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RESEARCH ESTABLISHMENT

LUCAS HEIGHTS

EXPERIMENTS IN EXTRUSION

PART 1. REVIEW OF THE THEORIES OF EXTRUSION

by

W. J. WRIGHT

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ABSTRACT

The theories of extrusion are examined; considerable limitations exist in the analysis of extrusion conditions using either the semi-empirical approach, which is based on the assumption of homogeneous straining, or the slip-line approach.

More information is required on the basic extrusion parameters such as yield stress and friction coefficient and on the variation of extrusion conditions with tooling geometry.

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Figure 1 Principles of Direct and Inverted Extrusion

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1. INTRODUCTION

This report is one of a series dealing with experiments on the extrusion of beryllium; the work was done at the Research Establishment of the Australian Atomic Energy Commission as part of a programme to investigate the suitability of beryllium as a material for application in high temperature gas–cooled reactor systems.

The major objectives were:

(i) To develop a satisfactory technique for the extrusion of beryllium metal which was required for related projects including irradiation testing (Hickman and Stevens 1963) and chemical studies (Draycott et al. 1961).

(ii) To determine the mechanical properties of beryllium metal and the variation of these properties with extrusion conditions.

(iii) To extend the technique to the tandem and co–extrusion of $(UTh)Be_{13}$–beryllium dispersions clad in beryllium, such as would be suitable for H.T.G.C.R. fuel elements and to determine the limitations of the extrusion technique in relation to this aim.

This report presents a survey of the theories of extrusion against which later work may be assessed.

2. THEORIES OF EXTRUSION OF METALS

2.1 Semi–empirical Relationships

The essential features of the extrusion process are shown in Figure 1. The billet is held in a container and a compressive load is applied by the ram to force the metal through the die. In the direct method the billet moves relative to the container and in the inverted method the die is forced through the billet with no movement at the billet–container interface. Direct extrusion is thus characterised by friction at the walls of the container and across the die face; in inverted extrusion only the latter is present.

If the load required to move the ram is plotted during each extrusion stroke, curves such as those given in Figure 2 are obtained, showing for direct extrusion a gradual fall during the stroke and for inverted extrusion a reasonably constant load. The reducing load required for direct extrusion is due to the progressively smaller frictional area on the container as the billet is pushed through. The sharp drop in load at D in Figure 2 is due to "coring" as the metal in the centre of the extrusion pulls away from the face of the ram, and the load is taken over a smaller billet area. There is no unique value for the load in direct extrusion; the load just before coring, (that is, at the point of minimum friction), and the mean load have both been used in the literature. The load in inverted extrusion is usually quoted as the mean between points C and D in Figure 2b.

By plotting the load or pressure on the ram as a function of the reduction in area of the billet, a simple logarithmic relationship of the form:

$$P = K \log R \tag{1.1}$$

is obtained, where P = unit pressure on the ram

R = ratio $\dfrac{\text{area of billet}}{\text{area of extrusion}}$

K = a material constant.

This relationship was confirmed analytically by Siebel and Fangemeier (1931) and by Fink (1874) assuming the strain field to be uniform and ignoring work losses due to friction and redundant straining, that is, strains that are reversed during passage through the die. Their analysis was based on the tensile straining of a billet of original length l_1 and area a_1, to a rod of length l_2 and area a_2. The work required to produce δl increase in length of the billet is:

$$\delta W = aY\,\delta l \,, \tag{1.2}$$

where a and l are the instantaneous dimensions of the billet and Y is the yield stress in simple tension. Thus, in straining the billet from length l_1 to l_2 the total work W is:

$$W = \int_{l_1}^{l_2} aY\,dl \,, \tag{1.3}$$

and for constancy of volume $V = al =$ constant.

Hence
$$W = \int_{l_1}^{l_2} YV\frac{dl}{l} \tag{1.4}$$

$$= YV \log_e \frac{l_2}{l_1} \tag{1.5}$$

The total work is also given by the product of force applied and distance travelled by the ram, that is,

$$W = Fl \tag{1.6}$$

$$= Pal$$

$$= PV \,, \tag{1.7}$$

and by combining (1.5) and (1.7):

Extrusion Pressure
$$= P = Y \log_e \left(\frac{l_2}{l_1}\right) \tag{1.8}$$

$$= Y \log_e \left(\frac{a_1}{a_2}\right) \tag{1.9}$$

$$= Y \log_e R \,. \tag{1.10}$$

Substitution of the yield stress Y into equation (1.10) gives values for the extrusion pressure which are below those observed experimentally even with due allowance for the effect of friction and strain hardening (Pearson and Parkins 1960). The constant K in Equation 1.1 may therefore be identified by the expression:

$$K = \beta Y \,,$$

where β is a dimensionless constant.

Equation 1.10 further predicts that at zero reduction no load is required and this is consistent with the model of simply pushing a billet through a parallel smooth container. If any small finite reduction is taken however, some volume of the material must be stressed to the yield point and a relatively large increase in pressure is noted. The analysis of Siebel and Fangemeier has no lower limit on validity but slip line solutions proposed by Hill suggest that below a reduction ratio of two, the pressure is proportional to the reduction, rather than its logarithm (Hill 1948). These solutions are discussed further in Section 1.2.

A further criticism of Equation 1.10 is that no allowance is made for the effects of friction across the die face present in both direct and inverted extrusion, and the fact that these frictional conditions are determined by the geometry of the tooling as well as the efficiency of lubrication.

Johnson (1956 a) proposed a modification of Equation 1.10 to overcome some of these criticisms; this is of the form:

$$\frac{P}{\bar{Y}} = a + b \log_e R \quad , \tag{1.11}$$

where a and b are constants and Y is the yield stress effective at the appropriate strain ($a + b \log_e R$) and strain rate. Johnson did not assign any physical meaning to the constants a and b although he later derived them analytically for a range of plane strain extrusion conditions, as follows (Johnson 1956 b, 1956 c):

(i) Smooth container and smooth die:

$$\frac{P}{\bar{Y}} = 0.63 + 0.95 \log_e R \tag{1.12}$$

(ii) Rough container and smooth die:

$$\frac{P}{\bar{Y}} = 1.19 + 0.83 \log_e R \tag{1.13}$$

(iii) Smooth container and rough die:

$$\frac{P}{\bar{Y}} = 0.37 + 1.24 \log_e R \tag{1.14}$$

(iv) Rough container and rough die:

$$\frac{P}{\bar{Y}} = 0.63 + 1.28 \log_e R \quad . \tag{1.15}$$

The "rough" and "smooth" surfaces were defined by the type of slip-line field at the interface. A "rough" surface was one where frictional forces were equivalent to the shear stress of the metal and movement occurred only by lamellar shear over the surface; for a "smooth" surface there were no frictional forces.

Johnson's analysis was based on the proportions of the slip-line fields for a square or shear die extruding in plane strain, and the shape of the plastic field was assumed to be independent of material properties. From this, the constants a and b should also be independent of the material, but dependent on the geometry of the tooling, the volume of "dead metal", and the frictional conditions across the die and container. In experiments with lubricated extrusion of pure lead, Johnson (1956 a) confirmed Equation 1.11, but derived values for a and b with a shear die such that:

$$\frac{P}{\bar{Y}} = 0.8 + 1.5 \log_e R \quad . \tag{1.16}$$

The values for $\bar{Y}$ (uniaxial compressive yield stress) were determined from compression tests at the mean strain and strain rate applying in extrusion.

Wilcox and Whitton (1958) confirmed the dependence of a and b on tooling geometry by extruding super-pure aluminium through dies of various shapes and semi-angles. They derived values for a and b such that:

$$a = 0.9 - 1.6 \cot \alpha \tag{1.17}$$

and

$$b = 1.5 + 0.8 \cot \alpha \quad , \tag{1.18}$$

where α is the die semi-angle.

Combining these results with Equation 1.11 gives

$$\frac{P}{\overline{Y}} = (0.9 - 1.6 \cot \alpha) + (1.5 + 0.8 \cot \alpha) \log_e R \quad , \tag{1.19}$$

and substituting $\alpha = 90°$; $\cot \alpha = 0$ in Equation 1.19 gives the relationship for shear dies extruding aluminium as:

$$\frac{P}{\overline{Y}} = 0.9 + 1.5 \log_e R \quad , \tag{1.20}$$

which is in remarkable agreement with Equation 1.16 for lead. Further, Equation 1.20 was shown to be applicable to a range of metals extruded with shear dies.

These empirical relationships are therefore extremely useful in the interpretation of extrusion behaviour in practice but they cannot be claimed to be complete solutions. In particular the value of the die angle alone is not sufficient to define frictional conditions on the die face; the extent of the dead metal region and thus the die condition depends on the die angle, die friction, and extrusion ratio as well as the method of extrusion. Both Johnson (1954 a, 1954 b) and Wilcox and Whitton (1958, 1959) remarked on the importance of dead metal regions and their effects on extrusion load but the conditions for lubricated slip across the die face or shear through a dead metal region have not been defined. The interaction of these variables could be of some importance; for example Johnson(1954 b) and Hill (1950) both noted an optimum die angle (giving a minimum extrusion load) with high friction coefficients on the die (μ = 0.1 – 0.2) but this effect is not apparent with better lubrication.

The literature refers to the close agreement between the extrusion pressures determined under plane strain and axial symmetry; the work of Thomsen (1956), Thomsen and Frisch (1958), and Dodeja and Johnson (1956) is particularly relevant. No theoretical justification has been put forward for this agreement; as discussed on page 7, the slip-line conditions for axial symmetry are insoluble at present.

The value of $\overline{Y}$ to be used in the equations is the mean yield stress effective at the strain and strain rate involved. For ideal plastic metals the yield stress is assumed to be constant and independent of strain rate; these conditions are approximated with soft metals such as lead where work hardening is not present. On the other hand, most metals under normal conditions of extrusion show work hardening or annealing effects and direct determination of $\overline{Y}$ at the appropriate strains and strain rates is impossible at present; engineering (logarithmic) strains between 3 and 6 and strain rates of 10 to 20 seconds^{-1} are commonly encountered. The most successful approach to date has been the work at the British Iron and Steel Research Association Laboratories by Cook (1957) using a cam-operated compression machine to achieve high strain rates; unfortunately the compression test is not suitable for achieving logarithmic strains above 0.7 (that is, 50 per cent. reduction in height) because of "barreling" of the specimen.

Similarly the mean strain in extrusion ($a + b \log_e R$) is an approximation to the actual conditions of varying strain which the metal experiences in the billet during extrusion. These approximations are necessary to allow reasonable analytical treatment of extrusion which is basically a non-homogeneous deformation process, but the limitations of these assumptions must not be overlooked.

2.2 Slip-line Solutions

The work discussed in the previous section was based essentially on the interpretation of experimental evidence using a simple model of tensile straining, modifying this, as required, to incorporate the effects of tooling geometry and to account for redundant work. Slip-line solutions are based on the examination of individual elements as they deform and these solutions incorporate the effects of unequal strain across the section, tooling geometry, and frictional conditions on the die and container.

Slip-lines are formally defined as the tangents to the instantaneous directions of maximum shear stress. Consider, for example, the extrusion of a rectangular billet of metal in plane strain, that is, where strain is confined to two of the principal directions. The slip-line field proposed by Hill (1948) for 50 per cent. reduction in area through a shear die is given in Figure 3; there are two families of slip-lines (α and β) shown as radii and concentric arcs about the point 0 and the field is symmetrical about the point B or the axis of the extrusion. The α and β slip-lines are orthogonal, and since friction is assumed to be absent the β slip-lines from 0 intersect the wall and axis at 45° thus defining a triangular dead metal region. The limiting α slip-line AB similarly defines the extent of the plastic region along the axis of the container. Intermediate slip-lines may be inserted as required by further subdivision of the net.

In defining the slip-line field, the material is assumed to be "ideally plastic" such that no work-hardening is experienced and infinite rates of shear are possible. Similarly elastic strains are neglected as being infinitely smaller than the plastic strains within the field. The plastic field is therefore bounded by a rigid region which is considered to remain stationary during steady state extrusion.

If the slip-line field is defined in this way, the stress and velocity throughout the plastic region may be calculated by applying the Hencky-Geiringer relationships (Hencky 1923, Geiringer 1937) which in finite form may be written:

$$(p_B - p_A) + 2k(\phi_B - \phi_A) = 0 \quad \text{on an } \alpha \text{ line} \tag{1.21}$$

$$(p_B - p_A) - 2k(\phi_B - \phi_A) = 0 \quad \text{on a } \beta \text{ line} \tag{1.22}$$

$$du - v.d\phi = 0 \quad \text{on an } \alpha \text{ line} \tag{1.23}$$

$$dv + u.d\phi = 0 \quad \text{on a } \beta \text{ line} \tag{1.24}$$

and

$$\sigma_x = -p - k \sin \phi \tag{1.25}$$

where:

p_A, p_B = mean compressive stress at positions A, B,

k = maximum shear stress of material

= $\frac{Y}{\sqrt{3}}$ (Von Mise's Yield Criterion)

or = $\frac{Y}{2}$ (Tresca Yield Criterion),

Y = Uniaxial yield stress,

ϕ = Anticlockwise rotation or position of slip-line considering x-direction positive,

u = velocity component of element along α line,

v = velocity component of element along β line.

(Velocities are measured relative to the ram or die moving at unit velocity).

Consider first the conditions on OB; the region OBMN is rigid and hence the maximum stress on the material along OB is the shear stress k; thus $p = k$ on OB. Application of Equation 1.22 also shows that the stress is uniform along the β slip-lines OA, OB, OC, OD, OE. The stress variation along the α slip-lines is calculated from Equation 1.21 as follows:

On OC where $\phi = -\frac{\pi}{8}$:

$$(p_C - p_B) + 2k\left(-\frac{\pi}{4} - \left(-\frac{\pi}{8}\right)\right) = 0$$

$$p_C = 1.79k = \left(1 + \frac{\pi}{4}\right)k .$$

On OD where $\phi = 0$:

$$(p_D - p_B) + 2k\left(-\frac{\pi}{4}\right) = 0$$

$$p_D = 2.57k = \left(1 + \frac{\pi}{2}\right)k \; .$$

On OE where $\phi = +\frac{\pi}{8}$:

$$p_E = 3.36k = \left(1 + \frac{3\pi}{4}\right)k \; .$$

On OA where $\phi = +\frac{\pi}{4}$:

$$p_A = 4.14k = (1 + \pi)k \; .$$

Thus the point stresses throughout the region may be calculated, and particularly the conditions along the limiting slip-line OA. Since the region OAX is considered to be rigid, stresses acting on OA are acting on the die face OX and these may be calculated from Equation 1.25 as:

$$\sigma_x = - p_A - k \sin 2\phi_A$$

$$= -(1 + \pi)k - k(\sin 90)$$

$$= -(2 + \pi)k \text{ or } (2 + \pi)k \text{ in } -x \text{ direction,}$$

and the extrusion pressure P, defined as the extrusion force divided by the original area of billet, is

$$P = \frac{(2 + \pi)k}{2} = (1 + \frac{\pi}{2})k \; .$$

Velocity conditions within the plastic region are determined by integration of Equations 1.23 and 1.24, such that:

$$u = \frac{1}{\sqrt{2}} - \sin\theta \text{ on } \alpha \text{ slip-line} \quad , \quad \text{and} \qquad (1.26)$$

$$v = \cos\theta \text{ on } \beta \text{ slip-line} \; . \qquad (1.27)$$

Equation 1.24 states that the velocity along the β slip-lines is constant; similarly for constancy of volume, a particle moving through the plastic region must accelerate from unit velocity on AB to a velocity of 2 in the extruded bar. The equation of the trajectory of a point crossing AB at θ_o is:

$$\frac{r}{a} = \frac{\frac{1}{\sqrt{2}} - \sin\theta_o}{\frac{1}{\sqrt{2}} - \sin\theta} \quad , \qquad (1.28)$$

where r = radius of α slip-line on the trajectory

a = radius of limiting α line AB

θ_o = location of the point as it crosses AB

θ = resultant location of point on radius r .

The time required for a point to travel along these trajectories between two points on radii θ_1 and θ_2 is:

$$t = a\left(\frac{1}{\sqrt{2}} - \sin\theta_0\right)\left[F(\theta_1) - F(\theta_2)\right] \tag{1.29}$$

where

$$F(\theta) = \frac{2\cos\theta}{\frac{1}{\sqrt{2}} - \sin\theta} - \left[4\coth^{-1}\left[(\sqrt{2}+1)\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right]\right], \tag{1.30}$$

so the velocity conditions within the plastic region are fully defined.

These equations also allow the distortion of a plane surface or grid to be calculated as shown in Figure 3.

Slip-line solutions to extrusion problems are therefore a means of tracing the strain history of each point in the billet and therein lies the solution to many features noted in extruded sections; residual stresses, unequal properties across the bar, and phase changes induced by strain (to mention a few problems) can be related to the strain history of the billet through slip-line solutions.

However the solutions have limitations. Definition of the geometry of the plastic field for all extrusion conditions is a complicated procedure; the boundary conditions of velocity and stress (that is, solutions to Equations 1.21 – 1.24) require a "trial and error" procedure from which the correct solution emerges only slowly. The geometry of the field is affected by frictional conditions on the container and the die and by the tooling arrangement and these factors must be known with some confidence. For "real" metals, infinite rates of shear, such as postulated on AB in the example, are impossible and the limiting slip-lines must be replaced by a band more or less diffuse depending on the strain hardening characteristics of the material. The slip-line patterns derived theoretically have been confirmed for conditions of plane strain on the limited class of metals with negligible strain hardening tendencies, but these experiments have not been extended to metals with pronounced strain hardening.

The mathematical techniques associated with plasticity and slip-line theory must also be mentioned as precluding their general acceptance as a practical solution to extrusion problems. At present the slip-line theory is applicable only to conditions of plane strain; for axial symmetrical extrusion, the velocity and stress equations are not uniquely soluble, and the partial solutions which have been achieved involve *a priori* assumptions and iterative techniques based on established physical behaviour (Hill 1950).

2.3 Effects of Friction

The total work (and hence pressure) involved in extrusion is the sum of the work required to deform the metal and the work required to overcome frictional forces between the billet and tooling. These frictional forces exist between the billet and container in direct extrusion (but not in inverted extrusion) and between the billet and die for slip over the die face, as with conical dies.

The empirical relationships between extrusion pressure and reduction in area, presented in Section 2.1 do not include any allowance for these frictional forces; slip-line solutions include the frictional forces within the plastic zone but forces outside the plastic zone must be estimated independently.

The frictional forces between the billet and container in direct extrusion may be determined (Stone 1953, Treco 1962) as shown in Figure 4a. For equilibrium of axial forces within the element dx:

$$(\mu\,\sigma_r)(\pi D\,dx) = A\,d\sigma_x$$

or

$$d\sigma_x = \frac{4\mu}{D}\,\sigma_r\,dx ,$$

which on integration between O and L becomes:

$$\sigma_{r_L} = \sigma_{r_O} \exp\frac{4\mu L}{D} . \tag{1.31}$$

For rotational equilibrium:

$$\sigma_{r_L} = \sigma_{x_L} - Y$$

and $$\sigma_{r_O} = \sigma_{x_O} - Y \quad ,$$

from which

$$\sigma_{x_L} = (\sigma_{x_O} - Y) \exp \frac{4\mu L}{D} + Y \quad .$$

Substituting σ_{x_L} = actual extrusion pressure with friction = P_L , and

σ_{x_O} = extrusion pressure without friction = P_O ;

Thus:

$$P_L = (P_O - Y) \exp \frac{4\mu L}{D} + Y \quad .$$

If now $$P_O = \bar{Y} (a + b \log_e R)$$

then $$P_L = \bar{Y} \left[\left((a + b \log_e R - 1) \exp \frac{4\mu L}{D} \right) + 1 \right] \quad . \qquad (1.32)$$

The solution to Equation 1.32 for a 4-inch diameter container is shown in Figure 5; a value of $P_O = \bar{Y}(0.8 + 1.5 \log_e R)$ is assumed although the ratio P_L/P_O is relatively insensitive to the value of P_O or reduction ratio as shown. Figure 5 shows that the effects of container friction are most important; if a typical value of $\mu = 0.04$ is assumed for lubricated extrusion, the frictional forces on a 10-inch long billet will be such as to increase the starting load, and hence the press capacity required, by about 40 per cent. over that for frictionless extrusion.

Similar equations have been developed (Pearson and Parkins 1960, Treco 1962) for the frictional forces on the die face for lubricated slip (Figure 4b). Thus the equilibrium condition for axial forces is:

$$D\, d\sigma_x + 2\sigma_x\, dD + 2P\, dD \left(1 + \frac{\mu_2}{\tan \alpha}\right) = 0 \quad , \qquad (1.33)$$

from which:

$$\sigma_{x_B} = \sigma_{x_O} \left(1 + \frac{\tan \alpha}{\mu_2}\right)\left(R^{\frac{\mu_2}{\tan \alpha}} - 1\right) + \sigma_{x_A}\left(R^{\frac{\mu_2}{\tan \alpha}}\right) \quad .$$

With no opposing stresses at the die opening $\sigma_{x_A} = 0$, and substituting σ_{x_B} = extrusion pressure with die friction = P_D and σ_{x_O} = extrusion pressure without die friction = P_O , then:

$$P_D = P_O \left(1 + \frac{\tan \alpha}{\mu_2}\right)\left[\left(R^{\frac{\mu_2}{\tan \alpha}}\right) - 1\right] \quad . \qquad (1.34)$$

The solution to Equation 1.33 given above is strictly only valid for low values of die semi-angle α as experienced in wire drawing; substitution of typical values of $R = 10$, $\alpha = 70°$, and $\mu = 0.04$ in Equation 1.34 gives a ratio P_D/P_O of 2.3 which is a major over-estimate of the actual conditions, as noted previously (Pearson and Parkins 1960).

A more realistic but approximate solution may be obtained by considering the force balance on the die in non-integrated form. Thus, using the notation in Figure 4b, the additional pressure required to overcome die friction is:

$$\Delta P = \frac{\mu_2 P_O}{\cos \alpha} \; \frac{(D^2 - d^2)}{(D^2)} \; \frac{1}{\sin \alpha}$$

$$= \frac{\mu_2 P_O}{\sin \alpha \cos \alpha} \left(1 - \frac{1}{R}\right) \quad . \qquad (1.35)$$

Substituting $\alpha = 70°$, $R = 10$, $\mu_2 = 0.04$ in this expression gives:

$$\Delta P = 0.112\, P_O \quad ,$$

that is, an increase in pressure of about 11 per cent.

Combining Equations 1.32 and 1.35, the estimated extrusion pressure with friction on the container and die face is therefore:

$$P_L = \overline{Y}\left[1 + (a + b\log_e R - 1)\ \exp\left(\frac{4\mu_1 L}{D}\right) + \left(\frac{\mu_2\,(a + b\log_e R)}{\sin\alpha\,\cos\alpha}\left(1 - \frac{1}{R}\right)\right)\right] \tag{1.36}$$

The major problem in estimating the loss due to container or die friction is to establish a true value for the coefficient of friction. For example values in the literature for the hot extrusion of copper with graphite lubricant range from 0.009 (Treco 1962) to 0.20 (Zholobov 1937); and controlled experiments (Bisson et al. 1957) have shown that the friction coefficient is dependent on such factors as the temperature, surface speed, atmosphere, and surface films which may be present. The coefficient tends to decrease with increasing temperature, loading, and surface speeds. The coefficient thus assumed or derived from experimental results in extrusion is therefore a mean value for a range of conditions.

3. CONCLUSIONS

Limitations exist in the analysis of extrusion problems using the established models. The semi-empirical analytical techniques, and the model of homogeneous straining are an excessive simplification of conditions; the corrections required to account for non-homogeneous straining are large (commonly about 50 per cent. increase in pressure) and moreover, no techniques exist to relate these corrections to conditions of tooling lubrication, geometry, or material.

The more formal slip-line model predicts the extrusion conditions in plane strain with some confidence, at least for materials without pronounced strain-hardening tendencies or where the stress-strain curve of the metal is well established at the conditions of extrusion. The analysis is not applicable to axial symmetric extrusion, which is the most common practical problem.

In both cases, further progress with the existing models depends on development of techniques to measure the basic extrusion parameters such as effective yield stress and friction coefficient.

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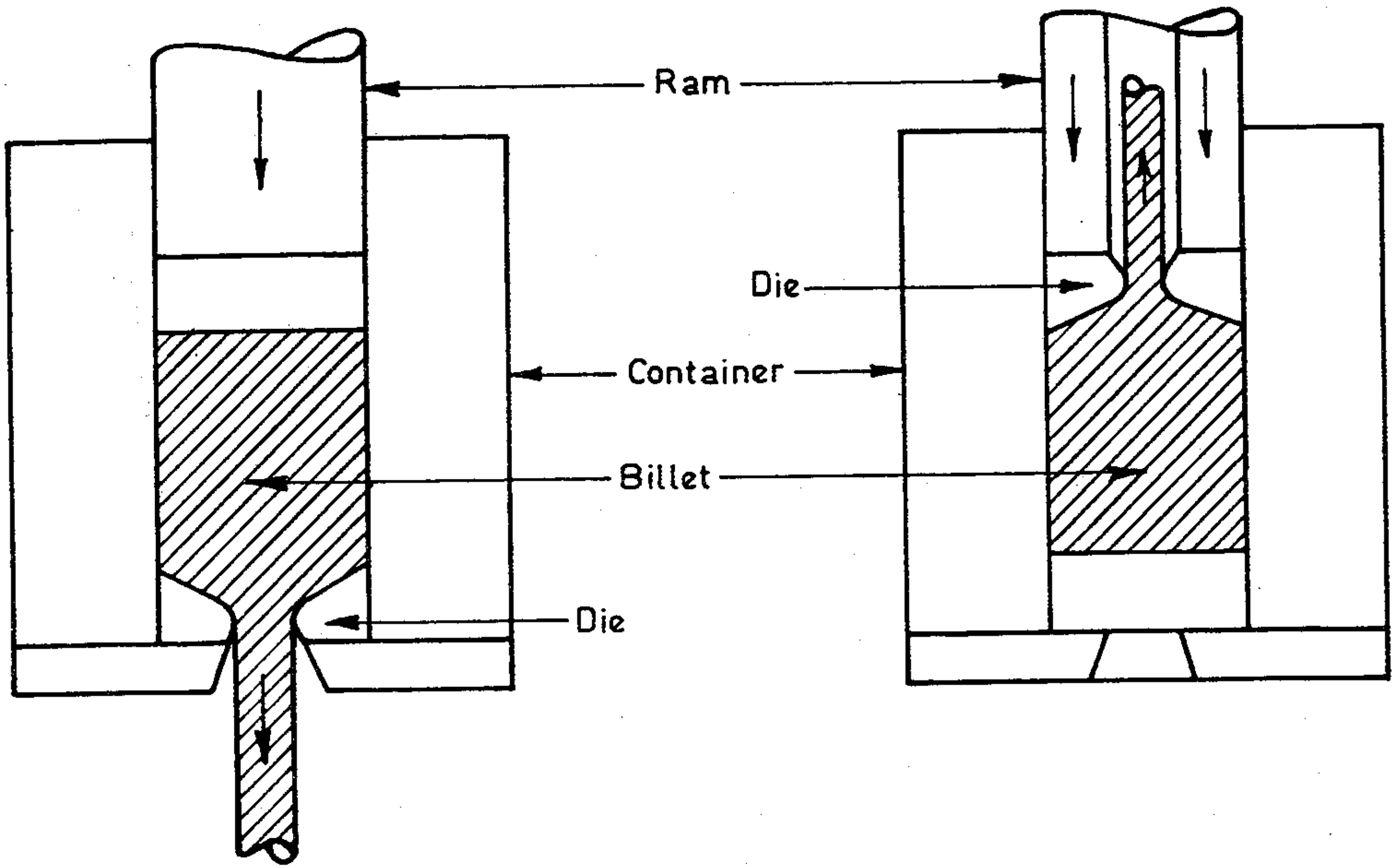

a. Direct Extrusion

b. Inverted Extrusion

FIGURE 1. PRINCIPLES OF DIRECT AND INVERTED EXTRUSION

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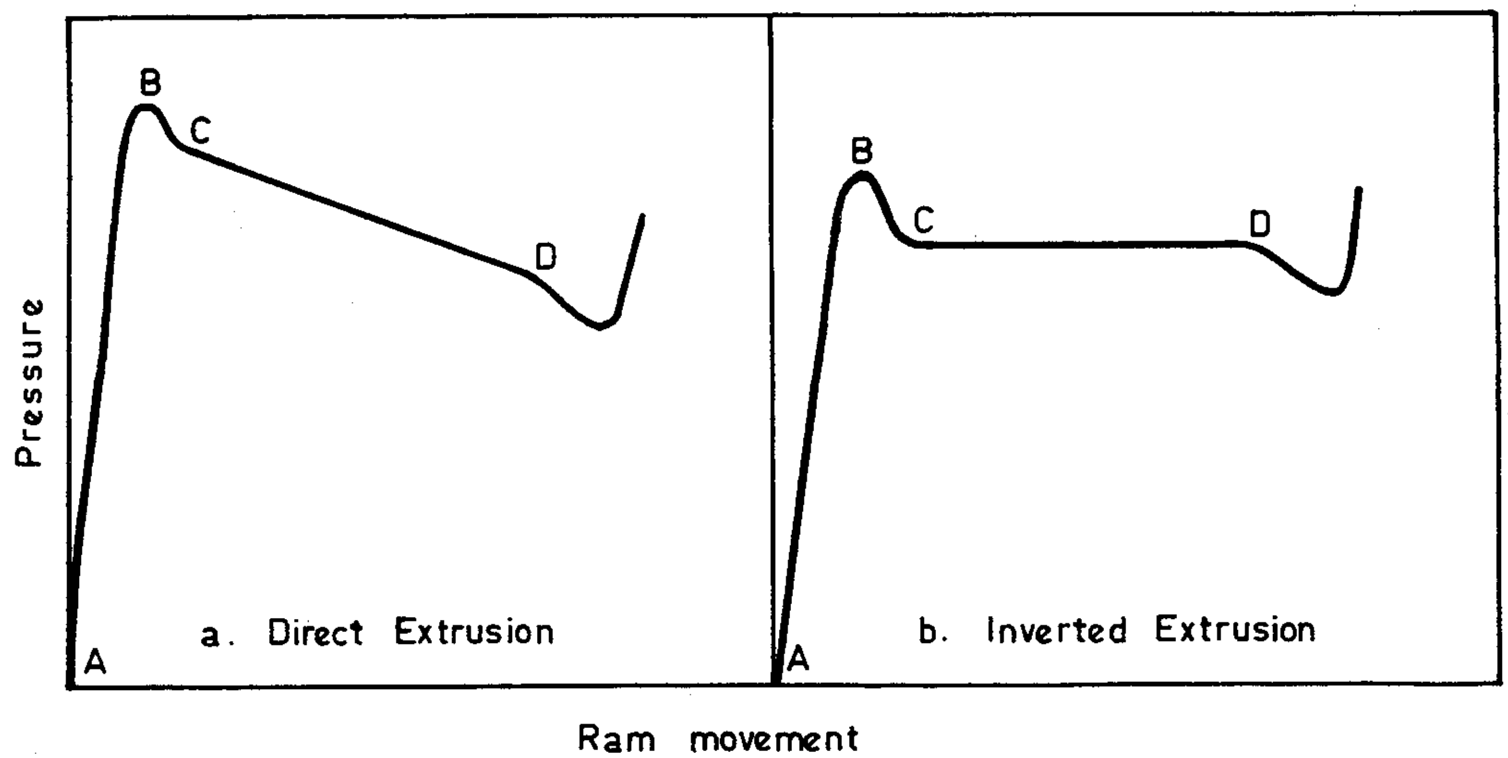

FIGURE 2. TYPICAL VARIATIONS IN PRESSURE DURING DIRECT AND INVERTED EXTRUSION

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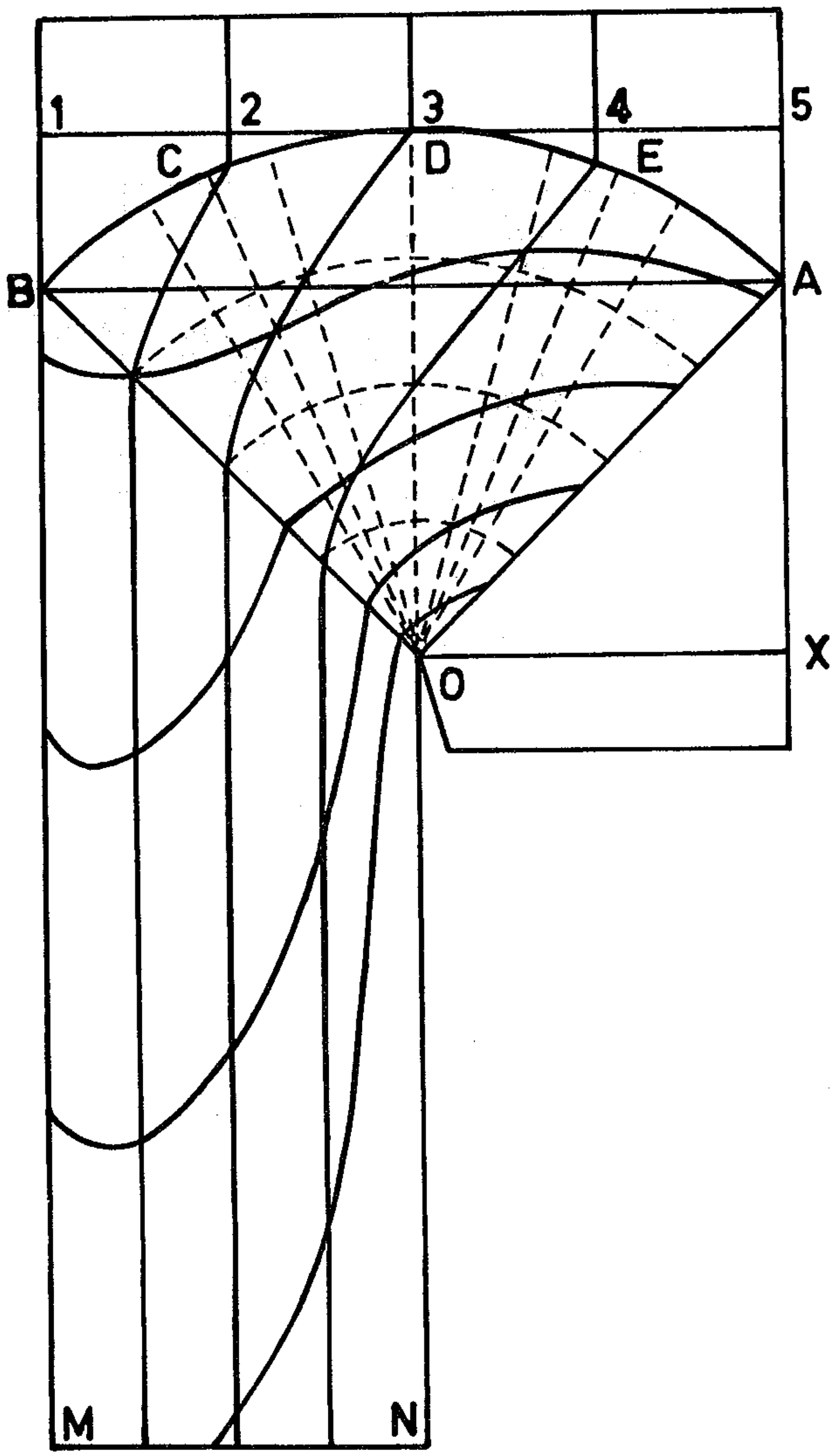
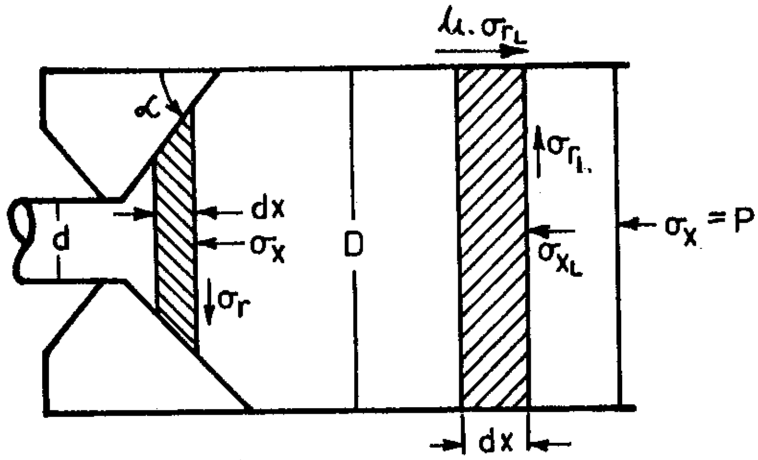
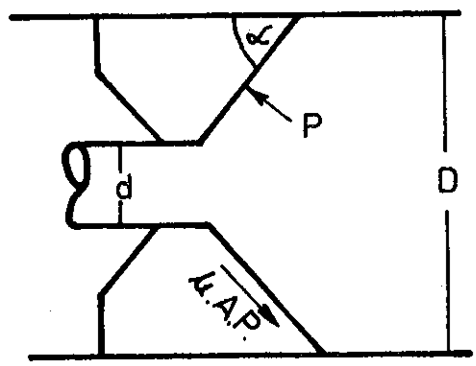


FIGURE 3. HILL'S SLIP LINE FIELD FOR 50 per cent
REDUCTION IN PLANE STRAIN



(a) Container friction



(b) Die friction

FIGURE 4. STRESS DIAGRAMS FOR CALCULATION OF FRICTIONAL FORCES ON CONTAINER AND DIE

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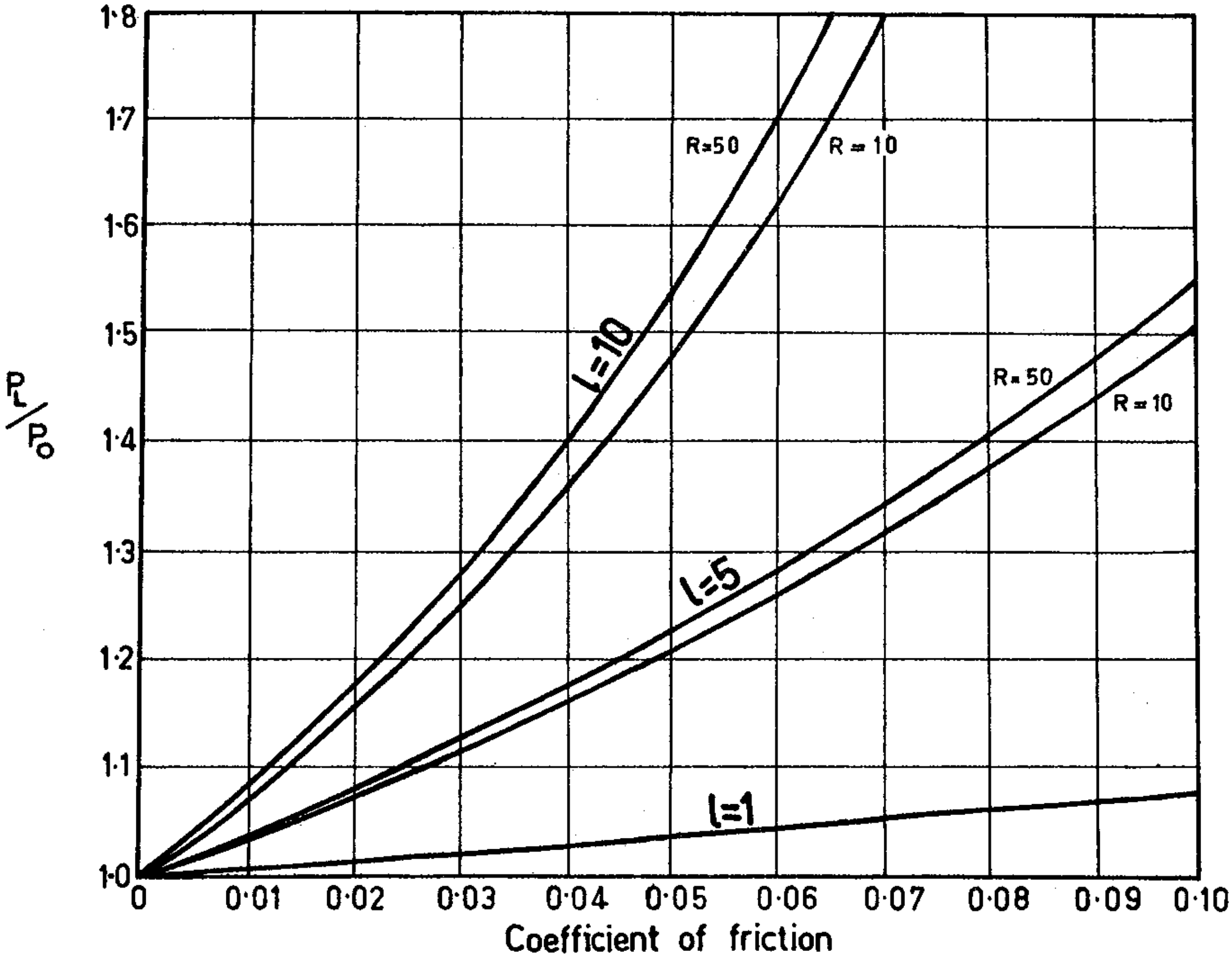

FIGURE 5. SOLUTIONS TO EQUATION 1.32

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