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RESEARCH ESTABLISHMENT
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ANALYSIS OF (n-2n) CROSS SECTIONS

by

W.K. BERTRAM

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ABSTRACT

The statistical model is used to derive an expression for the (n-2n) excitation function, taking into account (n-3n) competition. The (n-2n) cross sections thus obtained are found to agree with experiment over a wide range of energies extending well beyond the (n-3n) threshold.

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1. INTRODUCTION

The statistical model (Blatt and Weisskopf 1952) has been reasonably successful in describing the energy dependence of (n-2n) reaction cross sections. The method of Barr et al. (1961) which does not take into account (n-3n) competition, can only be used to calculate (n-2n) cross sections for incident neutron energies below the (n-3n) threshold. Vandenbosch et al. (1961) used a Monte Carlo method to describe the (n-2n) process with (n-3n) and higher order neutron emissions as competing processes. An analytical method to account for (n-3n) competition was developed by Pearlstein (1965). However, Pearlstein's calculations of (n-2n) cross sections for nuclei for which experimental data above the (n-3n) threshold was available, did not agree with the measured values at high energies. Menlove et al. (1967) also used Pearlstein's method to analyse their own experimental results, and found that the method underestimated the cross sections at high energies. It was suggested by Menlove et al. (1967) that this discrepancy might be the result of neglecting (n-2n) competition in the evaluation of the (n-3n) cross sections. The work in the present paper shows that this is indeed the case and the correct method to describe (n-3n) competition is given.

A curious feature, arising out of the analysis of (n-2n) cross sections with the statistical model, is that values obtained for the nuclear level density parameter are not in agreement with values obtained using other methods, such as the analysis of resonance data or the analysis of emission spectra of reactions such as (n,n'), (n,p) and (p,n).

2. THE EXCITATION FUNCTION

Following Pearlstein's procedure (Pearlstein 1965) we assume that for a given target nucleus the (n-2n) cross section, for incident neutron energy E_n, can be written as

$$\sigma_{n-2n}(E_n) = K(\sigma_{n-2n}/\sigma_{n-M}) \quad , \tag{1}$$

where

$$\sigma_{n-M} = \sigma_{n-n} + \sigma_{n-2n} + \sigma_{n-3n} ------ \quad .$$

The quantity K can be written as

$$K = \left(\frac{\sigma_{n-M}}{\sigma_{ne}} \right) \sigma_{ne} \quad , \tag{2}$$

where σ_{ne} is the non-elastic cross section. Both σ_{ne} and $(\sigma_{n-M}/\sigma_{ne})$ are assumed to be independent of E_n, so that the energy dependence of σ_{n-2n} is completely contained in the excitation function $\sigma_{n-2n}/\sigma_{n-M}$. The value of K for a particular nucleus can be calculated from the empirical formula of Pearlstein (1965) or, as is done in this paper, by normalizing $\sigma_{n-2n}/\sigma_{n-M}$ to the measured values of σ_{n-2n}.

According to the statistical model, if a compound nucleus with mass number A+1 has an excitation energy U, the probability of its emitting a neutron with energy in the range E to E+dE is

$$P(E)dE = G\,E\,\rho_A\,(U-E)dE \quad . \tag{3}$$

ρ_A is the density of states of the final nucleus A, and the quantity G is proportional to the capture cross section of A for neutrons with energy E. For the sake of simplicity it is assumed that G does not depend on E.

In order to obtain a simple expression for the excitation function $\sigma_{n-2n}/\sigma_{n-M}$, it is customary to assume that, if it is energetically possible for a compound nucleus to decay by neutron emission, then this process will always take place. In other words,

$$\int_0^U P(E)dE = 1 \quad , \tag{4}$$

which then gives

$$G^{-1} = \int_0^U E\,\rho_A(U-E)dE \quad . \tag{5}$$

When a neutron with energy E_n is absorbed by a target nucleus A, then if the energy E_1 of the subsequently emitted neutron lies in the range

$$E_n - \epsilon_A \leq E_1 \leq E_n \quad ,$$

where ϵ_A is the energy required to dislodge one neutron from A in its ground state, no further neutron emission is possible. The total probability for single neutron emission, which corresponds to inelastic scattering, is then

$$P_A(1) = G_A(E_n)\int_{E_n-\epsilon_A}^{E_n} E_1\,\rho_A(E_n-E_1)dE_1 \quad , \tag{6}$$

whereas the probability for multiple neutron emission is

$$P_A(2,3,\ldots.) = G_A(E_n)\int_0^{E_n-\epsilon_A} E_1\,\rho_A(E_n-E_1)dE_1 \tag{7}$$

After emission of one neutron with energy E_1, leaving a compound nucleus A with excitation energy E_n-E_1, the probability of emission of only one more neutron is

$$P_{A-1}(1) = G_{A-1}(U)\int_{U-\epsilon_{A-1}}^{U} E_2\,\rho_{A-1}(U-E_2)dE_2 \quad , \tag{8}$$

where

$$U = E_n - E_1 - \epsilon_A \quad .$$

Hence, the probability for the (n-2n) process is

$$P(n\text{-}2n) = G_A(E_n)\int_0^{E_n-\epsilon_A} E_1\,\rho_A(E_n-E_1)\;G_{A-1}(U)\left[\int_{U-\epsilon_{A-1}}^{U} E_2\,\rho_{A-1}(U-E_2)dE_2\right]dE_1 \tag{9}$$

The excitation function $\sigma_{n-1n}/\sigma_{n-M}$ is equal to the ratio P(n-2n)/P(n-M). However P(n-M) is normalized to unity, from equation (4), so that using (5) we find

$$\frac{\sigma_{n-2n}}{\sigma_{n-M}} = \frac{\displaystyle\int_0^{E_n-\epsilon_A} E_1\,\rho_A(E_n-E_1)\left[\int_0^U E_2\,\rho_{A-1}(U-E_2)dE_2\right]^{-1}\left[\int_{U-\epsilon_{A-1}}^{U} E_2\,\rho_{A-1}(U-E_2)dE_2\right]dE_1}{\displaystyle\int_0^{E_n} E_1\,\rho_A(E_n-E_1)dE_1} \tag{10}$$

3. RESULTS AND DISCUSSION

The (n-2n) cross sections were evaluated for twelve nuclei for which experimental data beyond the (n-3n) threshold was available, the constant K of equation (1) being determined by normalization of $\sigma_{n-2n}/\sigma_{n-M}$ to the experimental values. The binding energies ϵ_A and ϵ_{A-1} were taken from the nuclidic mass tables of Konig et al. (1962). The results of our calculations (Figures 1 to 2), the details of which are given below, no longer contain the discrepancies beyond the (n-3n) threshold which occurred in Pearlstein's analysis (Pearlstein 1964, 1965). In some cases, for example ^{232}Th and ^{238}U, these discrepancies were greater than a factor of 2.

For the nuclei with $A < 150$ (Figures 1 to 6) our method gives a good fit to the cross section over the entire energy range, although in most cases the experimental information is rather scant. For example for ^{63}Cu there are only two measurements available for $E_n > 20$ MeV, and for the nuclei ^{115}In, ^{116}Cd, ^{121}Sb, ^{127}I and ^{133}Cs, no data are available for energies up to about 3 MeV above the threshold. Furthermore, independent measurements of (n-2n) cross sections for the same nucleus often show large disagreements.

The results for ^{181}Ta (Figure 8) and ^{197}Au (Figure 9) are not as good above the (n-3n) threshold as they are for the other nuclei. The nuclei with $A > 200$ (Figures 10 to 12) again show excellent agreement between theory and experiment, although beyond the (n-3n) threshold experimental data is either scarce as for ^{203}Tl (Figure 10) and ^{238}U (Figure 12), or varies widely between different experiments.

For the evaluation of $\sigma_{n-2n}/\sigma_{n-M}$, two forms for the nuclear level density were used. One was the simple exponential as used by Pearlstein (1964)

$$\rho_A(E) \sim \exp(2\sqrt{aE}) \quad , \tag{11}$$

with

$$a = 0.154(\bar{j}_N+\bar{j}_z+1)A^{1/3} \quad . \tag{12}$$

The values of the effective spins $\bar{j}_N$ and $\bar{j}_z$ were obtained from the tabulation of Newton (1956).

Equation 12 was obtained by Pearlstein (1965) from values of a determined by Vandenbosch et al. (1961) and Barr et al. (1961) from (n-2n) excitation functions for a number of nuclei. This form for a does indeed fit the (n-2n) cross sections of most nuclei to a good approximation, as can be seen from the curves labelled (a) in Figures 1 to 12. However, the level density parameter (12) is not in agreement with values given by other authors. Newton (1956) gave

$$a = 2\alpha(\bar{j}_N+\bar{j}_z+1)A^{2/3} \quad . \tag{13}$$

Subsequent investigations vary somewhat in the magnitude of α. Lang (1961) gave $2\alpha = 0.0784$, whereas investigations of the work of Thomson (1963) and Seth et al. (1964) (see Cindro 1966) gave values $2\alpha = 0.102$ and 0.075 respectively.

Other forms for the level density formula which have factors E^{-1} or E^{-2} multiplying the exponential have also been used (see for example Cindro 1966), but the effects of these variations were found to be quite insignificant. All these investigations show that the dependence of a on A is more closely given by (13) than by (12). More recent analysis (e.g. Gilbert and Cameron 1965) indicates that a varies linearly with A, which is in conflict with the results obtained from (n-2n) cross sections.

A second set of calculations was made using the supposedly more refined level density formula

$$\rho(E) \sim U^{-3/2}\, e^{2\sqrt{aU}} \quad , \tag{14}$$

where $$U = E - P(N) - P(Z)$$

and $$a = (0.00917\,S + 0.192)A \quad , \quad S = S(N) + S(Z) \tag{15}$$

The values of pairing energies P(N), P(Z) and shell corrections S(N) and S(Z) were obtained from the tabulations of Gilbert and Cameron (1965).

As is evident from Figures 1 to 12, the level density parameters given by (15) do not give the correct energy dependence of (n-2n) cross sections. Table 1 shows a comparison of a values obtained from (n-2n) reactions with those obtained from other reactions. The discrepancy is difficult to explain, especially in view of the results of Facchini et al. (1968) who, by investigating values of a at 8 MeV and at 20 MeV excitation energies, found that a does not vary with excitation energy to any great extent.

4. CONCLUSION

We have shown that the statistical model gives a good description of the energy dependence of (n-2n) reaction cross sections over a wide range of energies extending well beyond the (n-3n) threshold. The discrepancies which occurred in Pearlstein's method at high energies were due to incorrect description of (n-3n) competition.

Values of the level density parameter obtained from (n-2n) reactions do not agree with those obtained from other reactions. At the moment the reason for this is not clear.

5. ACKNOWLEDGEMENT

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TABLE 1

VALUES OF THE LEVEL DENSITY PARAMETER a FROM ANALYSIS OF (n-n')

Nucleus	a Values				
	n-2n	(n-n') (Erba et al. 1961)	(n-n') (Lang 1961)	n-α (Cindro 1966)	Resonances (Gilbert and Cameron 1965)
^{63}Cu	3.5	11.3 12.8	11.2	9.5	8.9
^{115}In	7.3	20.8 to 22.6	22.3 ± 2.6		17.9
^{127}I	7.0	21.6 20.4	16.4	18.0	16.9
^{181}Ta	8.3	26.7	26.5		21.3
^{197}Au	6.1	20.2 22.0	19.0	20.0	20.0
^{203}Tl	4.1	10.2 to 14.3			13.6
^{232}Th	10.4				29.4
^{238}U	10.4			29.0	33.6

FIGURES 1 - 12

COMPARISON OF CALCULATED AND EXPERIMENTAL (n-2n) CROSS SECTIONS FOR VARIOUS NUCLEI

In each case Curve (a) has been calculated using the level density given by Equations (11) and (12) and Curve (b) has been calculated using the level density given by Equations (14) and (15)

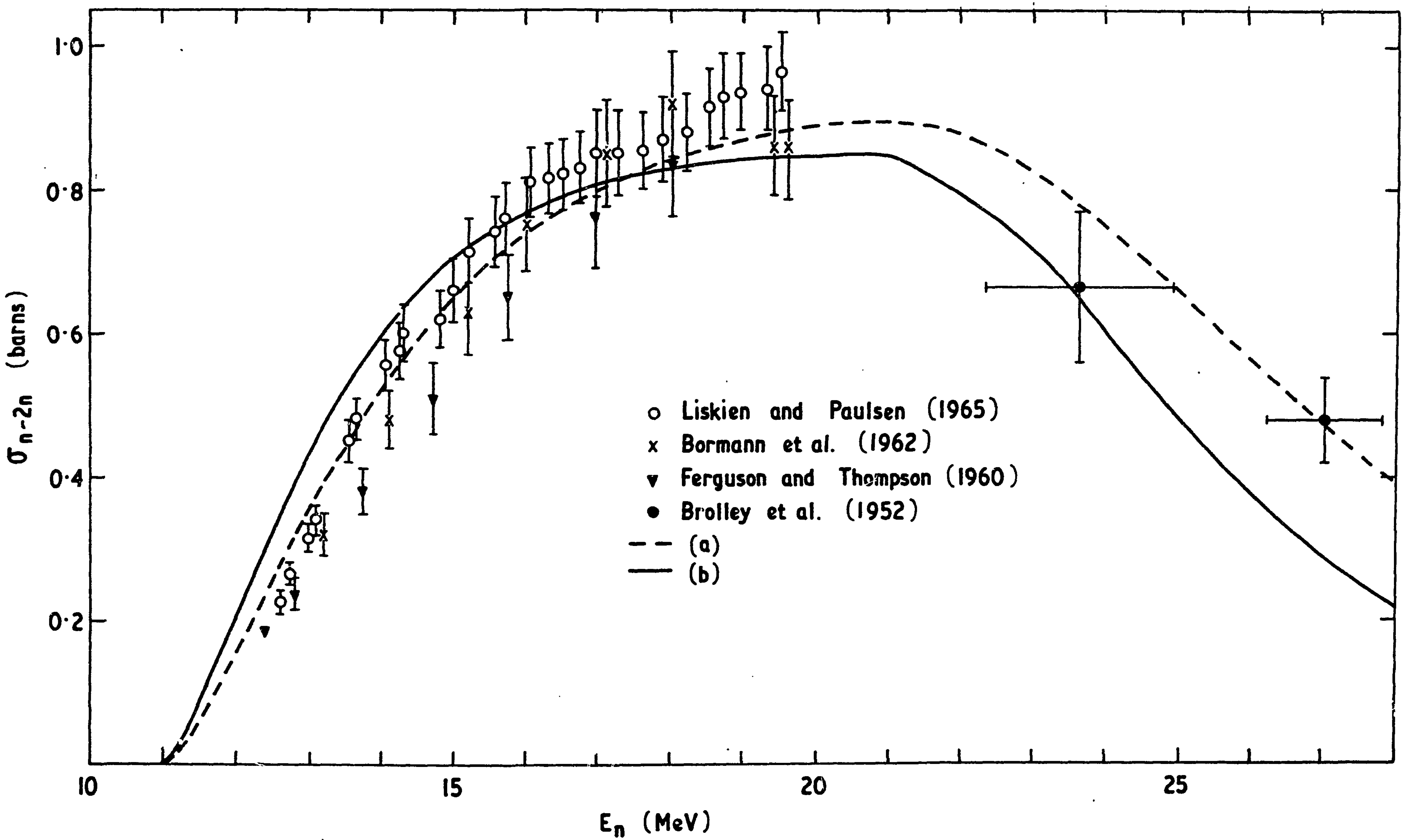

FIGURE 1. ^{63}Cu

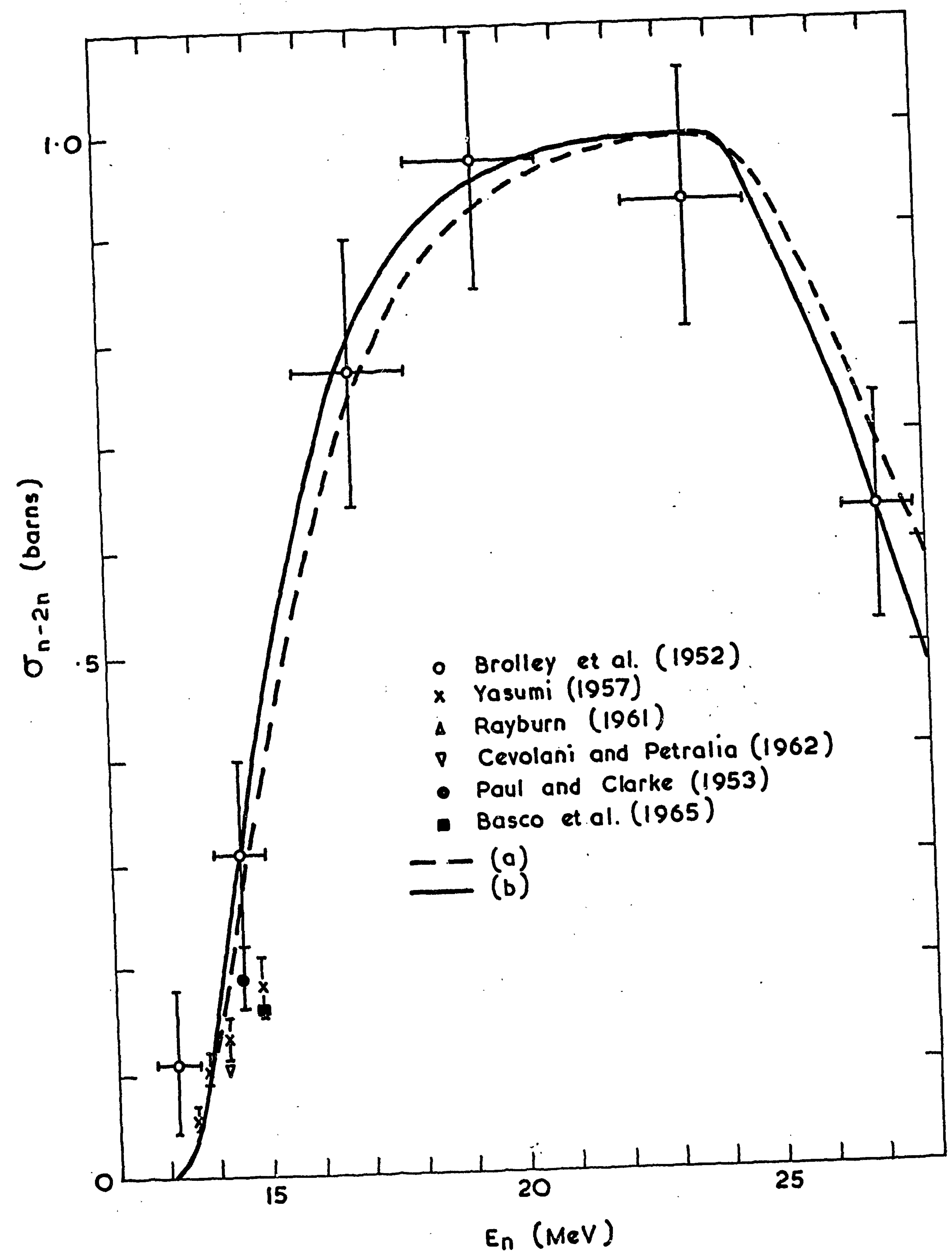

FIGURE 2. ^{92}Mo

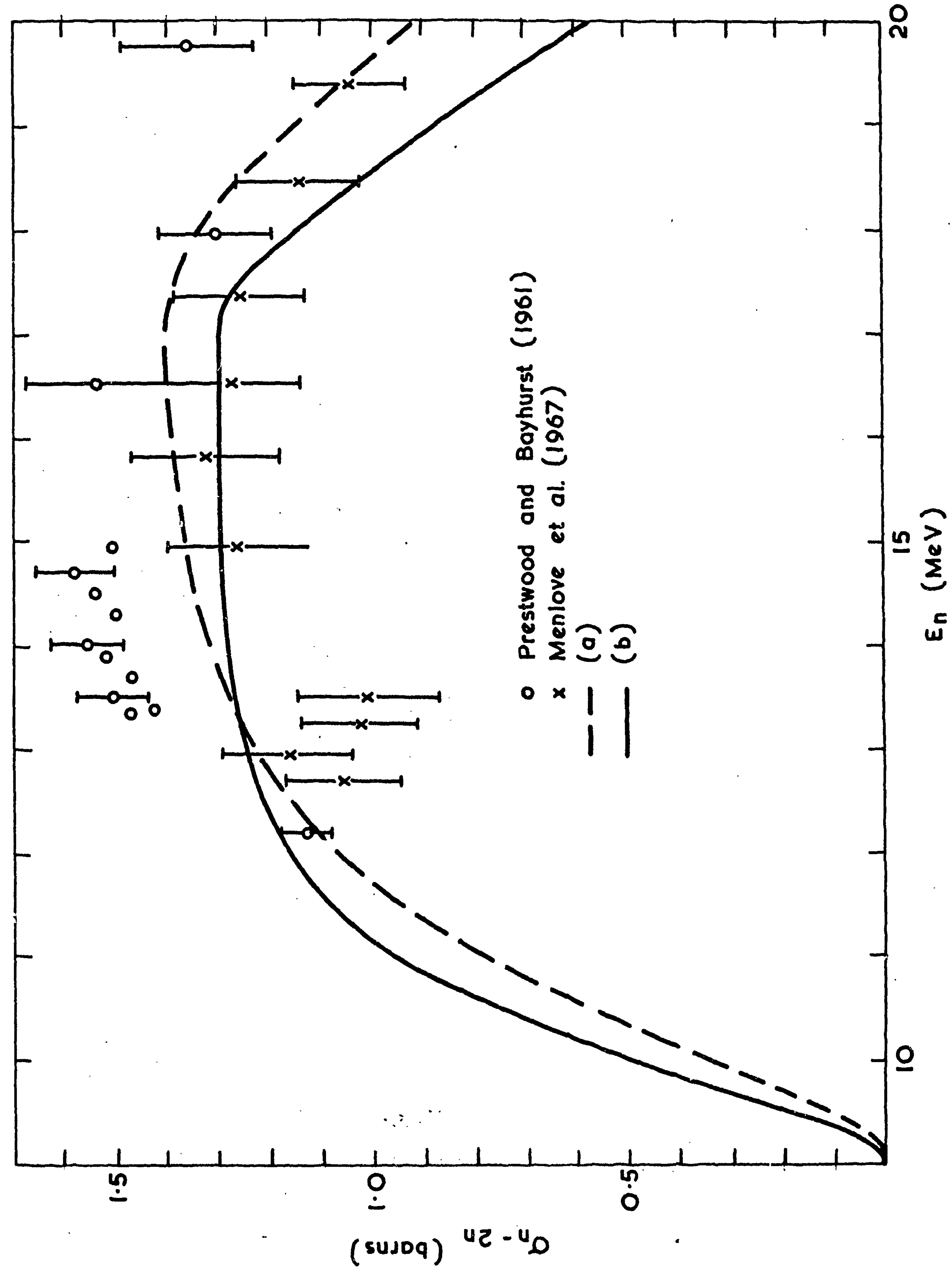

FIGURE 3. ^{115}In

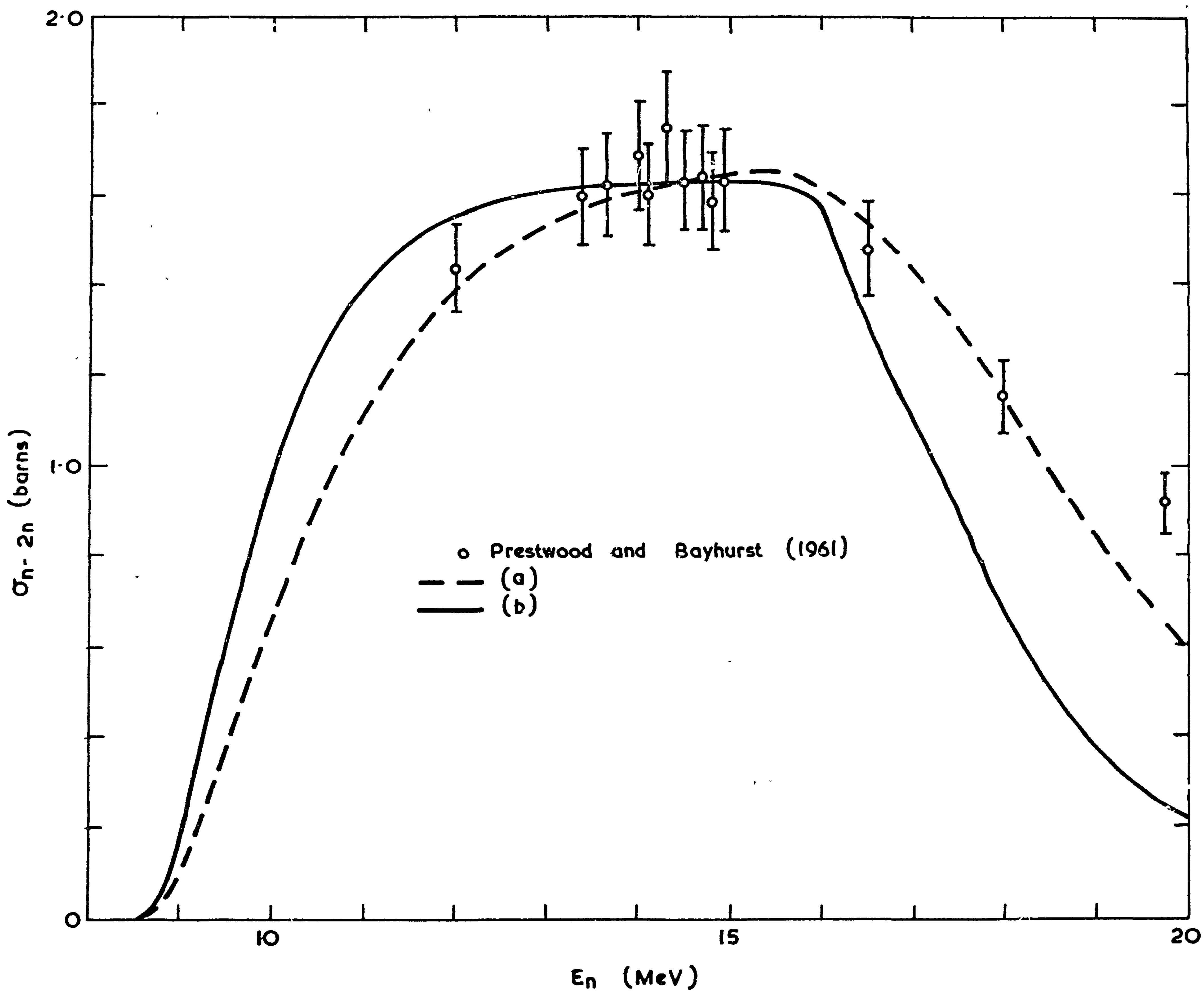

FIGURE 4. ^{116}Cd

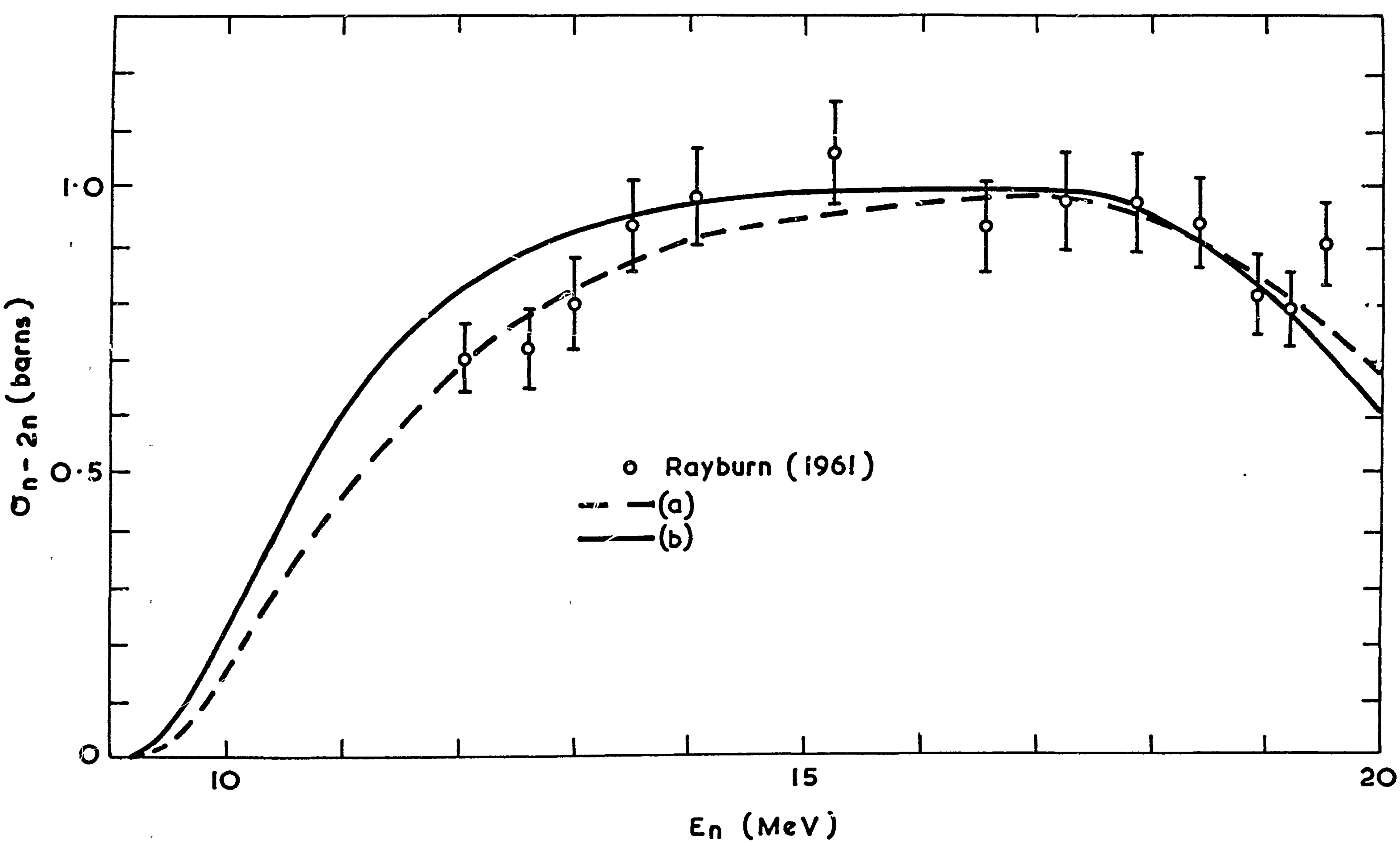

FIGURE 5. ^{121}Sb

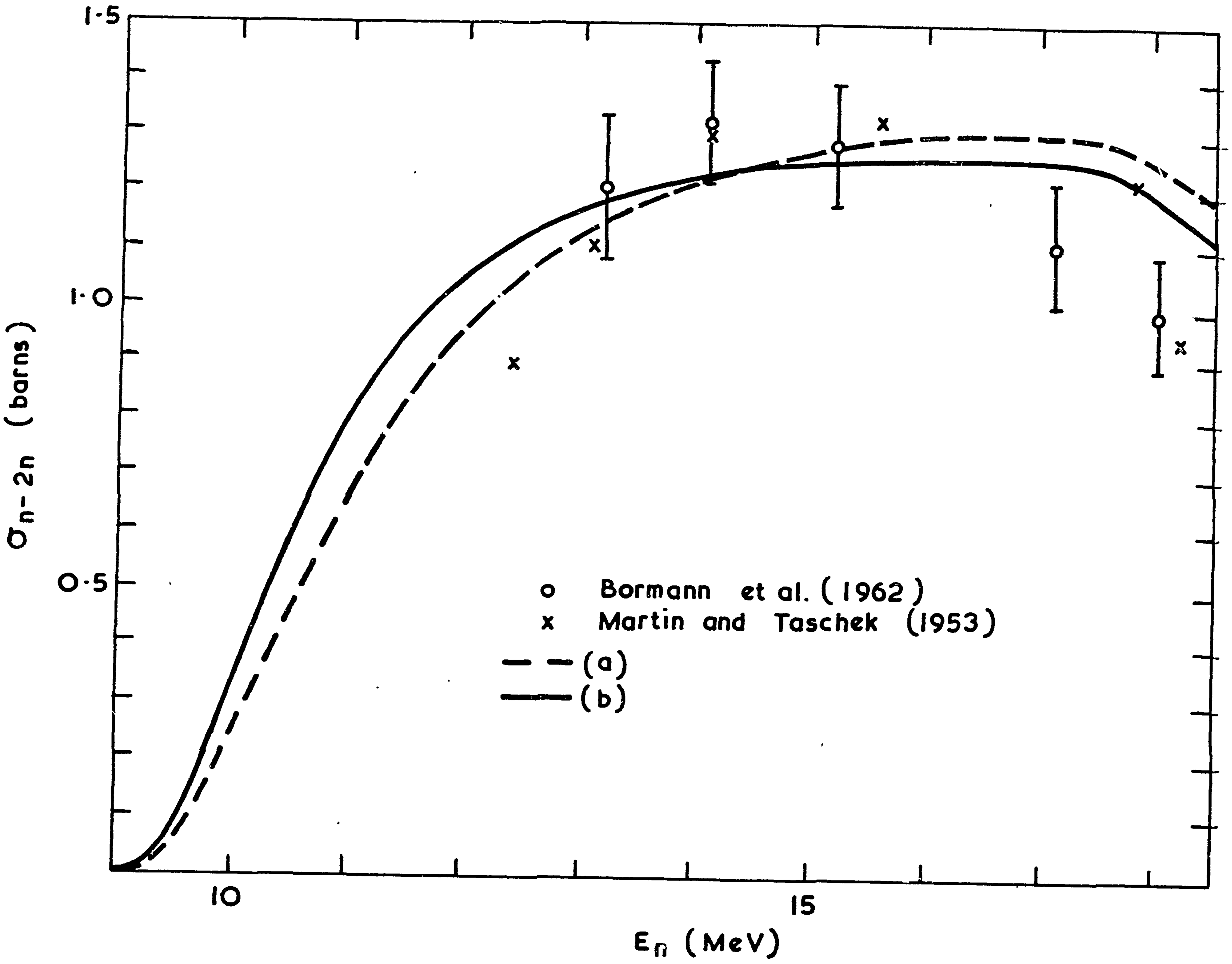

FIGURE 6. ^{127}I

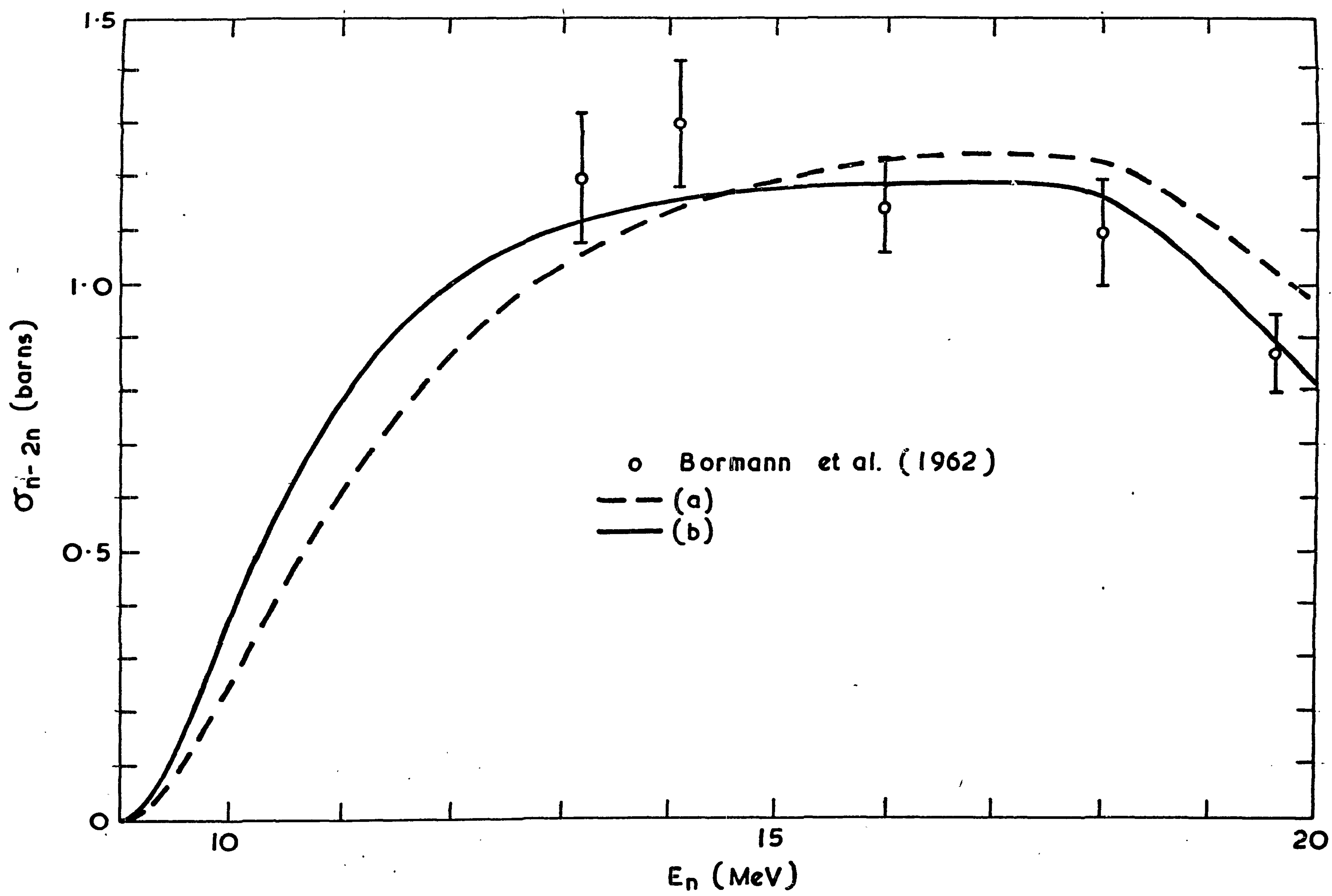

FIGURE 7. ^{133}Cs

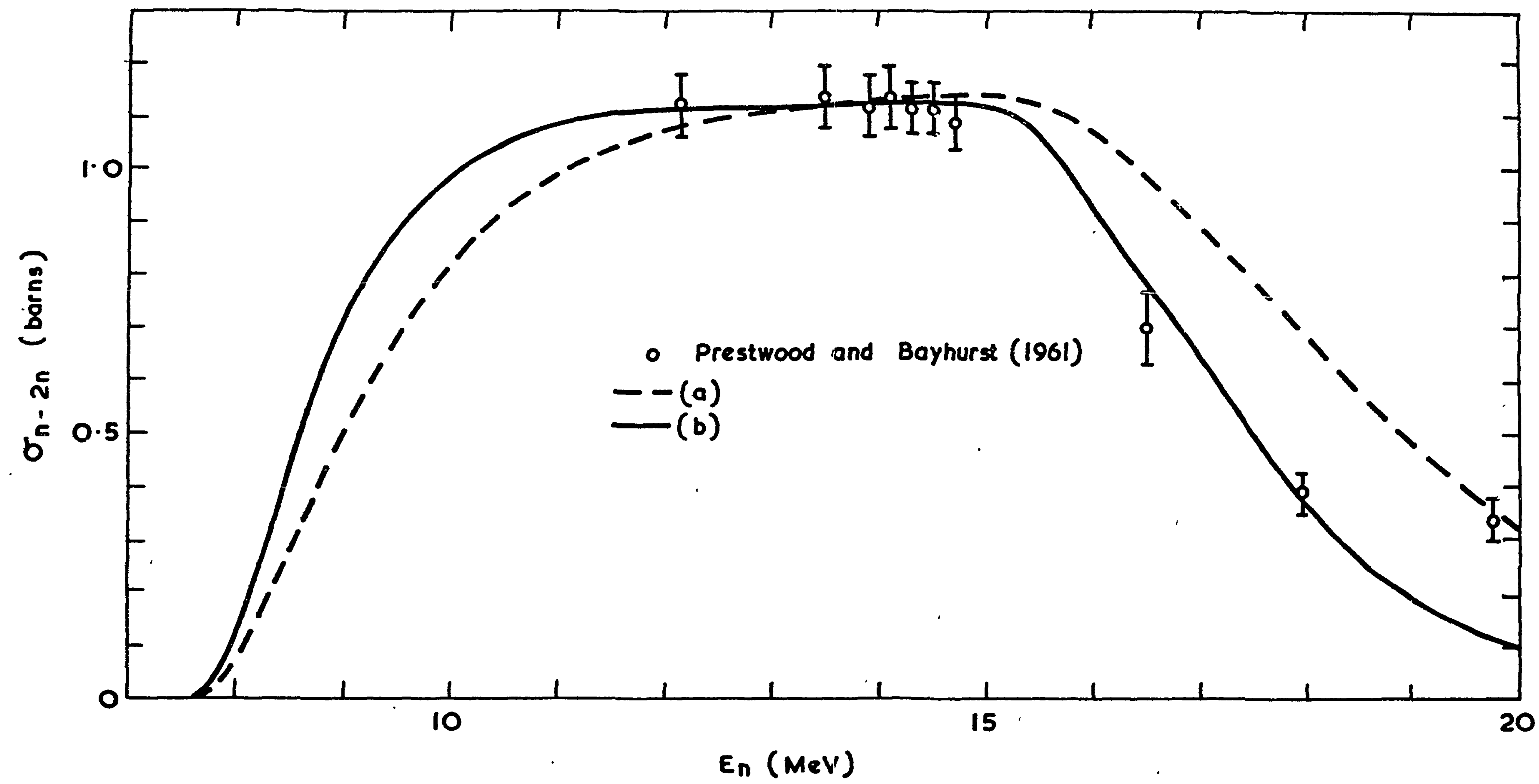

FIGURE 8. ^{181}Ta

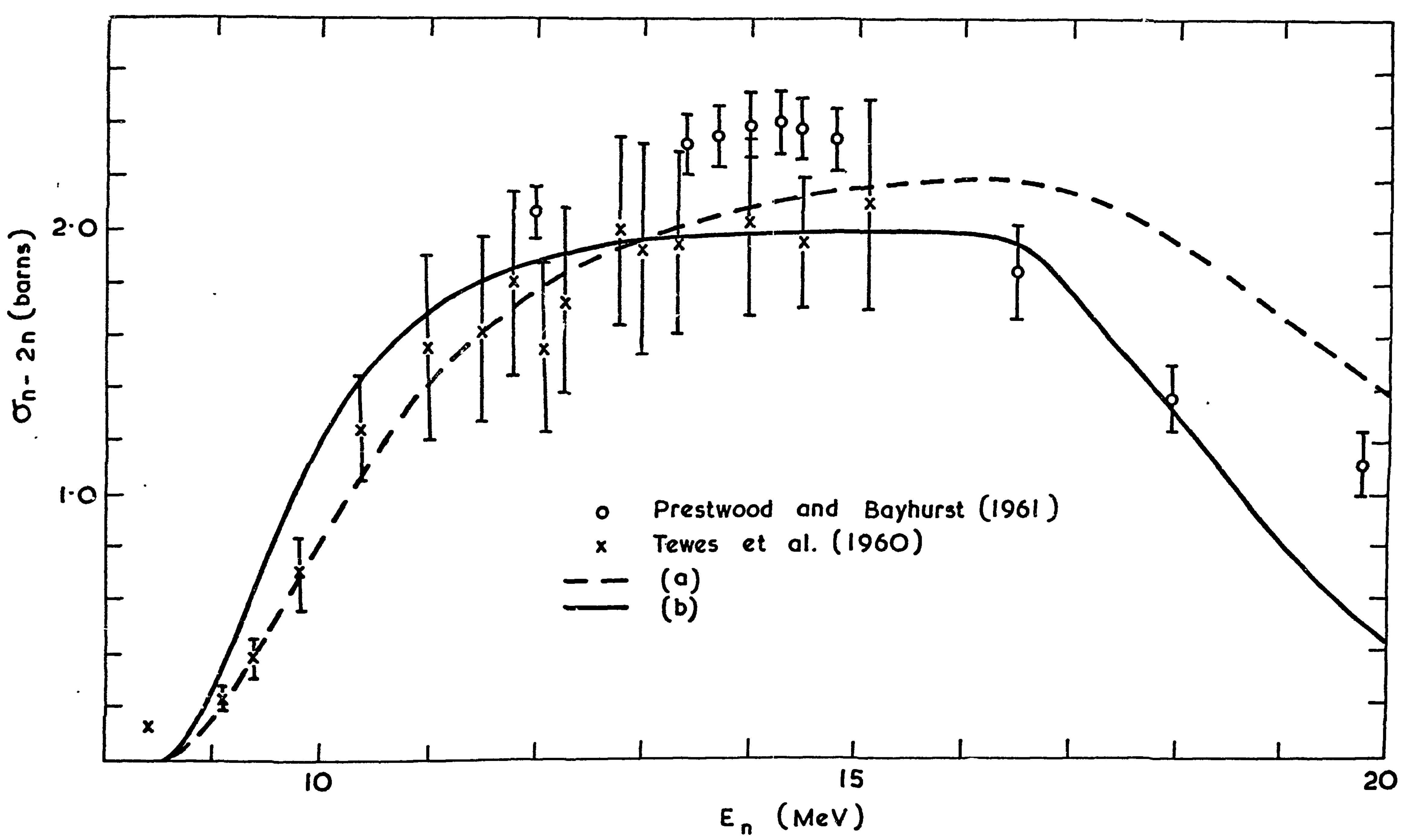

FIGURE 9. ^{197}Au

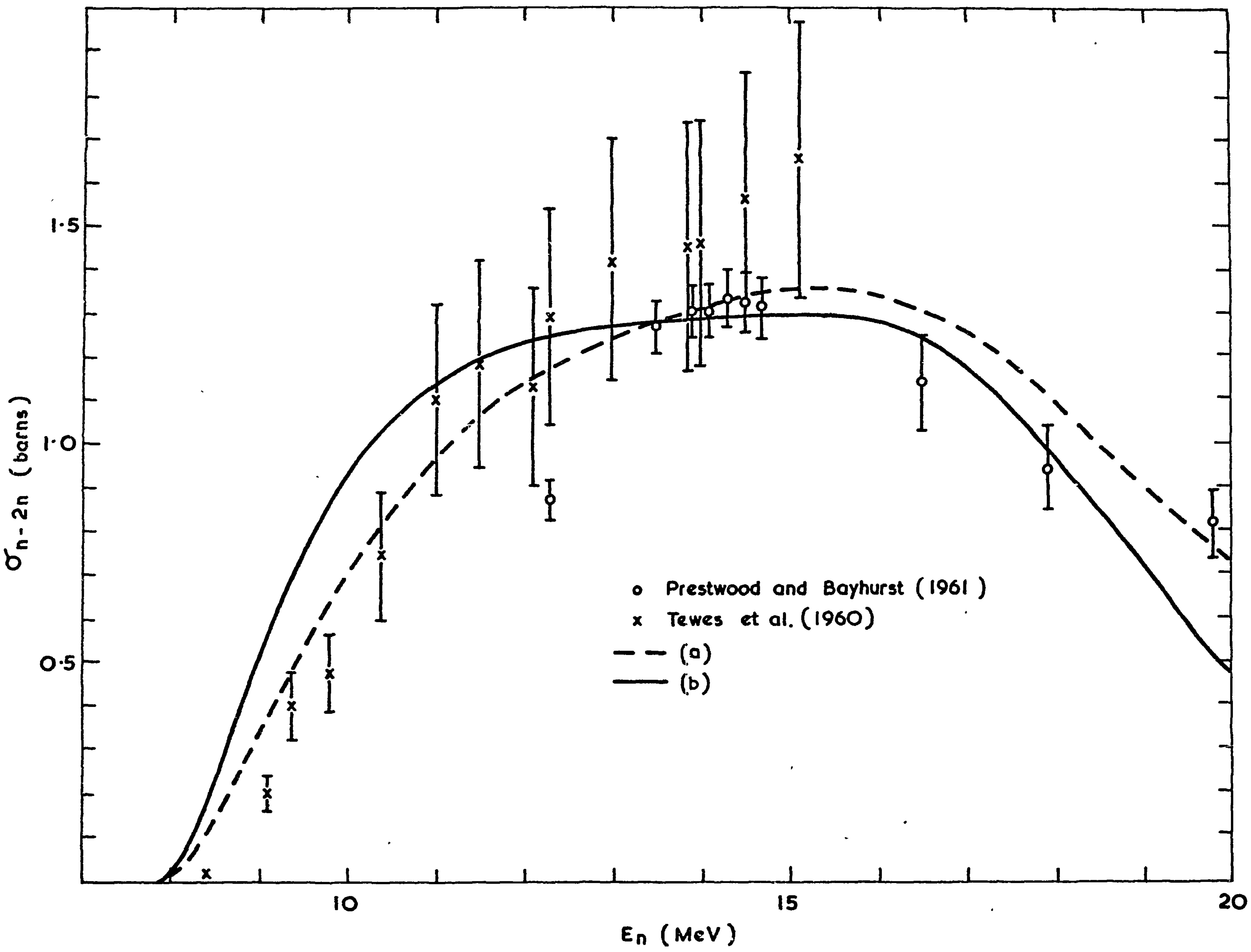

FIGURE 10. ^{203}Tl

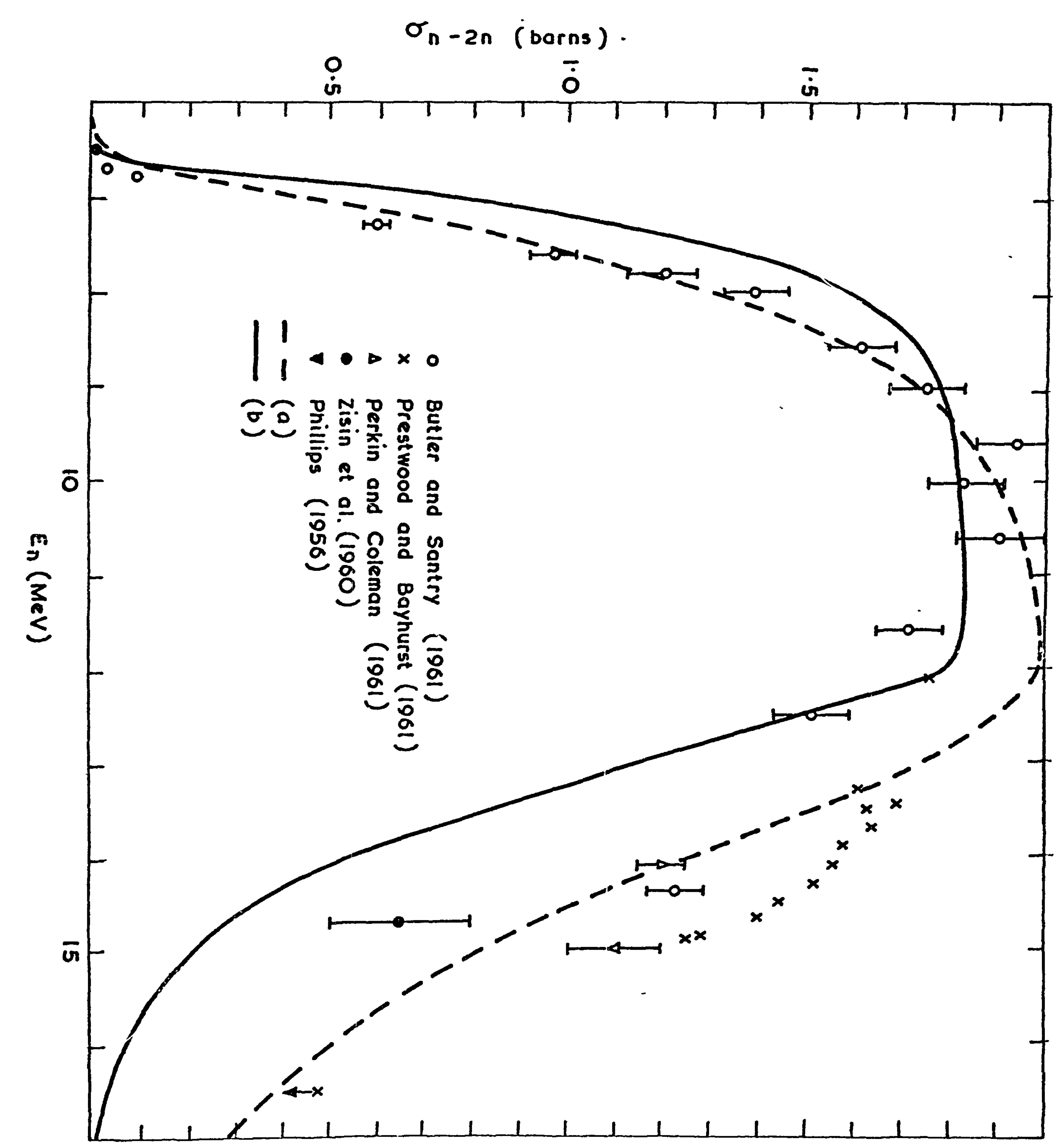

FIGURE 11. ^{232}Th

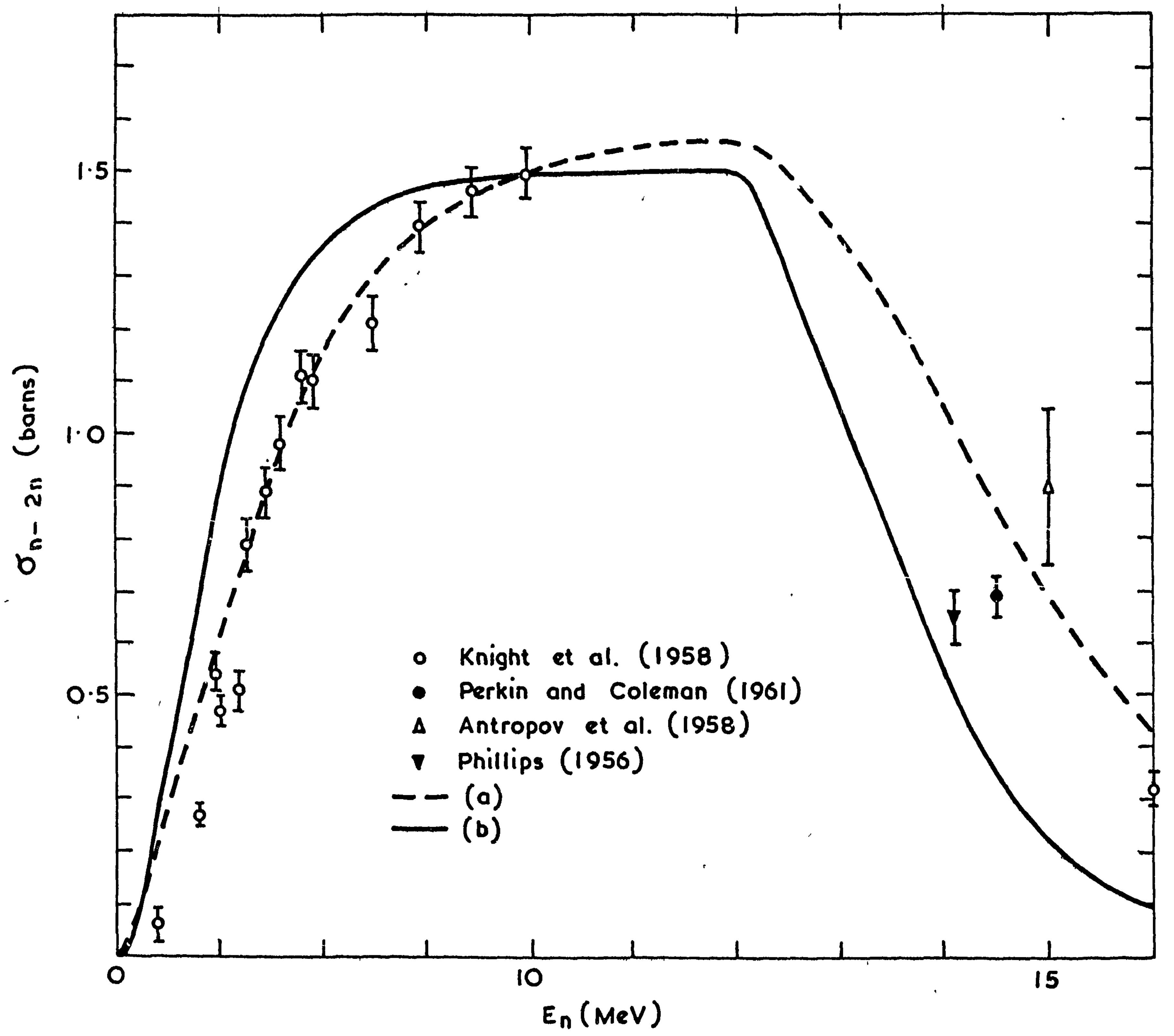

FIGURE 12. ^{238}U