	50	169575
S					E(L,M,N,S)					
0	286	4862	106722	1386	90090	2574	546	2310	450450	4290
1	208	7072	0	0	9240	528	168	0	92400	1760
2	-338	1326	216216	2664	137640	888	-888	3720	576600	1240
3	-352	-3264	0	0	72360	2160	-360	0	646416	6432
4	-8	-4488	225225	2325	37045	-4185	465	1195	57121	-2151
5	160	-3264	0	0	144500	-1800	300	0	982600	-4080
6	83	-1581	243100	1700	-49980	-1140	-140	-2380	209916	1596
7	-8	-528	0	0	135660	-5560	-140	0	488376	-6672
8	-22	-118	277134	646	-40698	1346	14	-5814	1098846	-12114
9	-8	-16	0	0	-34884	6696	36	0	23256	-1488
10	-1	-1	352716	-1292	71060	1276	4	-6460	1065900	6380
11			0	0	-326876	-2024	-4	0	1961256	4048
12			676039	-7429	-245157	-759	-1	7429	735471	759

	L=12 M= 4	L=12 M= 6	L=12 M= 6	L=12 M= 6	L=12 M= 6	L=12 M= 8	L=12 M= 8	L=12 M= 8	L=12 M=10	L=12 M=10
	N=12 K=24	N= 0 K=22	N= 4 K=23	N= 8 K=22	N=12 K=23	N= 0 K=22	N= 8 K=23	N=12 K=23	N= 0 K=22	N= 4 K=23
D=	36773550	121550	53550	1425	1682450	3464175	50	132825	4408950	532950
S	E(L,M,N,S)									
0	910	2310	7150	24310	2210	462	277134	8398	462	2730
1	560	0	2200	14960	2040	0	227392	10336	0	1400
2	-1240	2520	6200	4760	-2040	168	18088	-2584	-264	-1240
3	-1072	0	12328	43792	-3128	0	434112	-10336	0	1608
4	239	-2205	-1673	22491	-1071	-945	549423	-8721	-825	-1195
5	680	0	8500	-12600	900	0	114912	-2736	0	-6460
6	196	-5380	7532	20444	1076	-380	82308	1444	1100	-2940
7	-168	0	-6612	32248	348	0	618272	2224	0	3612
8	-126	-2166	6498	-25574	-114	1346	905858	1346	-638	3654
9	-8	0	2812	-64232	-148	0	645792	496	0	300
10	20	7980	-20900	-44660	-60	-812	259028	116	188	-940
11	8	0	-19228	-14168	-12	0	56672	16	0	-460
12	1	-3059	-4807	-1771	-1	161	5313	1	-23	-69

	L=12 M=10	L=12 M=10	L=12 M=12	L=12 M=12	L=14 M= 0	L=14 M= 2	L=14 M= 2	L=14 M= 2	L=14 M= 2	L=14 M= 4
	N= 8 K=22	N=12 K=23	N= 0 K=23	N=12 K=24	N= 0 K=25	N= 0 K=26	N= 4 K=27	N= 8 K=27	N=12 K=27	N= 0 K=26
D=	1925	3450	33801950	50	58	39585	16269	18734	42010995	44414370
S					E(L,M,N,S)					
0	25194	58786	462	1352078	736164	56628	180180	180180	4620	1716
1	25840	90440	0	2496144	0	0	13728	27456	1056	0
2	-2584	28424	-792	1961256	1486485	109989	296637	136653	-3333	2937
3	15504	-28424	0	1307504	0	0	111672	147312	792	0
4	43605	-53295	495	735471	1531530	99858	135458	-229086	-5874	1522
5	25992	-47652	0	346104	0	0	247000	39600	-6600	0
6	-21660	-29260	-220	134596	1616615	81719	-40641	-236049	7337	-589
7	-47816	-13244	0	42504	0	0	312816	-327712	8624	0
8	-39034	-4466	66	10626	1763580	52972	-121524	46412	-3116	-2964
9	-18600	-1100	0	2024	0	0	188784	-258336	-5328	0
10	-5452	-188	-12	276	2028117	7429	-24035	25645	155	-4807
11	-920	-20	0	24	0	0	-192280	586960	1672	0
12	-69	-1	1	1	2600150	-74290	170430	170430	274	-4370
13					0	0	-681720	-215280	-216	0
14					5014575	-334305	-596505	-94185	-63	6555

	L=14 M= 4	L=14 M= 4	L=14 M= 4	L=14 M= 6	L=14 M= 6	L=14 M= 6	L=14 M= 6	L=14 M= 8	L=14 M= 8	L=14 M= 8
	N= 4 K=27	N= 8 K=26	N=12 K=27	N= 0 K=26	N= 4 K=27	N= 8 K=27	N=12 K=27	N= 0 K=25	N= 8 K=26	N=12 K=26
D=	58	18183	163101510	46881835	4959	1914	476905	12785955	58	130065
S	E(L,M,N,S)									
0	3063060	92820	2380	1716	340340	587860	45220	1716	1763580	45220
1	466752	28288	1088	0	77792	268736	31008	0	1074944	41344
2	4443681	62033	-1513	2277	382789	304589	-22287	1353	542963	-13243
3	3499056	139872	752	0	500456	1140304	18392	0	3505728	18848
4	1158242	-59358	-1522	-558	-47182	137826	10602	-2574	1907334	48906
5	6422000	31200	-5200	0	595400	164880	-82440	0	95040	-15840
6	164331	28923	-899	-3337	103447	1037807	-96773	-3421	3191793	-99209
7	5630688	-178752	4704	0	16464	-29792	2352	0	3558016	-93632
8	3814668	-44148	2964	-3732	533676	-352052	70908	1132	320356	-21508
9	1384416	-57408	-1184	0	-319792	755872	46768	0	1492608	30784
10	8724705	-282095	-1705	69	-13915	25645	465	5129	5718835	34565
11	1153680	-106720	-304	0	313720	-1654160	-14136	0	6189760	17632
12	5624190	170430	274	6670	-953810	-1647490	-7946	-4370	3238170	5206
13	14997840	143520	144	0	-1184040	-645840	-1944	0	861120	864
14	6561555	31395	21	-3105	-345345	-94185	-189	1035	94185	63

	L=14 M=10	L=14 M=10	L=14 M=10	L=14 M=10	L=14 M=12	L=14 M=12	L=14 M=14	L=14 M=14	L=14 M=14	L=14 M=14
	N= 0 K=26	N= 4 K=27	N= 8 K=27	N=12 K=27	N= 0 K=26	N=12 K=27	N= 0 K=26	N= 4 K=27	N= 8 K=27	N=12 K=27
D=	58815393	2091045	6670	28275	75404350	58	145422675	190285095	5462730	5481
S	E(L,M,N,S)									
0	1716	18564	352716	208012	1716	5200300	1716	3060	19380	742900
1	0	7072	268736	237728	0	7131840	0	1632	20672	1188640
2	165	1513	13243	-7429	-1287	1448655	-3003	-4539	-13243	482885
3	0	21432	537168	66424	0	454480	0	-3384	-28272	-227240
4	-3630	-16742	537966	317262	-2574	5624190	2002	1522	-16302	-624910
5	0	-27560	-83952	321816	0	11840400	0	2600	2640	-657800
6	55	-93	-10263	7337	4147	13830245	-1001	279	10263	-476905
7	0	-25872	514976	-311696	0	11051040	0	-1008	6688	-263120
8	4460	-34788	252436	-389804	-2964	6476340	364	-468	1132	-113620
9	0	22128	-575328	-272912	0	2847520	0	144	-1248	-38480
10	-4091	45001	-912293	-126821	1209	936975	-91	165	-1115	-10075
11	0	10600	-614800	-40280	0	225264	0	24	-464	-1976
12	1550	-12090	-229710	-8494	-274	37538	14	-18	-114	-274
13	0	-7800	-46800	-1080	0	3888	0	-8	-16	-24
14	-225	-1365	-4095	-63	27	189	-1	-1	-1	-1

	L=16 M= 0	L=16 M= 2	L=16 M= 2	L=16 M= 2	L=16 M= 2	L=16 M= 2	L=16 M= 4	L=16 M= 4	L=16 M= 4	L=16 M= 4
	N= 0 K=31	N= 0 K=29	N= 4 K=28	N= 8 K=29	N=12 K=28	N=16 K=30	N= 0 K=30	N= 4 K=29	N= 8 K=29	N=12 K=29
D=	66	8415	285285	1124838	216315	1944671850	5819814	66	79695	10360350
S					E(L,M,N,S)					
0	41409225	920205	80223	85085	109395	7293	70785	10669659	323323	31977
1	0	0	4719	10010	19305	1716	0	1255254	76076	11286
2	83431920	1799512	138424	88088	-12584	-12584	125840	16736720	304304	-3344
3	0	0	39325	62062	51051	-4004	0	9831250	443300	28050
4	85357272	1673672	80344	-70840	-200200	8008	80344	6668552	-167992	-36520
5	0	0	92323	56210	-115115	4004	0	20126414	350108	-55154
6	88884432	1452360	8680	-135800	54600	-3640	10416	107632	-48112	1488
7	0	0	134113	-73402	-69433	-2548	0	22799210	-356524	-25942
8	94645460	1113476	-44436	-48188	121212	1092	-74060	5110140	158332	-30636
9	0	0	127995	-190750	274365	1092	0	13567470	-577700	63918
10	104006000	611800	-46920	40600	-78120	-168	-156400	20738640	-512720	75888
11	0	0	41745	-34650	-237545	-308	0	1085370	-25740	-13574
12	120349800	-157320	13800	-13320	-9528	-8	-207000	31395000	-865800	-47640
13	0	0	-130065	305370	90857	52	0	9104550	-610740	-13978
14	155117520	-1520760	80040	121800	19720	8	-160080	14567280	633360	7888
15	0	0	-310155	-134850	-13485	-4	0	56448210	701220	5394
16	300540195	-5892945	-310155	-67425	-4495	-1	310155	28224105	175305	899

	L=16 M= 4	L=16 M= 6	L=16 M= 6	L=16 M= 6	L=16 M= 6	L=16 M= 6	L=16 M= 8	L=16 M= 8	L=16 M= 8	L=16 M= 8
	N=16 K=30	N= 0 K=29	N= 4 K=28	N= 8 K=29	N=12 K=28	N=16 K=30	N= 0 K=30	N= 8 K=30	N=12 K=29	N=16 K=31
D=	931395465	415701	231	22770	740025	266112990	15935205	66	19305	173551950
S					E(L,M,N,S)					
0	10659	135135	2909907	617253	61047	20349	45045	23661365	780045	52003
1	5016	0	513513	217854	32319	14364	0	11134760	550620	48944
2	-16720	200200	3803800	484120	-5320	-26600	48048	13361712	-48944	-48944
3	-11000	0	3592875	1134042	71757	-28140	0	48272952	1018164	-79856
4	7304	19096	226424	-39928	-8680	1736	-42504	10220280	740600	-29624
5	9592	0	5518997	672038	-105869	18412	0	14708040	-772340	26864
6	-496	-191016	-281976	882312	-27288	9096	-97776	51937680	-535440	35696
7	-476C	0	2778055	-304094	-22127	-4060	0	13464024	326564	11984
8	-1380	-312340	3078780	667748	-129204	-5820	-48188	11847364	-764124	-6884
9	1272	0	-2029635	604950	-66933	-1332	0	59405000	-2190900	-8720
10	816	-225400	4269720	-738920	109368	1176	81200	30612400	-1510320	-3248
11	-88	0	-1529385	253890	133889	868	0	1474200	259140	336
12	-200	124200	-2691000	519480	28584	120	119880	57662280	1057608	888
13	-40	0	3251625	-1526850	-34945	-100	0	98939880	754812	432
14	16	560280	-7283640	-2216760	-27608	-56	-146160	66502800	276080	112
15	8	0	-12096045	-1051830	-8091	-12	0	21036600	53940	16
16	1	-310155	-4032015	-175305	-899	-1	40455	2629575	4495	1

	L=16 M=10	L=16 M=10	L=16 M=10	L=16 M=10	L=16 M=10	L=16 M=12	L=16 M=12	L=16 M=12	L=16 M=14	L=16 M=14
	N= 0 K=29	N= 4 K=28	N= 8 K=29	N=12 K=28	N=16 K=30	N= 0 K=30	N=12 K=29	N=16 K=30	N= 0 K=29	N= 4 K=28
D=	8580495	148005	150150	10395	3738042	110320650	66	148335	319929885	30045015
S					E(L,M,N,S)					
0	32175	124355	260015	557175	185725	19305	15043725	1002915	6435	10659
1	0	36575	152950	491625	218500	0	15928650	1415880	0	4389
2	17160	58520	73416	-17480	-87400	-2288	104880	104880	-5720	-8360
3	0	158125	491970	674475	-264500	0	12562830	-985320	0	1375
4	-58344	-124168	215832	1016600	-203320	-40040	31395000	-1255800	-8008	-7304
5	0	-41965	-50370	171925	-29900	0	23725650	-825240	0	-20383
6	-45864	-12152	374808	-251160	83720	13104	3229200	-215280	15288	1736
7	0	-279055	301098	474695	87100	0	4633590	170040	0	24395
8	52780	-93380	-199636	836940	37700	40404	28831140	259740	-12740	9660
9	0	-8533	25070	-60099	-1196	0	47269170	188136	0	-11607
10	62600	-212840	363080	-1164360	-12520	-52080	43590960	93744	6440	-9384
11	0	73953	-121014	-1382697	-8964	0	26648230	34552	0	1243
12	-100440	390600	-743256	-886104	-3720	28584	11347848	9528	-2040	3400
13	0	155295	-718794	-356439	-1020	0	3368698	1928	0	805
14	48024	-112056	-336168	-90712	-184	-7888	670480	272	376	-376
15	0	-94395	-80910	-13485	-20	0	80910	24	0	-217
16	-8091	-18879	-8091	-899	-1	899	4495	1	-31	-31

	L=16 M=14	L=16 M=14	L=16 M=14	L=16 M=16	L=16 M=16
	N= 8 K=29	N=12 K=28	N=16 K=30	N= 0 K=31	N=16 K=32
D=	5598450	4785	2046	9917826435	66
S			E(L,M,N,S)		
0	52003	1002915	9694845	6435	300540195
1	42826	1238895	15967980	0	565722720
2	-24472	52440	7603800	-11440	471435600
3	9982	123165	-1400700	0	347373600
4	29624	1255800	-7283640	8008	225792840
5	-57086	1753635	-8844420	0	129024480
6	-124936	753480	-7283640	-4368	64512240
7	-61418	-871455	-4637100	0	28048800
8	48188	-1818180	-2375100	1820	10518300
9	79570	-1716741	-990756	0	3365856
10	37352	-1078056	-336168	-560	906192
11	-4746	-488047	-91756	0	201376
12	-15096	-161976	-19720	120	35960
13	-8694	-38801	-3220	0	4960
14	-2632	-6392	-376	-16	496
15	-434	-651	-28	0	32
16	-31	-31	-1	1	1

	L=18 M= 0	L=18 M= 2	L=18 M= 2	L=18 M= 2	L=18 M= 2	L=18 M= 2	L=18 M= 4	L=18 M= 4
	N= 0 K=34	N= 0 K=34	N= 4 K=33	N= 8 K=34	N=12 K=33	N=16 K=35	N= 0 K=33	N= 4 K=32
D=	74	537795	14245	8510	1110555	21459624450	16564086	74
S				E(L,M,N,S)				
0	295488050	3476330	10808226	29059070	1385670	48906	316030	75657582
1	0	0	505648	2722720	194480	9152	0	7079072
2	594452430	6829966	19248086	35773738	429858	-47762	576290	125054930
3	0	0	4283136	18450432	768768	0	0	57108480
4	605260656	6454448	12825072	-12412400	-2482480	-45136	413712	63297936
5	0	0	10448256	24344320	-640640	-46592	0	125379072
6	624660036	5801796	4294836	-44864820	-900900	88452	159068	9066876
7	0	0	16409120	-3139136	-2018016	81536	0	164091200
8	655237800	4823560	-3445400	-34615784	1839544	-65912	-161000	8855000
9	0	0	18779040	-50621760	1118880	-72576	0	137712960
10	702040500	3429300	-7152540	745500	467180	25844	-503700	80894220
11	0	0	14087040	-60023040	3104640	39424	0	56348160
12	775587600	1440720	-4322160	11538288	-674640	-3888	-800400	184892400
13	0	0	0	17951232	-4126720	-13312	0	0
14	901620585	-1550775	4652325	-8967525	-464225	-935	-930465	214937415
15	0	0	-22331160	106801200	1923240	2592	0	104212080
16	1166803110	-6823410	10545270	55477290	521730	494	-620310	73816890
17	0	0	-42181080	-56485968	-324632	-224	0	421810800
18	2268783825	-23881935	-47453715	-31773357	-121737	-63	1550775	237268575

	L=18 M= 4 N= 8 K=32	L=18 M= 4 N=12 K=32	L=18 M= 4 N=16 K=33	L=18 M= 6 N= 0 K=34	L=18 M= 6 N= 4 K=33	L=18 M= 6 N= 8 K=33	L=18 M= 6 N=12 K=33	L=18 M= 6 N=16 K=34
D=	1638175	855127350	1365612465	235841034	464646	396825	23015850	330800535
S	E(L,M,N,S)							
0	529074	25194	48906	170170	1939938	2184126	312018	201894
1	99008	7072	18304	0	272272	613088	131376	113344
2	603694	7254	-44330	270270	2792790	2170602	78246	-159390
3	638976	26624	0	0	2013440	3627008	453376	0
4	-159120	-31824	-31824	82992	604656	-244720	-146832	-48944
5	758784	-19968	-79872	0	3589248	3497216	-276096	-368128
6	-246012	-4940	26676	-155932	-423244	1848908	111380	-200484
7	-81536	-52416	116480	0	2940000	-235200	-453600	336000
8	231080	-12280	24200	-348600	913000	3835928	-611544	401720
9	-964224	21312	-76032	0	142560	-160704	10656	-12672
10	-21900	-13724	-41756	-384300	2938980	-128100	-240828	-244244
11	-623616	32256	22528	0	-1837440	3273984	-508032	-118272
12	-1282032	74960	23760	-174000	1914000	-2136720	374800	39600
13	0	0	0	0	0	-1380864	952320	56320
14	-1076103	-55707	-6171	283185	-3115035	2510907	389949	14399
15	-1294560	-23312	-1728	0	2049720	-4099440	-221464	-5472
16	1008678	9486	494	728190	-4126410	-9078102	-256122	-4446
17	1467168	8432	320	0	-9169800	-5135088	-88536	-1120
18	412641	1581	45	-471975	-3438675	-962829	-11067	-105

	L=18 M= 8	L=18 M= 8	L=18 M= 8	L=18 M= 8	L=18 M=10	L=18 M=10	L=18 M=10	L=18 M=10
	N= 0 K=33	N= 8 K=33	N=12 K=32	N=16 K=34	N= 0 K=34	N= 4 K=33	N= 8 K=33	N=12 K=33
D=	61846425	74	9657	186605430	519509970	280830	3885	225330
S				E(L,M,N,S)				
0	170170	163809450	7800450	275310	121550	2645370	23401350	3343050
1	0	61308800	4379200	206080	0	618800	10948000	2346000
2	214214	129030230	1550430	-172270	101530	2002910	12231170	440910
3	0	316538880	13189120	0	0	3244800	45926400	5740800
4	-80080	17710000	3542000	64400	-167024	-2323152	7387600	4432560
5	0	203315200	-5350400	-389120	0	1223040	9363200	-739200
6	-332332	295538100	5934500	-582660	-244244	-1265628	43440540	2616900
7	0	1793792	1153152	-46592	0	-4790240	3011008	5806944
8	-323512	266989832	-14188312	508376	7000	-35000	-1155400	184200
9	0	298909440	-6606720	428544	0	-4348512	38515392	-2553888
10	10500	262500	164500	9100	295100	-4308460	1475500	2773940
11	0	305571840	-15805440	-200704	0	1822592	-25516288	3959424
12	427344	393583824	-23012720	-132624	109040	-2289840	20085168	-3523120
13	0	22093824	-5079040	-16384	0	0	14268928	-9840640
14	358701	238536165	12348385	24871	-430621	9043041	-57272593	-8894551
15	0	712008000	12821600	17280	0	5286120	-83067600	-4487560
16	-652674	610250190	5739030	5434	257114	-2781506	-48080318	-1356498
17	0	225943872	1298528	896	0	-3056600	-13449040	-231880
18	207669	31773357	121737	63	-49445	-687735	-1513017	-17391

	L=18 M=10	L=18 M=12	L=18 M=12	L=18 M=12	L=18 M=14	L=18 M=14	L=18 M=14	L=18 M=14
	N=16 K=34	N= 0 K=33	N=12 K=32	N=16 K=33	N= 0 K=35	N= 4 K=34	N= 8 K=34	N=12 K=34
D=	43541267	3587092650	74	3217335	66719923290	1767263190	498945	34410
S				E(L,M,N,S)				
0	589950	24310	96948450	1140570	24310	75582	4680270	19389690
1	552000	0	81640800	1280640	0	24752	3065440	19049520
2	-244950	7722	4862430	-180090	-7150	-20150	-861350	-900450
3	0	0	143431680	0	0	49920	4945920	17928960
4	402960	-48048	184892400	1120560	-48048	-95472	2125200	36978480
5	-268800	0	36748800	890880	0	-134784	-7223040	16536960
6	-1284660	-20020	31102500	-1017900	28028	20748	-4984980	-8708700
7	-1173120	0	193482432	-2605824	0	-14560	64064	3583008
8	-33000	51576	196791912	-2350392	37240	-2 600	-6146728	28418376
9	828288	0	38112960	-824064	0	165024	-10231488	19674528
10	767260	19740	26905620	496132	-67060	139868	-335300	-18280556
11	251392	0	213373440	903168	0	-104576	10248448	-46118016
12	-101520	-74960	351187600	674640	47920	-143760	8826864	-44901040
13	-158720	0	304742400	327680	0	0	1365504	-27310080
14	-89573	55707	166842465	112013	-18969	56907	-2522877	-11362431
15	-30240	0	60261520	27072	0	18984	-2088240	-3271576
16	-6422	-18414	14086710	4446	4114	-6358	-769318	-629442
17	-800	0	1947792	448	0	-4760	-146608	-73304
18	-45	2387	121737	21	-385	-765	-11781	-3927

	L=18 M=14 N=16 K=35	L=18 M=16 N= 0 K=34	L=18 M=16 N=16 K=35	L=18 M=18 N= 0 K=35	L=18 M=18 N= 4 K=34	L=18 M=18 N= 8 K=34	L=18 M=18 N=12 K=34	L=18 M=18 N=16 K=35
D=	6919	21585857535	74	167890003050	17557874550	1175614209	9008538	11655
S	E(L,M,N,S)							
0	106073010	24310	1166803110	24310	38038	152950	1900950	129644790
1	138949440	0	1746752560	0	16016	128800	2401200	218349120
2	15507750	-24310	579989850	-43758	-62062	-172270	-540270	115997970
3	0	0	0	0	-36608	-235520	-2561280	0
4	104212080	-24752	590535120	31824	31824	-46000	-2401200	-84362160
5	186416640	0	1877975040	0	34944	121600	-835200	-117373440
6	132530580	55692	2896739820	-18564	-6916	107900	565500	-107286660
7	-22439040	0	305170940	0	-20384	5824	977184	-76292736
8	-157828440	-52360	2441000760	8568	-3080	-46216	641016	-44381832
9	-197808768	0	1549329408	0	7392	-29760	171680	-21518464
10	-156746044	30940	795510716	-3060	3212	-500	-81780	-8741876
11	-90771968	0	332423168	0	-1408	8960	-120960	-2968064
12	-40109040	-12240	112693680	816	-1232	4912	-74960	-834768
13	-13655040	0	30638080	0	0	512	-30720	-191488
14	-3547203	3179	653$203	-153	231	-665	-8985	-34969
15	-683424	0	1057536	0	56	-400	-1880	-4896
16	-92378	-494	122018	18	-14	-110	-270	-494
17	-7840	0	8960	0	-8	-16	-24	-32
18	-315	35	315	-1	-1	-1	-1	-1

	L=20 M= 0	L=20 M= 2	L=20 M= 2	L=20 M= 2	L=20 M= 2	L=20 M= 2	L=20 M= 2	L=20 M= 4
	N= 0 K=38	N= 0 K=37	N= 4 K=38	N= 8 K=37	N=12 K=38	N=16 K=38	N=20 K=38	N= 0 K=38
D=	82	899745	96186	480930	21401625465	25432710	581073841725	1055400885
S				E(L,M,N,S)				
0	4266847442	20415538	126095970	29099070	293930	-127019750	102102	1200914
1	0	0	4803656	2217072	33592	1478048	19448	0
2	8574525960	40244984	229633144	39880776	182104	241906600	-182104	2229448
3	0	0	41164344	15943824	164424	2482272	-47736	0
4	8700621930	38457978	147241386	-2322762	-451438	-195570414	128622	1724138
5	0	0	103090000	25615200	-5200	-9547200	53040	0
6	8923714800	35377680	80255760	-38707344	-404144	116401584	-70992	922480
7	0	0	170155440	10946208	-397488	2079168	-39984	0
8	9266934600	30826440	-8067960	-43763208	194472	-41544600	29784	-110520
9	0	0	214792560	-29090016	-223344	15774912	22032	0
10	9772403760	24492240	-70052400	-17532240	353040	3790320	-8976	-1273680
11	0	0	206482320	-63606048	574320	-23660736	-8976	0
12	10518906825	15817905	-79655895	9879111	-59155	2518125	1683	-2413815
13	0	0	118506180	-44036616	399100	16834896	2652	0
14	11668031100	3721860	-23032380	5512668	-57260	95148	-84	-3290340
15	0	0	-60682500	43691400	-1011900	-6814800	-540	0
16	13612702950	-14267130	80019990	-9221942	-194258	-882818	-46	-3479130
17	0	0	-305354340	104046664	576164	1519664	68	0
18	17672631900	-45902940	113759460	68255676	188404	351500	12	-1995780
19	0	0	-492957660	-65727688	-109668	-146224	-4	0
20	34461632205	-149184555	-616197075	-41079805	-45695	-45695	-1	6486285

	L=20 M= 4 N= 4 K=39	L=20 M= 4 N= 8 K=38	L=20 M= 4 N=12 K=39	L=20 M= 4 N=16 K=39	L=20 M= 4 N=20 K=39	L=20 M= 6 N= 0 K=37	L=20 M= 6 N= 4 K=37	L=20 M= 6 N= 8 K=36
D=	82	3690	18245205	56394270	1288468083825	45734038350	7995	14391
S				E(L,M,N,S)				
0	8700621930	669278610	20281170	13123110	138138	92378	223092870	85804950
1	662904528	101985312	4635696	3999424	52624	0	25496328	19612560
2	14921695464	863825352	11833224	-5336552	-232024	153816	343163496	99329640
3	5461136304	705071328	21813584	6457152	-124176	0	195970424	126505840
4	8794827938	-40716182	-23740054	-17108754	132626	69186	117639262	-2723090
5	12576980000	1041684800	-634400	-22838400	126880	0	384763600	159339680
6	2454719280	-394636944	-12361232	7195344	-42576	-45488	-40347856	32432944
7	18036476640	386766016	-42133728	4321408	-83104	0	400519728	42942816
8	33929640	61348424	-817848	-547688	-2456	-154008	15760152	142480216
9	18185770080	-820984896	-18909792	26188416	36576	0	176607216	-39864096
10	4273196400	356488880	-21535440	16050320	10736	-212976	238178160	99349360
11	11975974560	-1229716928	33310560	-26908288	-10208	0	-123889392	63606048
12	14258405205	-589453623	10588745	-24945261	-5907	-180699	355796331	-73544493
13	3081160680	-381650672	10376600	8582496	1352	0	-187786716	116301832
14	23884578060	-1905545572	59378620	15561588	1708	-32364	78310092	-31238452
15	687735000	-165056400	11468200	1514400	120	0	105452700	-126543240
16	22889196270	-879292122	-55566234	-4383678	-258	210366	-461332638	88610834
17	16489134360	-1872839952	-31112856	-1609056	-72	0	183212604	-104046664
18	5801732460	1160346492	9608604	345876	12	399156	-386782164	-386782164
19	50281681320	2234741392	11186136	292448	8	0	-1160346492	-257854776
20	3142605C825	698356685	2330445	45695	1	-299367	-483477705	-53719745

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S				E(L,M,N,S)				
0	7800450	1682450	230230	277134	2316733650	23401350	45426150	478170
1	2674440	769120	131560	0	706052160	10697760	27688320	364320
2	4082040	-613640	-346840	387192	2250330120	10275480	-13902120	-604440
3	11741560	1158560	-289640	0	4096336320	42244320	37514880	-721440
4	-4763190	-1144230	115310	-23946	8482410	1648590	3564270	-27630
5	-291120	-3493440	252304	0	3882476160	-788160	-85121280	472896
6	3047696	-591344	45488	-444912	2855000304	29809104	-52054704	308016
7	-14034384	479808	-119952	0	373213568	-13552448	4169984	-80192
8	-5698296	-1271992	-74152	-599496	4991603528	-22181352	-44562536	-199832
9	-2754576	1271616	23088	0	1667827584	12805056	-53201664	-74304
10	-18005040	4473040	38896	-318768	1338294320	-26948720	60254480	40304
11	-5168880	1391808	6864	0	5682140288	-51305920	124334848	47168
12	3963385	-3112351	-9581	299367	1096581321	-6566205	46406547	10989
13	-9486300	-2615376	-5356	0	2127307296	-19279600	-47838528	-7536
14	2920260	255108	364	787524	6841220988	-71059660	-55868652	-6132
15	26376860	1161040	1196	0	1782609120	-41285520	-16355520	-1296
16	16799094	441766	338	400954	1520016614	32018786	7577986	446
17	-5185476	-89392	-52	0	9572293088	53007088	8224064	368
18	-9608604	-115292	-52	-1197468	10443118428	28825812	3112884	108
19	-3872124	-33744	-12	0	4469482784	7457424	584896	16
20	-537795	-3515	-1	432419	698356685	776815	45695	1

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S	E(L,M,N,S)							
0	277134	12168702	42122430	135727830	23951970	1260630	92378	21037813650
1	0	2317848	16046640	77558760	18249120	1200600	0	14425929360
2	291720	11833224	30826440	44902440	-5522760	-1200600	58344	4175926920
3	0	13721448	79719120	262256280	21172320	-2035800	0	38773582320
4	-261690	-8090186	1685430	104494830	20538090	-796050	-153582	28516810410
5	0	11355760	42324192	-2740848	-26910144	747504	0	14240160
6	-583440	-9409296	68071536	226724784	-35992944	1064880	-153296	27700433904
7	0	-12424048	-11988704	138874736	-3884608	373520	0	43799884128
8	-302280	562424	45761528	-64869288	-11847544	-265640	87912	8772650568
9	0	-26718768	54279072	132939312	-50211648	-350640	0	8749972704
10	382512	-7777744	-29198416	187558384	-38123696	-127504	211824	48296931120
11	0	-7580368	35026528	-100888880	22226752	42160	0	41230432800
12	684015	-24487737	45555399	-87017005	55908297	66195	-59155	3499314025
13	0	9996012	-55717416	161082900	36335952	28620	0	15551085000
14	-61380	2700348	-9694692	32122860	2295972	1260	-269940	65691248700
15	0	-4395180	47467944	-350697556	-12630096	-5004	0	85100924920
16	-1019590	40653834	-70277558	-472241198	-10160618	-2990	278718	60028331814
17	0	32992308	-168627352	-297876788	-4201424	-940	0	26124428088
18	757020	-13337316	-120035844	-105694644	-1037628	-180	-109076	7081541148
19	0	-19265012	-38530024	-20507916	-146224	-20	0	1107427464
20	-164021	-4816253	-4816253	-1708993	-9139	-1	15873	76904685

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D=	69003	6306184170	239194637550	65793315	13158663	322014	861	78686790
S				E(L,M,N,S)				
0	412506150	4342170	92378	5624190	6489450	176788350	1944671850	20470230
1	377148480	4962480	0	1499784	3461040	141430680	2074316640	27293640
2	-57068520	-2481240	12376	696072	604440	7443720	-57068520	-2481240
3	347805120	-6688560	0	4904952	9498960	264198120	1329513120	-25567560
4	622764270	-4827630	-188734	-8090186	561810	294485430	3607857810	-27967890
5	15534720	-86304	0	-6026800	-7487520	4099440	2508857280	-13938096
6	-488610096	2891184	-24752	-553488	1334736	37585392	-371927088	2200752
7	-136130176	2617888	0	-9491664	-3053024	299000208	-521333696	10025648
8	178023576	798312	208488	-537864	-14587736	174830328	1990335656	8925272
9	-367210624	-512864	0	8942832	-6055776	-125395248	2952245568	4123248
10	-1090775920	-729616	-17136	483120	604560	-32832720	415992720	278256
11	-1009271680	-382880	0	6201360	-9551520	232599600	-3194199360	-1211760
12	-249811565	-59155	-243355	12079815	-7490835	120971975	-4844815635	-1147245
13	389767200	61400	0	-6391580	11875480	-290268500	-4081383120	-642940
14	521695860	57260	259540	-15831940	18946420	-530759300	-2364668940	-259540
15	340538080	26984	0	-1976100	7113960	-444359020	-997535280	-79044
16	143505986	8446	-127554	7051914	-4063506	-230854518	-309610002	-18222
17	40941536	1832	0	3104892	-5289816	-79002252	-69457584	-3108
18	7724564	268	32116	-784548	-2353644	-17521572	-10722156	-372
19	877344	24	0	-767676	-511784	-2303028	-1023568	-28
20	45695	1	-3367	-137085	-45695	-137085	-45695	-1

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S				E(L,M,N,S)				
0	92378	13612702950	143291610	92378	230230	796950	21710850	238819350
1	0	16594533120	218349120	0	78936	546480	22331160	327523680
2	-40664	1312575960	57068520	-100776	-232024	-604440	-7443720	57068520
3	0	5250735360	-100975680	0	-20696	-120240	-3344280	-16829280
4	-171054	22889196270	-177435630	-75582	-132626	27630	14482890	177435630
5	0	28521745920	-158454144	0	-317200	-1182240	647280	396135360
6	137904	14505156432	-85829328	201552	184496	-1334736	-37585392	371927088
7	0	712065536	-13693568	0	540176	521248	-51048816	89008192
8	90984	6080066456	27264872	-209304	22104	1798488	-21554424	-245383848
9	0	25936292352	36223872	0	-432816	879264	18206672	-428649152
10	-243984	41460607760	27732848	139536	-161040	-604560	32832720	-415992720
11	0	41579800064	15773824	0	188848	-872608	21249840	-291815744
12	215373	30014165907	7107309	-64923	131923	-245421	3963385	-158729901
13	0	16441218432	2589984	0	-35828	199704	-4881300	-68634576
14	-109332	6972866964	765324	21204	-52948	190092	-5325180	-23725044
15	0	2293407360	181728	0	-4300	46440	-2900780	-6511920
16	33982	577388162	33982	-4674	10578	-18286	-1038858	-1393262
17	0	107985536	4832	0	3348	-17112	-255564	-224688
18	-6068	14180916	492	628	-628	-5652	-42076	-25748
19	0	1169792	32	0	-468	-936	-4212	-1872
20	481	45695	1	-39	-65	-65	-195	-65

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S				E(L,M,N,S)				
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1	3029594040	0	131282408400	0	0	79948960	806208	25395552
2	1744311720	-167960	113380261800	62451130742	2926845766	4680583310	18819918	227930118
3	227518920	0	88732378800	0	0	690987440	6046560	142094160
4	-966955410	125970	62852101650	63209646500	2819628500	3625106180	2659140	-325784940
5	-1547128656	0	40225345056	0	0	1763478000	10975200	71869200
6	-1547128656	-77520	23206929840	64537580250	2635922250	2102619330	-13524990	-461825910
7	-1203322288	0	12033222880	0	0	3011480640	8384640	-256925760
8	-773564328	38760	5586853480	66545416080	2367228240	442777104	-20195568	-44776368
9	-420866416	0	2311801440	0	0	4040580672	-4606848	-381506112
10	-195613968	-15504	847660528	69424785045	1999245645	-947011095	-14044023	336353517
11	-77824912	0	273438880	0	0	4393801368	-21785808	114089448
12	-26423661	4845	76904685	73508595930	1508081490	-1645660854	-1358358	181173042
13	-7600836	0	18643560	0	0	3622502520	-28338960	653420040
14	-1830612	-1140	3838380	79407433875	851914875	-1321813185	5368735	-109877285
15	-362748	0	658008	0	0	1397046000	-10093600	59984400
16	-57646	190	91390	88363159500	-49894500	85818540	-435860	7517660
17	-7068	0	9880	0	0	-2307121680	28078560	-954419440
18	-628	-20	780	103384896615	-1381578705	1990890339	-2687421	-270168561
19	-36	0	40	0	0	-6840735528	42888624	668463016
20	-1	1	1	134564468610	-3723127590	2004761010	34894034	257043514
21				0	0	-9851968392	-32354576	-142464504
22				263012370465	-11435320455	-13546456539	-22243771	-65296231

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S				E(L,M,N,S)				
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1	18811520	671840	0	3997448000	40310400	97675200	3808000	1768000
2	-3485170	-4879238	56392570	111375325750	447823350	417202950	-335750	-6110650
3	55090880	-335920	0	33443792096	292653504	529027488	10795136	-855712
4	-285280060	-2551700	45887420	72860045476	53445348	-503681916	-23213636	-2699260
5	-100776000	-2713200	0	79709205600	496079040	249883680	-18441600	-6454560
6	27335490	7509750	29068470	28636318746	-184201638	-483828534	1507254	5383050
7	-112869120	5891520	0	121663817856	338739456	-798446208	-18461184	12527232
8	241257744	-7395408	7028208	1623516048	-74050416	-12629232	3581424	-1427184
9	222513408	-6511680	0	137379742848	-156632832	-997785216	30629376	-11652480
10	-87224535	4398291	-18568845	10862774325	161093205	-296782515	4050675	-2655315
11	71287392	4689960	0	114238835568	-566431008	228178896	7503936	6417840
12	-126235734	-1652658	-45293418	61040429658	50383866	-516924918	18956694	3226314
13	-397531680	-2341560	0	59409041328	-464758944	824314512	-26394816	-2021136
14	100839935	333925	-69569115	133308338163	-541451053	852416411	-41173979	-1772485
15	39048800	813200	0	7264639200	-52486720	23993760	8220800	222560
16	3489260	4900	-85818540	182295742668	-925853812	1228385644	30007636	547820
17	-190166080	-189040	0	17534124768	-213397056	557968288	5851264	75616
18	-30458077	-22223	-84122127	149709345351	-202086489	-1562762673	-9272719	-87953
19	48285536	26600	0	150496181616	-943549728	-1131245104	-4300736	-30800
20	12797330	5446	-40913490	27207470850	473561890	268342130	703150	3890
21	-5133856	-1720	0	374374798896	1229473888	416434704	789824	3440
22	-1764763	-473	175928007	257382674241	422631649	95432953	135751	473

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S				E(L,M,N,S)				
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1	0	453492000	1632571200	33242400	1296000	4212000	0	4208405760
2	27335490	7712513250	11070873450	86671350	-69750	-8886150	1587222	18641922390
3	0	3585005424	11199438432	170127216	3471552	-1926288	0	26512956288
4	15072540	3418882716	895308876	-70904268	-3267828	-2659860	235620	405879012
5	0	7526478000	16722546400	70785200	-5224000	-12798800	0	31963879808
6	-2241810	-315497394	724503174	15991578	-49818	-1245450	-1308870	12266933242
7	0	9055811904	9001190528	-178292416	-4122368	19581248	0	9764192768
8	-20243856	-668047248	10877931312	15590096	-4421072	12332464	-2243952	34967504016
9	0	6258938688	-2547586944	-136375488	4186368	-11148480	0	1848881664
10	-33510225	2800497375	14826541275	-229537675	3132875	-14375725	-1927611	24733176741
11	0	373329528	-660836176	2237048	73568	440440	0	29076791744
12	-36360954	7000349382	2062822398	-177848766	6522078	7770126	-261702	430557946
13	0	-4151502888	11594413168	-172809256	5533408	2965976	0	37641655936
14	-24133655	6606415673	-9579349091	126730643	-6121427	-1844635	1977955	22768126979
15	0	-2594514000	6692056800	-25707600	-8808000	-1669200	0	3926006656
16	4656820	-1413145292	2562246596	-28567108	-697852	-89180	3051020	48682685324
17	0	3953773232	-17178463008	377449136	3958208	358064	0	26888029056
18	44006949	-11188242891	5391609993	350370279	2078937	138033	794871	2824176663
19	0	2280245176	-5103746256	-51420232	-195488	-9800	0	61244955072
20	68189150	-6477969250	-40252760650	-191672950	-502250	-19450	-4984862	85335852578
21	0	-26741057064	-31351584144	-89236008	-169248	-5160	0	41802112192
22	-58642669	-12256317821	-7184738033	-13633279	-19393	-473	2022161	7184738033

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S				E(L,M,N,S)				
0	225832860	6603300	17168580	1058148	171609900	1279719540	49005730620	1432916100
1	85691520	3340800	10857600	0	27132000	404654400	23243824800	906192000
2	145943370	-117450	-14963130	1284894	195205050	1160852310	25637385330	-20632050
3	402750144	8218368	-4560192	0	174466512	2257971552	96760722096	1974462912
4	-32143716	-1481436	-1205820	-596700	-72880020	-79067340	17664433020	814116420
5	135300544	-9985280	-24463936	0	202396560	1863003616	22246251248	-1641789760
6	270761574	-843494	-21087350	-2076750	-157375050	1497201850	93225445950	-290421950
7	-193405696	-4471808	21241088	0	-47917632	-197318528	11025641536	254928128
8	50114928	-14211696	39643152	-1847664	-32831568	2214780624	8954395632	-2539306224
9	98972928	-3038208	8090880	0	-341789760	576351360	87035757120	-2671768320
10	-382907637	5226165	-23981139	166689	-7501005	-164522043	7185240471	-98068695
11	-98430112	-3236992	-19379360	0	-295002840	2163354160	-20659138280	-679400480
12	-37121082	1361306	1621802	2327706	-241305522	-294584126	71647566582	-2627462806
13	-561031712	17964416	9629152	0	34584648	-400153616	16824728312	-538733216
14	-301212467	14549363	4384315	2123975	-313073915	1880688835	-70188453055	3390288895
15	-15081792	-5167360	-979264	0	97611920	-1043053088	11303460336	3872818880
16	-542775052	-13259188	-1694420	-1306100	213416740	-1603107140	50420945620	1231708780
17	-590789952	-6195456	-560448	0	-75147888	1352661984	-83842940688	-879238464
18	183527289	1088967	72303	-3796533	519735633	-1037621673	-190217692899	-1128664797
19	617042784	2345856	117600	0	570695400	-5291902800	-150404178600	-571802400
20	406346654	1064770	41234	3510338	-179567290	-4622575094	-62094368882	-162708910
21	118981344	225664	6880	0	-347028920	-1685569040	-13534127880	-25669280
22	13633279	19393	473	-848003	-95432953	-231765743	-1240628389	-1764763

	L=22 M=10	L=22 M=12	L=22 M=12	L=22 M=12	L=22 M=12	L=22 M=14	L=22 M=14	L=22 M=14
	N=20 K=43	N= 0 K=42	N=12 K=43	N=16 K=42	N=20 K=43	N= 0 K=42	N= 4 K=43	N= 8 K=42
0=	2006513145	40838108850	90	31635	236060370	2917007775	22088430	616590
S				E(L,M,N,S)				
0	40940460	3174444	147017191860	4298748300	122821380	12345060	286016500	878238900
1	32364000	0	83677769280	3262291200	116510400	0	63308000	388785600
2	-28884870	2746146	54793627470	-44095950	-61734330	5584670	121205750	296796150
3	-12039408	0	293403479904	5987081088	-36506592	0	267015056	1422955296
4	7281900	-4123860	122080859316	5626449036	50326020	-22802780	-397870924	-177737868
5	-44202032	0	27465758944	-2026993280	-54572896	0	-112954800	-428118880
6	-79786250	-6384690	286609159746	-892863426	-245292150	-15255630	-165151974	646959898
7	-13306688	0	183719046784	4247839232	-221727872	0	-583834944	-989946496
8	77838768	-710736	3444463568	-976788176	29942032	19338384	49089744	-1363574832
9	78187200	0	254083070592	-7799680512	228251520	0	-24803136	17222016
10	4945107	6595167	284289070313	-3880156585	195656621	21664545	-139272075	-1257817995
11	-44697400	0	15979437232	525502912	34572560	0	612311304	-1848940016
12	-34398322	4986414	153483485058	-5628553314	-73688118	-20818854	308317314	154584802
13	-3173272	0	401012255728	-12840541504	-75633968	0	1211304	-5770928
14	11226725	-5783015	191104357087	-9230848543	-30567365	-22623545	476387219	-1178363501
15	8065232	0	2778477408	951968640	1982496	0	-180545200	794398880
16	1729700	-7517660	290213250172	7089478468	9955820	41341580	-965030596	2984862076
17	-874032	0	622528501088	6528289664	6489632	0	-248772016	1843839648
18	-823503	11415573	571954454019	3393716157	2476143	-25222197	493264871	-405495321
19	-315000	0	298134505136	1133439424	624400	0	276470216	-1055613552
20	-69242	-5245786	92792708554	243149270	103474	7296770	-53322550	-565219030
21	-8600	0	16240953456	30803136	10320	0	-69405784	-138811568
22	-473	848003	1240628389	1764763	473	-848003	-13633279	-13633279

	L=22 M=14	L=22 M=14	L=22 M=14	L=22 M=16	L=22 M=16	L=22 M=16	L=22 M=18	L=22 M=18
	N=12 K=43	N=16 K=43	N=20 K=43	N= 0 K=41	N=16 K=42	N=20 K=42	N= 0 K=42	N= 4 K=43
D=	315	442890	16861455	39763421775	90	167895	516924483075	170639190
S	E(L,M,N,S)							
0	81676217700	2388193500	477638700	1763580	88363159500	2524661700	1058148	117417300
1	54235591200	2114448000	528612000	0	89410944000	3193248000	0	33415200
2	15518637950	-12810750	-125545350	-41990	24947250	34926150	-596258	-61979450
3	148088990448	3021848256	-128981328	0	85885598208	-523692672	0	42357744
4	96434719524	4444472604	27827646C	-3611140	182295742668	1630552260	-1815260	-151698412
5	-12415333840	916260800	172679920	0	105164300800	2831346560	0	-193175760
6	97832321106	-304773586	-586103050	377910	1955398718	537197450	1831410	94957002
7	134337915712	3106078976	-1134913472	0	69045803C08	-3604039168	0	85382976
8	-13388607856	3796769392	-814691408	3829488	194730741296	-5969189872	511632	6220368
9	6316045632	-193885952	39717440	0	168319031296	-4925724160	0	191267136
10	133409236465	-1820853425	642714835	-1710285	37230053975	-1877322835	-2549439	78495885
11	42880346168	1410171488	649421C80	0	12149C99776	799282880	0	-281743176
12	-91544466534	3357122822	307656398	-3474378	145106625086	1899712682	2705562	-191905038
13	589274776	-18868768	-777992	0	289044265216	1702542272	0	154284312
14	106801877623	-5158814647	-119581595	5307365	313449792481	1037968955	-1660505	167466299
15	-20907134160	-7163252800	-104423440	0	229294553600	477511040	0	-25356240
16	-227994088948	-5569556812	-54749660	-3489260	121749353476	170973740	659980	-73785764
17	-27755640161 6	-2910659648	-20253968	0	48127678976	47842688	0	-11854032
18	-180529676613	-1071180539	-5470927	1286961	14156142013	10328687	-169173	15845871
19	-72862468744	-277006496	-1068200	0	3029282048	1668800	0	6310392
20	-18438937790	-48316450	-143930	-261170	447906550	190610	25666	-898310
21	-2706825576	-5133856	-12040	0	41070848	13760	0	-888552
22	-177232627	-252109	-473	22919	1764763	473	-1763	-135751

	L=22 M=18	L=22 M=18	L=22 M=18	L=22 M=18	L=22 M=20	L=22 M=20	L=22 M=22	L=22 M=22
	N= 8 K=42	N=12 K=43	N=16 K=43	N=20 K=43	N= 0 K=42	N=20 K=43	N= 0 K=42	N= 4 K=43
0=	233403030	411255	1170	12915	6055401087450	90	11835556670925	23164440681690
S				E(L,M,N,S)				
0	51505740	2579248980	53017895700	19692361260	352716	269128937220	352716	508300
1	29315520	2202046560	60352387200	28020751200	0	425500296000	0	176800
2	-21681270	-626162490	354250950	6447367290	-411502	182433251910	-646646	-872950
3	32247072	1807076304	25922787264	-2054855088	0	11914008288	0	-427856
4	-9681012	2828322324	91637037012	10655469420	-226100	54414941700	497420	539852
5	-104595808	-1633288816	84738157760	29658355216	0	284408761952	0	461040
6	-53140122	-4326953526	9476163018	33843439350	726750	550627386250	-319770	-215322
7	20682112	-1511250496	-24564372224	16668681152	0	701886627968	0	-329664
8	-24683472	-130502064	26016657744	-10367540304	-821712	682686820624	170544	26928
9	-18972288	-3746585664	80852243712	-30759005760	0	537814121600	0	166464
10	101275083	-5783961459	55497037965	-36379644717	603687	353190902701	-74613	29835
11	121536272	-1517735384	-35088518432	-30009917080	0	196192396400	0	-58344
12	-13780962	4383057906	-112997195694	-19231435314	-318402	92792708554	26334	-24258
13	-105006512	5773552456	-129964318624	-9951786104	0	37415943344	0	12792
14	-59176403	2888037691	-98068428997	-4221715355	123025	12824073275	-7315	9581
15	15938208	-225865584	-54402788160	-1472835312	0	3710208736	0	-1040
16	32603012	-1340947364	-23028418148	-420407260	-34300	895813100	1540	-2236
17	12551328	-1017354864	-7500115776	-96924144	0	177444064	0	-272
18	-1860903	-446109201	-1860846609	-17650383	6573	28117103	-231	281
19	-3442032	-127928856	-341908512	-2448600	0	3430000	0	88
20	-1360298	-23895046	-44017190	-243514	-778	302642	22	-10
21	-253872	-2665656	-3554208	-15480	0	17200	0	-8
22	-19393	-135751	-135751	-473	43	473	-1	-1

	L=22 M=22	L=22 M=22	L=22 M=22	L=22 M=22
	N= 8 K=42	N=12 K=43	N=16 K=43	N=20 K=43
D=	646626422970	55828277505	158828670	21285
S		E(L,M,N,S)		
0	1560780	11165580	229514700	24466267020
1	1C8576C	11651040	319324800	42550029600
2	-2137590	-8819190	4989450	26061893130
3	-228C096	-18253296	-261846336	5957004144
4	241164	-10C65204	-326110452	-10882988340
5	1747424	3898064	-202239C40	-20314911568
6	843494	9811686	-21487898	-22025095450
7	-558976	5834944	54843136	-18470700736
8	-747984	-564944	112626224	-12880883408
9	-115584	-3260736	70367488	-7683058880
10	269451	-2198389	21C93515	-3968437109
11	176176	-314296	-7266208	-1783567240
12	-12194	554046	-14283554	-697689538
13	-6C944	478696	-10775584	-236809768
14	-23699	165229	-5610643	-69319315
15	4576	-9264	-2231360	-17337424
16	6916	-40636	-697852	-3656380
17	2016	-23344	-172096	-638288
18	-231	-7911	-32999	-89831
19	-336	-1784	-4768	-9800
20	-106	-266	-490	-778
21	-16	-24	-32	-40
22	-1	-1	-1	-1

3. TABLE 2

Relations between coefficients of associated Legendre polynomials in series representation of pole density function, $A_{\ell m} \equiv Q_{\ell m}/2\pi$, and coefficients of augmented Jacobi polynomials in series representation of crystallite orientation distribution function, $W_{\ell mn}$, for different sets of lattice planes (H K L).

Note that $A_{00} = W_{000}$ and $A_{2m} = W_{2m0} = 0$ for all (H K L), and that $A_{\ell m} = W_{\ell m0}$ for H K L = (1 0 0).

HKL = (1 1 0)

A4M = - 0.25000 W4M0

A6M = - 1.62500 W6M0

A8M = 0.56250 W8M0

A10M = - 0.03125 W10M0

A12M = 0.28906 W12M0 - 1.48306 W12M4

A14M = - 0.69531 W14M0

A16M = 0.40234 W16M0 + 0.16571 W16M4

A18M = 2.89258 W18M0 + 4.88429 W18M4

A20M = 0.06424 W20M0 - 0.83416 W20M4

A22M = - 0.57414 W22M0 - 0.17021 W22M4

HKL = (1 1 1)

A4M = - 0.66667 W4M0

A6M = 1.77777 W6M0

A8M = 0.29630 W8M0

A10M = - 1.58025 W10M0

A12M = 0.41152 W12M0 - 1.04160 W12M4

A14M = 1.05350 W14M0

A16M = - 0.70233 W16M0 + 1.24141 W16M4

A18M = - 1.95092 W18M0 - 2.63718 W18M4

A20M = 0.72416 W20M0 - 1.11096 W20M4

A22M = 1.30784 W22M0 + 3.02248 W22M4

HKL = (2 1 0)

A4M = 0.20000 W4M0

A6M = - 0.68000 W6M0

A8M = - 0.21600 W8M0

A10M = - 1.00640 W10M0

A12M = 0.13971 W12M0 + 0.70465 W12M4

A14M = - 0.20128 W14M0

A16M = 0.39525 W16M0 - 0.16377 W16M4

A18M = - 1.56507 W18M0 - 2.53793 W18M4

A20M = 0.02944 W20M0 - 0.69604 W20M4

A22M = - 1.59571 W22M0 - 2.18096 W22M4

HKL = (2 1 1)

A4M = - 0.25000 W4M0

A6M = 0.51389 W6M0

A8M = - 0.40046 W8M0

A10M = 0.80903 W10M0

A12M = 0.47296 W12M0 - 0.42518 W12M4

A14M = - 0.87002 W14M0

A16M = 0.00445 W16M0 + 0.15298 W16M4

A18M = - 1.97736 W18M0 - 2.98447 W18M4

A20M = - 0.00220 W20M0 - 0.23318 W20M4

A22M = 1.24866 W22M0 + 0.83011 W22M4

HKL = (3 1 0)

A4M = 0.55000 W4M0

A6M = 0.05500 W6M0

A8M = − 0.09350 W8M0

A10M = − 0.71765 W10M0

A12M = − 0.39505 W12M0 + 0.54586 W12M4

A14M = − 0.82783 W14M0

A16M = − 0.05431 W16M0 + 0.78402 W16M4

A18M = − 0.14549 W18M0 + 0.36102 W18M4

A20M = 0.32938 W20M0 + 0.50235 W20M4

A22M = − 0.28344 W22M0 − 0.63646 W22M4

HKL = (3 1 1)

A4M = 0.21488 W4M0

A6M = 0.13223 W6M0

A8M = − 0.57537 W8M0

A10M = 0.41090 W10M0

A12M = 0.02983 W12M0 − 0.87138 W12M4

A14M = 0.61797 W14M0

A16M = 0.40798 W16M0 − 0.34446 W16M4

A18M = − 1.25718 W18M0 − 1.64531 W18M4

A20M = 0.07327 W20M0 + 0.45589 W20M4

A22M = − 1.62559 W22M0 − 1.62576 W22M4

HKL = (3 2 1)

A4M = − 0.25000 W4M0

A6M = − 0.10969 W6M0

A8M = − 0.11972 W8M0

A10M = 0.56405 W10M0

A12M = − 0.05683 W12M0 + 0.45470 W12M4

A14M = 0.10953 W14M0

A16M = − 0.44354 W16M0 + 0.29623 W16M4

A18M = 1.24665 W18M0 + 1.91238 W18M4

A20M = 0.43903 W20M0 − 0.38062 W20M4

A22M = − 0.81348 W22M0 − 0.87882 W22M4

HKL = (3 3 1)

A4M = − 0.37119 W4M0

A6M = − 0.51553 W6M0

A8M = 0.33806 W8M0

A10M = 0.51761 W10M0

A12M = − 0.73678 W12M0 + 0.68897 W12M4

A14M = 0.34070 W14M0

A16M = 0.12563 W16M0 + 0.32841 W16M4

A18M = − 2.32628 W18M0 − 4.00433 W18M4

A20M = − 0.30388 W20M0 + 0.02120 W20M4

A22M = 0.90350 W22M0 + 1.78304 W22M4

HKL = (3 3 2)

A4M = - 0.58058 W4M0

A6M = 1.19525 W6M0

A8M = 0.22304 W8M0

A10M = - 0.54306 W10M0

A12M = - 0.18342 W12M0 + 0.12253 W12M4

A14M = 0.28016 W14M0

A16M = 0.34660 W16M0 - 0.42594 W16M4

A18M = 0.81281 W18M0 + 2.29286 W18M4

A20M = - 0.21278 W20M0 + 0.22089 W20M4

A22M = - 0.28057 W22M0 - 2.13633 W22M4

HKL = (4 3 1)

A4M = - 0.25000 W4M0

A6M = - 0.67871 W6M0

A8M = 0.13646 W8M0

A10M = 0.34051 W10M0

A12M = - 0.19829 W12M0 + 0.40420 W12M4

A14M = 0.33646 W14M0

A16M = - 0.44728 W16M0 + 0.32673 W16M4

A18M = - 0.09838 W18M0 - 0.61619 W18M4

A20M = 0.13891 W20M0 + 0.27365 W20M4

A22M = - 0.66614 W22M0 - 1.37141 W22M4

HKL = (5 2 1)

A4M = 0.28333 W4M0

A6M = - 0.07722 W6M0

A8M = - 0.43720 W8M0

A10M = - 0.15088 W10M0

A12M = - 0.11284 W12M0 - 0.18853 W12M4

A14M = 0.38827 W14M0

A16M = 0.22815 W16M0 - 0.02165 W16M4

A18M = 1.19661 W18M0 + 1.34282 W18M4

A20M = - 0.12467 W20M0 + 0.18948 W20M4

A22M = 0.95107 W22M0 + 1.87676 W22M4

HKL = (5 3 1)

A4M = - 0.05714 W4M0

A6M = - 0.61388 W6M0

A8M = - 0.17297 W8M0

A10M = - 0.03693 W10M0

A12M = 0.18282 W12M0 + 0.32304 W12M4

A14M = 0.45532 W14M0

A16M = - 0.41089 W16M0 + 0.13398 W16M4

A18M = 1.32332 W18M0 + 1.89377 W18M4

A20M = - 0.23396 W20M0 + 0.21714 W20M4

A22M = 0.11174 W22M0 - 0.25864 W22M4

4. TABLE 3

Relations between coefficients of crystallite orientation distribution functions before ($W_{\ell mn}$) and after ($W'_{\ell mn}$) transformation for various orientation relationships.

RELATIONS BETWEEN COEFFICIENTS, BAIN RELATION

w'4M0 = 0.16667 w4M0

w'6M0 = - 0.75000 w6M0

w'8M0 = 0.70833 w8M0

w'10M0 = 0.31250 w10M0

w'12M0 = 0.52604 w12M0 - 0.98870 w12M4

w'12M4 = - 0.24718 w12M0 - 0.01563 w12M4

w'14M0 = - 0.13021 w14M0

w'16M0 = 0.60156 w16M0 + 0.11047 w16M4

w'16M4 = 0.40700 w16M0 - 0.61979 w16M4

w'18M0 = 2.26171 w18M0 + 3.25618 w18M4

w'18M4 = - 1.23207 w18M0 - 2.03385 w18M4

w'20M0 = 0.37616 w20M0 - 0.55011 w20M4

w'20M4 = - 0.06252 w20M0 - 0.56301 w20M4

w'22M0 = - 0.04943 w22M0 - 0.11347 w22M4

w'22M4 = 0.36414 w22M0 + 0.74441 w22M4

RELATIONS BETWEEN COEFFICIENTS, KURODUMOV-SACHS RELATION

w'4M0 = 0.09722 w4M0

w'6M0 = - 0.58386 w6M0

w'8M0 = 0.44481 w8M0

w'10M0 = 0.21465 w10M0

w'12M0 = - 0.04574 w12M0 - 0.32739 w12M4

w'12M4 = - 0.16387 w12M0 + 0.03175 w12M4

w'14M0 = - 0.11937 w14M0

w'16M0 = - 0.06709 w16M0 + 0.22639 w16M4

w'16M4 = 0.07246 w16M0 + 0.09027 w16M4

w'18M0 = - 0.16932 w18M0 - 0.34481 w18M4

w'18M4 = 0.07034 w18M0 + 0.19482 w18M4

w'20M0 = - 0.24707 w20M0 - 0.01354 w20M4

w'20M4 = - 0.12422 w20M0 + 0.19644 w20M4

w'22M0 = - 0.30858 w22M0 - 0.45946 w22M4

w'22M4 = 0.03123 w22M0 + 0.08121 w22M4

RELATIONS BETWEEN COEFFICIENTS, NISHIYAMA RELATION

$$w'_{4M0} = 0.09722\ w_{4M0}$$

$$w'_{6M0} = -0.62847\ w_{6M0}$$

$$w'_{8M0} = 0.50723\ w_{8M0}$$

$$w'_{10M0} = 0.25052\ w_{10M0}$$

$$w'_{12M0} = -0.02082\ w_{12M0} - 0.45377\ w_{12M4}$$

$$w'_{12M4} = -0.21168\ w_{12M0} + 0.03792\ w_{12M4}$$

$$w'_{14M0} = -0.16913\ w_{14M0}$$

$$w'_{16M0} = 0.04422\ w_{16M0} + 0.30842\ w_{16M4}$$

$$w'_{16M4} = 0.14642\ w_{16M0} + 0.09241\ w_{16M4}$$

$$w'_{18M0} = 0.05387\ w_{18M0} - 0.04344\ w_{18M4}$$

$$w'_{18M4} = -0.06814\ w_{18M0} - 0.03045\ w_{18M4}$$

$$w'_{20M0} = -0.31825\ w_{20M0} - 0.14293\ w_{20M4}$$

$$w'_{20M4} = -0.19384\ w_{20M0} + 0.11567\ w_{20M4}$$

$$w'_{22M0} = -0.41369\ w_{22M0} - 0.49410\ w_{22M4}$$

$$w'_{22M4} = 0.03907\ w_{22M0} + 0.01619\ w_{22M4}$$

NOTE: TRANSFORMATIONS

Davies, Kallend & Morris [1976] have shown how the Euler angles relating to crystal axes of parent and product phases can be calculated for any transformation. The transformation can be specified either in the form $(h\ k\ l)_a$ // $(H\ K\ L)_b$ and $[u\ v\ w]_a$ // $[U\ V\ W]_b$, where a and b denote after and before transformation, or by specifying the invariant axis and the rotation angle about this axis. The latter method is simpler if enumeration of all the crystallographically equivalent variants of the transformation are required. On account of symmetry there is more than one invariant axis for a given transformation but there is generally one description that is obviously simpler than the rest. Thus the Bain transformation which can be described as a rotation of 45° about [0 0 1] can also be described as a rotation of approximately 98.42° about an axis with direction cosines (0.35741, 0.86286, 0.35741). This rotation would bring [1 0 0] to [0 0 1] and [0 1 0] to [1 1 0], which is crystallographically equivalent to the former rotation, but obviously the description in terms of a rotation about [0 0 1] is to be preferred. Similarly the Kurdjumov-Sachs relation can be described as a rotation of 90° about $[\bar{1}\ \bar{1}\ 2]$, which would give $(1\ 1\ 1)_a$ // $(\bar{1}\ 1\ 0)_b$ and $[1\ 1\ 0]_a$ // $[1\ 1\ 1]_b$. There is no equally simple invariant axis description of the Nishiyama transformation, but the description which best shows its relation to the Kurdjumov-Sachs transformation is a rotation of approximately 90.1207° about an axis with direction cosines (-0.4133407, -0.3484624, 0.8412629). This axis is approximately 3.7° from $[\bar{1}\ \bar{1}\ 2]$ and the rotation would give $(1\ 1\ 1)_a$ // $(\bar{1}\ 1\ 0)_b$ and $[2\ \bar{1}\ \bar{1}]_a$ // $[1\ 1\ 0]_b$. Of course, since the alternative descriptions are crystallographically equivalent, it does not matter which is chosen as the basic transformation: the relations between the coefficients of the crystallite orientation distribution functions will always be the same.

5. REFERENCES

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