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BEHAVIOUR OF PARTICLES OF U, UO_2 AND UC_2 IN A VERTICAL TUBE THROUGH WHICH LIQUID SODIUM IS FLOWING

by

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SUMMARY

This paper sets out to calculate the relation between the particle velocity and the fluid velocity in a vertical tube without making any assumptions as to the "apparent" viscosity of the suspension of particles in liquid sodium.

The theory and calculations method is developed for suspensions of U, UO_2 and UC_2 in liquid Na in the temperature range 200° – 800°C. At the moment, it does not help in assessing the performance of a circulating suspension in the primary coolant stream of a nuclear power reactor.

CONTENTS

1. DATA USED IN CALCULATIONS

(a) Density of U = 19.00 g/cm3
Density of UO_2 = 10.90 g/cm3
Density of UC_2 = 13.63 g/cm3

These, the generally accepted figures at normal temperatures, are used without any correction at elevated temperatures, since the expansion coefficients of UO_2 and UC_2 are not available.

(b) Properties of Liquid Na

Temp oC	Density g/cm3	μ centipoises	k watts/(cm)(°C)	C cal/(gm)(°C)
100	0.927	0.705		0.3305
200	0.904	0.450	0.815	0.3200
300	0.882	0.345	0.757	0.3116
400	0.859	0.284	0.712	0.3055
500	0.834	0.243	0.668	0.3015
600	0.809	0.210	0.627	0.2998
700	0.783	0.186	0.590	0.3003
800	0.757	0.165	0.547	0.3030

2. NOMENCLATURE

U_p	=	Velocity in vertical direction of particle with respect to a fixed horizontal plane.
U_s	=	Slip velocity = velocity of particle in vertical direction relative to bulk fluid.
U_{so}^{∞}	=	Slip velocity of single particle in infinite fluid system with zero fraction solids.
U_{po}	=	Particle velocity in vertical direction in infinite fluid system with zero fraction solids.
$U_{s\lambda}^{\infty}$	=	Slip velocity in multiple particle – infinite fluid system of λ, volume fraction solids.
$U_{p\lambda}^{\infty}$	=	Particle velocity in vertical direction in multiple particle – infinite fluid system of λ, volume fraction solids.
U_{Lo}^{∞}	=	L = liquid throughput rate, $cm^3/(cm^2$ of cross section)(sec).
$U_{L\lambda}^{\infty}$	=	Average liquid velocity, cm/sec., in multiple particle – infinite fluid system of λ, volume fraction solids.
G	=	Solids throughput rate, $cm^3/(cm^2$ of cross section) (sec).
d	=	Diameter of particle, cm.
g	=	Acceleration due to gravity.
m	=	Mass of particle.
w	=	Mass of fluid displaced by particle.
F1	=	Resultant force tending to accelerate particle.

F_R^I	=	Resisting force due to friction effects.
(fD)	=	Friction factor.
ρ_ℓ	=	Density of fluid, gm/cm^3.
ρ_s	=	Density of particle, gm/cm^3.
V	=	Velocity of particle, cm/sec.
μ	=	Viscosity of fluid, gm/(cm)(sec).
(Re)	=	Reynold's No.

3. THEORY

For a single particle in an infinite system, we can write

$$U_{po}^{\infty} = U_{so}^{\infty} + U_{Lo}^{\infty} \qquad \ldots\ldots\ldots\ldots \quad (1)$$

and

$$U_{po}^{\infty} = U_{so}^{\infty} + L \qquad \ldots\ldots\ldots\ldots \quad (2)$$

For a multiple particle system, we have

$$U_{p\lambda}^{\infty} = U_{s\lambda}^{\infty} + U_{L\lambda}^{\infty} \qquad \ldots\ldots\ldots\ldots \quad (3)$$

Now the volume fraction liquid $(1-\lambda)$ = average fractional free area for liquid flow through the particle system, so

$$U_{L\lambda}^{\infty} = \frac{L}{1-\lambda} \qquad \ldots\ldots\ldots\ldots \quad (4)$$

This gives $U_{p\lambda}^{\infty} = U_{s\lambda}^{\infty} + \frac{L}{1-\lambda}$ .... (5)

Dividing throughout by U_{so}^{∞}, we have

$$\frac{U_{p\lambda}^{\infty}}{U_{so}^{\infty}} = \frac{U_{s\lambda}^{\infty}}{U_{so}^{\infty}} + \frac{1}{(1-\lambda)} \cdot \frac{L}{U_{so}^{\infty}} \qquad \ldots\ldots\ldots \quad (6)$$

Where we can define

$\frac{U_{p\lambda}^{\infty}}{U_{so}^{\infty}}$ = Particle velocity No.

$\frac{U_{s\lambda}^{\infty}}{U_{so}^{\infty}}$ = Slip-velocity No.

$$\frac{L}{U_{so}^{\infty}} = \text{Liquid throughput No.}$$

A single particle falling in an infinite liquid at rest, will have a slip velocity, U_{so}^{∞}. It will displace from its path liquid which passes round and to the rear of the particle. The effects of this local liquid movement are included in the value of U_{so}^{∞} and the area for back flow is unrestricted. For an increase in particle concentration, the free area for back flow is reduced and the liquid back flow has then a greater effect on the particle. The slip velocity is reduced from U_{so}^{∞} to $U_{s\lambda}^{\infty}$.

If we assume we have n spherical particles of diameter d per cm^3, then the average number of particles in a horizontal section through the centre of one particle will be $n^{2/3}$.

$$\text{The min. free area} = 1 - n^{2/3} \cdot \frac{\pi d^2}{4}$$

but the volume fraction solids, $\lambda = \frac{n \pi d^3}{6}$

or $n^{2/3} = \frac{6^{2/3} \lambda^{2/3}}{\pi^{2/3} d^2}$

$$\text{So min. free area} = 1 - \frac{6^{2/3} \lambda^{2/3}}{\pi^{2/3} d^2} \cdot \frac{\pi d^2}{4}$$

$$= 1 - \frac{6^{2/3} \pi^{1/3}}{4} \lambda^{2/3}$$

$$= 1 - 1.209 \lambda^{2/3}$$

and so $$\frac{U_{s\lambda}^{\infty}}{U_{so}^{\infty}} = 1 - 1.209 \lambda^{2/3} \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

also solids throughput $G = \lambda U_{p\lambda}^{\infty}$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (8)

or solids throughput No. $\frac{G}{U_{so}^{\infty}} = \frac{\lambda U_{p\lambda}^{\infty}}{U_{so}^{\infty}}$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (9)

combining (6) and (9), we have $$\frac{G}{U_{so}^{\infty}} = \lambda \left[\frac{U_{s\lambda}^{\infty}}{U_{so}^{\infty}} + \frac{1}{(1-\lambda)} \frac{L}{U_{so}^{\infty}} \right] \ldots\ldots\ldots \quad (10)$$

Then substituting (7) we have for spherical particles

$$\frac{G}{U^{\infty}_{so}} = \lambda \left[1 - 1.209 \;\; \lambda^{2/3} + \frac{1}{(1-\lambda)} \cdot \frac{L}{U^{\infty}_{so}} \right] \quad \ldots\ldots\ldots \quad (11)$$

On the basis of this Fig. 1 has been drawn with solids throughput No. G/U_{so} plotted against volume fraction solids, λ with liquid-throughput No. L/U_{so} as parameter.

There are three main regions -

(i) to the right of $L/U^{\infty}_{so} = O$. concurrent downflow.

(ii) between $G/U^{\infty}_{so} = O$ and $L/U^{\infty}_{so} = O$, countercurrent.

(iii) to the left of $G/U^{\infty}_{so} = O$, concurrent upflow.

It should be noted that $G/U^{\infty}_{so} = O$ represents the fluidisation line.

Fig. 2 shows the countercurrent region in more detail. If the maximum solids throughput for each curve of constant liquid upflow, we have the maximum throughput line. This line divides the countercurrent region into two regions -

(a) the one above the line called the N-phase where the volume fraction solids decreases as the solids throughput No. increases.

(b) the other below the line called the P-phase where the volume fraction solids increases as the solids - throughput No. increases.

It should be noted that at certain solids - throughput Nos., one can have a low concentration moving slowly (N-phase).

In the above, we have assumed an infinite system; provided the tube/particle diameter ratio exceeds 100, no correction need be applied.

In Mertes and Rhodes paper (Chem. Eng. Prog. 51, 429 - 432, 517 - 522 (1955), a further line $L/G = -1.0$ is plotted and called the sedimentation line. It is doubtful whether this line gives the conditions for sedimentation in a quiet liquid, since there must be flow of liquid upwards to make room for the particles which are settling.

If we have a particle in a fluid, three forces will act on the particle

(i) that of gravity
(ii) the bouyancy effect resulting from the displacement of fluid by the solids
(iii) the frictional resistance from the relative motion of the solid and fluid.

The frictional resistance increases with the increasing velocity until the accelerating and resisting forces are equal. Then the solid will attain a constant maximum velocity called the terminal velocity. Provided

(a) the solid is a non-porous, incompressible spherical particle
(b) the fluid is incompressible and the effect of the confining walls may be neglected
(c) the accelerating force is derived from a uniform gravitational field.
(d) the particle is freely moving, i.e. not interfered with by other particles.

We can write

$$F^1 = m.\frac{dv}{dt} = mg - wg - F^1_R \quad \ldots\ldots\ldots\ldots (12)$$

For a sphere of diameter, d

$$F^1_R = (f_D) \cdot \frac{\pi d^2 \rho_\ell v^2}{8} \quad \ldots\ldots\ldots\ldots (13)$$

(12) can then be rewritten

$$\left(\frac{\pi d^3}{6}\right)(\rho_s) \frac{dv}{dt} = \frac{\pi d^3}{6} \cdot g\ (\rho_s - \rho_\ell) - f_D \frac{\pi d^2 \rho_\ell v^2}{8} \quad \ldots (14)$$

$$\text{or } \frac{dv}{dt} = \frac{(\rho_s - \rho_\ell)}{(\rho_s)} g \quad \frac{3(f_D)\ \rho_\ell \quad v^2}{4.d.\rho_s} \quad \ldots\ldots\ldots\ldots (15)$$

for V_m = max. velocity, $\frac{dv}{dt} = 0$

$$\text{and so } \frac{(\rho_s - \rho_\ell)}{(\rho_s)} g = \frac{3(f_D)\ \rho_\ell\ v_m^2}{4\,d\,\rho_s} \quad \ldots\ldots\ldots\ldots (16)$$

$$\text{or } v_m^2 = \frac{4(\rho_s - \rho_\ell).\ g.\ d.}{3\ \rho_\ell\ (f_D)} \quad \ldots\ldots\ldots \quad (17)$$

$$\text{and } v_m = \sqrt{\frac{4(\rho_s - \rho_\ell).\ g.\ d.}{3\ \rho_\ell\ (f_D)}} \quad \ldots\ldots\ldots \quad (18)$$

For viscons or laminar flow, $F_R^1 = 3\pi\, d\, \mu\, v$ .... (19)

$$\text{and we get } \frac{dv}{dt} = \frac{(\rho_s - \rho_\ell)}{\rho_s}.\ g - \frac{18\mu v}{d^2\ \rho_s} \quad \ldots\ldots\ldots \quad (20)$$

$$\text{and so } v_m = \frac{(\rho_s - \rho_\ell).\ g.d.^2}{18\ \mu} \text{ (Stokes' Law)} \quad \ldots\ldots\ldots \quad (21)$$

$$\text{Then the friction factor, } (f_D) = \frac{4(\rho_s - \rho_\ell)\ g.\ d.}{3\ v_m.\ \rho_\ell}\ \frac{18\mu}{(\rho_s - \rho_\ell)\ g.d.}$$

$$= \frac{24\mu}{d\ v_m.\ \rho_\ell} = \frac{24}{(Re)} \quad \ldots\ldots\ldots \quad (22)$$

$$\text{From (17), Also } \log(f_D) = \log\left[\frac{4g\ (\rho_s - \rho_\ell)\ d}{3\ \rho_\ell}\right] - 2\log v_m \quad \ldots\ldots\ldots \quad (23)$$

$$\text{and } \log(Re) = \log\left[\frac{d.\ \rho_\ell}{\mu}\right] + \log v_m \quad \ldots\ldots\ldots \quad (24)$$

Eliminating v_m, we have

$$\log(f_D) = -2\log(Re) + \log\left[\frac{4g\ (\rho_s - \rho_\ell).\ d^3.\ \rho_\ell}{3\ \mu^2}\right] \quad \ldots\ldots\ldots \quad (25)$$

This equation may be plotted on a chart of log (f_D) versus log (Re) as a straight line with a slope of -2 passing through the points

$$(f_D) = \frac{4g\ (\rho_s - \rho_\ell)\ d^3.\ \rho_\ell}{3\mu^2} \quad (Re) = 1$$

$$\text{and } (f_D) = 1, \quad (Re) = \sqrt{\frac{4g\,(\rho_s - \rho_\ell)\,.\,d^3 \rho_\ell}{3\mu^2}}$$

Data is available for (f_D) versus (Re) for spheres flowing through fluids (see Fig. 69, p. 76 Brown "Unit Operations" or Fig. 112, p. 1018 Perry "Chemical Engineers' Handbook"). It is therefore possible to calculate v_m for any set of conditions regardless of whether the flow is streamline or turbulent provided the particles are spheres. It is also possible to allow for shape factor but it is not proposed to do this at the moment.

This terminal velocity $v_m = U_{so}^{\infty}$, the slip velocity of single particle in infinite fluid system with zero fraction solids.

4. CALCULATIONS

(a) The terminal velocity, $v_m = U_{so}^{\infty}$, slip velocity of single particle in infinite fluid system with zero fraction solids, assuming spherical particles.

Particle Diameter	2μ			5μ		
TEMP. °C	U	UO_2	UC_2	U	UO_2	UC_2
200	8.77×10^{-3}	4.84×10^{-3}	6.12×10^{-3}	5.06×10^{-2}	1.76×10^{-2}	3.56×10^{-2}
300	1.14×10^{-2}	6.33×10^{-3}	8.06×10^{-3}	7.16×10^{-2}	3.96×10^{-2}	5.04×10^{-2}
400	1.39×10^{-2}	7.70×10^{-3}	9.80×10^{-3}	8.70×10^{-2}	4.82×10^{-2}	6.13×10^{-2}
500	1.63×10^{-2}	9.02×10^{-3}	1.15×10^{-2}	0.102	5.64×10^{-2}	7.18×10^{-2}
600	1.89×10^{-2}	1.05×10^{-2}	1.33×10^{-2}	0.118	6.55×10^{-2}	8.33×10^{-2}
700	2.16×10^{-2}	1.19×10^{-2}	1.51×10^{-2}	0.134	7.41×10^{-2}	9.44×10^{-2}
800	2.36×10^{-2}	1.31×10^{-2}	1.66×10^{-2}	0.147	8.18×10^{-2}	0.104

Particle Diameter	10μ			20μ		
TEMP °C	U	UO_2	UC_2	U	UO_2	UC_2
200	0.219	0.121	0.154	0.877	0.484	0.615
300	0.286	0.158	0.201	1.11*	0.610*	0.788*
400	0.348	0.193	0.245	1.33*	0.728*	0.944*
500	0.408	0.226	0.287	1.40*	0.785*	0.983*
600	0.472	0.262	0.333	1.65*	0.975*	1.25*
700	0.534	0.297	0.376	1.86*	1.12*	1.39*
800	0.589	0.327	0.416	2.00*	1.14*	1.53*

* For these conditions the flow is not streamline.

(b) λ, Volume fraction solids for U, UO_2 and UC_2 for atomic ratios U:Na of 1:50, 1:100 and 1:200 at Na temperature of 500°C.

Atomic Ratio	U	UO_2	UC_2
1:50	0.00889	0.0174	0.0136
1:100	0.00447	0.00880	0.00684
1:200	0.00224	0.00530	0.00344

Fig. 4 shows L/U_{so}^{∞} versus G/U_{so}^{∞} with λ as parameter and on this figure are drawn the volume fraction solids lines for U, UO_2 and UC_2 with atomic ratio U:Na of 1:100.

Fig. 5 shows L versus G with λ as parameter and on this are drawn the volume fraction solids lines for U with atomic ratios of 1:50, 1:100 and 1:200 for a temperature of 500°C and a particle size of 10μ.

5. DISCUSSION

The data presented will only be of use in the Australian programme, if, in order to avoid carry over of particles into the primary heat exchanger, the velocity in the suspensions is restricted to prevent this and the suspension in the core is surrounded by sodium to carry away the heat. It shows that the sodium velocity to prevent carry over is very small, but enables the velocity for particle removal from the core for chemical processing to be predicted.

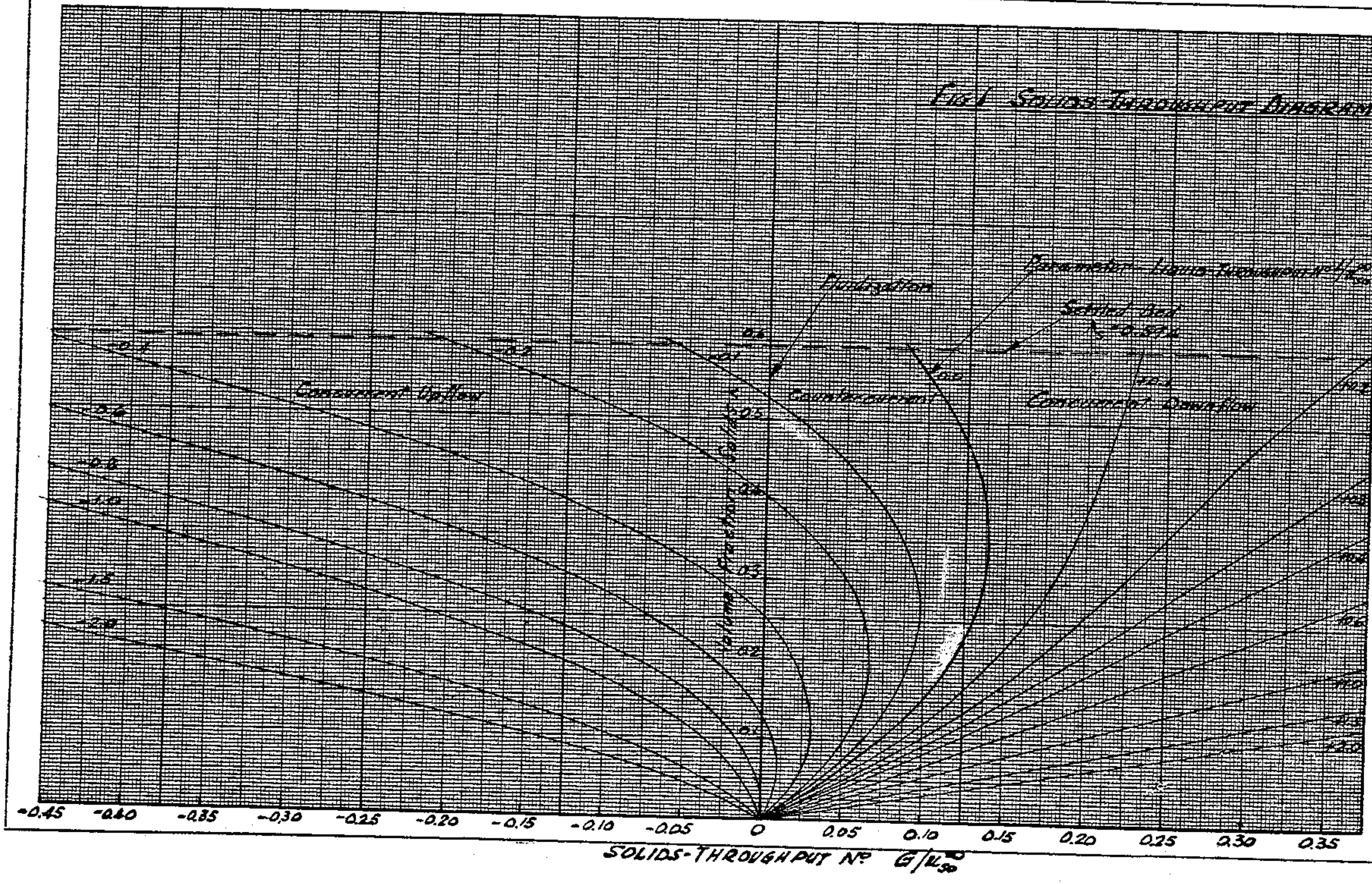

Fig 1 Solids-Throughput Diagram

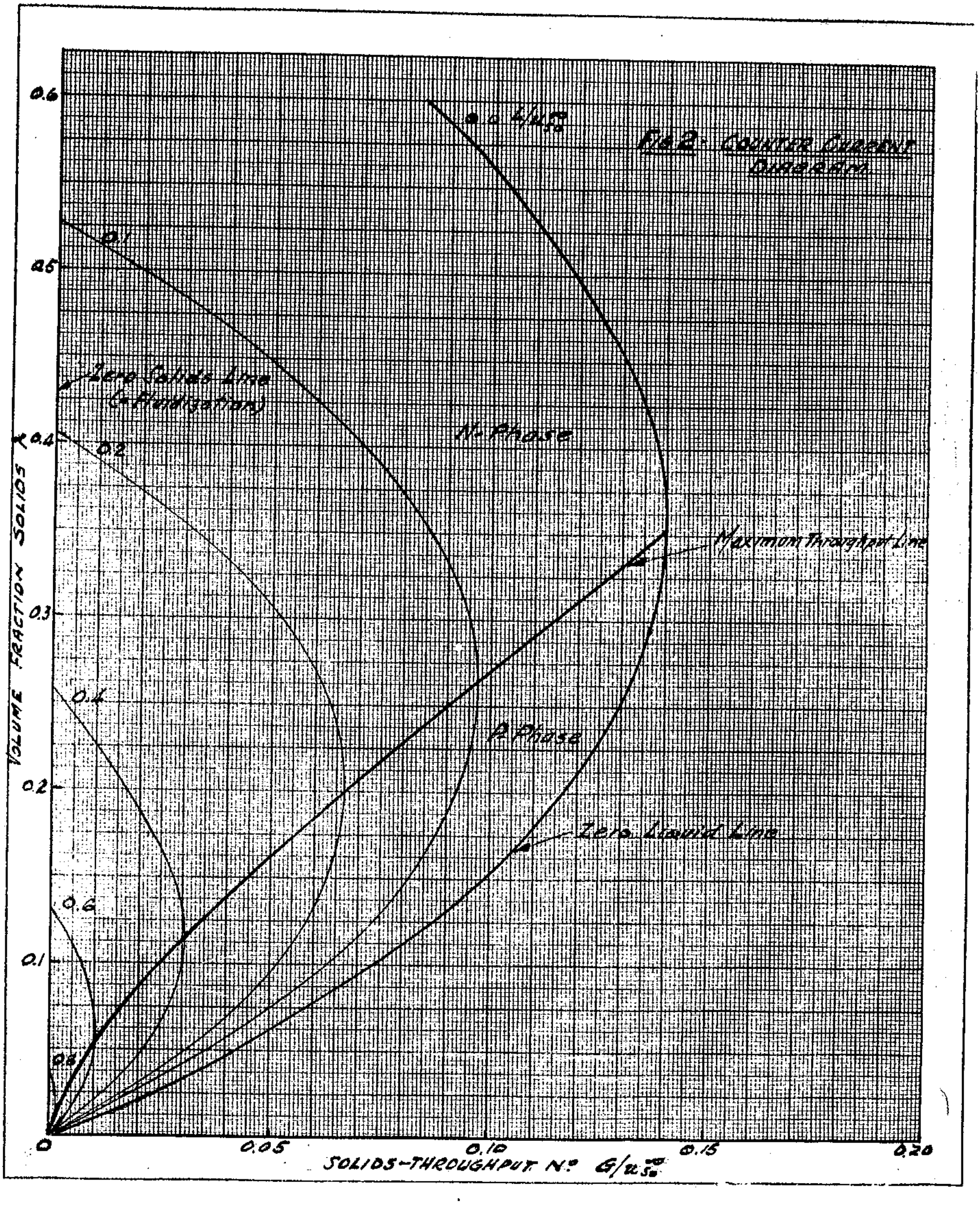

Fig 2. Counter Current Diagram
N-Phase
P-Phase
Zero Solids Line
(= Fluidization)
Maximum Throughput Line
Zero Liquid Line
0.1
0.2
0.4
0.6
0.8
0.6
0.5
0.4
0.3
0.2
0.1
0
0.05
0.10
0.15
0.20
VOLUME FRACTION SOLIDS
SOLIDS-THROUGHPUT No

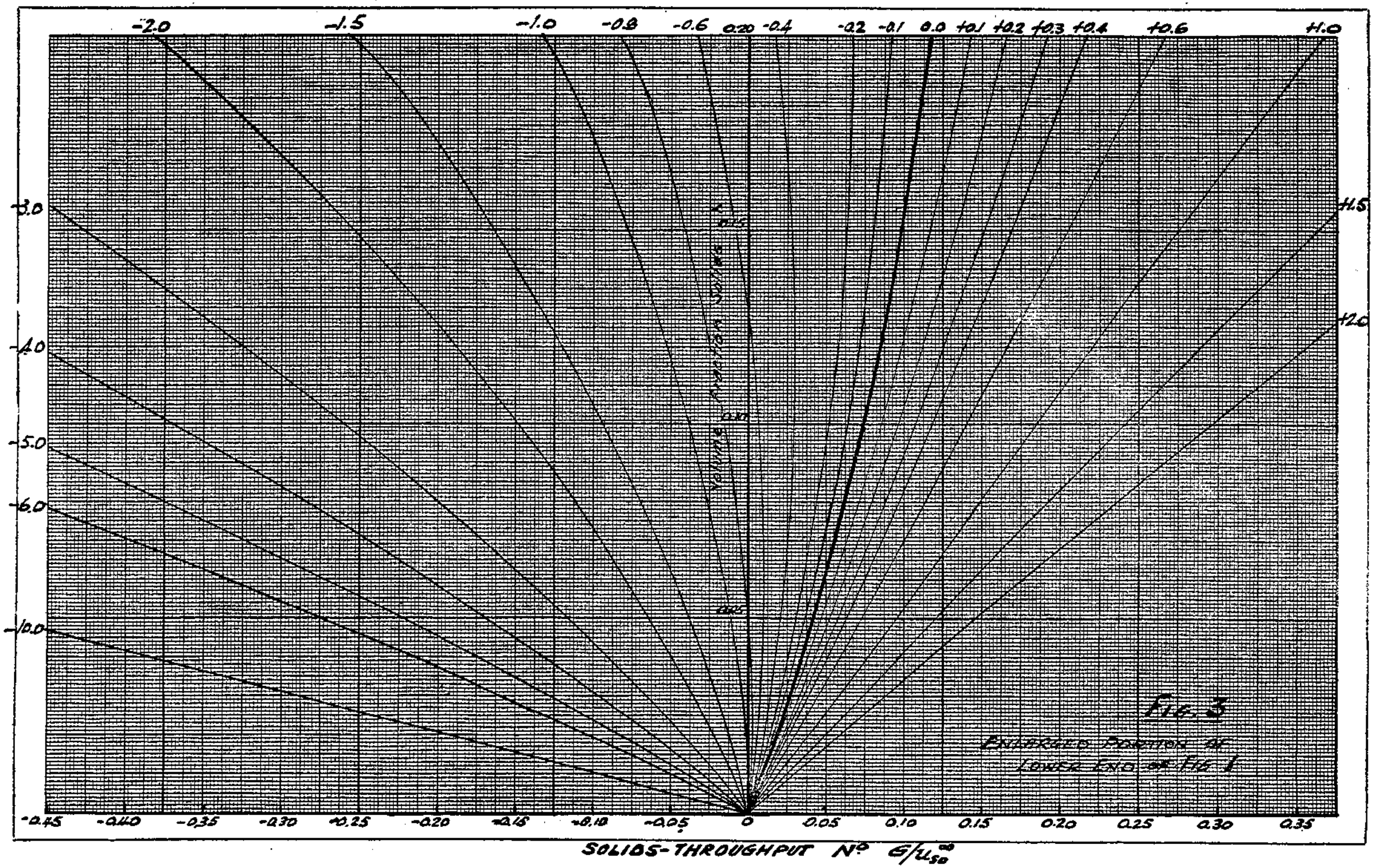

FIG. 3

ENLARGED PORTION OF LOWER END OF FIG. 1

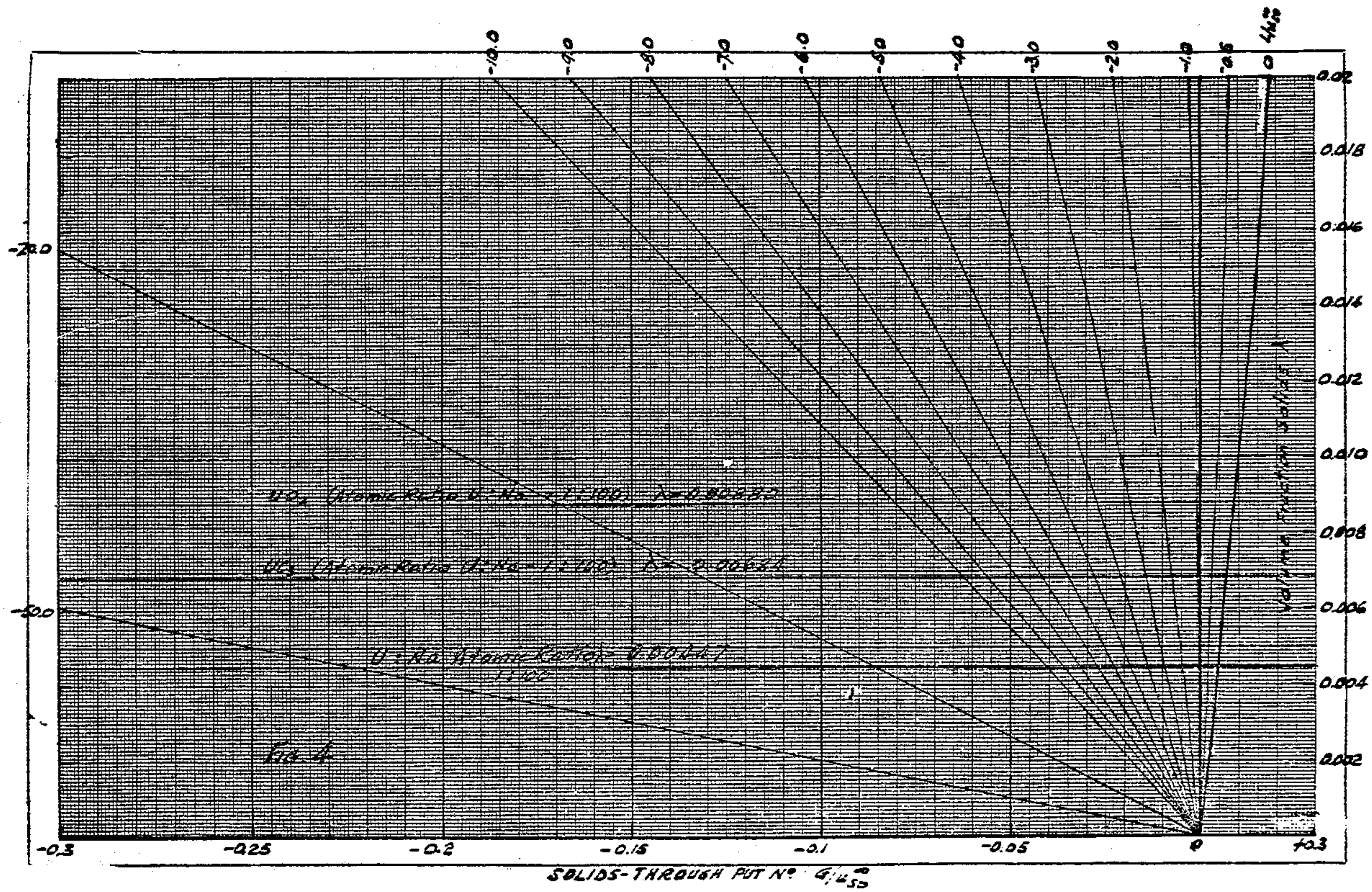
Fig. 4
-10.0
-9.0
-8.0
-7.0
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
-0.5
0
-20.0
-50.0
0.02
0.018
0.016
0.014
0.012
0.010
0.008
0.006
0.004
0.002
-0.3
-0.25
-0.2
-0.15
-0.1
-0.05
0
+0.3
Volume Fraction Solids λ
SOLIDS-THROUGH PUT No

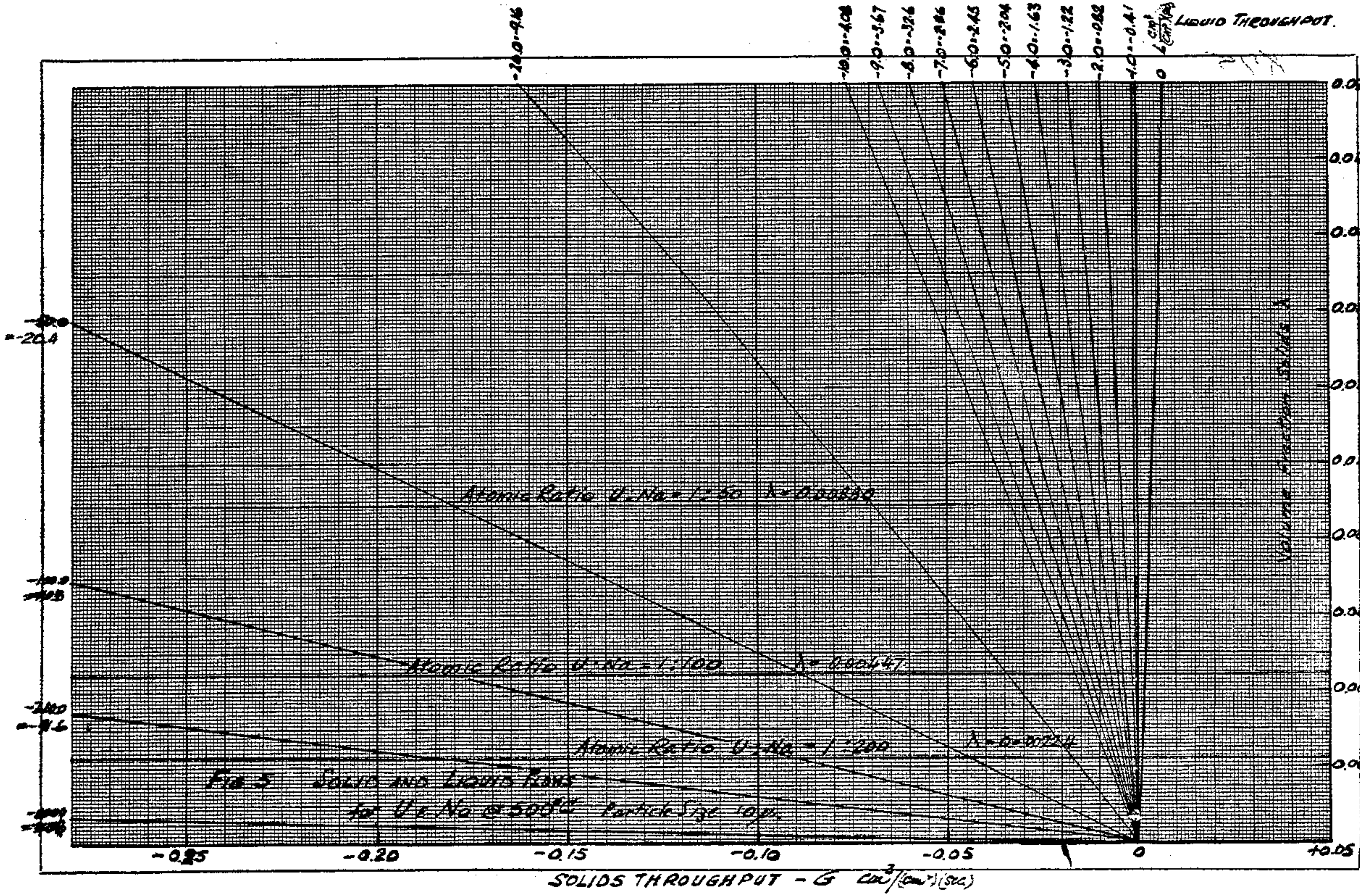

Fig 5 Solid and Liquid Flows for U + Na @ 500°C Particle Size 10μ