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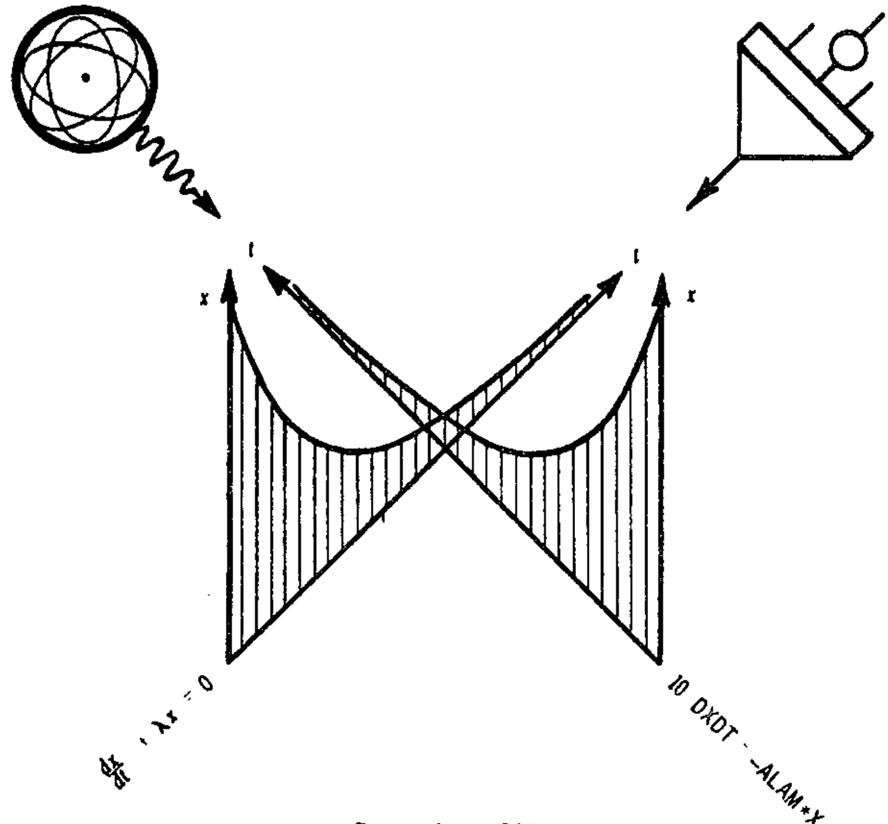
AUSTRALIAN ATOMIC ENERGY COMMISSION
RESEARCH ESTABLISHMENT
LUCAS HEIGHTS

ANALOGUE AND HYBRID COMPUTERS

REACTOR PHYSICS, MATHEMATICS AND COMPUTERS

SUMMER SCHOOL FOR TEACHERS

Lecture by C.P. GILBERT



December, 1972

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ABSTRACT

The use of analogue methods of addition and integration in the solution of differential equations is introduced. A circuit for the solution of a reactor problem is then developed and the way in which an operator would use it is described.

The essential features of a Hybrid computer are briefly reviewed.

CONTENTS

1. ANALOGUES

When examining physical systems such as the reactor problem already introduced, we use mathematical equations to describe their behaviour. In some instances the situation is reversed, and we use carefully chosen physical processes to help us understand, or 'solve' a set of equations. Such processes are called analogues.

Figure 1a shows how addition can be performed using a liquid: if the contents of the smaller containers are emptied into a sufficiently large container the final volume of liquid is the sum of the initial volumes.

$$V = v_1 + v_2 + v_3 + v_4 + v_5 \quad .$$

A more important process is integration, and this can be achieved as shown in Figure 1b. The height H of the fluid in a container of base area A, is the integral, with respect to time, of the fluid flow F (volume/sec) as determined by the tap.

$$H = \frac{1}{A} \int_0^t F \, dt \quad .$$

These simple analogues would be of no use in practice: they are inaccurate, slow, unsuitable for interconnection, and probably wet. However, there are much better analogue processes available, and the most important of them is described in the following section.

2. OPERATIONAL AMPLIFIER CIRCUITS

An operational amplifier has the following properties, illustrated in Figure 2:

(a) Very high voltage amplification, or 'gain' K (10^6 say).

(b) Negative gain (a positive input produces a negative output, and vice versa).

(c) Works at d.c. as well as at a.c. for all frequencies up to perhaps 10^6 Hz.

For computing purposes, such amplifiers are used in a feedback circuit which exchanges the high voltage gain for other more desirable properties. The circuit of Figure 3a is arranged so that the voltages v_1, v_2, v_3 and V_0 are all of the order of a few volts. Then, if $K = 10^6$, the amplifier input voltage u is equal to $-V_0/K$, which never exceeds a few micro volts and can usually be neglected. For instance, if the combined effect of all the inputs is positive, u tries to go positive: this causes V_0 to go negative by a very much larger amount, which opposes the rise in u because of R_f. Finally u ends up very slightly positive, causing a negative output V_O.

If we assume that u is zero, the current i is the sum of the input currents:

$$i = \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} .$$

This current cannot enter the amplifier, but is drawn through R_f by V_O, and so

$$i = -\frac{V_O}{R_f} .$$

Eliminating i,

$$-\frac{V_O}{R_f} = \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3}$$

leading to

$$V_O = -\left[v_1 \frac{R_f}{R_1} + v_2 \frac{R_f}{R_2} + v_3 \frac{R_f}{R_3}\right] .$$

Thus the output is minus the sum of the input voltages. The resistance ratios $\frac{R_f}{R_1}$ are normally fixed at convenient values, such as 1 or 10, and variable coefficients are introduced using potentiometers (see Figure 3b). It is a pity that the amplifier gives a reversal in sign, but it is unavoidable, and causes little difficulty.

The complete <u>adder</u> circuit is conventionally drawn as in Figure 3c, the values of the resistance ratios being marked only if they are other than unity. Then

$$V_O = -[0.3\ V_1 + 1.6\ V_2 + V_3] .$$

Note that all voltages are measured with respect to earth, although the earth connection itself is omitted.

In the <u>integrator</u> circuit of Figure 4a, a feedback capacitor C is used.

Assuming as before that u = o,

$$i = \frac{v}{R} .$$

Again, the current is constrained to flow into the feedback component, but in this case the voltage is proportional to the integral of the current i with respect to time t, namely

$$V_o = -\frac{1}{C}\int_0^t i\,dt$$

and substituting for i shows that

$$V_o = -\frac{1}{CR}\int_0^t v\,dt \quad .$$

A constant value of v causes V_o to change at a constant rate, a sine input gives a cosine output and so on (Figure 4b). Unless otherwise shown, the time constant CR can be assumed to be unity, and the integrator circuit is conventionally drawn as in Figure 4c, for which

$$V_o = -0.6\int_0^t V_1\,dt$$

or

$$-\frac{dV_o}{dt} = 0.6\ V_1 \quad .$$

Accuracies better than 0.1% can be obtained without difficulty for adders and integrators of the type shown.

3. ELECTRONIC ANALOGUE COMPUTERS

An electronic analogue computer consists of a number of operational amplifiers which can be used for addition, integration, multiplication and a range of other functions: facilities are provided which permit the interconnection and switching of the computing circuits, and which allow accurate measurements to be made on them. The problem variables in which we are interested (flux, velocity or force, for instance) are all represented in the computer by voltages. These voltages may vary quite slowly, and can then be read on a voltmeter, or they may change so quickly that an oscilloscope is required to observe them.

A medium sized machine might have about 100 amplifiers, including perhaps 30 integrators, and could thus perform 30 integrations at the same time.

Consider the circuit of Figure 5. The extra input on top of the integrator is inverted, and supplies a fixed voltage b to the output as an 'initial condition' before the integration starts, but has no other effect. When switch A is closed, this defines the instant which the computer regards as t = 0; at this time, V = b. We have now made the integrator input equal to V, and so the circuit obeys the equation

$$-\frac{dV}{dt} = V \qquad \text{or} \qquad \frac{dV}{dt} = -V \qquad \ldots(1)$$

[Simple analysis shows that if we let $V = ke^{-t}$, where k is an unknown constant, then differentiating we get

$$\frac{dV}{dt} = -ke^{-t} = -V \quad .$$

This demonstrates that $V = ke^{-t}$ is a solution of Equation (1). Since we have made V = b at t = o, substitution shows that k = b, and so the solution is $V = be^{-t}$.]

The circuit 'solves' Equation (1) by producing a voltage proportional to be^{-t} each time switch A is closed. V starts off positive, and, via the integrator, forces itself to get smaller. As it does so, it falls more slowly, reaching zero exponentially.

Switches such as A, and many other controls which the computer needs, are usually omitted from the computing circuit - their presence is assumed.

4. PROBLEM SOLUTION

In the previous section we started with a computer circuit and analysed its behaviour. The usual process is the other way round - we are given an equation and have to design a circuit which will solve it, resulting in the process illustrated in Figure 6. The equivalence between the physical system and the analogue circuit is very marked, and examination of the behaviour of the latter, used as a working model, provides considerable insight into the operation of the original system. In some cases simulators behave so much like the original system that they are used to train operators.

Our example problem (Pollard, 1972)* has been given as

$$\frac{dp}{dt} = -[p - 1 - be^{-t}]\,p$$

or

$$\frac{dp}{dt} = -qp \qquad \ldots(2)$$

where

$$q = [p - 1 - be^{-t}]$$

and

$$p = 1 \text{ at } t = 0 \quad .$$

We wish to find out how large b may be without causing p to exceed 1.2.

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* Pollard, J.,(1972). Numerical mathematics and FORTRAN. AAEC/S8

Consider the circuit of Figure 7. Apart from a reversal in sign, integrator 1 is connected as in Figure 5, so we know it produces $-be^{-t}$, which can be used to represent the input disturbance. Adder 2 combines the terms which make up q, and multiplier 3 forms the product qp, which Equation (2) shows to be equal to $-\frac{dp}{dt}$. Thus integrator 4 provides p. Once started with the correct initial conditions, this circuit behaves in exactly the same way as the reactor, (as described by Equation (2)). Unit 5 is a comparator which triggers when its total input passes through zero. It is used to indicate if p exceeds a peak of 1.2, and so tells us if the fuel would have started to melt or not.

As shown, the circuit would be adjusted and observed by an operator. He would take a number of solutions with a range of values of b until he found the value which just failed to trigger the comparator by trial and error. Voltages would be plotted on a chart, or displayed on an oscilloscope, and the wanted result (the size of b) would be read on a digital voltmeter.

Notice that all the computing operations occur simultaneously (in parallel) not in sequence as in a digital computer, and that the solution arises at a definite speed - the same speed as the reactor in our case. By using smaller capacitors in the integrators, the solution can be speeded up by up to 10^4 times, and if many integrations are involved the overall operation is far faster than can be achieved by a digital computer. However the high speed cannot be properly utilised by a human operator.

A major drawback to pure analogue computers is their inability to store information, and in general their usefulness is limited to the solution of ordinary differential, and straightforward algebraic equations. However, within this field they provide valuable insight as well as useful answers.

5. HYBRID COMPUTERS

A recent development, whose full impact has not yet been felt, is the Hybrid Computer. This consists of an analogue computer, a general purpose digital computer, and an interface. The latter provides the facilities required for the two machines to cooperate effectively (Figure 8).

The analogue computer allows high speed, parallel computation. The digital computer can be programmed to ;

(a) Perform the scaling calculations and check the analogue circuit.

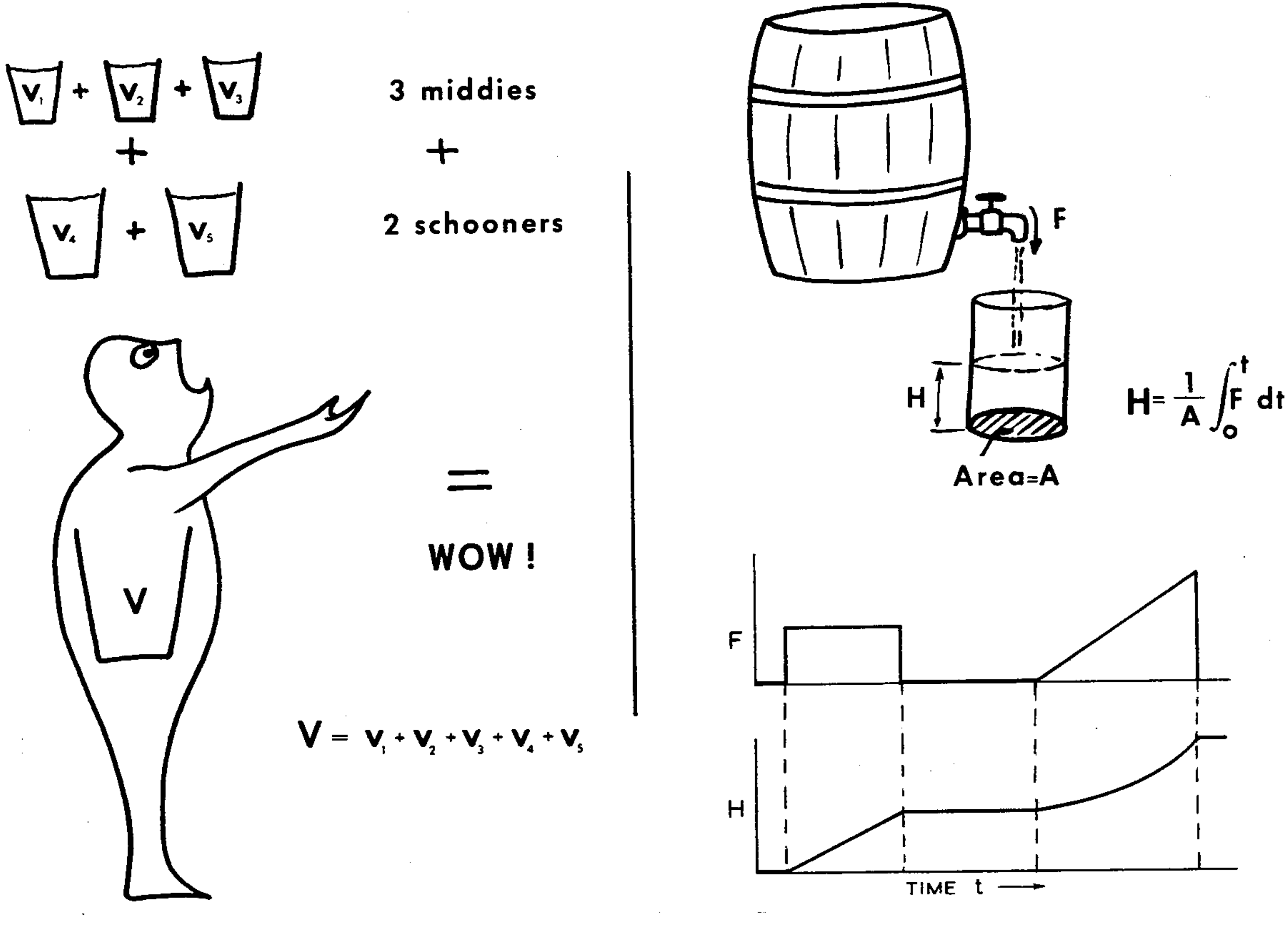
(b) Act as a very high speed operator, who readjusts the computer before each solution, as determined by the preceding solution.

(c) Perform parts of the computation which the analogue computer finds difficult.

(One might also express the same idea by saying that the analogue computer becomes one of the peripherals upon which the digital computer can call when required.)

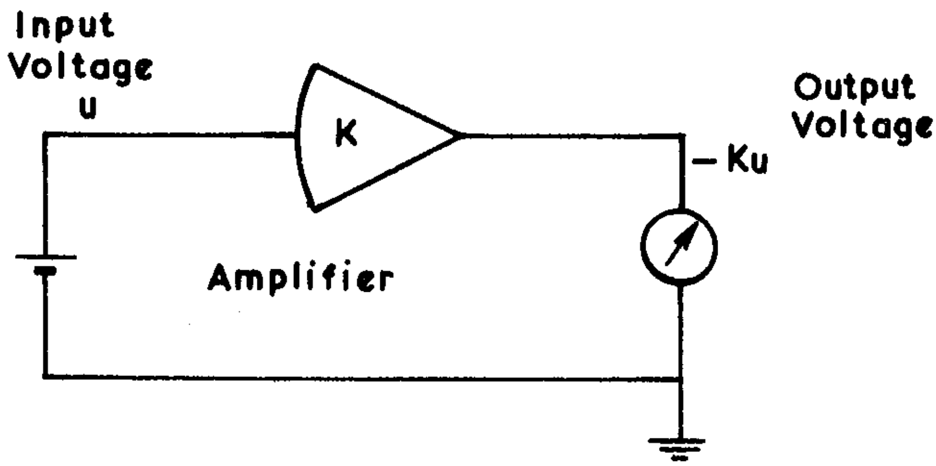
In the problem of Figure 7 for instance, having written an executive programme, the operator would only have to type in the permissible power peak (1.2 in our case), and the computer would then print out the size of disturbance b which just avoids this peak. The programme would go through almost the same trial-and-error process as the human operator using the circuit, but would do it perhaps 10^5 times faster.

There are few problems which a hybrid computer cannot profitably undertake, but much more experience is required in using these machines on large problems. We are doing our best to gain that experience now.



(a) (b)

FIGURE 1. Liquid analogues for (a) addition, using volumes, and (b) integration



$10^4 < K < 10^8$

FIGURE 2. A circuit demonstrating the properties of an operational amplifier. The amplifier itself is represented by the triangular symbol with a curved side at the input

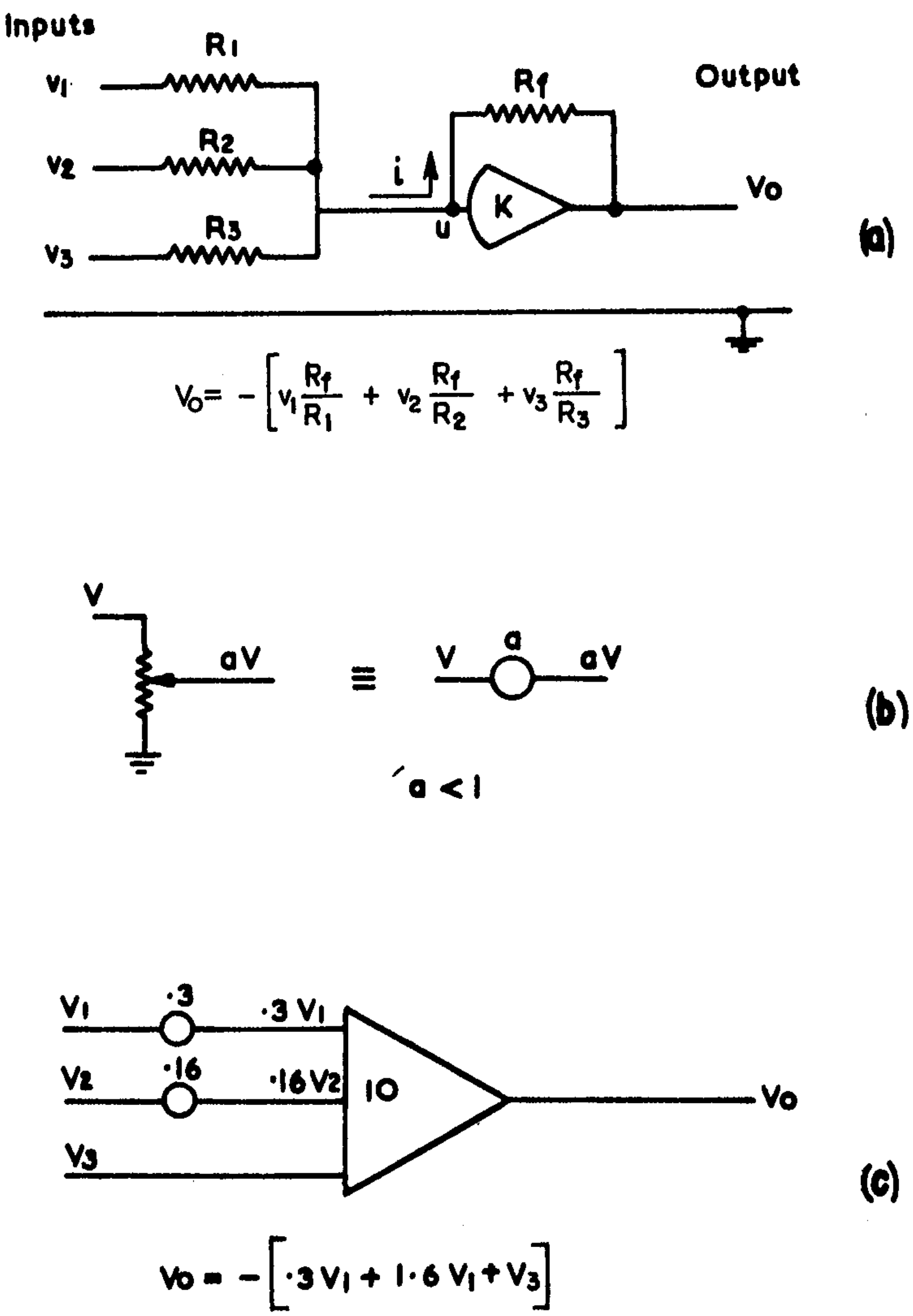


FIGURE 3. A circuit for addition: (a) Circuit details (b) Potentiometers (c) Circuit using conventional symbols: the whole of the circuit (a) is contained within the triangle

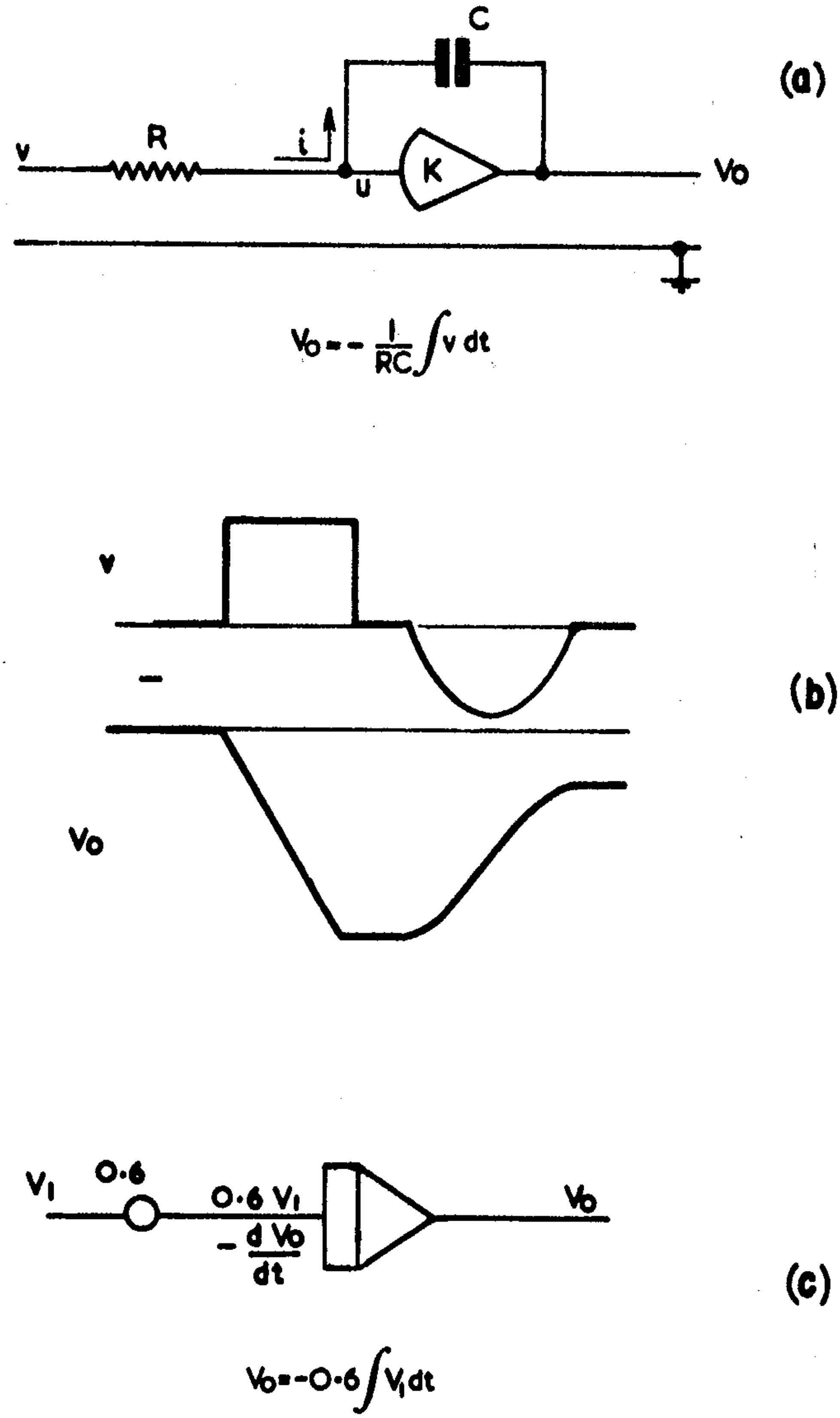


FIGURE 4. A circuit for integration: (a) Circuit details (b) Typical waveforms (c) Circuit using conventional symbols: the whole of circuit (a) is contained within the special triangular symbol. More than one input circuit is permitted

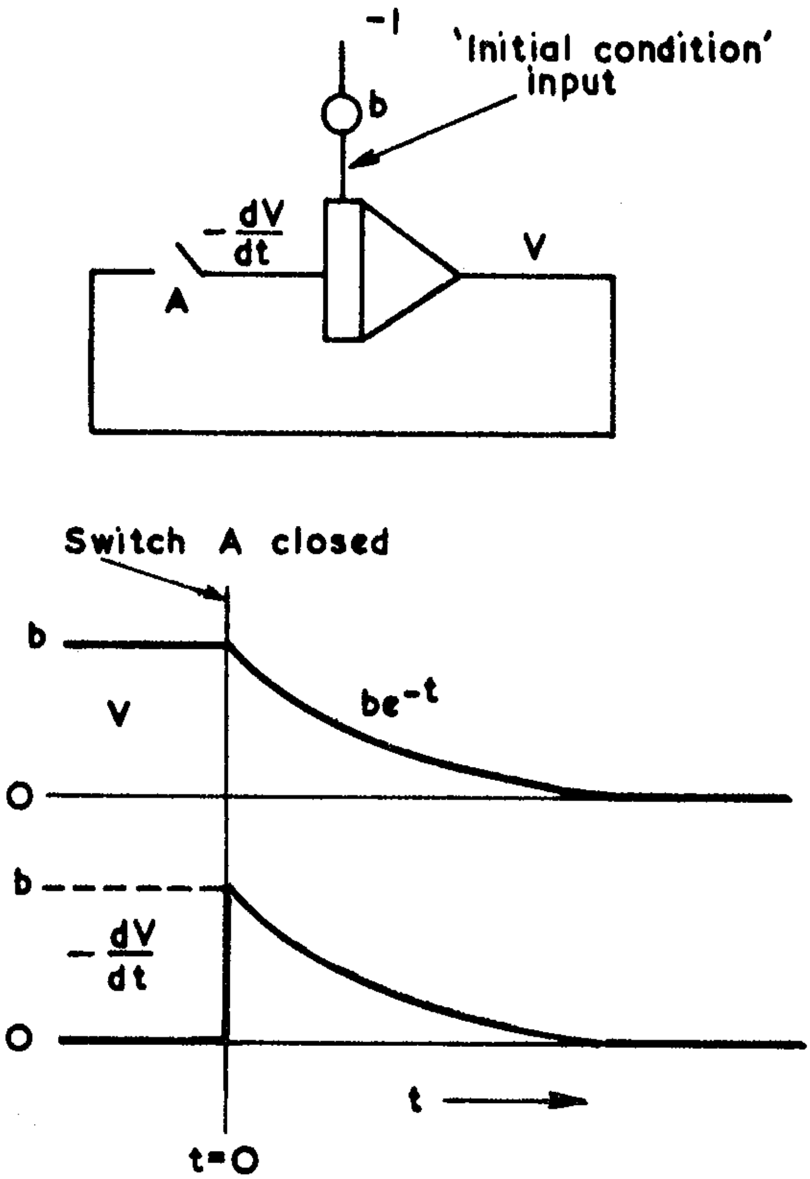


FIGURE 5. A circuit for the solution of $\frac{dV}{dt} = -V$ and typical waveforms. The input via b only provides the starting voltage

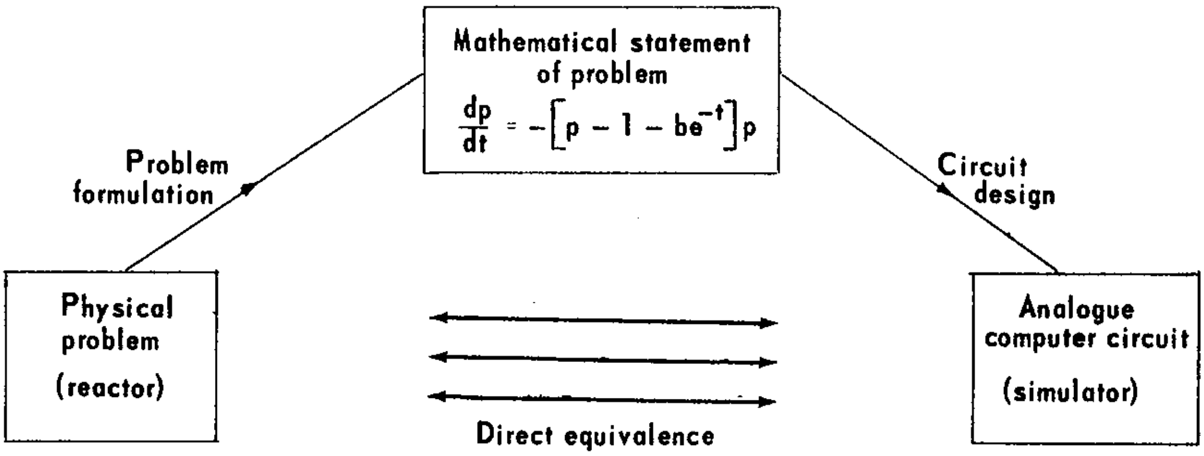


FIGURE 6. Pictorial representation of the analogue method of problem solving

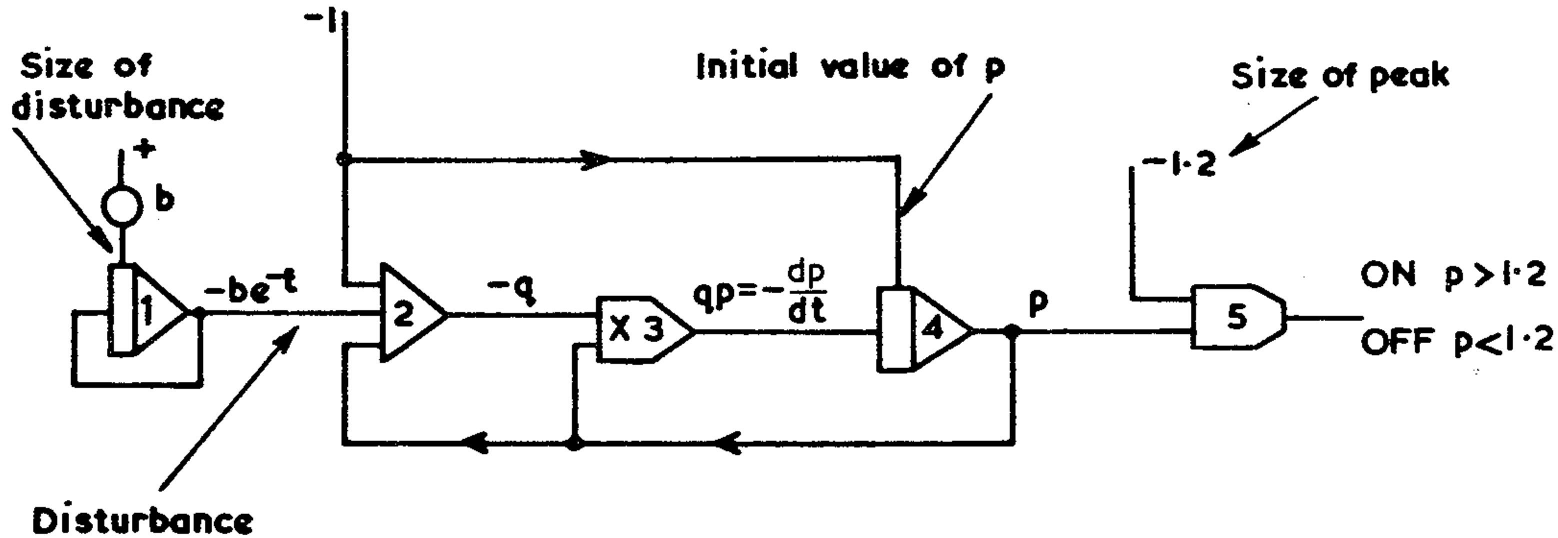


FIGURE 7. A Circuit for the solution of $\frac{dp}{dt} = -\left[p - 1 - be^{-t}\right]p$ with an indication when $p > 1.2$

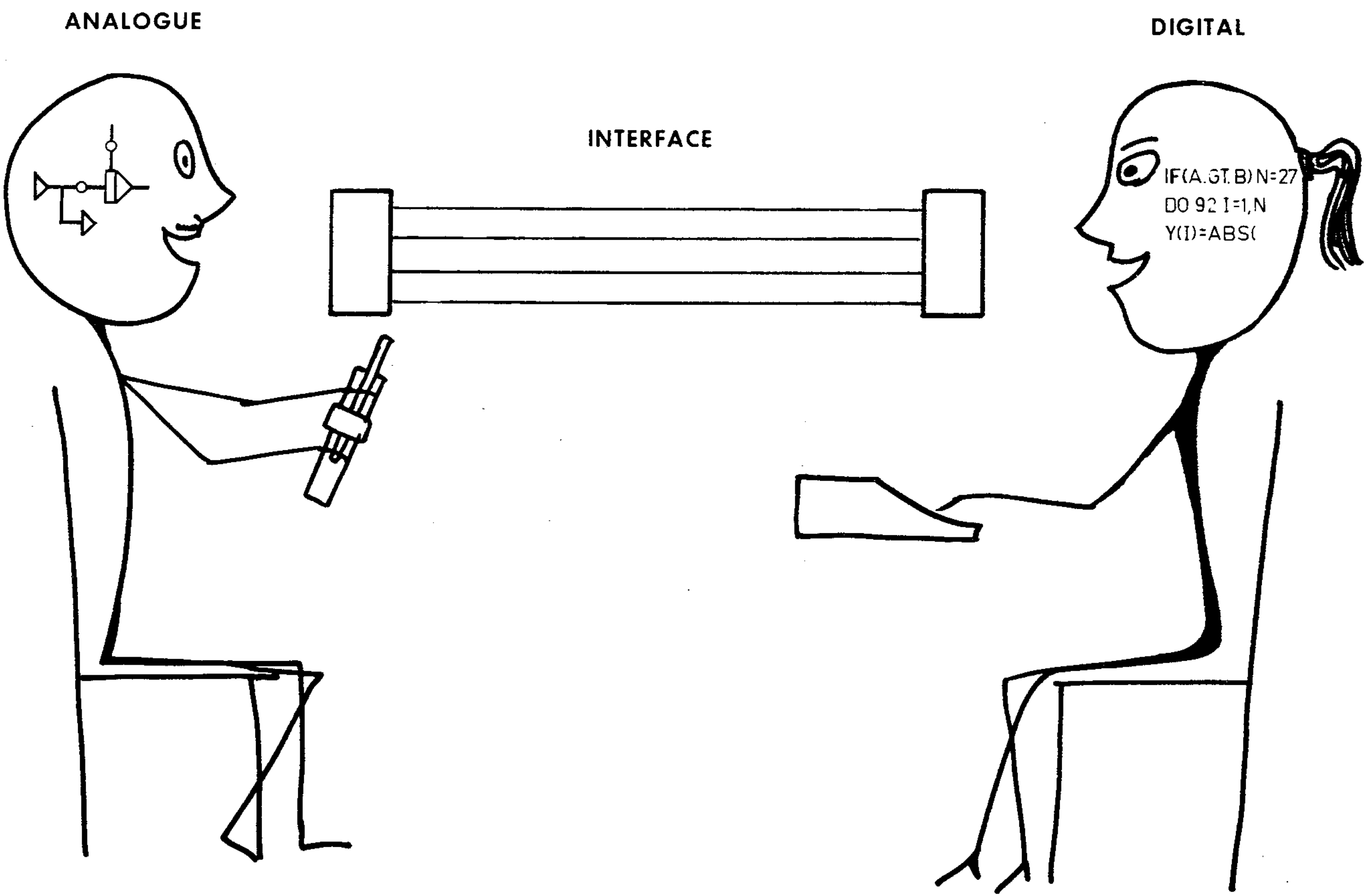


FIGURE 8. A hybrid computer