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LUCAS HEIGHTS RESEARCH LABORATORIES

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ABSTRACT

If a neutron probe lies in or near a water density gradient, the probe may register a water density different to that at the measuring point. The effect of a thin stratum of soil containing an excess or depletion of water at various distances from a probe in an otherwise homogeneous system has been calculated, producing an 'importance' curve. The effect of these strata can be integrated over the soil region in close proximity to the probe resulting in the net effect of the presence of a water density gradient.

In practice, the probe is scanned through the point of interest and the count rate at that point is corrected for the influence of the water density on each side of it. An example shows that the technique can reduce an error of 10 per cent to about 2 per cent.

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CORRECTIONS; ERRORS; DENSITY; HUMIDITY; MEASURING METHODS; MOISTURE GAGES; NEUTRON PROBES; SOILS; WATER

EDITORIAL NOTE

The Australian Nuclear Science and Technology Organisation replaced the Australian Atomic Energy Commission on 27 April 1987. Reports issued after April 1987 have the prefix ANSTO with no change of the symbol (E, M, S or C) or numbering sequence.

CONTENTS

1. INTRODUCTION

Wilson [1987b] has shown that if a neutron moisture probe is used in or in close proximity to a water density gradient, the indicated water density can be greater or less than that at the point of measurement. This effect is illustrated in **figure 1** which shows the error in the flux caused by a water gradient. The left hand scale, ϕ_G/ϕ_H, is the ratio of the calculated flux at the detector in the presence of a water gradient to the calculated flux in the same soil system with a constant water density the same as that at the detector in the gradient. Near point A (**figure 1**) the proximity of higher water densities increases the number of neutrons returning to the detector and near point B, the proximity of lower water densities reduces the number returning. Wilson [*ibid*] also showed that the magnitude of the introduced error depends on the dry soil density ρ, the mass absorption coefficient ($S_a = \Sigma_a/\rho$), the magnitude of the water density gradient, and the geometry of the neutron source and detector.

A technique is described which may be used to reduce this error to the level of the errors introduced by inaccuracies in the soil parameter data, which can range from 1.5 to 3.5 per cent [Wilson 1987a].

2. METHOD

From the above comments it may be concluded that if a sequence of water density measurements is made by scanning a point of interest (POI), there will be a relationship between the water density at that POI and the densities at the points above and below it. If this relationship can be modelled, a means of correcting for the influence of neighbouring perturbations can be found.

2.1 The Importance of the Amplitude and Position of the Perturbing Water Density

By keeping the basic system homogeneous and introducing perturbations of various magnitudes at different distances from the source/detector (S/D), the importance of the amplitude and the position of the disturbance can be ascertained (**figure 2**). The perturbation is accomplished by introducing into the otherwise homogeneous system, a one centimetre thick plane containing water at a different effective density to that in the homogeneous region. The thermal neutron flux arising from a source in the system is calculated with the source at different distances from the perturbing stratum. Results of these calculations are listed in **table 1**.

Table 2 and **figure 3** show the effect of introducing a perturbing layer into a homogeneous system. The table, derived in part from **table 1**, compares the thermal flux at the S/D position in the homogeneous system (which contains water at a density of 0.25 g cm^{-3} throughout) to that of a perturbed system. **Figure 3** is a graph of two of these 'importance' functions in which the ratio ϕ_P/ϕ_H (where P = perturbed and H = homogeneous) is plotted against the distance from the source for the point, where the perturbing layer contains more water than the unperturbed region, and the reciprocal is plotted when the perturbing layer contains less water. This illustrates the fact that a reduction in water density has a relatively greater effect on the flux than an increase. Because of this, a detector at the centre of a linear water density gradient does not register the correct water density.

The ratio ϕ_P/ϕ_H is closely approximated by the function

$$\phi_P/\phi_H = 1 + B\exp(Cx) \qquad (1)$$

where x is the distance between the mid point of the S/D and the mid plane of the perturbing stratum. The constants B and C, determined by fitting exponentials to the data of **table 2**, are given in **table 3**. To obtain values for B and C when using a computer program to correct for a gradient, the values in **table 3** have been fitted to a polynomial in P (perturbation), where P is the ratio of the water content in the perturbing layer to that in the homogeneous base system:

$$B' = 0.2103 - 0.2569P + 0.04808P^2 \qquad (2)$$
$$C' = -0.2598 - 0.02764P + 0.00909P^2 \qquad (3)$$
$$B'' = -0.1784 + 0.1860P - 0.00728P^2 \qquad (4)$$
$$C'' = -0.266 - 0.01461P + 0.00096P^2 \qquad (5)$$

where B' and C' refer to layer density perturbations less than the base and B" and C" to layer density perturbations greater than the base.

2.2 The Effect of the Magnitude of the Base Water Density on the Ratio ϕ_G/ϕ_H

This is a non-linear effect with a maximum occurring when the base water density is between 0.2 and 0.25 g cm^{-3}. **Table 4** and **figure 4** show the extent of the effect. The effect may be fitted to a polynomial of the form

$$\text{Effect} = A + B(\text{base water density}) + C(\text{base water density})^2 + D(\text{base water density})^3 \quad (6)$$

where the constants A,B,C,D depend on the perturbation. From the data of **table 4**, a set of equations can be constructed to provide the following constants:

$$A = 0.9810 + 0.01495P + 0.005269P^2 - 0.001217P^3 \quad (7)$$
$$B = -0.3574 + 0.4995P - 0.1785P^2 + 0.03696P^3 \quad (8)$$
$$C = 0.9558 - 1.1258P + 0.2038P^2 - 0.03355P^3 \quad (9)$$
$$D = -0.7615 + 0.6711P + 0.1529P^2 - 0.06461P^3 \quad (10)$$

Note that the effect described here is calculated for a perturbation at a distance of 5 cm from the S/D. The effect at other positions will be this effect multiplied by the 'importance' of those positions relative to the 'importance' at 5 cm.

3. CORRECTING FOR THE PRESENCE OF A LINEAR WATER DENSITY GRADIENT

The assumption is made that the effect of a gradient is made up of sequential perturbations about the base water density which is defined as the water density at the position of the S/D.

Consider the situation in **figure 5**, graph E, in which the S/D is at the top of a water density gradient of 0.0075 g cm^{-4} and the water density is 0.25 g cm^{-3}. At -20.0 cm, the gradient ends at a water density of 0.1 g cm^{-3} and remains at this density for an effectively infinite distance. In this particular case, a small perturbation at -20.0 cm has little effect on the detector and it is unnecessary to extend the perturbations to greater distances. This may not be the case in lighter soils containing less water.

Using the definition of perturbation given in **section 2.1**, the perturbation at -20.0 cm will be

$$P = (0.25 - 20.0 \times 0.0075)/0.25 = 0.4 \; ;$$

insertion of this value for P into **equations 7-10** will produce the following values for the constants:

$$A = 0.987719 \;, \quad C = 0.53908$$
$$B = 0.183770 \;, \quad D = 0.42772$$

Using these constants in **equation 6** with the base water density of 0.25 g cm^{-3}, the effect is 0.967884. Since A-D are determined for a perturbation at 5 cm from S/D, the 'effect' must be normalised using ϕ_P/ϕ_H - 1.0 from **equation 1** which, at 20.0 cm, is

$$0.11529 \times \exp(-0.2694 \times 20.0) = 0.000527 \;.$$

This value is marked as 'Importance' in **appendix A** and, relative to the 5 cm value of 0.029974 (see **table 5** for convenience), is 0.0176. The change in the thermal flux at the detector due to the 1 cm thick stratum containing 0.1 g cm of water at 20.0 cm distance is, therefore,

$$(0.967884 - 1.0) \times 0.0176 = -0.000565 \;.$$

If the base water density had been other than 0.25 g cm^{-3}, this value would need to be multiplied by the base effect (**table 4**), relative to the base effect at 0.25 g cm^{-3}.

Continuing with the sequence by examining the effect of the next 1 cm stratum nearer the S/D, *i.e.* at 19.0 cm distance,

$$P = (0.25 - 19.0 \times 0.0075)/0.25 = 0.43 \quad ,$$

and

$$A = 0.988280 \ , \qquad C = 0.506690$$
$$B = -0.172657 \ , \qquad D = -0.449835$$

Using **equation 6**, these values produce an 'effect' of 0.969755. For a perturbation of 0.43, **table 5** gives B' as 0.108782 and C' as -0.27003 from which the importance is calculated to be 0.000643. Relative to the 5 cm value of 0.02282, the effect of P = 0.43 at 19.0 cm is (0.969755 - 1.0) × 0.02282 = -0.00069. The detailed calculations given here are the first two of case 5, **appendix A**, in which the S/D is at the top end of the gradient. This sequence is continued for each perturbation from 18 cm away from the S/D until there is no perturbation (P = 1.0). Finally all of the perturbation effects are summed to produce the total effect of the gradient. Several complete cases of these calculations are given in **appendix A**. Case 1 has the S/D at 20 cm from the high end of the gradient, where the gradient has an insignificant effect. Case 2 calculates the effect of the gradient with S/D at 15 cm from the high end, when only the last 5 cm of the gradient has any effect. Further calculations are made through position of greatest effect to case 12 where the S/D is 15 cm from the low end of the gradient. At 20 cm from the low end of the gradient there would be no significant effect. Although this technique is tedious to calculate by hand, it is ideally suited to computer solution; additional computation above that necessary to calculate the effects of the soil parameters is trivial. **Figure 5, points A-D**, represents cases 9,8,7 and 6 (see **appendix A**). Note how the water density at S/D is 0.25 g cm^{-3} in each case.

Figure 6 compares the full scale multigroup calculations with the perturbation calculations on a system having a water density gradient at various positions relative to the S/D (see **figure 5** for the gradient positions).

4. PRACTICAL APPLICATION AND CONCLUSIONS

A moisture probe scan is made throughout the borehole at regular spacing. The count rates are stored and processed to produce a first estimate of the water density profile. This profile is then used to correct the neutron flux, and hence the count rate, for the presence of any density gradient which may be present.

A point of interest (POI) in the scan is chosen and the effect on the measured flux at this point caused by the water density 1 cm above it is calculated (by interpolation if necessary). This calculation is repeated for another point 2 cm above it and then another until the effect becomes negligible at about 20 cm. This set of calculations is repeated for points up to 20 cm below the POI. These effects are then summed and used to correct the flux to what it would be in the absence of a water density gradient. The next POI is then chosen and the process repeated. When all the POIs have been processed, a corrected water density profile has been produced. Second and further iterations can be made but the corrections obtained are generally less than the errors introduced by inaccuracies in the soil parameters.

The choice of the number of points used to correct the POIs is determined by the 1 cm thickness of the perturbing layers used in the original calculations; this was chosen as a reasonable interval for the subsequent integration of the effect. No attempt has been made to optimise this interval or to produce an analytical integration.

Figure 7 shows the error introduced by a linear gradient for the soil and water mixtures given in **table 1** and a gradient shown in **figure 5**. The errors after correction by this technique have been reduced to about two per cent.

Figures 8a and **b** show the error caused by a non-linear gradient. The two graphs represent two gradients which are the same in all respects except that the gradient in **figure 8a** is half that in **figure 8b**. The outer points are not greatly affected because the gradient, although steep, is much the same over the 20 cm on each side of the POI. In **figure 8a**, at 20 cm below the first point, the true water density is 0.23 g cm^{-3} whereas in **figure 8b** it is 0.216 g cm^{-3}. The greater perturbation effect in **figure 8b** is shown up by approximately twice the error in the water density at the POI. The second point has a greater error because there is a larger positive than negative effect caused by the large difference in the gradient on each side of the POI. At the centre of the gradient system, although the effect on both sides is positive, the gradients become very small close to the POI where the importance is large and the total effect is therefore small.

The example examined is a non-linear density gradient; however, there is nothing in the technique which precludes its use to correct for any type of gradient, the perturbation P being derived from the initial sequence of count rates as in **figures 8a** and **8b**. It should be noted, however, that the equations and constants given are for a point source coincident with a point detector. A probe consisting of a scintillation detector close to its neutron source would be corrected satisfactorily but for an extended detector such as a BF counter, the variation in the thermal flux over the counter length may significantly change the shape of the importance curve.

5. REFERENCES

Wilson, D.J. [1987a] - Neutron probes: the minimum error attainable. AAEC/E651.

Wilson, D.J. [1987b] - The error introduced into a water density measurement due to the presence of a water density gradient. ANSTO/E669.

TABLE 1

RELATIVE THERMAL FLUX AT A SOURCE/DETECTOR IN A SYSTEM CONTAINING 0.25 g cm^{-3} WATER WHEN PERTURBED BY A 1 cm THICK LAYER CONTAINING WATER AT A DIFFERENT DENSITY

Soil parameters: Dry soil density 1.333 g cm^{-3};
$S_a = 6.305 \times 10^{-3}$ cm^2 g^{-1}; $S_s = 0.12$ cm^2 g^{-1}.

Distance from Source/Detector (cm)	Perturbation Layer Water Density (g cm^{-3})	Perturbed Flux ϕ_P	ϕ_P/ϕ_H* in Flux	Fract. Change
	0.1	494517	0.896545	0.103455
	0.15	515245	0.934124	0.065876
0.0	0.2	533971	0.968074	0.031976
	0.3	568521	1.030712	0.030712
	0.35	585037	1.060655	0.060655
	0.4	601276	1.090096	0.090096
	0.1	533861	0.967874	0.032160
	0.15	540495	0.979901	0.020099
5.0	0.2	546298	0.990422	0.009578
	0.3	556506	1.008929	0.008929
	0.35	561168	1.017381	0.017381
	0.4	565625	1.025461	0.025461
	0.1	546872	0.991463	0.008537
	0.15	548674	0.994730	0.005270
10.0	0.2	550212	0.997518	0.002482
	0.3	552831	1.002266	0.002266
	0.35	553991	1.004369	0.004369
	0.4	555079	1.006342	0.006342
	0.1	550421	0.997899	0.002101
	0.15	550876	0.998722	0.001278
15.0	0.2	551254	0.999407	0.000593
	0.3	551873	1.000529	0.000529
	0.35	552136	1.001006	0.001006
	0.4	552378	1.001445	0.001445
	0.1	551318	0.999520	0.000480
	0.15	551425	0.999717	0.000283
20.0	0.2	551510	0.999871	0.000129
	0.3	551641	1.000109	0.000109
	0.35	551694	1.000205	0.000205
	0.4	551740	1.000290	0.000290

*In a homogeneous system with the same soil parameters and a water density of 0.25 g cm^{-3},the thermal flux ϕ_H is 551581

TABLE 2

RELATIVE THERMAL FLUX AT SOURCE/DETECTOR WITH 1 cm PERTURBING LAYER AT VARIOUS DISTANCES FROM SOURCE/DETECTOR

Soil parameters: dry soil density 1.3333 g cm^{-3};
$S_a = 6.305 \times 10^{-3}$ cm^2 g^{-1}; $S_s = 0.12$ cm^2 g^{-1}
ϕ_P (0.3) is flux at detector when perturbing layer contains 0.3 g cm^{-3} water

Distance from Centre of Plane to Source/Detector (cm)	Water Density in the Perturbing Layer with Base Water Density 0.25 g cm^{-3}			
	ϕ_P (0.3)	ϕ_P/ϕ_H	ϕ_P (0.2)	$1/\left(\frac{\phi_P}{\phi_H}\right)$
0	568521	1.0307	533971	1.0330
2	562364	1.0195	-	-
5	556506	1.0089	546298	1.0097
10	552831	1.0023	550212	1.0025
15	551873	1.0005	551254	1.0006
20	551641	1.0001	551510	1.0001

TABLE 3

FITTING INCREASE OR DECREASE OF DETECTOR FLUX TO AN EXPONENTIAL FUNCTION

Increase or decrease = B exp(Cx)

x is the distance in cm between the centre of the source/detector and the centre of the 1 cm thick layer containing the perturbation

Perturbation Layer Water Density (g cm^{-3})	B	C	Regression
0.1	0.115290	-0.269490	0.999040
0.15	0.073538	-0.273111	0.998998
0.2	0.035632	-0.276095	0.999009
0.3	0.034377	-0.282164	0.998929
0.35	0.067838	-0.284586	0.998955
0.4	0.100725	-0.286931	0.998961

TABLE 4

EFFECT OF DIFFERENT PERTURBATION LAYER WATER DENSITIES WITH DIFFERENT BASE WATER DENSITY CONDITIONS

All calculations are for a perturbation in a 1 cm thick layer
5 cm from the source/detector

Base Water Density 0.1 g cm^{-3}							
+Pert (g cm^{-3})	0.04	0.06	0.08	0.10	0.12	0.14	0.16
Rel. ϕ	167556	169126	170591	171987	173331	174635	175910
Rel. to ϕ_H	0.974236	0.983365	0.991889	1.000000	1.007815	1.015397	1.022810
Base Water Density 0.15 g cm^{-3}							
Pert (g cm^{-3})	0.06	0.09	0.12	0.15	0.18	0.21	0.24
Rel ϕ	270299	273273	275978	278507	280913	283227	285470
Rel. to ϕ_H	0.970529	0.981207	0.990919	1.000000	1.008639	1.016948	1.025001
Base Water Density 0.20 g cm^{-3}							
Pert (g cm^{-3})	0.08	0.12	0.16	0.20	0.24	0.28	0.32
Rel. ϕ	392551	397243	401418	405269	408894	412349	415677
Rel. to ϕ_H	0.968618	0.980196	0.990500	1.000000	1.008945	1.017470	1.025682
Base Water Density 0:25 g cm^{-3}							
Pert (g cm^{-3})	0.10	0.15	0.20	0.25	0.30	0.35	0.40
Rel. ϕ	533861	540495	546298	551581	556506	561168	565625
Rel. to ϕ_H	0.967874	0.979901	0.99042	1.000000	1.008929	1.017381	1.025461
Base Water Density 0.30 g cm^{-3}							
+Pert (g cm^{-3})	0.12	0.18	0.24	0.30	0.36 ¦	0.42	0.48
Rel. ϕ	693791	792511	710020	716781	723029 ¦	728901	734479
Rel. to ϕ_H	0.967926	0.980092	0.990568	1.000000	1.008717 ¦	1.016908	1.024691
Base Water Density 0.40 g cm^{-3}							
Pert (g cm^{-3}	0.16	0.24	0.32	0.40 ¦	0.48	0.56	0.64
Rel. ϕ	1067896	1080908	1091838	1101476 ¦	1110243	1118373	1126012
Rel. to ϕ_H	0.969514	0.981327	0.991250	1.000000 ¦	1.007959	1.015340	1.022276

+Pert = ω/ρ in perturbing layer; steps are changes of ±20%. Perturbations to the right of broken line are likely to be above saturation.

Perturbations in vertical columns represent the same ratio to the base (homogeneous) water density and the central data column is that for the homogeneous system.

TABLE 5

CONSTANTS FOR THE IMPORTANCE FUNCTION

Constants B', C', B'', C'' to calculate the importance at 5 cm for various perturbation layer water densities in a base water density of 0.25 g cm^{-3}

P	B'	C'	5 cm value
0.40	0.115290	-0.269400	0.029978
0.43	0.108782	-0.270003	0.028200
0.46	0.102361	-0.270590	0.026458
0.49	0.096026	-0.271160	0.024750
0.52	0.089777	-0.271714	0.023075
0.55	0.083615	-0.272251	0.021434
0.58	0.077540	-0.272773	0.019825
0.61	0.071551	-0.273277	0.018247
0.64	0.065649	-0.273766	0.016701
0.67	0.059833	-0.274238	0.015186
0.70	0.054104	-0.274694	0.013701
0.73	0.048462	-0.275133	0.012245
0.76	0.042906	-0.275556	0.010818
0.79	0.037436	-0.275963	0.009420
0.82	0.032053	-0.276353	0.008050
0.85	0.026756	-0.276727	0.006707
0.88	0.021546	-0.277085	0.005391
0.91	0.016423	-0.277426	0.004102
0.94	0.011386	-0.277751	0.002840
0.97	0.006436	-0.278060	0.001603
1.00	0.001572	-0.278353	0.000391
1.00	0.000342	-0.279665	0.000084
1.03	0.005484	-0.280045	0.001352
1.06	0.010613	-0.280423	0.002612
1.09	0.015729	-0.280799	0.003863
1.12	0.020832	-0.281174	0.005107
1.15	0.025922	-0.281546	0.006343
1.18	0.031000	-0.281918	0.007572
1.21	0.036064	-0.282287	0.008792
1.24	0.041115	-0.282651	0.010005
1.27	0.046154	-0.283020	0.011211
1.30	0.051179	-0.283385	0.012409
1.33	0.056192	-0.283747	0.013996
1.36	0.061192	-0.284108	0.014783
1.39	0.066179	-0.284467	0.015959
1.42	0.071153	-0.284824	0.017128
1.45	0.076114	-0.285179	0.018290
1.48	0.081062	-0.285533	0.019444
1.51	0.085997	-0.285885	0.020592
1.54	0.090919	-0.286236	0.021732
1.57	0.095829	-0.286584	0.022866
1.60	0.100725	-0.286931	0.023992

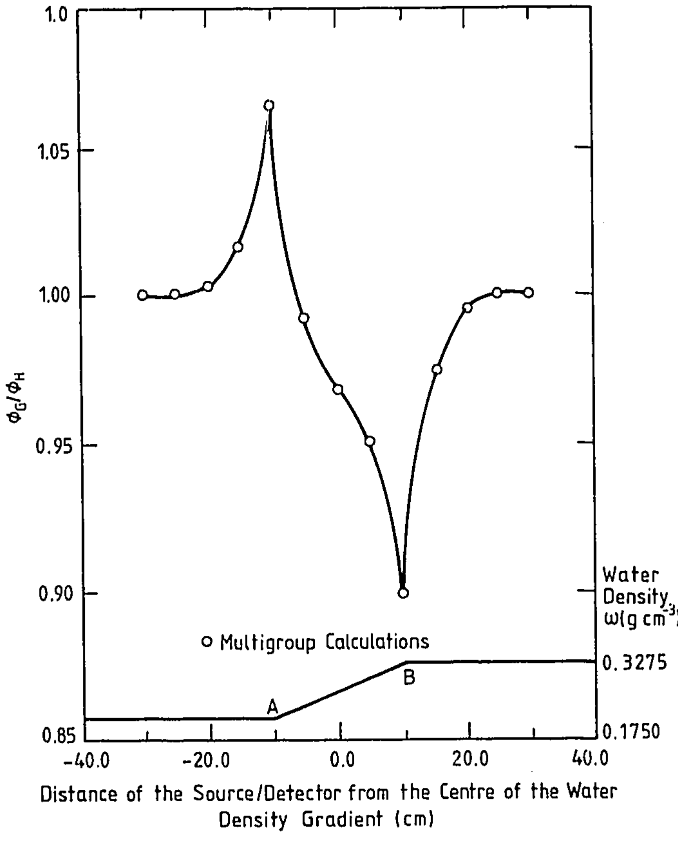


Figure 1 The extended effect of the water density gradient

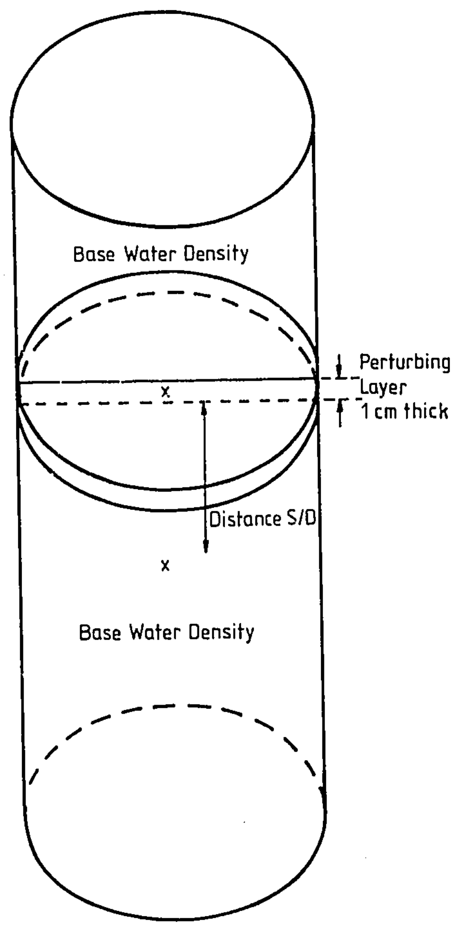


Figure 2 Geometry for calculating the effect of a perturbing layer at a distance from the source/detector

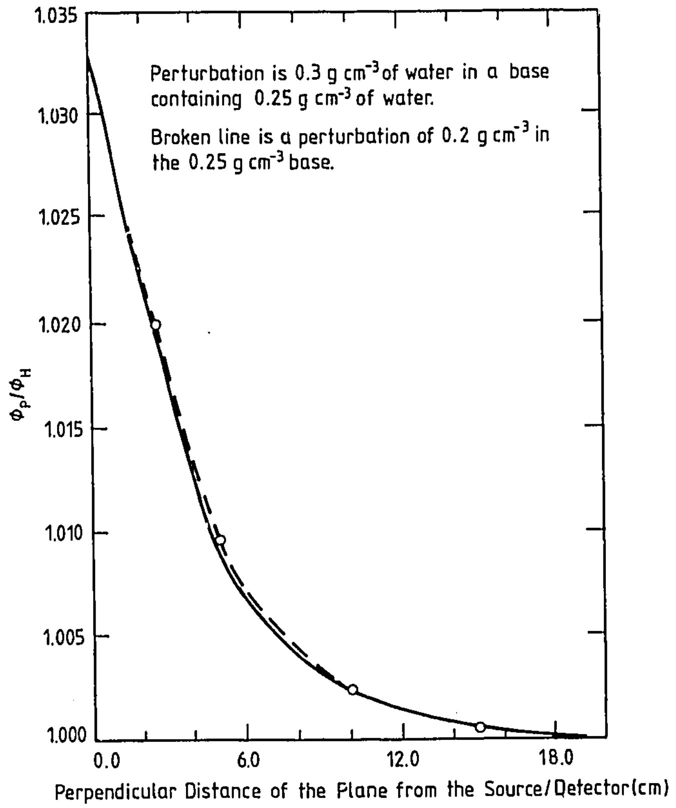


Figure 3 The effect of a 1 cm thick perturbing plane on the thermal flux at the detector

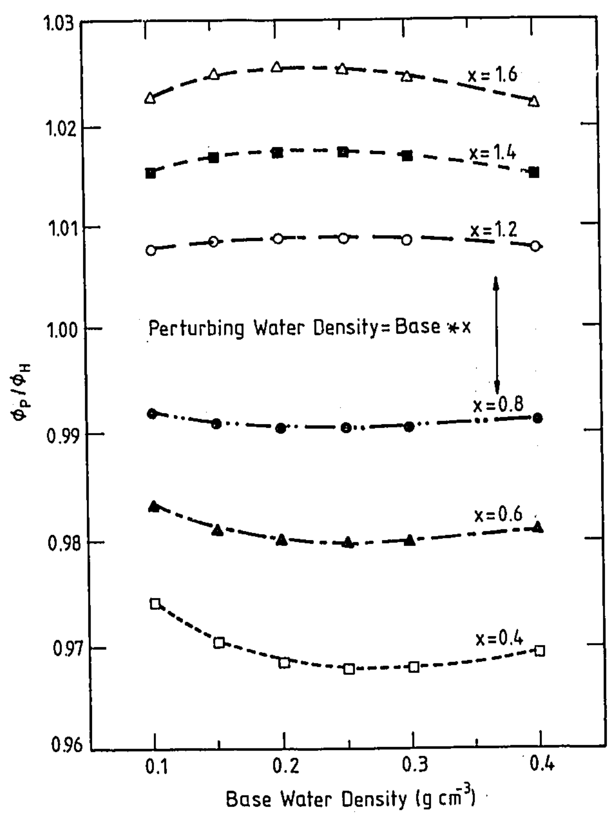


Figure 4 The effect of varying the water density in a 1 cm thick plane from a source/detector in various water densities

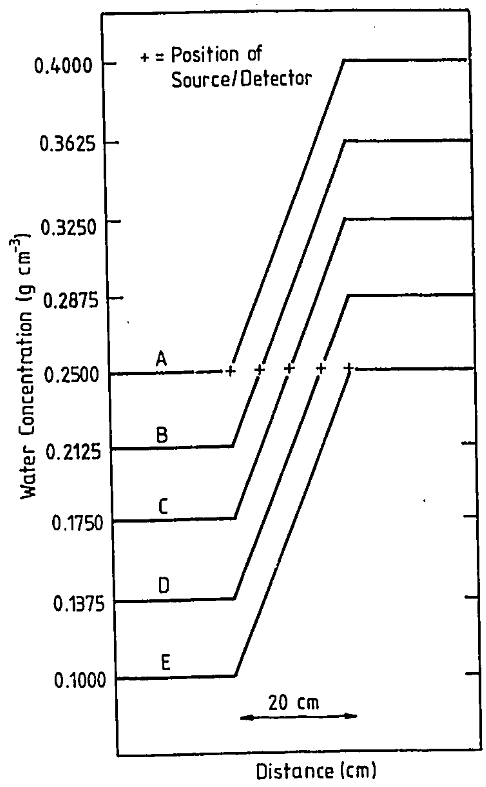


Figure 5 The position of the source relative to the water density gradient

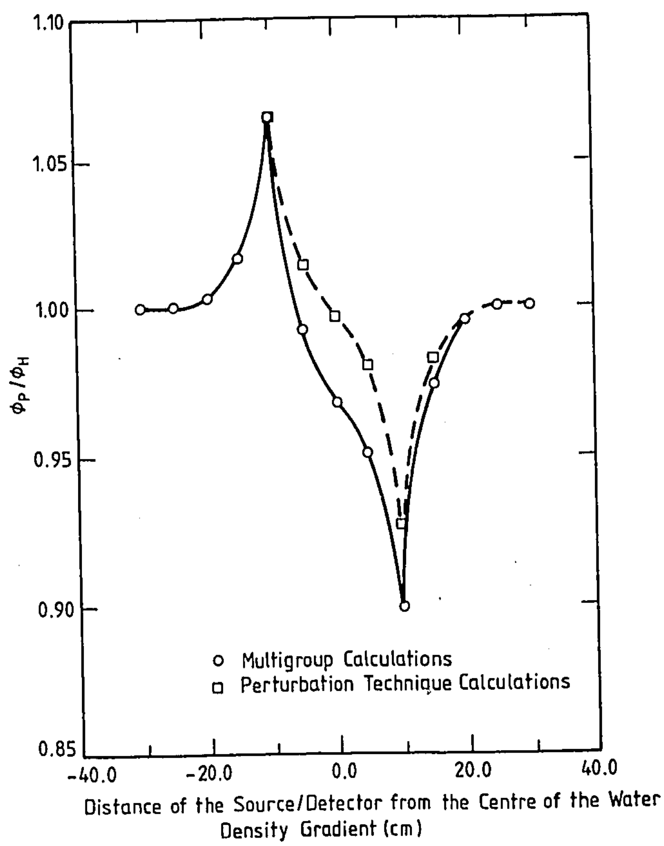


Figure 6 Comparison of multigroup and perturbation calculations

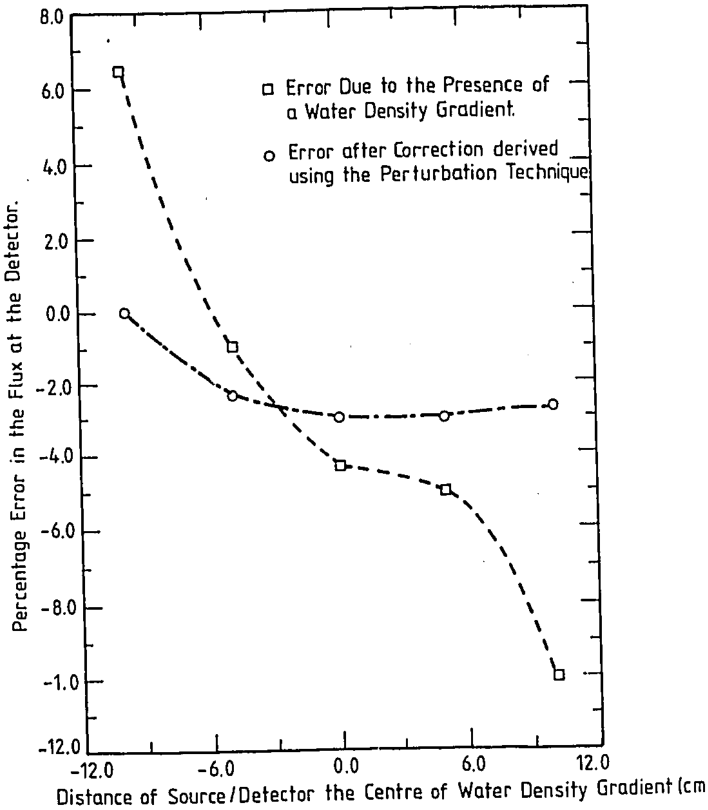


Figure 7 The correction of errors arising from the presence of a water density gradient

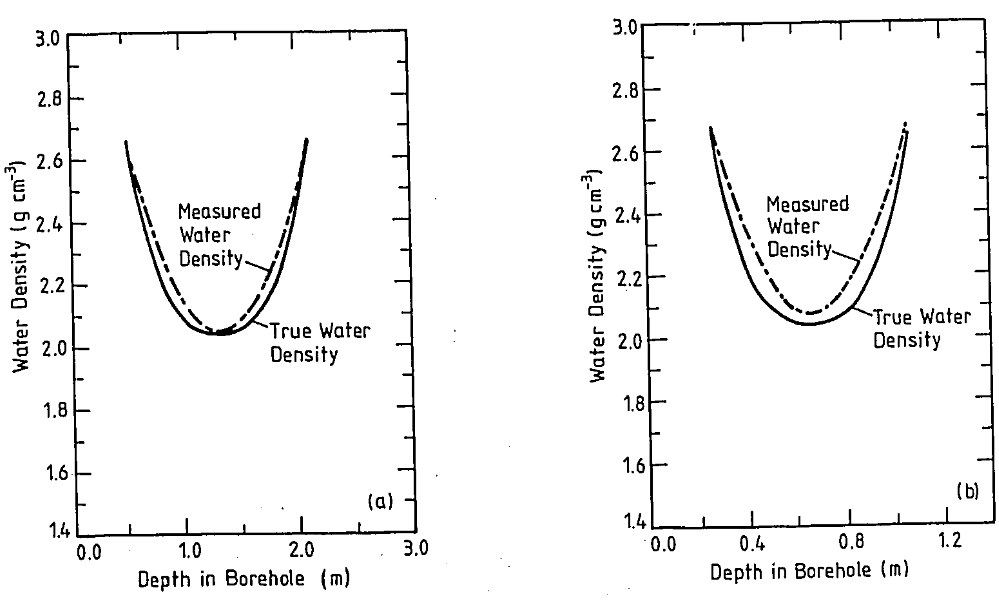


Figure 8 The effect of a water density gradient on a neutron probe measurement

APPENDIX A

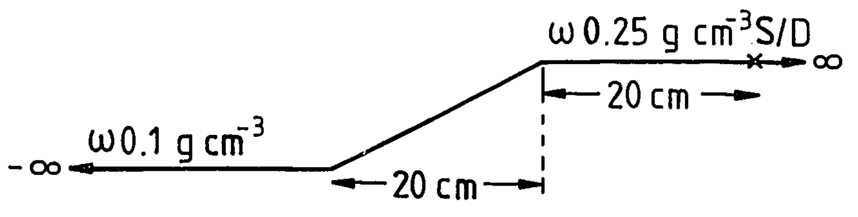
THE EFFECT OF A WATER DENSITY GRADIENT AT VARIOUS DISTANCES FROM A SOURCE/DETECTOR

A complete set of calculations is given here to show the effect of the gradient following the techniques described in **section 3**. In case 1, the source/detector (S/D) is sufficiently far removed from the gradient for the effect to be trivial. In case 2, the S/D is 15 cm from the end of the gradient and the 5 cm A-B has a small effect which is calculated at 1 cm intervals and then summed. Case 3 calculates the effect of the gradient for the S/D at 10 cm from the end and the 10 cm A-B is the perturbing factor. This sequence is continued until case 12 in which the S/D is 20 cm from the low end of the gradient, where the effect is again negligible. These calculations are used to produce graphs such as that in **figure 6**.

A.1 CALCULATION OF THE EFFECT OF SOIL MOISTURE GRADIENTS ON THE THERMAL FLUX AT THE SOURCE/DETECTOR USING THE PERTURBATION TECHNIQUE

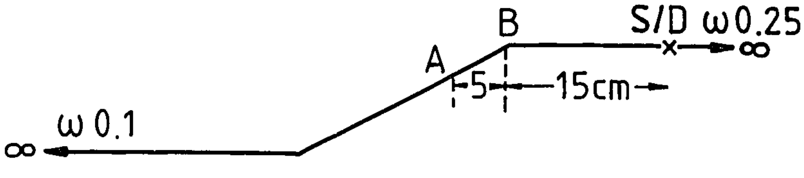
Gradient, constant slope = $0.0075 \text{ g cm}^{-3} \text{ cm}^{-1}$.
S/D is always at 0.25 g cm^{-3} H_2O
Dry soil density lineup = 1.667 g cm^{-3}
Soil mass absorption coefficient = $6.305 \times 10^{-3} \text{ cm}^2 \text{ g}^{-1}$
Soil mass scattering coefficient = $0.109 \text{ cm}^2 \text{ g}^{-1}$

Case 1



For the given soil parameter, the gradient is sufficiently removed at 20 cm from the S/D to have negligible effect.

Case 2



At A(-20 cm), the perturbation is (0.25 - 5 × 0.0075)/ 0.25 = 0.85. Putting 0.85 into **equations 7-10.**

> A = 0.99674; B = -0.039084; C = 0.125503; D = 0.120329;
> Effect (Eff) = 0.99293 ; Eff-1 = -0.00707 from **equation 6.**

This is the effect that such a perturbation would have if it occurred at 5 cm from S/D and must be corrected by the relative importance between 20 cm and 5 cm.

From **table 5**, the coefficients B' and C' (because the perturbation is less than 1.0) for a perturbation of 0.85 are B' = 0.026756 and C' = 0.276727 and from **equation 1**, the importance (Imp) at 20 cm is 0.000106 whereas at 5 cm it is 0.006707, *i.e.* Imp (20 cm) = 0.0157 × Imp(5 cm).

Therefore the actual effect of the perturbation at 20 cm is

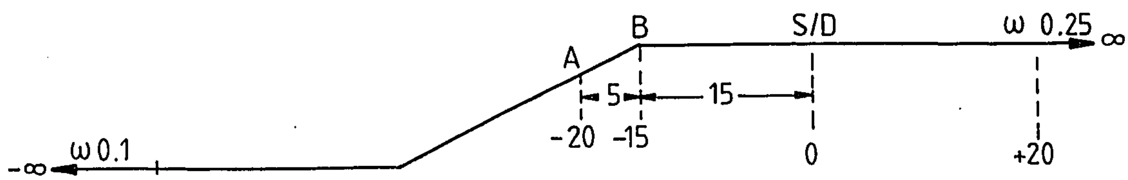
$$0.0157 \times -0.00707 = 0.000111$$

This is the increase in the thermal flux at S/D due to the perturbation.

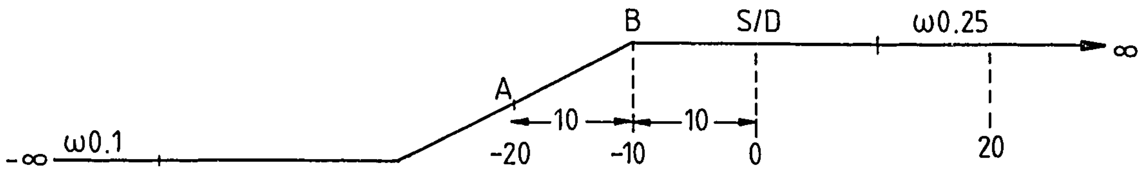
$$\text{The perturbed flux} = (1 - 0.000111) \times \text{unperturbed flux.}$$

At 19 cm, perturbation = (0.25 - 4 × 0.0075)/0.25 = 0.88; A = 0.997381; B = -0.030876; C = 0.100049; D = -0.096613; Eff = 0.99441; (Eff-1) = -0.00559; B' = 0.021546; C' = 0.277085. Relative importance at 19 cm = 0.000111/0.005391 = 0.02059. Effect of perturbation at 20 cm is -0.00559 × 0.02059 = 0.000115. Continuing in tabular form:

Distance from S/D	Pert.	A	B	C	D	Eff	Eff-1	B'	C' (Table 5)	Imp. at Dist. of Col. 1	Imp. at 5 cm (Table 5)	Rel. Imp.	Eff. at Dist. of Col. 1
No perturbation from -15 to +20 (B to S/D) (water is constant at 0.25 g cm^{-3})													
-18	0.91	0.998024	-0.022813	0.0748023	-0.072929	0.99586	-0.00414	0.016423	-0.277426	0.00011138	0.004102	0.02715	-0.000112
-17	0.94	0.99867	-0.01489	0.049758	-0.049287	0.99729	-0.00271	0.011386	-0.277751	0.0001013	0.002840	0.03568	-0.000097
-16	0.97	0.99932	-0.0070642	0.024909	-0.025698	0.99871	-0.00129	0.006436	-0.278060	0.000075241	0.001603	0.04694	-0.000061
										Total perturbation from A to S/D =			-0.000496

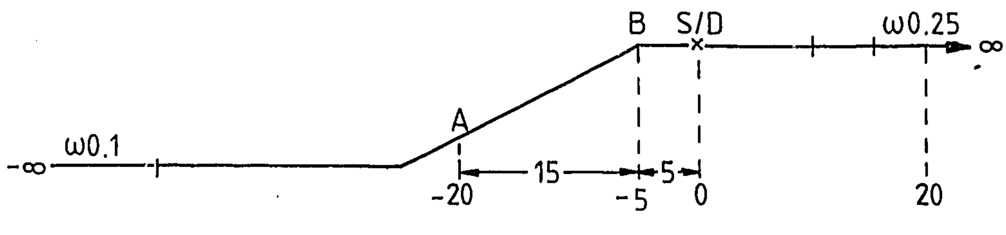


CASE 3



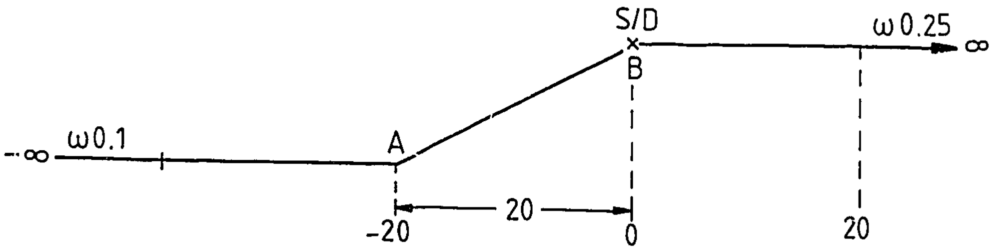
Distance from S/D	Pert.	A	B	C	D	Eff	Eff-1	B' (Table 5)	C' (Table 5)	Imp. at Dist. of Col. 1	Imp. at 5 cm (Table 5)	Rel. Imp.	Eff. at Dist. of Col. 1
	No perturbation from -10 to +20 (water density constant at 0.25 g cm^{-3})												
-20	0.70	0.993603	-0.082522	0.256077	-0.239021	0.98524	-0.01476	0.054104	-0.274694	0.0002225	0.013701	0.01624	-0.000240
-19	0.73	0.994222	-0.0734946	-0.229504	-0.215303	0.986828	-0.013132	0.048462	-0.275133	0.0026008	0.012245	0.02124	-0.000280
-18	0.76	0.994845	-0.064643	0.203165	-0.191564	0.988389	-0.01161	0.042905	-0.275556	0.0003009	0.010818	0.02781	-0.000323
-17	0.79	0.995473	-0.055962	0.177056	-0.167815	0.989926	-0.01007	0.037436	-0.275963	0.0003434	0.009420	0.03646	-0.000367
-16	0.82	0.996105	-0.047444	0.151170	-0.144067	0.991441	-0.008559	0.032053	-0.276353	0.0003851	0.008050	0.04784	-0.000409
-15 to 11			Same as -20 to -16 above				-0.00707	0.026756	-0.276727	0.0004214	0.006707	0.06283	-0.000444
-14							-0.00559	0.021546	-0.277085	0.0004453	0.005391	0.08260	-0.000462
-13							-0.00415	0.016423	-0.277426	0.0004458	0.004102	0.10868	-0.000451
-12							-0.00271	0.011386	-0.277751	0.0004063	0.002840	0.14307	-0.000388
-11							-0.00129	0.006436	-0.278060	0.0003022	0.001603	0.18851	-0.000243
								Total perturbation from A to S/D					-0.003607

CASE 4



Distance from S/D	Pert.	A	B	C	D	Eff	Eff-1	B'	C' (Table 5)	Imp. at Dist. of Col. 1	Imp. at 5 cm (Table 5)	Rel. Imp.	Eff. at Dist. of Col. 1
	No perturbation from -5 to +20 (water density constant at 0.25 g cm^{-3})												
-20	0.55	0.990588	-0.130467	0.392682	-0.3569	0.976935	-0.023065	0.083615	-0.272251	0.0003610	0.021434	0.01684	-0.000389
-19	0.58	0.991180	-0.120472	0.364824	-0.333479	0.978653	-0.021347	0.077540	-0.272773	0.0004352	0.019825	0.02195	-0.000469
-18	0.61	0.991778	-0.110682	0.337258	-0.309948	0.980343	-0.019657	0.071551	-0.273277	0.0005228	0.018247	0.02865	-0.000563
-17	0.64	0.992385	-0.101092	0.309949	-0.286354	0.982006	-0.017994	0.065649	-0.273768	0.0006252	0.016701	0.03743	-0.000674
-16	0.67	0.992990	-0.091696	0.282890	-0.262708	0.983642	-0.016358	0.059833	-0.274238	0.0007436	0.015186	0.04897	-0.000801
-15	0.70	As for previous calculations with					0.01476	0.054104	-0.274694	0.0008785	0.013701	0.06412	-0.000946
-14	0.73	the same perturbation					-0.013172	0.048462	-0.275133	0.0010293	0.012245	0.08406	-0.001107
-13	0.76						-0.01161	0.042906	-0.275556	0.0011934	0.010818	0.11031	-0.001281
-12	0.79						-0.01007	0.037436	-0.275963	0.0013649	0.009420	0.14489	-0.001459
-11	0.82						-0.008559	0.032053	-0.276353	0.0015334	0.008050	0.19049	-0.001630
-10	0.85						-0.00707	0.026758	-0.276727	0.0016812	0.006707	0.25066	-0.001772
-9	0.88						-0.00559	0.021546	-0.277085	0.0017800	0.005391	0.33012	-0.001845
-8	0.91						-0.00415	0.016423	-0.277420	0.0017847	0.004102	0.43509	-0.001806
-7	0.94						-0.00271	0.011386	-0.277751	0.0016293	0.002840	0.57368	-0.001555
-6	0.97						-0.00129	0.006486	-0.278060	0.0012135	0.001603	0.75702	-0.000976
-5	1.00						0.0						
								Total perturbation from A to S/D =					-0.017273

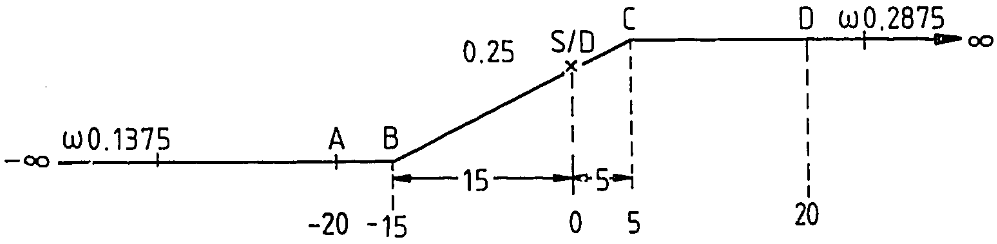
CASE 5



Distance from S/D	Pert.	A	B	C	D	Eff	Eff-1	B' (Table 5)	C' (Table 5)	Imp. at Dist. of Col. 1	Imp. at 5 cm (Table 5)	Rel. Imp.	Eff. at Dist. of Col. 1
	No perturbation from 0 to +20 (water density constant at 0.25 g cm^{-3})												
-20*	0.40	0.987719	-0.183736	0.555908	-0.472772	0.967884	-0.032116	0.11529	-0.269400	0.000527	0.029978	0.01758	-0.000563
-19*	0.43	0.988280	-0.172623	0.504690	-0.449855	0.969755	-0.030245	0.108782	-0.270003	0.000644	0.028200	0.02282	-0.000690
-18	0.46	0.988847	-0.161745	0.477760	-0.426772	0.971602	-0.028398	0.102361	-0.270590	0.000785	0.026458	0.02967	-0.000845
-17	0.49	0.989421	-0.151097	0.445115	-0.403595	0.973410	-0.026590	0.096062	-0.271160	0.000956	0.024750	0.03864	-0.001027
-16	0.52	0.990002	-0.140673	0.420747	-0.380313	0.975188	-0.024812	0.089777	-0.271714	0.001162	0.023075	0.05035	-0.001249
-15	0.55						-0.023065	0.083615	-0.272251	0.001408	0.021434	0.06571	-0.001516
-14	0.58		As for previous calculations				-0.021347	0.077540	-0.272773	0.001702	0.019825	0.0859	-0.001834
-13	0.61		with the same				-0.019657	0.071551	-0.273277	0.002050	0.018247	0.1123	-0.002207
-12	0.64		perturbation				-0.017994	0.065649	-0.273766	0.002454	0.016701	0.1469	-0.002643
-11	0.67						-0.016383	0.059833	-0.274238	0.002930	0.015186	0.1929	-0.003155
-10	0.70						-0.014760	0.054104	-0.274634	0.003469	0.013701	0.2532	-0.003737
-9	0.73						-0.013172	0.048462	-0.275133	0.004074	0.012245	0.3327	-0.004383
-8	0.76						-0.011610	0.042906	-0.275556	0.004733	0.010818	0.4375	-0.005080
-7	0.79						-0.010070	0.037436	-0.275963	0.005424	0.009420	0.5758	-0.005801
-6	0.82						-0.008559	0.03053	-0.276363	0.006106	0.008650	0.7585	-0.006492
-5	0.85						-0.00707	0.026756	-0.276727	0.006707	0.006707	1.0000	-0.007070
-4	0.88						-0.00559	0.021546	-0.277085	0.007112	0.005391	1.3193	-0.007375
-3	0.91						-0.00415	0.016423	-0.277420	0.007145	0.004102	1.7419	-0.007229
-2	0.94						-0.00271	0.011386	-0.277751	0.006533	0.002840	2.3004	-0.006234
-1	0.97						-0.00129	0.006436	-0.278060	0.004874	0.001603	3.0403	-0.003922
0	1.000						0.0						
								Total perturbation from A to S/D =					-0.073029

* See section 3.

CASE 6

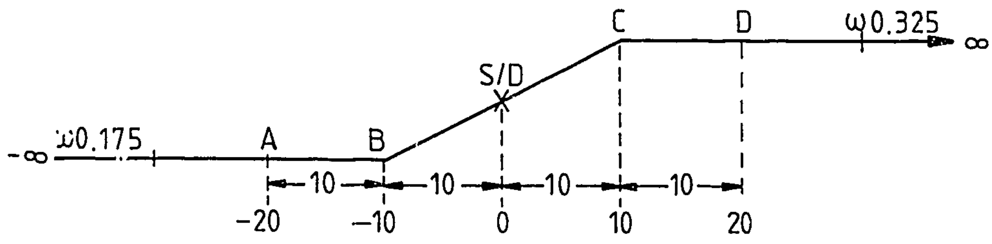


Distance from S/D	Pert.	A B C D Eff	Eff-1	B' (Table 5)	C'	Imp. at Dist. of Col. 1	Imp. at 5 cm (Table 5)	Rel. Imp.	Eff. at Dist. of Col. 1
-20	0.55		-0.023065	0.083615	-0.272251	0.000361	0.021434	0.01684	-0.000389
-19	0.55		"	"	"	0.000474	"	0.02211	-0.000510
-18	0.55		"	"	"	0.000622	"	0.02903	-0.000685
-17	0.55		"	"	"	0.000817	"	0.03812	-0.000879
-16	0.55		"	"	"	0.0010727	"	0.05005	-0.001154
-15	0.55		"	"	"	0.0014094	"	0.065708	-0.001516
-14	0.58		-0.021347	0.077540	-0.272773	0.0017023	0.019825	0.085868	-0.001833
-13	0.61		-0.019657	0.071551	-0.273277	0.0020500	0.018247	0.112344	-0.002208
-12	0.64		-0.017944	0.065649	-0.273766	0.0024575	0.016701	0.147145	-0.002640
-11	0.67		-0.016358	0.059833	-0.274238	0.0029298	0.015186	0.192929	-0.003156
-10	0.70		-0.014760	0.054104	-0.274694	0.0034694	0.013701	0.253219	-0.003738
-9	0.73		-0.013172	0.048462	-0.275133	0.0040738	0.012241	0.332693	-0.004382
-8	0.76		-0.01161	0.042906	-0.275556	0.004733	0.010818	0.437513	-0.005080
-7	0.79		-0.01007	0.037436	-0.275963	0.0054243	0.009420	0.57583	-0.005799
-6	0.82		-0.008559	0.032053	-0.276353	0.0061060	0.008050	0.75851	-0.006492
-5	0.85		-0.00707	0.026758	-0.276727	0.0067068	0.006707	1.0000	-0.007070
-4	0.88		-0.00559	0.021546	-0.277085	0.0071125	0.005391	1.31932	-0.007375
-3	0.91		-0.00415	0.016423	-0.277426	0.007145	0.004102	1.7419	-0.007229
-2	0.94		-0.00271	0.011386	-0.277751	0.006533	0.002840	2.3004	-0.006234
-1	0.97		-0.00129	0.006436	-0.278060	0.004874	0.001603	3.0403	-0.003922
0	1.00								
1	1.03	1.000632	0.008134	-0.024219	0.021281	1.001484	0.001484	0.005484	-0.280045
2	1.06	1.001292	0.015558	-0.048510	0.044649	1.002847	0.002847	0.010813	-0.280423
3	1.09	1.001953	0.022872	-0.072623	0.067923	1.004193	0.004193	0.015729	-0.280799
4	1.12	1.002617	0.030082	-0.096573	0.091091	1.005525	0.005525	0.020832	-0.281174
5	1.15	1.003284	0.037195	-0.120356	0.114144	1.006844	0.006844	0.025922	-0.281546
6						0.004787		0.7545	0.005165
7						0.003612		0.5694	0.003897
8						0.002726		0.4298	0.002942
9			As above			0.002057		0.3243	0.002220
10						0.001552		0.2447	0.001675
11						0.001171		0.1846	0.001263
12						0.000884		0.1394	0.000954
13						0.000667		0.1052	0.000720
14						0.000503		0.0793	0.000543
15						0.000380		0.0599	0.000410
16						0.000287		0.0452	0.000309
17						0.000216		0.0341	0.000233
18						0.000163		0.0257	0.000176
19						0.000123		0.0194	0.000133
20						0.000093		0.0147	0.000101

Perturbation A to S/D = -0.072291; S/D to D = +0.053408

Total Perturbation = -0.018883

CASE 7

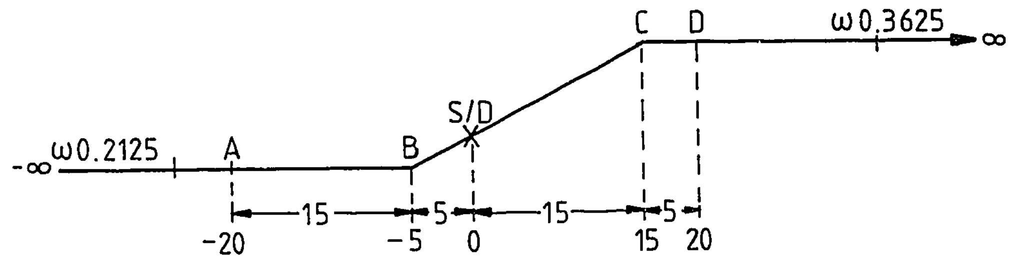


Distance from S/D	Pert.	A	B	C	D	Eff	Eff-1	B' (Table 5)	C' (Table 5)	Imp. at Dist. of Col. 1	Imp. at 5 cm (Table 5)	Rel. Imp.	Eff. at Dist. of Col. 1
-20	0.70	0.993603	-0.082522	0.2506077	-0.239021	0 98524	-0.01476	0.054104	-0.274694	0.0002225	0.013701	0.0162	-0.000240
-19	"									0.000293	"	0.0214	-0.000316
-18										0.000385	"	0.0281	-0.000415
-17										0.000507	"	0.0370	-0.000546
-16					As Above					0.000668	"	0.0488	-0.000720
-15										0.000879	"	0.0642	-0.000948
-14										0.001156	"	0.0844	-0.001246
-13										0.001522	"	0.1111	-0.001640
-12										0.002003	"	0.1462	-0.002158
-11										0.002636	"	0.1924	-0.002840
-10										0.003469	"	0.2532	-0.003737
-9	0.73						-0.013172	0.048462	-0.275133	0.004074	0.012245	0.3327	-0.004382
-8	0.76						-0.01161	0.042906	-0.275556	0.004713	0.010818	0.4375	-0.005079
-7	0.79						-0.01007	0.037436	-0.275963	0.005424	0.009420	0.5758	-0.005798
-6	0.82						-0.008559	0.032053	-0.276353	0.006106	0.008050	0.7585	-0.006492
-5	0.85						-0.007070	0.026756	-0.276727	0.006707	0.006707	1.0000	-0.007070
-4	0.88						-0.005590	0.021546	-0.277085	0.007112	0.005391	1.3192	-0.007374
-3	0.91						-0.00415	0.016423	-0.277426	0.007145	0.004102	1.7418	-0.007228
-2	0.94						-0.00271	0.011386	-0.277751	0.006533	0.002840	2.3004	-0.006234
-1	0.97						-0.00129	0.006436	-0.278060	0.004874	0.001603	3.0405	-0.003922
0	1.00						0.0	0.001572	-0.278353	0.001572	0.00391	4.0205	0
1	1.03						0.001484	0.005484	-0.280054	0.004145	0.001352	3.0655	0.004549
2	1.06						0.002847	0.010613	-0.280423	0.006057	0.002612	2.3190	0.006602
3	1.09						0.004193	0.015729	-0.280799	0.006774	0.003863	1.7536	0.007353
4	1.12						0.005525	0.020832	-0.281174	0.006765	0.005107	1.3247	0.007319
5	1.15						0.006844	0.025922	-0.281546	0.006343	0.006343	1.000	0.006844
6	1.18	1.003952	0.044215	-0.143980	0.137071	1.008149	0.008149	0.031000	-0.281918	0.005711	0.007572	0.7542	0.006146
7	1.21	1.004622	0.051150	-0.167451	0.159861	1.009442	0.009442	0.036064	-0.282287	0.004999	0.008792	0.5686	0.005653
8	1.24	1.005293	0.058006	-0.190774	0.182505	1.010723	0.010723	0.041115	-0.282651	0.004285	0.010005	0.4283	0.004593
9	1.27	1.005966	0.064785	-0.213955	0.204991	1.011993	0.011993	0.046154	-0.283020	0.003614	0.011211	0.3224	0.003867
10	1.30	1.006640	0.071498	-0.237000	0.227309	1.013254	0.013254	0.051179	-0.283385	0.003009	0.012409	0.2150	0.002850
11										0.002342	"	0.1673	0.002217
12										0.001770	"	0.1265	0.001677
13										0.001337	"	0.0955	0.001266
14					As Above					0.001010	"	0.0722	0.000957
15										0.000763	"	0.0545	0.000722
16										0.000577	"	0.0412	0.000546
17										0.000436	"	0.0312	0.000414
18										0.000329	"	0.0235	0.000311
19										0.000249	"	0.0178	0.000236
20										0.000188	"	0.0134	0.000178

Perturbation A to S/D = -0.068385; S/D to D = +0.064300

Total Perturbation = -0.004085

CASE 8

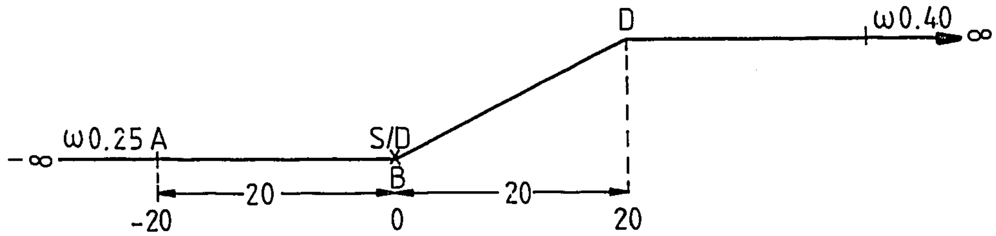


Distance from S/D	Pert.	A	B	C	D	Eff	Eff-1	B' (Table 5)	C' (Table 5)	Imp. at Dist. of Col. 1	Imp. at 5 cm (Table 5)	Rel. Imp.	Eff. at Dist. of Col. 1
-20	0.85						0.00707	0.021756	-0.276727	0.000106	0.006707	0.0157	-0.000111
-19										0.000139	"	0.0207	-0.000146
-18										0.000184	"	0.0274	-0.000194
-17										0.000242	"	0.0361	-0.000255
-16										0.000320	"	0.0477	-0.000337
-15										0.00421	"	0.0628	-0.000444
-14										0.000556	"	0.0829	-0.000586
-13										0.000733	"	0.1093	-0.000773
-12										0.000967	"	0.1442	-0.001019
-11										0.001275	"	0.1901	-0.001344
-10										0.001681	"	0.2506	-0.001772
-9										0.002217	"	0.3306	-0.002337
-8										0.002924	"	0.4360	-0.003083
-7										0.003856	"	0.5749	-0.00465
-6										0.005086	"	0.7583	-0.005361
-5										0.006707	"	1.0000	-0.007070
-4	0.88						-0.00559	0.021546	-0.277085	0.007112	0.005391	1.3193	-0.007375
-3	0.91						-0.00414	0.016423	-0.277420	0.007145	0.004102	1.7419	-0.007211
-2	0.94						-0.00271	0.011386	-0.277751	0.006533	0.002840	2.3004	-0.006234
-1	0.97						-0.00129	0.006436	-0.278060	0.004874	0.001603	3.0405	-0.003922
0	1.00							0.001572	-0.278353	0.001572	0.000391		0.0
1	1.03						0.001484	0.005484	-0.280045	0.004145	0.001352	3.0655	0.004549
2	1.06						0.002847	0.010613	-0.280423	0.006057	0.002612	2.3190	0.006602
3	1.09						0.004193	0.015729	-0.280799	0.006774	0.003863	1.7536	0.007353
4	1.12						0.005525	0.020832	-0.281174	0.006765	0.005107	1.3247	0.007319
5	1.15						0.006844	0.025922	-0.281546	0.006343	0.006343	1.0000	0.006844
6	1.18						0.008149	0.031000	-0.281918	0.005711	0.007572	0.7542	0.006146
7	1.21						0.009942	0.036064	-0.282287	0.004999	0.008792	0.5686	0.005653
8	1.24						0.010723	0.041115	-0.282651	0.004285	0.010005	0.4283	0.004593
9	1.27						0.011993	0.046154	-0.283020	0.003614	0.011211	0.3224	0.003867
10	1.30						0.013254	0.051179	-0.283385	0.003009	0.012409	0.2425	0.003214
11	1.33	1.007314	0.078150	-0.259913	0.249449	1.014505	0.014505	0.056192	-0.283747	0.002478	0.013996	0.1771	0.002569
12	1.36	1.008666	0.084745	-0.282700	0.271399	1.015748	0.015748	0.061192	-0.284108	0.002023	0.014783	0.1368	0.002154
13	1.39	1.009342	0.091290	-0.305366	0.293151	1.016984	0.016984	0.066179	-0.284467	0.001639	0.015959	0.1027	0.001744
14	1.42	1.010019	0.097791	-0.327918	0.314693	1.018212	0.018212	0.071153	-0.284824	0.001320	0.017128	0.0771	0.001404
15	1.45	1.010696	0.104255	-0.350360	0.336014	1.019434	0.019434	0.076114	-0.285179	0.001056	0.018290	0.0577	0.001121
16										0.000794	"	0.0434	0.000844
17					As for distance 10					0.000597	"	0.0326	0.000634
18										0.000449	"	0.0245	0.000477
19										0.000338	"	0.0185	0.000359
20										0.000254	"	0.0139	0.000270

Perturbation A to S/D = -0.053639; and S/D to D = 0.067716

Total Perturbation = 0.014077

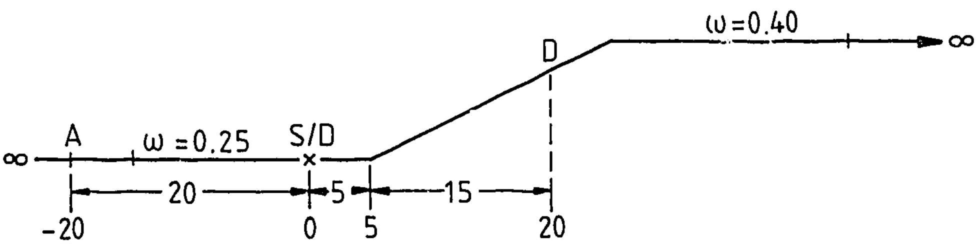
CASE 9



No perturbation between -20 and 0 (water density constant at 0.25 g cm^{-3})

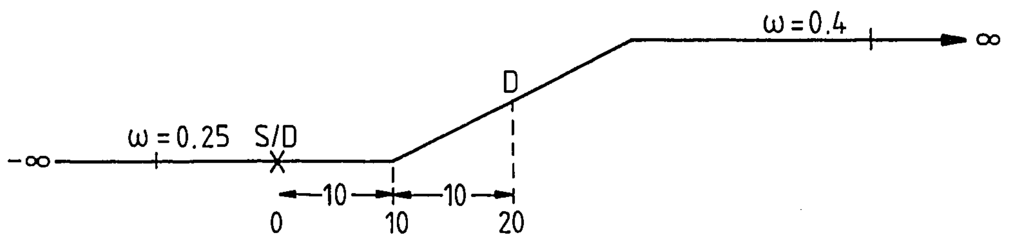
Distance from S/D	Pert.	A	B	C	D	Eff	Eff-1	B' (Table 5)	C' (Table 5)	Imp. at Dist. of Col. 1	Imp. at 5 cm (Table 5)	Rel. Imp.	Eff. at Dist. of Col. 1
1	1.03	As for similar perturbations					0.001484	0.005484	-0.280048	0.004145	0.001352	3.0655	0.004549
2	1.06						0.002847	0.010613	-0.280423	0.006057	0.002612	2.3190	0.006602
3	1.09						0.004193	0.015729	-0.280789	0.006774	0.003863	1.7536	0.007353
4	1.12						0.005525	0.020832	-0.281174	0.006765	0.005107	1.3247	0.007319
5	1.15						0.006844	0.025922	-0.281546	0.006343	0.006343	1.0000	0.006844
6	1.18						0.008149	0.031000	-0.281918	0.005711	0.007572	0.7542	0.006146
7	1.21						0.009942	0.036064	-0.282287	0.004999	0.008792	0.5686	0.005653
8	1.24						0.010723	0.041115	-0.282651	0.004285	0.010005	0.4283	0.004593
9	1.27						0.011993	0.046154	-0.283020	0.003614	0.011211	0.3224	0.003867
10	1.30						0.013254	0.051179	-0.283385	0.003009	0.012409	0.2425	0.003214
11	1.33						0.014505	0.056192	-0.283747	0.002478	0.013996	0.1771	0.002569
12	1.36						0.015748	0.061192	-0.284108	0.002023	0.014783	0.1368	0.002154
13	1.39						0.016984	0.066179	-0.284467	0.001639	0.015959	0.1027	0.001744
14	1.42						0.018212	0.071153	-0.284824	0.001320	0.017128	0.0745	0.001358
15	1.45						0.019434	0.076114	-0.285179	0.001056	0.018290	0.0577	0.001121
16	1.48	1.010696	0.110686	-0.372698	0.357105	1.020654	0.020654	0.081062	-0.285533	0.000841	0.019444	0.04325	0.000893
17 -	1.51	1.011372	0.117091	-0.394936	0.377955	1.021867	0.021867	0.085997	-0.285885	0.000666	0.020592	0.03237	0.000708
18	1.54	1.012048	0.123477	-0.417082	0.398553	1.023077	0.023077	0.090917	-0.286236	0.000526	0.021732	0.02421	0.000559
19	1.57	1.012723	0.129848	-0.439140	0.418889	1.024284	0.024284	0.095829	-0.286584	0.000414	0.022866	0.01811	0.000440
20	1.60	1.013397	0.136213	-0.461115	0.438952	1.025489	0.025489	0.100725	-0.286931	0.000324	0.023992	0.01352	0.000344
										Total Perturbation =			0.067888

CASE 10



Distance from S/D	Pert.	A	B	C	D	Eff	Eff-1	B'	C' (Table 5)	Imp. at Dist. of Col. 1	Imp. at 5 cm (Table 5)	Rel. Imp.	Eff. at Dist. of Col. 1
	No perturbation between -20 and +5 (water density constant at 0.25 g cm^{-3})												
6	1.03	As for similar perturbation					0.001484	0.005484	-0.280045	0.001022	0.001352	0.7558	0.001122
7	1.06						0.002847	0.010613	-0.280423	0.001491	0.002612	0.5706	0.001625
8	1.09						0.004193	0.015729	-0.280789	0.001664	0.003863	0.4307	0.001806
9	1.12						0.005525	0.020832	-0.281174	0.001659	0.005107	0.3248	0.001795
10	1.15						0.006844	0.025922	-0.281546	0.001552	0.006343	0.2447	0.001675
11	1.18						0.008149	0.031000	-0.281918	0.001395	0.007572	0.1842	0.001501
12	1.21						0.009942	0.036044	-0.282287	0.001219	0.008792	0.1386	0.001378
13	1.24						0.010723	0.041115	-0.282651	0.001043	0.010005	0.1042	0.001118
14	1.27						0.011993	0.046154	-0.283020	0.000878	0.011211	0.0783	0.000939
15	1.30						0.013254	0.051179	-0.283385	0.000729	0.012409	0.0588	0.000779
16	1.33						0.014505	0.056192	-0.283747	0.000600	0.013996	0.0429	0.000622
17	1.36						0.015748	0.061192	-0.284108	0.000489	0.014783	0.0331	0.000521
18	1.39						0.016984	0.066179	-0.284467	0.000395	0.015959	0.0248	0.000421
19	1.42						0.018212	0.071153	-0.284824	0.000318	0.017128	0.0185	0.000338
20	1.45						0.019434	0.076114	-0.285579	0.000254	0.018290	0.0139	0.000270
									Total Perturbation =				+0.015910

CASE 11

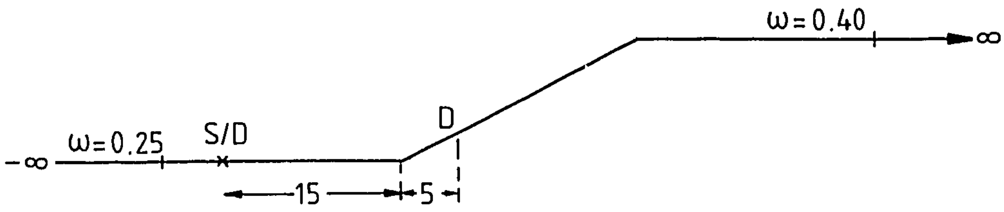


No perturbation between -20 and +10 (water density constant at 0.25 g cm^{-3})

Distance from S/D	Pert.	A	B	C	D	Eff	Eff-1	B' (Table 5)	C' (Table 5)	Imp. at Dist. of Col. 1	Imp. at 5 cm (Table 5)	Rel. Imp.	Eff. at Dist. of Col. 1
11	1.03						0.001484	0.005484	-0.280045	0.000252	0.001352	0.1863	0.000277
12	1.06						0.002847	0.010613	-0.280423	0.000367	0.002612	0.1402	0.000400
13	1.09						0.004193	0.015729	-0.280799	0.000409	0.003863	0.1058	0.000444
14	1.12						0.005525	0.020832	-0.281174	0.000407	0.005107	0.0796	0.000440
15	1.15						0.006844	0.025922	-0.281546	0.000380	0.006343	0.0599	0.000410
16	1.18						0.008149	0.031000	-0.281918	0.000341	0.007572	0.0450	0.000367
17	1.21						0.009942	0.036064	-0.282287	0.000297	0.008792	0.0338	0.000336
18	1.24						0.010723	0.041115	-0.282651	0.000254	0.010005	0.0254	0.000272
19	1.27						0.011993	0.046154	-0.283020	0.000213	0.011211	0.0190	0.000228
20	1.30						0.013254	0.051179	-0.283385	0.000177	0.012409	0.0143	0.000189

Total Perturbation = +0.003363

CASE 12



Distance from S/D	Pert.	A	B	C	D	Eff	Eff-1	B' (Table 5)	C' (Table 5)	Imp. at Dist. of Col. 1	Imp. at 5 cm (Table 5)	Rel. Imp.	Eff. at Dist. of Col. 1
	No perturbation between -20 and +15 (water density constant at 0.25 g cm^{-3})												
16	1.03						0.001484	0.005484	-0.280045	0.000062	0.001352	0.0459	0.000068
17	1.06						0.002847	0.010613	-0.280423	0.000090	0.002612	0.0346	0.000098
18	1.09						0.004193	0.015729	-0.280799	0.000100	0.003863	0.0260	0.000109
19	1.12						0.005525	0.020832	-0.281174	0.000100	0.005107	0.0195	0.000108
20	1.15						0.006844	0.025922	-0.281546	0.000093	0.006343	0.0147	0.000100

Total Perturbation = +0.000483