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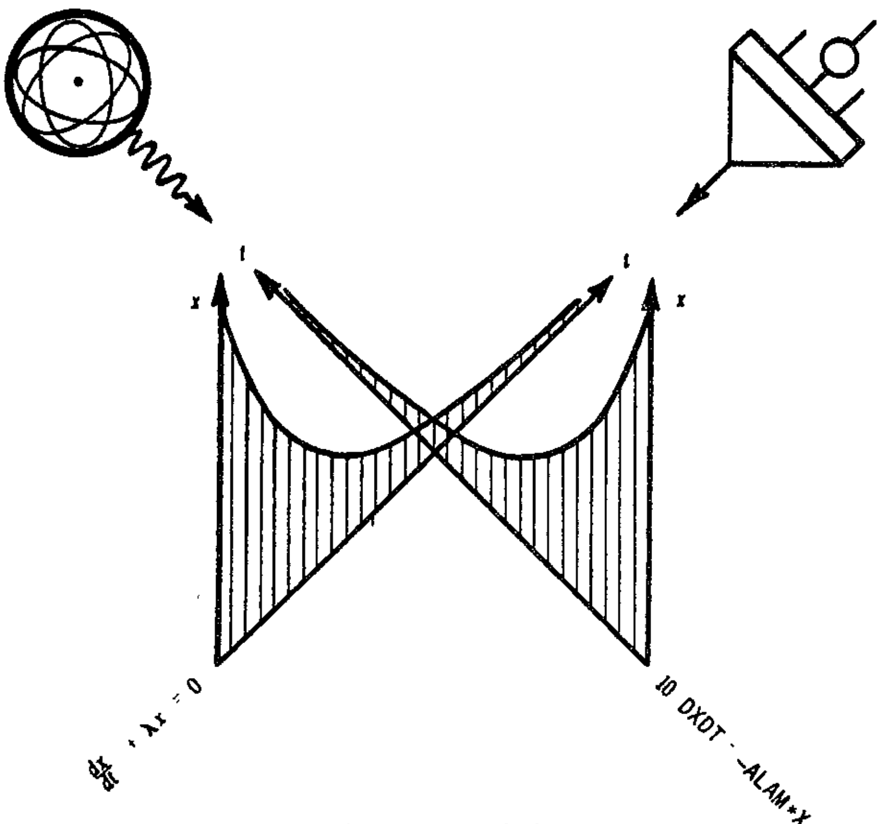
AUSTRALIAN ATOMIC ENERGY COMMISSION
RESEARCH ESTABLISHMENT
LUCAS HEIGHTS

COMPUTER ANALYSIS OF RESULTS AS THEY ARE OBTAINED FROM A NUCLEAR EXPERIMENT

REACTOR PHYSICS, MATHEMATICS AND COMPUTERS

SUMMER SCHOOL FOR TEACHERS

Lecture by R. WALSH



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ABSTRACT

The equivalence of mass and energy and the conservation of momentum can be demonstrated by measuring the energy of gamma rays emitted when positrons are annihilated. This measurement can be made using a small computer for data storage and analysis and for equipment control.

CONTENTS

1. THE NUCLEAR EXPERIMENT

1.1 Annihilation Radiation

Many radioactive isotopes emit positrons (positive electrons) which can be observed directly with detectors that measure the amount of ionisation they produce. Positrons can also be detected as the result of a second process which usually takes place after they have lost their initial energy. They eventually come sufficiently close to an ordinary electron so that the two attract and annihilate each other. In their place are observed two gamma rays which are emitted in opposite directions. Each annihilation gamma ray has an energy (E) which is given by the well known equation expressing the equivalence of mass and energy:

$$E = m c^2 \qquad \ldots(1)$$

where m is mass of positron or electron and c is the velocity of light. (Note: The production of two gamma rays with equal energy travelling in opposite directions, satisfies the principle of conservation of momentum).

Annihilation radiation is also observed when high energy gamma rays (energies greater than twice the equivalent electron mass) interact with matter. The first step in such an interaction is pair production in which a positron-electron pair is produced and the original gamma ray disappears. After energy loss by ionisation the positron interacts with another electron to produce two annihilation gamma rays.

1.2 Measurements

A gamma ray detector is used which can measure accurately the energies of gamma rays from a radioactive source. Several gamma rays with known energy are used to calibrate the detector and measuring equipment so that the energy of annihilation radiation can be determined. This energy and values for the electron mass and velocity of light are used to confirm equation (1).

Using previously measured values of the strength (disintegrations per second) of the calibration sources, the count rate from the detector can be used to establish the detector efficiency and confirm that two gamma rays are emitted in each annihilation event.

If two detectors are used the number of coincidences can be measured. A coincidence is a count in each detector at the same time. With a source of annihilation radiation, coincidences are observed only when the two detectors are opposite each other thus verifying that momentum is conserved in the annihilation process.

Question: What is the momentum of a gamma ray?

1.3 Equipment

The equipment needed for these measurments is shown schematically in Figure 1.

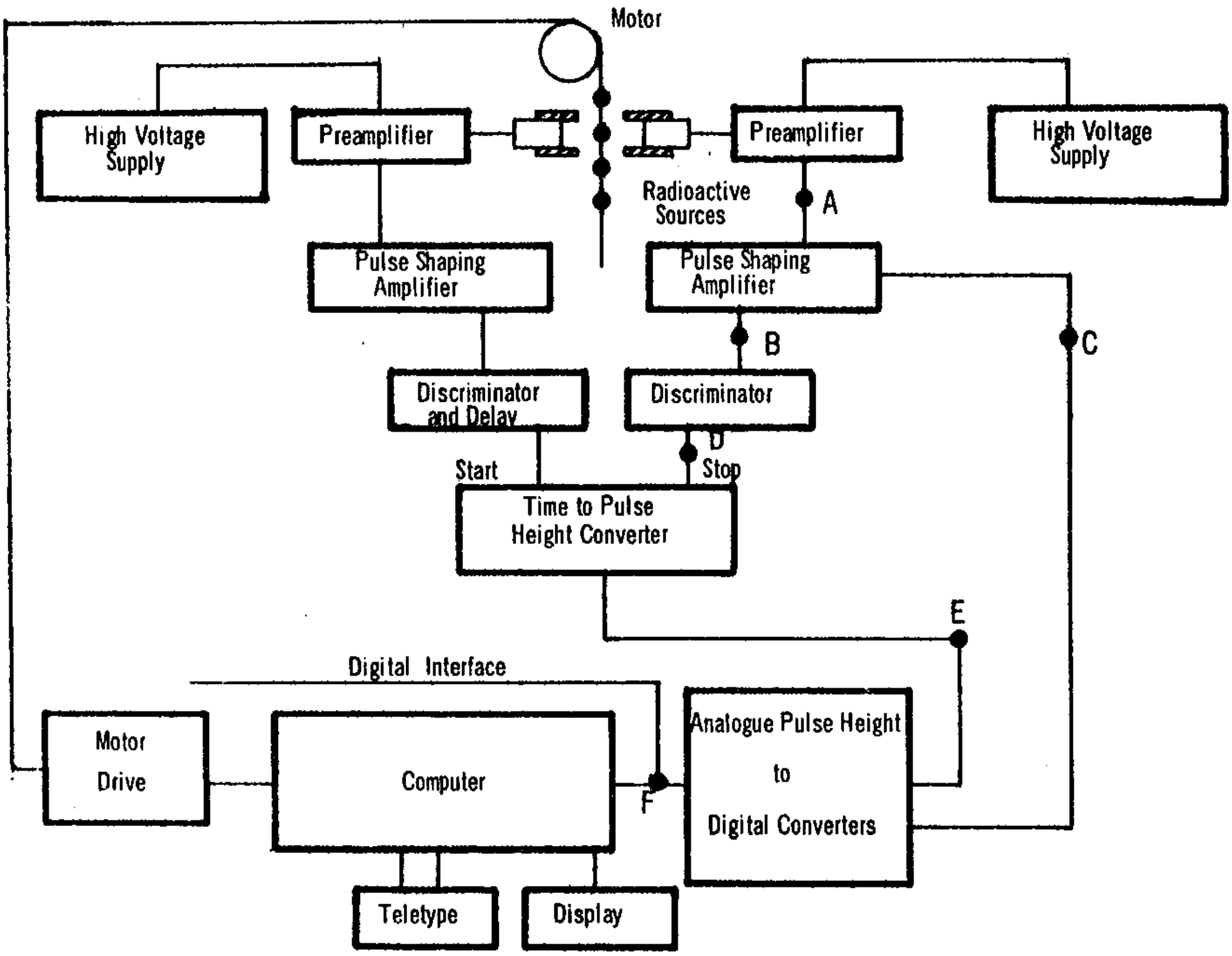

FIGURE 1 MEASURING EQUIPMENT

Gamma ray detectors respond to the ionisation caused by electrons which are the result of gamma ray interactions in the detector material. Two types of detector are commonly used:

(a) A crystal of sodium iodide (NaI) converts the ionisation energy into a flash of light (scintillation) and this produces an electrical pulse in a light sensitive device (photomultiplier). The size of the electrical pulse is proportional to the amount of ionisation energy and hence to the energy of the original gamma ray.

(b) A specially treated crystal of germanium is mounted in a liquid nitrogen cooled cryostat. Electrodes on each side of the crystal are connected to the high voltage supply (1000V), and collect directly the ions formed by interaction of a gamma ray within the crystal. The positive and negative ions are attracted to opposite electrodes and produce an electrical pulse. Lead shielding around the detectors reduces the number of background gamma rays.

The following radioactive sources are used:

TABLE I

LIST OF RADIOACTIVE SOURCES

SOURCE	HALF LIFE	GAMMA RAY ENERGY	SOURCE STRENGTH
^{133}Ba	7.2 years	0.360 MeV	
^{137}Cs	30 years	0.662 MeV	
^{54}Mn	314 days	0.835 MeV	
^{22}Na	2.6 years	Annihilation Radiation and 1.275 MeV	

A computer controlled automatic source positioner can cycle the sources so that each is in front of the detectors for a fixed time interval. The positioner responds to a series of fixed size (logic) pulses and moves a distance determined by the number of such pulses which arrive from the computer.

1.4 Pulse Height Measurments

The detector pulses, which are proportional in height (voltage) to the energy released in the detector, are called analogue pulses. They are amplified and shaped (Pulse shaping amplifier) before being passed to an analogue-to-digital converter (ADC). This converter measures the height of each pulse and presents the results to the computer as a binary number. The binary number (n) specifies that the pulse height was between (n-1) and n times a small voltage increment (ΔV). ΔV is usually called a 'channel' and can be chosen to suit a particular experiment.

When the computer receives such a binary number from the ADC, it uses that number as the address to find a particular storage location in its memory and it adds one to the number stored at that location. At the end of an experiment the set of locations corresponding to the channel numbers contains a record of the number of times that each value of pulse height has been observed in the experiment.

1.5 Time Measurements

When two detectors are used, the discriminators shown in Figure 1 give output pulses when gamma rays interact in the detectors. When a pair of such pulses occur close together, the time to pulse height converter measures the time interval between them, again converted to a binary number. The small time delay included in the connections for one detector allows coincidences to be observed as events with that value of time difference. The output of the time converter can be used by the computer to prepare a list of the number of times that coincidences or other time intervals were observed.

2. THE COMPUTER

2.1 Description

A small computer is used (PDP7 or PDP15) and its characteristics are compared in Table II with those of an IBM 360/50.

TABLE II

COMPARISON OF COMPUTER CHARACTERISTICS

	IBM 360 50	PDP 15	PDP 7
Word Size	32 bits	18 bits	18 bits
Fast Memory Size	1152K	8 K	8 K
One-line direct access storage size	5×10^7	nil	nil
Cycle Time	2 μS	0.8 μS	1.75 μS
Add/Subtract Time	4 μS	1.6 μS	3.5 μS
Mult./Divide Time	28 μS	100 μS	10 μS
No. of Registers	16	3	2

Although mathematical operations may be carried out in the PDP systems, the main function of the system we will be using is to:

(a) Accept the results of experimental measurements and control input and output equipment,

(b) Provide the results of calculations which will help the experimenter to understand the progress of his measurements,

(c) Allow interactive processing using a display unit to communicate with experimenter.

Auxiliary equipment used for this work includes:

Input Units:

Teletype - typed characters are read by the computer which then carries out the operation which has been associated with those characters in a list stored in the computer memory;

Tape Reader - for program or data input;

Light pen - the computer carries out a listed operation when a bright dot on the oscilloscope screen appears in front of the light pen;

ADC - links experimental measuring equipment to the computer;

Scalers - pulses are counted and when required the results can be read by the computer;

Status Register - used to record the position or other features of experimental equipment.

Output Units:

Teletype - prints output information, including data and programs;

Display - an oscilloscope used to give a continuously changing graph of data;

Tape Punch - paper tape record of data or program;

Status Register - sends out pulses which can be used to control electric motors and other devices.

2.2 Mode of Operation

There are two numbers associated with each memory location in the computer. Firstly the address which distinguishes that location from all others and secondly the contents of that address. For the PDP7 and 15, numbers stored at each address are 18 bit binary words and these can be interpreted either as data or as program instructions. The difference between these uses is established by the person writing the program who must ensure that the computer follows the required sequence of instruction words, using data words only when needed for the execution of instructions. There is another distinction which is also important - between the address of a location and an 'operand address' which can be included with the program instruction as the contents of an address (see Section 4).

When using the computer for a nuclear experiment, a large part of the memory is set aside for storing the results of measurements such as pulse height, time or number of events. For example, 2048 locations (out of a total of 8192) can accomodate a control program and the remaining locations are then available for experimental results. The control program is designed to operate as a continuous loop which causes the computer to periodically refresh a display and to respond according to pre-determined sub-routines when a teletype command or other form of 'interrupt' occurs (e.g. arrival of a signal from the ADC).

2.3 Control Program for Pulse Height Measurement

This program receives binary words from the ADC and maintains a record in memory of the number of times each work occurs during an experiment. This record is the number of gamma rays observed with each possible value of pulse height. A graph of number of gamma rays versus pulse height is continuously maintained on the oscilloscope. The program responds to typed commands and in this way an experiment can be started or stopped, the oscilloscope display can be changed and results can be printed or punched at any time. Typical commands are:

X1 - Start storing data in 'the first memory region'

F11 - Full scale deflection in the display to be 2^{11} counts

W1,321,350 - Print the number of counts in channels 321 to 350 in the first region.

2.4 Control Program for Data Analysis

Some data analysis can be carried out using the Pulse Height Measurement Program, but more elaborate analysis uses a special program. Simple analysis consists of adding or subtracting numbers and this is done in response to teletype commands. More elaborate analysis uses special least squares fitting methods to calculate the constants in a mathematical expression which will make that expression the best fit to the observed data points. For example, the best straight line can be found to fit the values of channel number which correspond to particular gamma ray energies. This line can then be used to calculate the energy of unknown gamma rays. This is all done in response to teletype commands.

In most of this analysis both the original data and calculated results can be displayed on the oscilloscope so that the experimenter can see whether mistakes have been made or whether results are good enough for his purposes.

He can also use the light pen to point to places on the oscilloscope screen where, for example, he thinks the calculated curve should go, or to point to special dots which are equivalent to typed commands.

3. INSTRUCTIONS

3.1 Detector and Electronics

Using an oscilloscope, observe the pulses at the points A to F shown in Figure 1. Sketch these pulses and mark voltage and time scales and label each sketch according to whether they are LOGIC or ANALOG pulses.

Each 'box' in Figure 1 represents an electrical circuit designed to perform a specific function. The lines joining the boxes represent coaxial cables which join the individual circuits into one or more large circuits around which the electrical pulses flow. Make your own notes on the purpose of each unit and the operation of front panel controls, but only to the extent necessary to understand the way in which very small detector pulses are processed so that a binary measurement of pulse height is presented to the computer.

3.2 Data Collection

A list of control commands is available for each type of computer control program. Using the appropriate list, make changes to the display until you are familiar with the method of operation.

With one radioactive source positioned in front of the detector, store pulse height measurements for a known time. The spectrum obtained should be similar to Figure 2. Using markers, locate the positions of any peaks so that these can be found later. Repeat this process for each radioactive source, using a new region of computer memory for each spectrum.

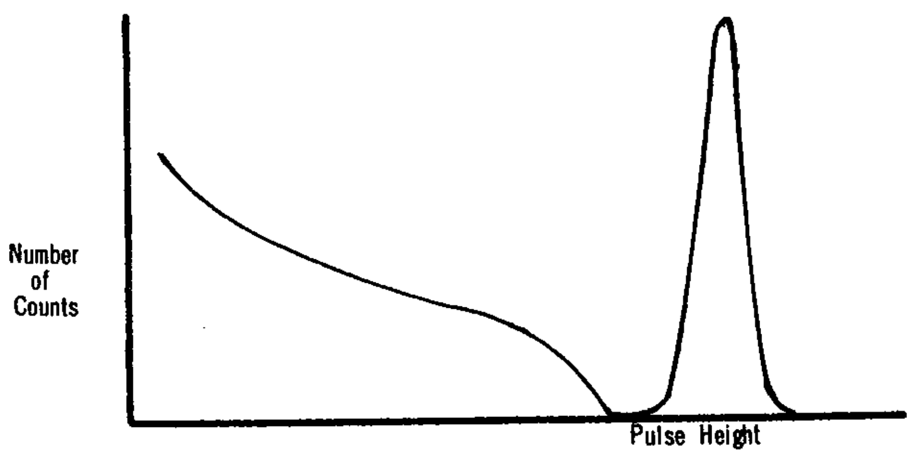

FIGURE 2 TYPICAL PULSE HEIGHT SPECTRUM

The measurements can be sequenced automatically by entering suitable instructions to the control program, or alternatively the sequence (including time measurements) can be controlled by the experimenter. After a set of measurements is complete, prepare a sum spectrum by adding each measured spectrum into one unused region of memory. Punch on paper tape this final sum spectrum.

3.3 Data Analysis

The aim of the data analysis is to determine the position and area of each peak. If a peak is superimposed on a smooth curve it is necessary to make an estimate of the location of the curve under the peak and to subtract it before further analysis. This can be done by using the light pen to 'draw' where the curve should go or by fitting a straight line or a polynomial to points on either side of the peak. Using the list of control commands apply one of these methods to obtain the required parameters for each peak.

Construct a table of channel numbers for the centre of each peak and the appropriate gamma ray energy. Type this table for the computer and fit a straight line to the data. If the differences between observed centres and fitted values are small, type the centre position for the peak from annihilation radiation to obtain a calculated value of its energy. If the differences are not small it may be necessary to fit a polynomial to the data.

3.4 Calculations

Compare the energy of annihilation radiation derived in the previous section with the value calculated from equation (1).

electron mass (m)	=	9.108×10^{-31} kg
velocity of light (c)	=	2.998×10^{8} m/s
1 eV	=	1.602×10^{-19} joules

Calculate the ratio of measured peak areas (number of counts in the fixed time interval) to the strength of each source (disintegrations in the same time interval). Make a graph of this ratio against the energy of the gamma ray involved. Except for annihilation radiation the results should lie on a smooth curve which defines the efficiency of the detector used in these measurements. From the smooth curve read an expected efficiency at the energy of annihilation radiation and then calculate the number of gamma rays per disintegration for the annihilation radiation.

3.5 Errors

Two kinds of errors can be observed in this type of experiment. Statistical errors are chance fluctuations in the number of radioactive disintegrations occurring during the period of measurement. If the measurements for one source and method of analysis are repeated many times the results obtained will have a standard deviation (σ) determined by the square root of the number of gamma rays observed.

$$\sigma = \frac{1}{i} \sum_{1}^{i} (N_i - \bar{N})^2 \qquad \ldots(2)$$

(Note: When the results of two measurements are subtacted, their standard deviations should be added as the sum of squares i.e. $\sigma^2 = \sigma_1^2 + \sigma_2^2$).

If one measurement is made and then analysed by a number of different methods the results will vary in a way which depends on the methods used and the skill and bias of the operator. These 'systematic' errors can often be larger than the statistical errors, but careful design and use of the analysis methods can help to avoid this situation. Both statistical and systematic errors can be observed by appropriate repetition of steps in Sections 3.3 and 3.4.

4. PROGRAMMING LANGUAGES

4.1 Machine Language

During the running of a programme, the computer's 'central processor' reads one word at a time from memory and interprets that word as an instruction (which normally includes an operand address). For example, the following is a typical instruction which means "load the word from operand address 04521_8 into the accumulator".

0 1 0	0 ¦ 0 0	1 0 0	1 0 1	0 1 0	0 0 1	(Binary)
2	0	4	5	2	1	(Octal)

Instruction Code ¦ Operand Address

(Note: the computer uses binary numbers, but the equivalent octal numbers are used for convenience in program listings.)

A program counter is used to indicate the address of the next instruction to be executed and this counter is normally advanced by one at each step. However, if an interrupt occurs the contents of the program counter and other temporary registers are stored and each input/output device is checked to find the origin of the interrupt. When this is found, an appropriate sub-routine is executed and the processor then retrieves the contents of the temporary registers and continues with its former task.

A set of binary instructions can be loaded into the computer manually, using the switches on the control panel. This is a convenient method for changing a small part of a program but is too tedious for extended programs. These can be loaded from a paper tape or from the teletype keyboard but the computer must then be already operating under a program which accepts information from these devices.

4.2 Symbolic Languages

It is much easier for the programmer to remember a set of mnemonic symbols than a list of octal numbers. A set of symbols is therefore defined using abbreviations of the descriptions of operations which they represent.
e.g. LAC X = Load accumulator with contents of address X;

ADD Y = Add the contents of address Y to contents of accumulator;

JMP Z = Change the address in the program counter to Z;

An 'Assembler' program is needed to interpret instructions written using these symbols. The assembler substitutes the corresponding machine language instruction for each symbol (including special symbols defined by the programmer to represent specific addresses) and reserves locations for constants and variables used by the programmer. PDP assembler programs are available on the PDP computers or the IBM360 and these produce a machine language version of a program, usually punched on paper tape ready for use at any time.

4.3 Fortran

Fortran II and IV compilers are available for use on the PDP7 and these convert each Fortran statement into a set of symbolic instructions. The latter must then be converted to machine language by the Symbolic Assembler program. A comparison of the simple operation of adding three numbers together when carried out in each of the three languages, is shown in Table III.

TABLE III

COMPARISON OF PROGRAMMING LANGUAGES
ADDITION OF THREE NUMBERS

FORTRAN LANGUAGE : ISUM = J + K + L

MACHINE LANGUAGE(OCTAL)			SYMBOLIC LANGUAGE		
Address	Contents		Address	Contents	
	Instruction Code	Operand Address	Address Symbol	Instruction Symbol	Operand Address
1000	20	2257	JKLSUM	LAC	J
1001	30	2260	JKLSUM +1	ADD	K
1002	30	2261	JKLSUM +2	ADD	L
1003	04	2256	JKLSUM +3	DAC	ISUM
EXAMPLE OF DATA STORAGE FOR THIS OPERATION					
2257	01	3742	J	13755	
2260	00	7355	K	7355	
2261	00	0077	L	77	
2256	02	3416	ISUM	23416	

Although Fortran greatly simplifies program writing, especially for mathematical operations, the resultant machine language program is usually much longer than one which is written and optimised in machine language. This is an important problem for a computer with a small memory, most of which is needed for storage of experimental data. For this reason Fortran is seldom used in the PDP computers described here.