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# A NOTE ON THE RESPONSE OF A CIRCULATING FUEL REACTOR TO RANDOM FLUCTUATION IN FUEL CONCENTRATION

by

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Summary

A simplified model of a Uranium – Sodium – Beryllium circulating fuel reactor has been analyzed to determine the root mean square power and temperature fluctuations due to random variations in the inlet fuel concentration.

The results indicate that limits of ± 2% on the fuel concentration should reduce the mean square power excursion, due to this cause, to less than ½%.

The calculations are based on assumptions only, as regards the specification of the statistical nature of the fuel concentration, but the method can be used to obtain more realistic estimates when experimental evidence becomes available.

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# NOTATION

| | |
|---|---|
| Po | Power level |
| $p$ | Power excursion |
| $\tau$ | Mean neutron life time |
| c | Fuel density coefficient of reactivity |
| $\rho$ | Density variation from mean (fractional) |
| $\mathcal{J}$ | Temperature excursion |
| $\alpha$ | Temperature coefficient of reactivity |
| S | (mean fluid density X mean specific heat)$^{-1}$ |
| k | Power transfer coefficient, due to heat transfer to moderator |
| t | Time |
| $z$ | Distance along channel from inlet |
| $v$ | Fluid velocity |
| L | Channel length |
| Y($q$) | Transfer Function – ratio of Laplace Transforms |
| $\omega$ | Angular frequency |
| $\chi(\omega)$ | Spectral Density |
| R($\tau$) | Correlation Function |
| $n$ | Number of channels |
| $T = 1/\lambda$ | Mean duration of constant density |
| $\bar{\ell} = vT$ | Mean constant density length |
| $\overline{p^2}$ | Mean square power excursion |
| $\overline{\rho_{in}^2}$ | Mean square inlet density variation |

| | | |
|---|---|---|
| Suffix | m | Denotes mean over a channel |
| " | c | Denotes channel |
| " | i or in | Denotes inlet to channel |
| " | O or out | Denotes outlet to channel |

## INTRODUCTION

It is to be expected that the concentration of the fuel suspension entering the moderator channels in a Uranium — Sodium circulating fuel reactor will fluctuate in a random fashion about a mean value, because of the various physical processes possible in the external circuit. Until experimental evidence is available, there is no possibility of useful calculations, but the necessary techniques can be established and some idea of the size of the effect obtained by the analysis of a simple model. Together with the results of calculations on the effect of sudden density changes for the same model, they may help in the choice of permissible limits.

## THEORY

Consider first a reactor with constant power density along all channels, all physical properties constant, constant moderator and inlet temperatures and no delayed neutrons. Assuming that the weighting functions for reactivity changes due to fuel concentration and fuel carrier temperature changes are independent of position, and that the power and temperature excursions from the mean steady state are small, the governing equations are

$$\frac{dp}{dt} = \frac{Po}{\tau}\left(c\,\rho m - \alpha\,\mathcal{J}_m\right) \qquad (1)$$

$$S(p - k\,\mathcal{J}) = \frac{\partial \mathcal{J}}{\partial t} + v\,\frac{\partial \mathcal{J}}{\partial z} \qquad (2)$$

The Transfer Functions relating power and temperature excursions to the mean density change are therefore

$$Y(q,p) = \frac{cPo}{\tau}\,/D \qquad (3)$$

$$Y(q,\mathcal{J}m) = \frac{cSPo}{\tau}(q + kS)^{-1}\left(1 + \frac{v}{L}(q + kS)^{-1}\left[\exp\left(-(q + kS)\,L/v\right) - 1\right]\right)/D \qquad (4)$$

$$Y(q,\mathcal{J}out) = \frac{cSPo}{\tau}(q + kS)^{-1}\left(1 - \exp\left(-(q + kS)\,L/v\right)\right)/D \qquad (5)$$

with

$$D = q + \frac{\alpha PoS}{\tau}(q + ks)^{-1}\left[1 + \frac{v}{L}(q + kS)^{-1}\left(\exp\left(-(q + kS)\,L/v\right) - 1\right)\right] \qquad (6)$$

If the density fluctuations at inlet and outlet to each channel are stationary random processes, then since

$$\left(\frac{d\rho m}{dt}\right)_c = \frac{v}{L}\left((\rho_{in})_c - (\rho_{out})_c\right) \qquad (7)$$

the power spectral density of a channel mean fuel density fluctuation can be written in terms of the inlet and outlet power spectral densities.

$$\chi_c(\omega) = \left(\frac{v}{L\omega}\right)^2\left\{\chi_{ii}(\omega) + \chi_{oo}(\omega) - \chi_{io}(\omega) - \chi_{oi}(\omega)\right\} \qquad (8)$$

where

$$\mathcal{X}_{io}(\omega) = \frac{2}{\pi} \int_0^\infty R_{io}(\tau) \cos(\omega\tau)\, d\tau$$

etc., the correlation functions being defined by

$$R_{io}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} \rho_{in}(t)\, \rho_{out}(t+\tau)\, dt$$

For a reactor with n channels, the power spectral density of the overall mean fuel density fluctuation is

$$\mathcal{X}(\omega) = \frac{1}{n^2} \sum_1^n \mathcal{X}_c(\omega) \tag{9}$$

As all channels are fed from the one source it is reasonable to assume that $\mathcal{X}_{ii}(\omega)$ etc., are the same for all channels. Thus

$$\mathcal{X}(\omega) = \frac{1}{n}\left(\frac{v}{L\omega}\right)^2 \left\{ \mathcal{X}_{ii}(\omega) + \mathcal{X}_{oo}(\omega) - \mathcal{X}_{io}(\omega) - \mathcal{X}_{oi}(\omega) \right\} \tag{10}$$

The mean square power and temperature excursions are now calculated from

$$\overline{p^2} = \int_0^\infty |Y(i\omega, p)|^2 \mathcal{X}(\omega)\, d\omega \tag{11}$$

etc., when, from (3) (4) and (5)

$$|Y(i\omega, p)|^2 = \left(\frac{cP_o}{\tau}\right)^2 (k^2 S^2 + \omega^2)^2 / Z \tag{12}$$

$$|Y i\omega, \mathcal{T}_m)|^2 = \left(\frac{cSP_o}{\tau}\right)^2 \left\{ \left(kS - \frac{v}{L}(1 - \exp(-kSL/v)\cos(\frac{\omega L}{v}))\right)^2 + \left(\omega - \frac{v}{L}\exp(-kSL/v)\sin(\frac{\omega L}{v})\right)^2 \right\} / Z \tag{13}$$

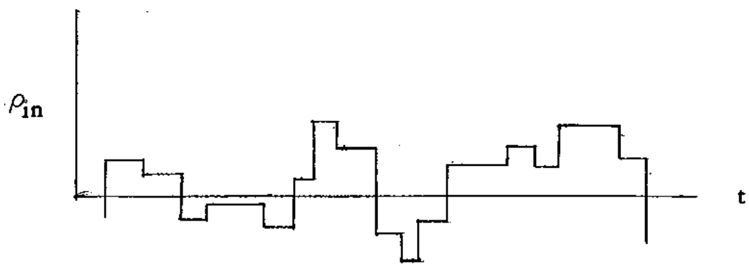
$$|Y(i\omega, \mathcal{T}_{out})|^2 = \left(\frac{ScP_o}{\tau}\right)^2 (k^2 S^2 + \omega^2) \left\{ (1 - \exp(-kSL/v)\cos(\frac{\omega L}{v}))^2 + (\exp(-kSL/v)\sin(\frac{\omega L}{v}))^2 \right\} / Z \tag{14}$$

and

$$Z = \left[\frac{\alpha P_o S}{\tau}\left(kS - \frac{v}{L}(1 - \exp(-kSL/v)\cos(\frac{\omega L}{v}))\right) - 2\omega^2 kS\right]^2 + \left[\frac{\alpha P_o S}{\tau}\left(\omega - \frac{v}{L}\exp(-kSL/v)\sin(\frac{\omega L}{v})\right) + \omega(k^2 S^2 - \omega^2)\right]^2 \tag{15}$$

## APPLICATION

It is necessary to assume some form for the spectral densities to perform the integrations indicated by equations (II) etc., in the absence of experimental evidence. For this purpose it will be assumed that an ever changing density pattern moves continuously along a channel with velocity $v$. This pattern consists of random lengths of constant independent random fuel concentrations. It is therefore generated by random inlet fuel density changes occurring at random intervals as shown below.




Thus

$$\rho_{out}(t) = \rho_{in}\left(t - \frac{L}{v}\right)$$

and from (8) and the definition of the correlation functions,

$$\chi_c(\omega) = \frac{2}{n}\left(\frac{v}{L\omega}\right)^2 \left(1 - \cos\left(\frac{\omega L}{v}\right)\right)\chi_{ii} \qquad (16)$$

If it is further assumed that the density changes are independent, and that the probability of a change in time dt is proportioned to dt, ( = $\lambda$dt), then the changes have a Poisson distribution. The constant $\lambda$ is equal to 1/T where T is the mean time duration of a constant inlet density length. This distribution is chosen for simplicity, and because it applies to traffic and telephone calls, which have some similarity to the problem in hand.

Thus

$$\chi_{ii} = \frac{2T}{\pi(1 + \omega^2 T^2)} \quad \overline{\rho_{in}^{\,2}} \qquad (17)$$

and the integrals become

$$\overline{p^2}/\overline{\rho_{in}^{\,2}} = \frac{4T}{n\pi}\left(\frac{v}{L}\right)^2 \int_0^{\infty} |Y(i\,\omega, p)|^2 \; \frac{1 - \cos\left(\frac{\omega L}{v}\right)}{\omega^2(1 + \omega^2 T^2)} \; d\omega \qquad (18)$$

etc.

These have been evaluated for a 1:100:2000 Uranium – Sodium – Beryllium system using the following values, leaving n unspecified.

$P_o$ = 1200 cal/cc/sec.

$c$ = 0.2

$\tau$ = $2.5 \times 10^{-4}$ sec.

$\alpha$ = $53 \times 10^{-6}$

$k$ = .5505 cal/cc/sec/$^o$C

$S$ = 3.3 (cal/cc/$^o$C)$^{-1}$

$L$ = 80 cm

$v$ = 800 cm/sec.

The results are shown in Figures 1 and 2 for a 1% root mean square inlet density fluctuation, but are applicable only if $p/P_o$ is small.

For the system chosen, the mean temperature is 500$^o$C and a fuel channel radius of 3 cms makes $n^{1/2}$ about 7. Thus from Figure 1, a 1% root mean square density fluctuation will never cause the root mean square power excursion nor the root mean square mean temperature excursion to exceed about 3% and 1% of their mean operating values respectively. These figures can be considerably reduced if the mean constant density length is made small, and 1 cm length gives values of 0.6% and 0.2%. The outlet temperature refers only to the exit from the channels and is not immediately significant from the point of view of the external heat exchangers.

An upper limit of ± 2% on the peak density fluctuations would almost certainly imply an r.m.s. value less than 1%, as the r.m.s. value for a rectangular probability density function between ± 2% is 1.16%. Thus the conclusion can be drawn that, for the model chosen, limits of ± 2% on the density fluctuations would keep the r.m.s. power and mean temperature excursions below ½% and 1/6% of their mean values, if the mean constant density length is not greater than 1 cm. It is believed that these values are pessimistic, since the appropriate value of $n^{1/2}$, for 3 cm. radius channels is almost certainly greater than 7.

Although the probability distributions of the magnitudes have not been specified, an assumed normal distribution for the power fluctuations, with a r.m.s. value of ½%, indicates that the power excursion would exceed 2% for only 1/10000 of the operating time.

## CONCLUSION

The results apply only to a particular system under one set of operating conditions, but the forms of the functions involved will enable the trends to be assessed. The analysis puts on a quantitative basis the intuitive idea that the effect of density fluctuations can be minimised by increasing their frequency and the number of fuel channels.

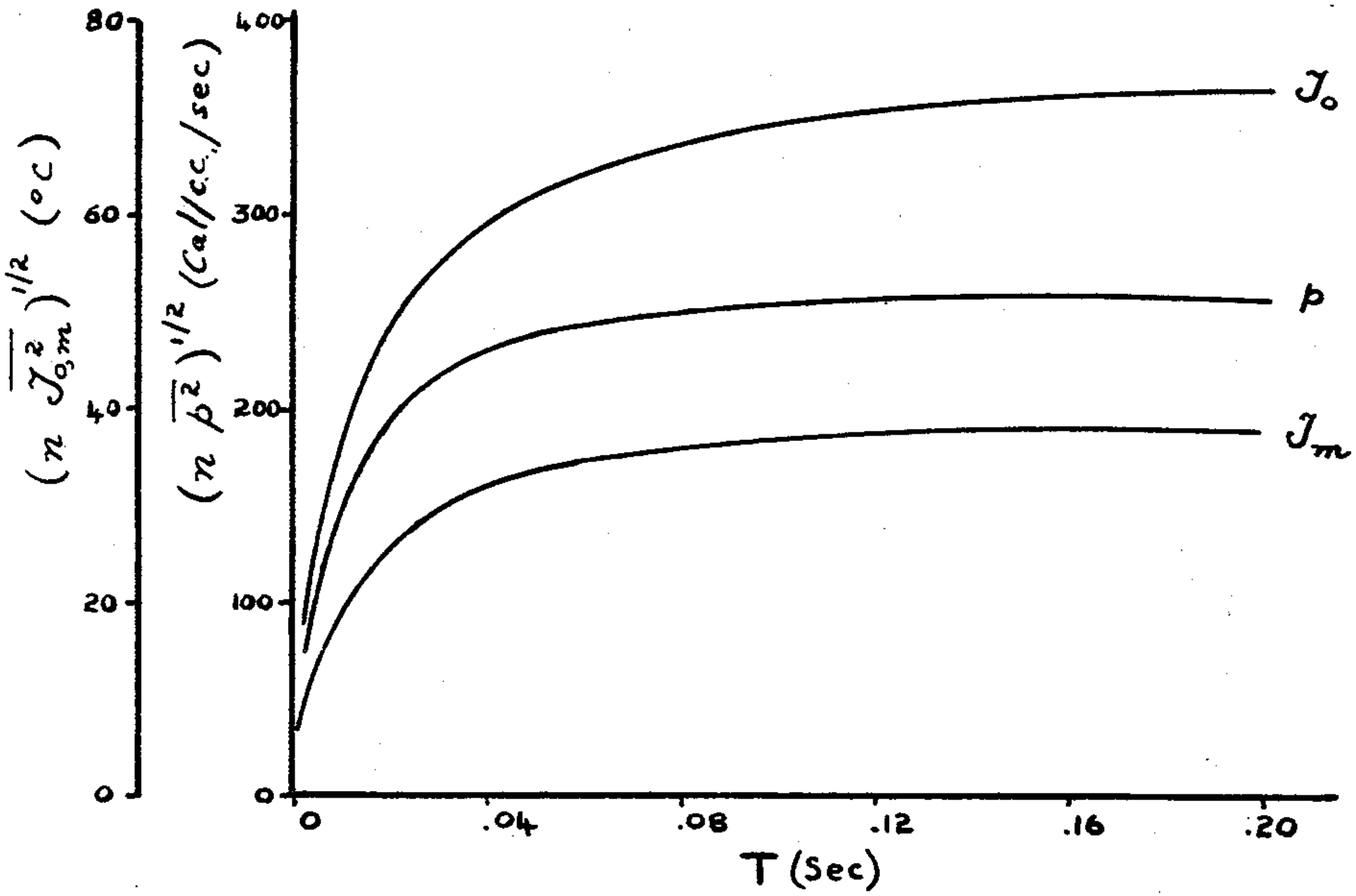



FIG.1 Response to 1% Fluctuation.

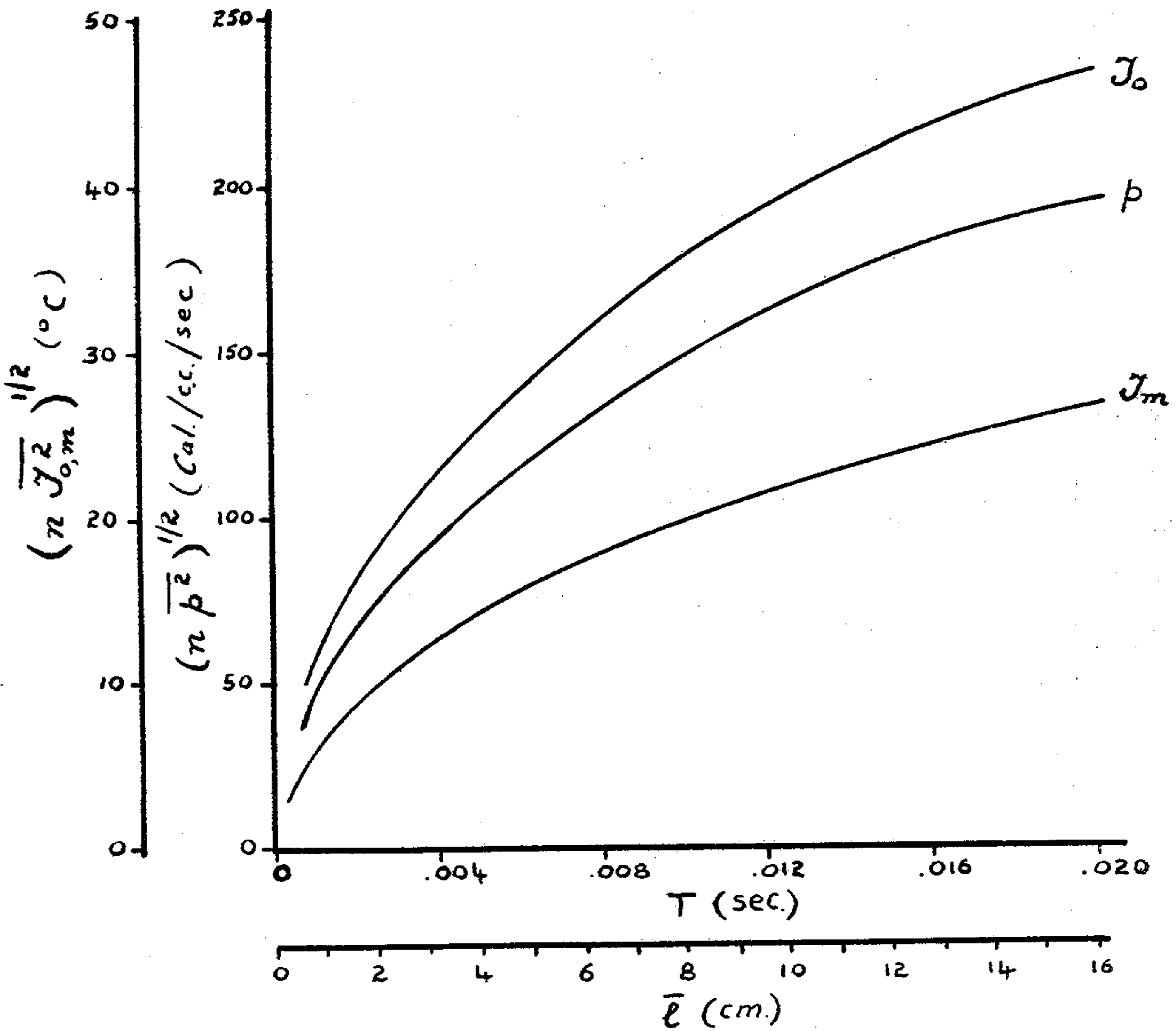



FIG. 2 Response to 1% Fluctuation